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# DETERMINATION OF THE POTENTIAL COEFFICIENTS OF THE BARYONS AND THE EFFECT OF SPIN AND ISOSPIN POTENTIAL ON THEIR ENERGY

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**Abstract**

Spin-and isospin-dependent terms are of great importance in the study of baryons for several reasons. On the other hand, the relativistic energy spectra has some highly important features which makes it much superior to the nonrelativistic one. The present work investigates the effects of spin-spin, spin-isospin and isospin-isospin interactions on the relativistic energy spectra of baryons. Our study reveals that both confining and non confining terms must be taken into account in order to obtain more exact energy spectra.

## 1 Introduction

In the relativistic limit, provided that the sizes of the particles under study are in quantum size, we have to use Dirac equation to study a baryonic system with total spin$\frac{1}{2}$ [1–3]. For such a system we represent the total wave function by $\psi_{\nu\gamma} = \begin{pmatrix} \Phi_{\nu\gamma} \\ \chi_{\nu\gamma} \end{pmatrix}$, ($\gamma$ and $\nu$ are introduced after Eqs. (7) and (9) respectively) Just similar to QED, considering non-linear terms in the potential relation in QCD leads to more exact calculations as well [4, 5]. When one single gluon is exchanged in a short distance, as both gluon and photon are massless, the interactions can be taken the same apart from their coupling constants. In many practical applications a harmonic oscillator potential yields spectra not much different from those found from potentials such as columbic plus linear that QCD prejudice would favor [6, 7]. Since harmonic oscillator models have nice mathematical properties, they have often been used as the confining potential [8, 9]. On the other hand, the

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columbic term alone is not sufficient, because it would allow free quarks to ionize from the system.

By making use of Jacobi coordinates [10]

$$\overrightarrow{\rho} = \frac{\overrightarrow{r_1} - \overrightarrow{r_2}}{\sqrt{2}}, \overrightarrow{\lambda} = \frac{\overrightarrow{r_1} + \overrightarrow{r_2} - 2\overrightarrow{r_3}}{\sqrt{6}}, \overrightarrow{R} = \frac{\overrightarrow{r_1} + \overrightarrow{r_2} + \overrightarrow{r_3}}{3} \tag{1}$$

as well as introducing the hyperradius quantity $x$, $x = \sqrt{\rho^2 + \lambda^2}$ the potential relation can be written as

$$2U_0(x) = 2V_0(x) = ax^2 + bx - \frac{c}{x}. \tag{2}$$

where $a$, $b$ and $c$ are constants. Note that in writing the potential relation in the above form we have neglected the center of mass motion. In terminology, the potentials $V_0(x)$ and $U_0(x)$ are respectively called vector and scalar potentials [11, 12]. The reason is that the so-called scalar potential is bracketed with the mass and the so-called vector potential goes with the energy in Dirac equation. We have considered the case $U_0(x) = V_0(x)$ since the potential relation has in general the form $U(x) = \frac{1}{2}(1 + e\gamma_0)A(x)$, where $A(x) = ax^2 + bx - \frac{c}{x}$. For $e$ we use the typical values $e = 0$ and $e = 1$. The latter is most important since it implied by $SU(2)$ symmetry as well as duality of angular momentum studied by Bell and Ruegg [13, 14]. It should be noted that the value $e = 0$ results in a scaler potential.

## 2 Exact analytical solution of Dirac equation

By representing the mass and energy of one quark respectively by $m_1$ and $\varepsilon_1$, the corresponding Dirac equation is written as

$$[\overrightarrow{\alpha} \,.\, \overrightarrow{p} + \beta(m_1 + U_0(x))]\psi_{\nu\gamma}(x) = (E_{\nu\gamma} - V_0(x))\psi_{\nu\gamma}(x), \tag{3}$$

where $m = 3m_1$, $E_{\nu\gamma} = 3\varepsilon_{\nu\gamma}$. Note that in obtaining the above equation the three quarks have been assumed to be of both equal rest mass and energy. For the up component of the wave function, we have

$$(p_1^2 + p_2^2 + p_3^2)\Phi_{\nu\gamma} = \sum_{i=1}^{3}(\varepsilon_i^2 - m_i^2)\Phi_{\nu\gamma} - 2U_{01}(x)\sum_{i=1}^{3}(\varepsilon_i + m_i)\Phi_{\nu\gamma}. \tag{4}$$

