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CHARMONIUM EXCITED STATES FROM LATTICE QCD

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Abstract

We apply the variational method with a large basis of interpolating operators to demonstrate the feasibility of extracting multiple excited states in charmonium from lattice QCD. The calculation is performed in the quenched approximation to QCD, using the clover fermion action on an anisotropic lattice. A crucial element of our approach is a knowledge of the continuum limit of the interpolating operators, providing important additional information on the spin assignment of the states, even at a single value of the lattice spacing. Though we find excited-state masses that are systematically high with respect to the quark potential model, and the experimental masses where known, we attribute this as most likely an artifact of the quenched approximation.

1 Introduction

Interest in the charmonium system has been rekindled by the wealth of new experimental results, with the promise of yet more at future facilities. For the lattice community, the charmonium system is not only fascinating in its own right, but an important theater in which to hone our skills for studies of light-quark systems, such as the search for the photoproduction of exotic mesons by the GlueX Collaboration at the future 12 GeV upgrade of Jefferson Laboratory. These motivations have spurred us to pursue a program of investigations of charmonium [1, 2]. In this talk, I will describe our recent efforts aimed at extracting the resonance spectrum in charmonium, including states with exotic quantum numbers [3], as a precursor to investigating their properties. I will conclude this introduction with a description of how the spectrum is determined from a lattice calculations, before proceeding to our results and discussion.

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1.1 Excited states and lattice QCD

The spectrum in Euclidean-space lattice QCD is obtained by measuring time-sliced correlation functions between interpolating operators $\mathcal{O}_i$ and $\mathcal{O}_j$:

$$C_{ij}(t) = \sum_{\mathbf{x}} \langle 0|\mathcal{O}_i(\mathbf{x},t)\mathcal{O}_j^\dagger(0)|0\rangle \tag{1}$$

We now use the resolution of unity to insert a complete set of states between the two operators. The effect of the time-sliced sum in is to put the intermediate states at rest, and we obtain

$$C_{ij}(t) = \sum_{\alpha} \frac{Z_i^{\alpha *} Z_j^{\alpha}}{2M_\alpha} e^{-M_\alpha t} \tag{2}$$

where the sum is over all states having non-zero overlap factors $Z_{i,j} = \langle \alpha|\mathcal{O}_{i,j}^\dagger|0\rangle$. We delineate the states contributing through the variational method [4,5], by finding the eigenvalues $\lambda_1 > \lambda_2 > \ldots$ of $C^{-1/2}(t_0)C(t)C^{-1/2}(t_0)$, which satisfy

$$\lambda_\alpha(t,t_0) = e^{-M_\alpha(t-t_0)}\left(1 + \mathcal{O}(e^{-\Delta M_\alpha(t-t_0)})\right),$$

where ΔM_α is the minimum absolute energy difference between α and any other state.

The effectiveness of the method depends on a suitable basis of interpolating operators, which must be constructed so as to lay in given irreducible representations of the symmetry group; we use an extension of those in ref. [6]. For states at rest, their symmetry properties on a cubic lattice are classified according to the irreducible representations of the cubic group, with full rotational symmetry recovered only in the continuum limit. The irreps. Λ, their dimension d_Λ, and their continuum content, are as follows:

Λ	d_Λ	J
A_1	1	$0, 4, 6, \ldots$
A_2	1	$3, 6, 7, \ldots$
E	2	$2, 4, 5, \ldots$
T_1	3	$1, 3, 4, \ldots$
T_2	3	$2, 3, 4, \ldots$

Each irrep. acquires additional subscripts associated with parity (P) and charge conjugation (C). Within a given irrep., the spectrum contains energies corresponding to each of the continuum angular momenta listed above. Thus, for example, the T_1^{-+} channel contains energies corresponding not only to spin 1, but also to spins 3, 4 and higher.

