MENU 2007
11th International Conference
on Meson-Nucleon Physics and
the Structure of the Nucleon
September10-14, 2007
IKP, Forschungzentrum Jülich, Germany

EVALUATION OF THE πN SIGMA TERM USING DISPERSION RELATIONS - PRESENT STATUS

M. Hadzimehmedovic, H. Osmanovic, J. Stahov
Faculty of Science
University of Tuzla, Bosnia and Herzegovina

Abstract

The dispersion relations are used to perform an analytical continuation of the πN scattering amplitude $\overline{D}^+$ to the Cheng-Dashen point and to calculate the πN sigma term. The dispersion relations were evaluated along two different families of dispersion curves passing through different kinematical regions in the s-channel physical region. The obtained results for the sigma term are still within the error bars of the previous Karlsruhe result.

1 Introduction

A low energy theorem [1] relates the πN sigma term to the isoscalar, crossing symmetric scattering amplitude $\overline{D}^+(\nu, t)$ at the Cheng-Dashen point (CD) $\nu = 0$, $t = 2m_\pi^2$:

$$\Sigma = F_\pi^2 \overline{D}^+(0, 2m_\pi^2), \tag{1}$$

where m_π is a charged pion mass and F_π is the pion decay constant. The bar indicates that a pseudovector Born term is subtracted. For more details about πN kinematics we refer to [2]. To apply the low energy theorem one has to perform analytic continuation of $\overline{D}^+$ amplitude to the CD point which lies outside of the πN physical region. An analytic continuation may be performed by means of dispersion relations along different curves. In our applications the dispersion curves pass through the regions where the s-channel partial wave expansion converges and the nearby parts of the t-channel cut may be calculated from the t-channel partial waves.

There are two the most frequently used methods of determination of $\overline{D}^+$ amplitude at the CD point. In the first method [2], $\overline{D}^+$ amplitude is represented by the subthreshold expansion:

$$\overline{D}^+(\nu^2, t) = \sum_{m,n} \overline{d}^+_{mn} \nu^{2m} t^n, \quad \overline{D}^+(0, 2m_\pi^2) = \overline{d}^+_{00} + \overline{d}^+_{01} \cdot 2m_\pi^2 + \overline{d}^+_{02} \cdot (2m_\pi^2)^2 + \ldots$$

The validity of expansion is limited by the nearest singularities, s- and u-channel thresholds and the t-channel pseudothreshold $t = 4m_\pi^2$.

In the second approach, the dispersion relations are applied along curves which pass through the CD point [3–6], and $\overline{D}^+$ amplitude at the CD point is calculated directly.

2 Continuation into the unphysical region

More technical details about our method may be found in [7]. Once subtracted FTDR were performed at 40 t-values $t \in [-3m_\pi^2, 0]$. Using the discrepancy function method, $\overline{D}^+$ amplitude was extrapolated at 20 equidistant values $\nu^2 \in [0, \nu_{max}^2 = 0.5m_\pi^2]$ inside Mandelstam triangle. An analytic continuation into the part of the Mandelstam triangle $0 < t < 4m_\pi^2$ was performed along hyperbolas in the (s,u) plane [8]:

$$(s - a)(u - a) = b,$$

where a is asymptote and b is a parameter. For $b = (m^2 - a - m_\pi^2) - 4m_\pi^2$ (m is a proton mass) and $a < 0$ hyperbolas remain inside the s-channel physical region and pass through the s-channel threshold (interior hyperbolas). Furthermore, if $a \geq -1\ GeV^2$ the nearby parts of the t-channel cut ($t \leq 30m_\pi^2$) may be represented by expansion in terms of the $\pi\pi \to N\overline{N}$ partial waves [9]. If $a = -m^2 + \frac{1}{2}m_\pi^2 \approx -0.871\ GeV^2$, interior hyperbola passes through the CD point. The interior dispersion relations along this single curve were recently used in [4] to calculate the πN sigma term. In our approach once subtracted IDR for $\overline{D}^+$ amplitude were evaluated along a set of hyperbolas crossing $\nu = 0$ line at 40 equidistant points $t \in [m_\pi^2, 3m_\pi^2]$. Along each hyperbola $\overline{D}^+$ amplitude was evaluated at 20 equidistant values $\nu^2 \in [0, \nu_{max}^2 = 0.5m_\pi^2]$. The coefficients $\{\overline{d}^+_{mn}\}$ were determined by fitting of obtained 1600 values of $\overline{D}^+$ to the subthreshold expansion. In Ref. [5] R. Koch proposed an analytic continuation of the $\overline{D}^+$ amplitude into the subthreshold region along hyperbolas in a (ν^2, t) plane:

$$(\nu^2 - \nu_0^2)(t - t_0) = \frac{a}{2}.$$

Varying parameters ν_0^2 and t_0 hyperbolas cover a wide angular range in the s-channel physical region, remain within or close to the s-channel physical region, and pass through the t-channel domain where the t-channel partial wave expansion converge. Hyperbolas in the (ν^2, t) plane are very convenient for calculation of the sigma term because a set of hyperbolas with $a = 2(t - 2m_\pi^2) \cdot \nu_0^2$ passes through the CD point.