By using the above equation as well as the relation [15–17]

$$\nabla^2 = \nabla_\rho^2 + \nabla_\lambda^2 = -[\frac{d^2}{dx^2} + \frac{5}{x}\frac{d}{dx} - \frac{L^2(\Omega_\rho, \Omega_\lambda, \xi)}{x^2}], \tag{5}$$

where $L$ is the total angular moment operator, one can find that

$$-[\frac{d^2}{dx^2}+\frac{5}{x}\frac{d}{dx}-\frac{L^2(\Omega_\rho,\Omega_\lambda,\xi)}{x^2}]\Phi_{\nu\gamma}$$
$$=\frac{1}{3}(E_{\nu\gamma}^2-9m_1^2)\Phi_{\nu\gamma}-(ax^2+bx-\frac{c}{x})(E_{\nu\gamma}+3m_1)\Phi_{\nu\gamma}. \qquad (6)$$

It should be noted that [18]

$$\Phi_{\nu\gamma}=\phi_{\nu\gamma}(x)Y_{\gamma,l_\rho,l_\lambda}(\Omega_\rho,\Omega_\lambda,\xi), \qquad (7)$$

where $\Omega_\rho=(\theta_\rho,\phi_\rho)$ and $\Omega_\lambda=(\theta_\lambda,\phi_\lambda)$ are solid angles corresponding to and coordinates respectively, $\xi=Arctg\frac{\rho}{\lambda}$ is called the hyperangle and $\gamma$ is the grand angular quantum number given by $\gamma=2n+l_\rho+l_\lambda$, where $n$ is a non negative integer, $l_\rho$ and $l_\lambda$ are the angular momentum corresponding to the coordinates $\rho$ and $\lambda$ respectively. On the other hand, as [15–17]

$$L^2(\Omega_\rho,\Omega_\lambda,\xi)Y_{\gamma,l_\rho,l_\lambda}(\Omega_\rho,\Omega_\lambda,\xi)=\gamma(\gamma+4)Y_{\gamma,l_\rho,l_\lambda}(\Omega_\rho,\Omega_\lambda,\xi). \qquad (8)$$

By choosing $\phi_{\nu\gamma}(x)$ (see equation 7) as

$$\phi_{\nu\gamma}(x)=x^{-\frac{5}{2}}\varphi_{\nu\gamma}(x), \qquad (9)$$

where $\nu$ determines the number of nodes the wave function and in fact represents the distinct solutions corresponding to a particular grand angular quantum number, and $\varphi_{\nu\gamma}(x)$ as

$$\varphi_{\nu\gamma}(x)=f_\nu(x)e^{g_\gamma(x)}. \qquad (10)$$

After considering appropriate ansatzs for $f_\nu(x)$ and $g_\gamma(x)$ [19–21] and taking $a=(E_{\nu\gamma}+3m_1)\omega^2$, it can be proved that, for the case $\nu=0$, is given by [26]

$$E_{0\gamma}=\frac{3(2\gamma+5)^2[m_1+\omega(2\gamma+6)]-9m_1c^2}{(2\gamma+5)^2+3c^2}. \qquad (11)$$

In the energy spectra obtained from Schrödinger equation two important points are not taken into account: first, the spin of the particles and second, their rest mass energy. On the contrary, in the energy spectra obtained from Dirac equation both spin and rest mass are taken in to account. Finally it is proved that the total wave function in the ground state and in the case $\gamma=0$ is [26]

$$\psi_{00}=N_{00}\begin{pmatrix}1\\ \frac{i\vec{\sigma}.\hat{x}}{E_{00}+3m_1}(\alpha x+\beta)\end{pmatrix}\exp[-\frac{1}{2}(E_{00}+3m_1)\omega x^2-\frac{b}{2\omega}x]Y_{[0],0,0}(\Omega_\rho,\Omega_\lambda,\xi), \qquad (12)$$

