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RENORMALIZATION GROUP EQUATION FOR NUCLEAR CURRENT OPERATORS

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Abstract

In order to eliminate the present ambiguity in the renormalization group equation for nuclear current operators, we introduce a new condition specifying the cutoff independence of the five point Green function corresponding to the two-body propagator with current operator insertion. The resulting effective current operator is then shown to obey a modified Ward-Takahashi identity which differs from the usual one, but that nevertheless leads to current conservation.

1 Current Operator RG Equation

The use of the Wilsonian renormalization group (RG) method [1] to impose a cutoff Λ on the momenta of virtual states is an important tool for studying various aspects of nuclear effective field theory (EFT) [2]. In this context, our main goal here is to provide an unambigous derivation of the recently proposed RG equation for nuclear current operators [3]. At the same time we examine the questions of current conservation and application to EFT.

Defining projection operators [3]

$$\eta = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\, |\mathbf{k}\rangle\langle\mathbf{k}|\, \theta(\Lambda - k), \tag{1}$$

$$\lambda = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\, |\mathbf{k}\rangle\langle\mathbf{k}|\, \theta(\overline{\Lambda} - k)\theta(k - \Lambda), \tag{2}$$

where $\overline{\Lambda} > \Lambda$, the RG equation for the reduced space effective potential V_Λ can then be written as [2]

$$\frac{\partial V_\Lambda}{\partial \Lambda} = V_\Lambda G_0 \frac{\partial \lambda}{\partial \Lambda} V_\Lambda \tag{3}$$

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where $G_0 = (E - H_0)^{-1}$ is the two-body free propagator. Eq. (3) can be derived from the reduced space Lippmann-Schwinger equation

$$T = V_\Lambda + V_\Lambda \eta G_0 T \tag{4}$$

by requiring the off-shell scattering amplitude, T, not to depend on Λ. Recently NA have derived the corresponding RG equation for the reduced space effective current operator O^μ_Λ [3]. Writing this current operator as

$$O^\mu_\Lambda = \eta \Gamma^\mu_\Lambda \eta, \tag{5}$$

the RG equation derived by NA can be expressed as

$$\frac{\partial \Gamma^\mu_\Lambda}{\partial \Lambda} = V_\Lambda G_0 \frac{\partial \lambda}{\partial \Lambda} \Gamma^\mu_\Lambda + \Gamma^\mu_\Lambda \frac{\partial \lambda}{\partial \Lambda} G_0 V_\Lambda. \tag{6}$$

However, Eq. (6) was derived as only a sufficient condition for the Λ invariance of the physical matrix element of O^μ_Λ:

$$\frac{\partial \langle O^\mu_\Lambda \rangle}{\partial \Lambda} = \frac{\partial}{\partial \Lambda} \overline{\psi}_\beta \eta \Gamma^\mu_\Lambda (E_\beta, E_\alpha) \eta \psi_\alpha = 0. \tag{7}$$

That is, the equation used to define Γ^μ_Λ was chosen to be[2]

$$\langle O^\mu_\Lambda \rangle \equiv \overline{\psi}_\beta \eta \Gamma^\mu_\Lambda (E_\beta, E_\alpha) \eta \psi_\alpha = \overline{\psi}_\beta \Gamma^\mu (E_\beta, E_\alpha) \psi_\alpha \tag{8}$$

where

$$\Gamma^\mu (E_\beta, E_\alpha) \equiv O^\mu_{\overline{\Lambda}} (E_\beta, E_\alpha) = \eta \Gamma^\mu_\Lambda (E_\beta, E_\alpha) \eta |_{\Lambda = \overline{\Lambda}} \tag{9}$$

can be identified with the current vertex function of the full space [4] in the limit $\overline{\Lambda} \to \infty$. The sandwiching two-body wave functions $\overline{\psi}_\beta$ and ψ_α include bound states, and scattering states whose relative momenta, p' and p, respectively, are smaller than the cutoff parameter: $p', p < \Lambda$.

We have eliminated the ambiguity in the validity of Eq. (6) by showing that this RG equation is a sufficient and *necessary* condition for Λ independence of the five-point function $\eta G \eta \Gamma^\mu_\Lambda \eta G \eta$ [5]. Specifically we chose Γ^μ_Λ to be defined by

$$\eta G(E') \eta \Gamma^\mu_\Lambda (E', E) \eta G(E) \eta = \eta G^\mu (E', E) \eta \tag{10}$$

for all $\Lambda < \overline{\Lambda}$, where G^μ is the five-point function defined as

$$G^\mu (E', E) = G(E') \eta \Gamma^\mu_\Lambda (E', E) \eta G(E) |_{\overline{\Lambda} = \Lambda} = G(E') \Gamma^\mu (E', E) G(E). \tag{11}$$

[2] To save on notation we suppress total momentum variables from the argument of Γ^μ_Λ.

