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APPLICATION OF χPT WITH VECTOR MESONS: KAON ELECTROMAGNETIC FORM FACTORS AND $K\overline{K}$ CONTRIBUTION TO MUON ANOMALOUS MAGNETIC MOMENT

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Abstract

In framework of Chiral Perturbation Theory with vector and axial-vector mesons we develop a model for charged and neutral kaon electromagnetic form factors. Regions of time-like and space-like photon momentum are considered. Beyond the tree level the model includes certain loop corrections, such as self-energy operators in vector-meson (ρ, ω, ϕ) propagators and "dressed" photon-meson vertices. The contribution of $\rho' = \rho(1450)$, $\omega' = \omega(1420)$ and $\phi' = \phi(1680)$ resonances is included. Results are compared with experimental data on form factors and good agreement is achieved without making a fit. We evaluate $K\overline{K}-$channel contribution to the muon anomalous magnetic moment as well.

1 Introduction

We review here results of our recent paper [1] devoted to application of Chiral Perturbation Theory [2] (ChPT) to calculation of kaon electromagnetic (EM) form factors (FF)s. The particles of interest are K^0 and $K^\pm$. Quark composition of mass eigenstates reads: $K^+ = u\overline{s}$, $K^0 = d\overline{s}$ (strangeness = +1) and $K^- = \overline{K^+} = \overline{u}s$, $\overline{K}^0 = \overline{d}s$ (strangeness = −1), and the corresponding masses are $m_{K^+} = 493.7$ MeV and $m_{K^0} = 497.6$ MeV.

FFs $F_K(q^2)$ for the time-like momentum $q^2 > 0$ enter the matrix element of EM current operator between vacuum and kaon-antikaon states: $\langle K(p_1)\overline{K}(p_2)|j^\mu_{em}(x=0)|0\rangle \equiv (p_1 - p_2)^\mu F_K(q^2)$. Here p_1 and p_2 are kaon momenta, photon invariant mass q^2 is equal to kaon pair invariant mass $q^2 = (p_1 + p_2)^2 \equiv s$, and quark EM current is $j^\mu_{em}(x) = \frac{2}{3}\overline{u}(x)\gamma^\mu u(x) - \frac{1}{3}\overline{d}(x)\gamma^\mu d(x) - \frac{1}{3}\overline{s}(x)\gamma^\mu s(x)$.

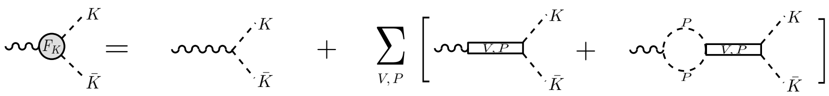


Figure 1: Illustration of the form factor.

We mainly focus on the region above the $K\overline{K}$ threshold $(2\ m_K)^2 < s < 4$ GeV^2. It covers the set of low-lying vector ($J^P = 1^-$) resonances V: $\rho(770)$, $\omega(782)$, $\phi(1020)$, and their excitations V': $\rho' = \rho(1450)$, $\omega' = \omega(1420)$, $\phi' = \phi(1680)$, which directly couple to the $K\overline{K}$ pair. The extension to the space-like region is straightforward through analyticity properties of $F_K(q^2)$. We limit our consideration in the space-like region to the interval -0.16 GeV$^2 < s < 0$, where data are available.

The approach [2] is appropriate for this problem, it gives rise to an effective hadronic model with coupling structures guided by the chiral symmetry. Among interesting features of the model let us note vector-meson dominance of the EM interaction. At the same time vector mesons ρ, ω, ϕ are not considered as gauge bosons of local chiral symmetry. In addition Wess-Zumino-Witten Lagranzian and some odd-intrinsic-parity interactions are incorporated in the model in order to describe EM vertex dressing and vector meson self energies.

Our results for FFs allow also for calculation of the $K\overline{K}$ contribution to the muon anomalous magnetic moment (AMM). This contribution and other hadronic contributions are presently the main source of uncertainty in theoretical prediction for muon AMM [3].

2 Model

Starting from χPT with vector mesons [2] one can obtain general expression for the charged and neutral kaon FFs:

$$F_{K^+}(s) = 1 - \sum_{V=\rho,\omega,\phi} \frac{g_{VK^+K^-}}{f_V(s)} A_V(s), \qquad F_{K^0}(s) = - \sum_{V=\rho,\omega,\phi} \frac{g_{VK^0\overline{K}^0}}{f_V(s)} A_V(s),$$

where $A_V(s) \equiv \frac{s}{s-m_V^2-\Pi_V(s)}$, m_V is the mass of V, and $\Pi_V(s)$ is self-energy. g_{VKK} is the constant of strong coupling of vector meson V to kaon pair, it can be found from data on strong decays of V (using $SU(3)_f$ symmetry if necessary). $f_V(s)$ is the EM coupling of V to photon, and its dependence on energy accounts for loop modification of electromagnetic vertex. The normalization of $f_V(s)$ at $s = m_V^2$ follows from decay widths of $V \to e^+e^-$ (for details see [1]). The FF is illustrated in Fig. 1.