Table 1:

| $Baryon$ | $I(J^p)$ | $a(fm^{-3})$ | $b(fm^{-1})$ | c |
|---|---|---|---|---|
| $N(938)$ | $\frac{1}{2}(\frac{1}{2})^+$ | 0.7182 | 1.5294 | 2.0757 |
| $\Lambda(1115)$ | $0(\frac{1}{2})^+$ | 0.7024 | 1.5118 | 2.0831 |
| $\Sigma(1189)$ | $1(\frac{1}{2})^+$ | 0.6935 | 1.4781 | 2.0894 |
| $\Sigma(1192)$ | $1(\frac{1}{2})^+$ | 0.6927 | 1.4706 | 2.0907 |
| $\Sigma(1197)$ | $1(\frac{1}{2})^+$ | 0.6919 | 1.4693 | 2.0913 |
| $\Xi(1314)$ | $\frac{1}{2}(\frac{1}{2})^+$ | 0.5856 | 1.3852 | 2.1014 |
| $\Xi(1321)$ | $\frac{1}{2}(\frac{1}{2})^+$ | 0.5819 | 1.3809 | 2.1032 |
| $N(1440)$ | $\frac{1}{2}(\frac{1}{2})^+$ | 0.5226 | 1.3771 | 2.1085 |

where $\hat{x}$ is the unit vector in the hypersphere space and $N_{00}$ is the normalization constant. On the other hand, the charge radius of each baryon is defined as [22]

$$\langle x^2\rangle^{\frac{1}{2}} = \frac{\int_0^\infty [1+\frac{((E_{0\gamma}+3m_1)\omega x+\frac{b}{2\omega})^2}{E_{0\gamma}^2(1+\frac{3m_1}{E_{0\gamma}})^2}]\exp(-(E_{0\gamma}+3m_1)\omega x^2-2\frac{b}{2\omega}x)x^7dx}{\int_0^\infty [1+\frac{((E_{0\gamma}+3m_1)\omega x+\frac{b}{2\omega})^2}{E_{0\gamma}^2(1+\frac{3m_1}{E_{0\gamma}}^2)^2}]\exp(-(E_{0\gamma}+3m_1)\omega x^2-2\frac{b}{2\omega}x)x^5dx}, \qquad (13)$$

where the parameters $b$ and $\omega$ are not yet determined. By using the Eq. (11) as well as substituting the corresponding value of the charge radius of each baryon in Eq. (13) we have calculated the parameters $b$, $\omega$ and thereby the parameters $a$, $c$ in each case, and have listed them in Table 1.

Now, by knowing the values of parameters $a$, $b$ and $c$, the wave function is a function of the hyperradius quantity $x$ only.

# 3 Spin and isospin effects

Within the algebraic approach, the quark energy is written in terms of symmetry groups to introduce an isospin-dependent terms, which turns out to be important for the description of spectrum [23,24]. Furthermore, the splittings within the multiplets are not all adequately described by the hyperfine interaction. One should remind that the form often assumed for the hyperfine interaction contains a $\delta$-like term, which is troublesome from the theoretical point of view. On the other hand, the splitting can be originated also by other terms, for instance isospin dependent ones. The standard hyperfine interaction is used to reproduce the splitting with in the SU(6)-multiplets.