We note that G^μ corresponds to the two-body Green function G with all possible insertions of a current [4]. In the five-point function $\eta G^\mu \eta$, neither the incoming nor outgoing two-body states are on the energy shell; by contrast, both these states are on the energy shell in $\overline{\psi}\Gamma^\mu\psi$. At the same time, such five-point Green functions are necessary ingredients for three-body currents where two-body subsystems are off shell. In this sense the use of $\eta G^\mu \eta$ for the RG approach to the current operator, is naturally related to the RG approach to the two-body interaction, where the cutoff independence of the fully off-shell two-body scattering amplitude is used [2].

1.1 Solution of the Current Operator RG Equation

We find, unambiguously, that the solution to Eq. (6) is given by[3]

$$
\begin{aligned}
O^\mu_\Lambda &= \eta(1 - V_{\overline{\Lambda}} G_0 \lambda)^{-1} O^\mu_{\overline{\Lambda}} (1 - \lambda G_0 V_{\overline{\Lambda}})^{-1} \eta \\
&= \eta \left[1 + V_{\overline{\Lambda}} \lambda (E' - H_0 - V_{\overline{\Lambda}}\lambda)^{-1}\right] O^\mu_{\overline{\Lambda}} \left[1 + (E - H_0 - \lambda V_{\overline{\Lambda}})^{-1} \lambda V_{\overline{\Lambda}}\right] \eta
\end{aligned}
\tag{12}
$$

Unfortunately Eq. (12) cannot be used as a basis of a perturbation theory for O^μ because V, V_Λ and O^μ enter Eq. (12) not only at low momenta; moreover, only high momentum intermediate states are involved due to projectors λ. Nevertheless, Eq. (12) is useful for a range analysis of O^μ_Λ, showing that the long range part of O^μ_Λ is not affected by cutoff Λ - only the short range part is different from the one of $O^\mu_{\overline{\Lambda}}$.

2 Current Conservation

In order to avoid the well known problems of current conservation in theories with a finite cutoff, we consider the simple case where the starting cutoff is taken to infinity, $\overline{\Lambda} = \infty$. Then in the best case we will have the usual two-body Ward-Takahashi (WT) identities [9]

$$
q_\mu G^\mu(E', E) = \Gamma^0_0 G(E) - G(E')\Gamma^0_0, \tag{13a}
$$

$$
q_\mu \Gamma^\mu(E', E) = G^{-1}(E')\Gamma^0_0 - \Gamma^0_0 G^{-1}(E) \tag{13b}
$$

where Γ^0_0 is the zero'th component of the current operator Γ^μ_0 of two non-interacting particles, and is specified for initial (final) total four-momentum $P = p_1 + p_2$ ($P' = p'_1 + p'_2$) and relative momentm $\mathbf{p}$ ($\mathbf{p}'$) as

$$
\begin{aligned}
\langle \mathbf{p}' | \Gamma^0_0(P', P) | \mathbf{p} \rangle &= i(2\pi)^3 \left[e_1 \delta(\mathbf{p}'_2 - \mathbf{p}_2) + e_2 \delta(\mathbf{p}'_1 - \mathbf{p}_1)\right] \\
&= i(2\pi)^3 \left[e_1 \delta(\mathbf{p}' - \mathbf{p} - \mathbf{q}/2) + e_2 \delta(\mathbf{p}' - \mathbf{p} + \mathbf{q}/2)\right]
\end{aligned}
\tag{14}
$$

[3] The solutions in Refs. [3, 6, 7] are misquoted; however, the calculations are correct [8].

The WT identities of Eqs. (13) are damaged after the introduction of a finite momentum cutoff Λ: introducing the cutoff into Eq. (13b) gives [5]

$$\begin{aligned} q_\mu \eta \Gamma^\mu_\Lambda(E',E)\eta = \eta[G_0^{-1}(E') - V_\Lambda(E')]\eta\Gamma^0_0[1+\lambda G_0(E)V_\Lambda(E)]\eta \\ - \eta[1+V_\Lambda(E')G_0(E')\lambda]\Gamma^0_0\eta[G_0^{-1}(E) - V_\Lambda(E)]\eta. \end{aligned} \tag{15}$$

Although Eq. (15) is not a usual WT identity, it still leads to a conserved current:

$$q_\mu \overline{\psi}_{\mathbf{p}'}\eta\Gamma^\mu_\Lambda(E',E)\eta\psi_{\mathbf{p}} = 0. \tag{16}$$

It is important to note that the modified WT identity, Eq. (15), relates the reduced space effective current vertex Γ^μ_Λ, only to the corresponding effective potential V_Λ (the initial potential V is not involved), and that it enters the WT identity only with relative momenta below Λ for all physically interesting low energy transitions. These properties are indispensable for constructing a self-contained effective field theory (EFT) in the reduced momentum space [10]. The longitudinal part of the effective current operator is fully determined by just the effective potential. This is exactly the situation one has from the beginning, on the level of the EFT Lagrangian. The predictive power of the EFT with cutoff is the same as of the underlying EFT, i.e. as it would be had one used the EFT Lagrangian in the renormalization scheme.

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