3 Results, discussion and conclusions

Partial wave solution GWU06 [10] was used as an input from the s-channel. The nearby parts of the t-channel cut were evaluated using our results for the $\pi\pi \to N\overline{N}$ s-wave [6] available up to $t = 30m_\pi^2$. Higher energy contribution from the s-channel and the t-channel, and contribution from the d and higher partial waves in the t-channel as well, were included in discrepancy function.

Analytic continuation of $\overline{D}^+$ amplitude to the CD point was performed along a set of 50 hyperbolas in the (ν^2, t) plane. Parameters ν_0^2 and t_0 assumed values within the range $(-10m_\pi^2, -30m_\pi^2)$. This set of hyperbolas covers the whole of the angular region in the s-channel where new data on πN scattering exist. The following values for $\overline{D}^+$ amplitude (in m_π units) at CD point and the Σ term were obtained:

$$\overline{D}^+(0, 2m_\pi^2) = 1.185 \pm 0.033; \quad \Sigma = (72 \pm 2)MeV.$$

From our second approach, the following values for the coefficients in the subthreshold expansion(in m_π units) and corresponding value of the Σ term were obtained:

$$\overline{d}^+_{00} = -1.377 \pm 0.01, \quad \overline{d}^+_{01} = 1.176 \pm 0.01, \quad \overline{d}^+_{02} = 0.039 \pm 0.001,$$
$$\overline{d}^+_{03} = 0.004 \pm 0.001; \quad \overline{D}^+(0, 2m_\pi^2) = 1.163 \pm 0.033; \quad \Sigma = (71 \pm 2)MeV.$$

Quoted errors are only due to numerical procedures used in our calculations. Using the same input from the s-channel and taking into account contribution from the nearby parts of the t-channel cut in an implicit way, authors in Ref. [4] obtained significantly higher value for the πN sigma term, $\Sigma = (81 \pm 6)MeV$. We do think that our value obtained by analytic continuation to the unphysical region along many curves is more reliable than the value obtained by continuation along a single curve. It is important to point out that experimental value of the πN sigma term depends on the πN scattering data, especially in the low energy region. The CHAOS collaboration [11] significantly increased the amount of the high quality data at the low energy. These data allow a direct determination of the $\overline{D}^+$ forward amplitude which makes the analytic continuation of the $\overline{D}^+$ amplitude into

the unphysical region more reliable. To our knowledge, these data were not included in any of new PWA.
Conclusions: Two methods used in our evaluation of the πN sigma term give mutually consistent values which are still inside the error bars of Karlsruhe value [5]. CHAOS data should be used in forthcoming PWA to get more reliable partial waves close to the threshold.

Supported by Ministry of Education and Science of Tuzla Kanton.
JS gratefully acknowledges support from FZ Juelich.

References

[1] L. S. Brown *et al.*, *Phys. Rev.* **D4**, 2801 (1971).

[2] G. Höhler, *Pion-Nucleon Scattering, Landolt-Börnstein*, **Vol.I/9b2** (Springer, 1983).

[3] D. Moir, R. Jacob, G. E. Hite, *Nucl. Phys.* **B103**, 477 (1976).

[4] G. E. Hite, W. B. Kaufman, R. J. Jacob, *Phys. Rev.* **C71**, 065201 (2005).

[5] R. Koch, *Z. Physik* **C15**, 161 (1982).

[6] M. Hadzimehmedovic, H. Osmanovic, J. Stahov, *Int. J. of Mod. Phys.* **20**, No 8, 9, 1876 (2005).

[7] J. Stahov, πN *Newsletter,* **15**, 13 (1999).

[8] G. Hite, F. Steiner, *Nuovo Cim.* **18A**, 237 (1973).

[9] E. Borie, F. Kaiser, *Nucl. Phys.* **B126**, 173 (1977).

[10] SAID πN database, http://gwdac.phys.gwu.edu.

[11] H. Denz *et al.*, Phys. Lett. **B633**, 209 (2006).

[12] E. Borie, F. Kaiser, *Nucl. Phys.* **B126**, 173 (1977).