As this interaction contains a term it is modified by introducing a smearing factor given by a Gaussian factor of the hyper radius $x$

$$H_S = A_S(\frac{1}{\sqrt{2}\sigma_S})^3 \exp(-\frac{x^2}{\sigma_S^2}) \sum(S_i.S_j), \qquad (14)$$

Where $A_S = 67.4(fm^2)$ and $\sigma_S = 2.87(fm)$ and

$$\sum(S_i.S_j) = \frac{1}{2}(S^2 - \frac{9}{4}). \qquad (15)$$

There are different multiples for the introduction of a flavour dependent term in the three-quark interaction. The well known Gueresey-Radicati mass formula contains flavour dependent terms, which is essential for the description of the strange baryon spectrum [24]. For the non strange baryons this formula implies isospin dependence [24]. It has also been pointed recently that an isospin dependence of the quark potential can be obtained by means of quark exchange [25]. More generally, one can expect that the quark-quark pair production can lead to an effective quark interaction containing an isospin dependent term. With these motivations in mind, we have calculated the effects of two isospin terms as well as the introduced spin dependent one. The first one depends on the isospin only and has the form

$$H_I = A_I(\frac{1}{\sqrt{2}\sigma_I})^3 \exp(-\frac{x^2}{\sigma_I^2}) \sum(I_i.I_j), \qquad (16)$$

Where $A_I = 51.7(fm^2)$ and $\sigma_I = 3.45(fm)$. The second one is a spin-isospin interaction, Given by:

$$H_{SI} = A_{SI}(\frac{1}{\sqrt{2}\sigma_{SI}})^3 \exp(-\frac{x^2}{\sigma_{SI}^2}) \sum(S_i.S_j)(I_i.I_j), \qquad (17)$$

Where $A_{SI} = -106.2(fm^2)$ and $\sigma_{SI} = 2.31(fm)$. Finally, by using the first order time-independent perturbation theory we have calculated the obtained results and have listed them in table-2. A look at table-2 reveals that the first-order energy shift due to spin-spin interaction has the greatest value in comparison with the two other mentioned perturbations. It also shows that the heavier the baryon, the less the energy shifts due to spin-isospin and isospin-isospin perturbations. The reason is that the heavier the baryon, the less $u$ and $d$ quarks there are in its quark content.

**Table -2** .

| $Baryon$ | $I(J^p)$ | $\Delta_S^{(1)}(MeV)$ | $\Delta_I^{(1)}(MeV)$ | $\Delta_{SI}^{(1)}(MeV)$ |
|---|---|---|---|---|
| $N(938)$ | $\frac{1}{2}(\frac{1}{2})^+$ | $-30.15$ | $-20.73$ | $-12.07$ |
| $\Lambda(1115)$ | $0(\frac{1}{2})^+$ | $-23.17$ | $-18.59$ | $-11.83$ |
| $\Sigma(1189)$ | $1(\frac{1}{2})^+$ | $-22.85$ | $-18.06$ | $11.13$ |
| $\Sigma(1192)$ | $1(\frac{1}{2})^+$ | $-21.13$ | $-17.76$ | $11.05$ |
| $\Sigma(1197)$ | $1(\frac{1}{2})^+$ | $-20.72$ | $-17.64$ | $10.85$ |
| $\Xi(1314)$ | $\frac{1}{2}(\frac{1}{2})^+$ | $-17.38$ | $-13.29$ | $9.31$ |
| $\Xi(1321)$ | $\frac{1}{2}(\frac{1}{2})^+$ | $-17.19$ | $-12.47$ | $8.65$ |
| $N(1440)$ | $\frac{1}{2}(\frac{1}{2})^+$ | $-15.42$ | $-19.04$ | $11.36$ |

# 4 Conclusion

By using the energy spectra obtained from the exact analytical solution for the three-body baryonic system we calculated the potential coefficients for each baryon and thereby the effects of spin-spin, spin-isospin and isospin-isospin interactions by considering the corresponding potentials as perturbations. Our study reveals that the first-order energy shift due to spin-spin interaction has the greatest value in comparison with the two other mentioned perturbations. We also found that the heavier the baryon, the less the energy shift due to spin-isospin and isospin-isospin perturbations. The reason is that the heavier the baryon, the less $u$ and $d$ quarks there are in its quark content. Also, one can see that the shift due to spin-spin term has the greatest value among the three considered terms.

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