Analogously, for inclusion of higher resonances we add the terms:

$$\Delta F_{K^+}(s) = -\sum_{V'} \frac{g_{V'K^+K^-}}{f_{V'}} A_{V'}(s), \qquad \Delta F_{K^0}(s) = -\sum_{V'} \frac{g_{V'K^0\overline{K}^0}}{f_{V'}} A_{V'}(s).$$

For these resonances one may assume constant (and imaginary) self-energy: $\Pi_{V'} = -\imath\, m_{V'}\Gamma_{V'}$, as it is difficult to unambiguously find and test the energy dependence of $\Pi_{V'}$. We impose $SU(3)_f$ relations for ratios of the strong and EM couplings for the "primed" resonances, similarly to those for ρ, ω and ϕ, and use available data on branching ratios from [4]. This leads to: $g_{\rho' K^+K^-}/f_{\rho'} = -0.063$, $g_{\omega' K^+K^-}/f_{\omega'} = -0.021$, and $g_{\phi' K^+K^-}/f_{\phi'} = -0.036$.

The normalization conditions $F_{K^+}(0) = 1$ and $F_{K^0}(0) = 0$ are fulfilled automatically in the present model.

3 Results and conclusion

Fig. 2a and Fig. 2b show comparison of our results for kaon FFs with experiments at $2\ m_K < \sqrt{s} < 1.75$ GeV. Small deviations from the data at $\sqrt{s} > 2$ GeV may be addressed to $\rho(1700)$ and $\omega(1650)$ resonances which are not included. Experimental information in the time-like region comes from measurements (CMD-2, SND, KLOE) of electron-positron annihilation $e^+e^- \to K\overline{K}$ cross section $\sigma(e^+e^- \to K\overline{K}) = \frac{\pi\alpha^2}{3q^2}\big(1 - \frac{4m_K^2}{q^2}\big)^{3/2}|F_K(q^2)|^2$.

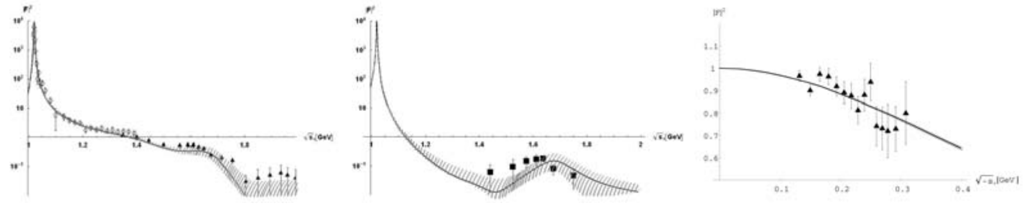
FF also agrees with the data at $-q^2 < 0.16$ GeV^2 (Fig. 2c). Available information[1] comes from kaon scattering on atomic electrons (SPS). Theoretical error corridors (hatched) shown in Fig. 2 are caused by uncertainties in the model parameters.

The $K\overline{K}$ channel contribution to muon AMM [5] can be calculated via the dispersion integral [10] technique: $a_\mu^{had,K\overline{K}} = \frac{\alpha^2}{3\pi^2}\int_{4m_K^2}^{\infty} W(s)R(s)\frac{\mathrm{d}s}{s}$, where $\alpha = 1/137$, $R(s) = \frac{\sigma(e^+e^-\to K\overline{K})}{\sigma(e^+e^-\to\mu^+\mu^-)}$, $W(s) = \int_0^1 \frac{x^2(1-x)}{x^2+(1-x)s/m_\mu^2}\mathrm{d}x$. Our estimate is

$$\begin{aligned} a_\mu^{\mathrm{had},K^+K^-} &= (19.06 \pm 0.57) \times 10^{-10}, \\ a_\mu^{\mathrm{had},K^0\overline{K}^0} &= (15.64 \pm 0.44) \times 10^{-10}, \end{aligned}$$

resulting in $a_\mu^{\mathrm{had,total}K\overline{K}} = (34.70 \pm 1.01) \times 10^{-10}$. This value agrees with result [11] obtained from e^+e^- annihilation.

[1]Data analysis at large momentum transfer up to $-s \sim 3$ GeV^2 from $ep \to e\Lambda K^+$ and $ep \to e\Sigma^0 K^+$ (JLab experiment E98) is not completed yet.



a b c

Figure 2: a: Charged kaon FF and b: neutral kaon FF in the time-like region; c: charged kaon FF in the space-like region. Data for a: diamonds from [6], triangles from [7]; for b: boxes from [8]; and for c: triangles from [9].

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