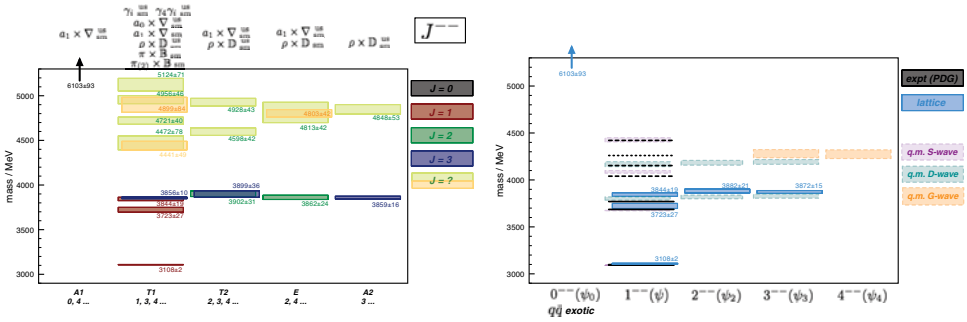

Figure 1: The left-hand figure shows the spectrum according to the lattice irreps., with color coding indicating our spin assignments. The right-hand figure shows the spectrum after the continuum spin assignments, together with quark-model expectations and the experimental values, where known.

The implicit assumption in most calculations is that the lightest state in a given irrep. corresponds to the state of lowest spin. The classical way of identifying a spin is to observe continuum degeneracies between the energies in the different lattice irreps. a procedure requiring the calculation be performed at several lattice spacings; thus a spin-4 state should have degenerate energies in the continuum limit between the E, T_1 and T_2 irreps.. An important realization in ref. [3], which we exploit below, is that a knowledge of the continuum form of the overlaps of operators is an important additional source of information that can aid the identification of the spin.

2 Results

We work in the quenched approximation to QCD, using an anisotropic action with temporal and spatial inverse lattice spacings $a_t^{-1} \simeq 6$ GeV and $a_s^{-1} \simeq 2$ GeV respectively. Begining with the $J^{PC} = J^{--}$ sector, we show in Figure 1 the spectrum, together with the spin assignments using the methods outlined above. Our ability to perform such a precise extraction of the low-lying excited states is very encouraging, though the masses are high with respect to quark potential model predictions and, where they exist, experiment, which we attribute to the use of the quenched approximation.

Particularly intriguing is the J^{-+} channel, where our analysis admits two equally plausible spin assignments, shown in Figure 2. In particular, we cannot assert whether the lowest-lying energy in the T_1^{-+} channel corresponds to the exotic 1^{-+}, or the non-exotic 4^{-+}, and consequently whether the lightest exotic is around 4300 MeV, or around 4700 MeV.

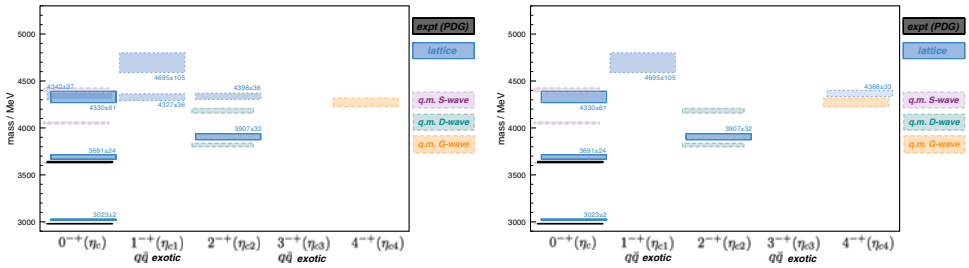

Figure 2: The left- and right-hand panels show the two possible continuum spin assignments in the J^{-+} channel.

We have shown how the variational method, exploiting the continuum behavior of the operators, greatly aids the identification of the continuum spins. Future work will focus on studies at a finer lattice spacing, to better resolve the continuum spin assignments in the T_1^{-+} channel, and on calculations employing 2 and 3 flavors of sea quarks.

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