DEUTERON SPIN DICHROISM IN CARBON TARGET

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Abstract

Birefringence phenomenon for unpolarized deuteron with energy up 20 MeV in carbon target is considered. The estimation for spin dichroism of deuterons is presented. It is shown that magnitude of the phenomenon strongly depends on behavior of the deuteron wave functions on small distance between nucleon in deuteron.

It was shown in [1] that birefringence effect arise for deuterons passing through unpolarized isotropic matter. According to [1] this phenomenon is caused by difference of forward scattering amplitudes for deuterons with spin projection \( m = 0 \) and \( m = \pm 1 \) on coordinate axis parallel to deuteron wave vector \( (m \) is magnetic quantum number). For unpolarized deuteron beam because of spin dichroism caused by birefringence the tensor polarization is appear after target passing. The first experimental study of deuteron spin dichroism in carbon target was carried out at the electrostatic HVEC tandem Van-de-Graaf accelerator with deuterons of up to 20 MeV (Institut für Kernphysik of Universität zu Köln) [2]. As a result spin dichroism of deuteron beam passing through unpolarized carbon target was discovered [2]. Later in 2007 spin dichroism was measured for 5.5 GeV/c deuterons in carbon target on Nuclotron in Dubna [3].

Let us discuss a possible magnitude of the birefringence effect for deuteron with energy up to 20 MeV in carbon target on the base of eikonal approximation. Introducing the deuteron center-of-mass coordinate \( \mathbf{R} \) and the relative distance between the proton and neutron in the deuteron \( \mathbf{r} = \mathbf{r}_p - \mathbf{r}_n \), the Hamiltonian \( H \) can be written as

\[
H = -\frac{\hbar^2}{2m_D} \Delta(\mathbf{R}) + H_D(\mathbf{r}) + H_N(\{\xi_i\}) + V_{DN}^N(\mathbf{R}, \mathbf{r}, \{\xi_i\}) + V_{DN}^C(\mathbf{R}, \mathbf{r}, \{\xi_i\}) \tag{1}
\]

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where $H_D$ and $H_N$ are the deuteron and nuclear Hamiltonian, $V_{DN}$ stands for the energy of deuteron-nucleus nuclear and Coulomb interaction, $\{\xi_i\}$ is a set of coordinates of the nucleons. Let us consider scattering of deuterons with energy above deuterons binding energy $\varepsilon_d$. In that case we can apply the impulse approximation i.e. neglect of $H_D(\mathbf{r})$ in (1). In that approximation $r$ is a parameter, and forward scattering amplitude should be averaged over that parameter. For fast deuterons the scattering amplitude can be found in the eikonal approximation [4], [5]. The forward scattering amplitude can be written in this approximation as follows

$$f(0) = \frac{k}{2\pi i} \int \left( e^{i\xi_0(\mathbf{b},\mathbf{r})} - 1 \right) d^2b |\varphi(\mathbf{r})|^2 d^3r$$

where $k$ is the deuteron wave number, $\mathbf{b}$ is the impact parameter, $\varphi(\mathbf{r})$ is deuterons wave function. The phase shift due to the deuteron scattering by a carbon is $\chi_D = \chi_{pN} + \chi_{nN} + \chi_{C}^D = -\frac{1}{3m} \int_{-\infty}^{+\infty} \{V_{pN} + V_{nN} + V_{C}^D\} dz'$, where $v$ is the deuteron speed. For deuteron with magnetic quantum number $m = \pm 1$, the wave function is $|\varphi_{\pm 1}(\mathbf{r})|^2$, whereas for $m = 0$, it is $|\varphi_0(\mathbf{r})|^2$. Owing to the additivity of phase shifts, equation (2) can be rewritten as

$$f(0) = F_{pN}(0) + F_{nN}(0) + F_{ppN}^C(0) + 2i\frac{k}{\pi} \int t_{nN}(\mathbf{b} - \frac{r_1}{2}) t_{pN}^C(\mathbf{b} + \frac{r_1}{2}) \times$$

$$\times |\varphi(\mathbf{r}_{\perp}, z)|^2 d^2b d^2r_1 dz + \frac{2ik}{\pi} \int t_{pN}(\mathbf{b} - \frac{r_1}{2}) t_{nN}(\mathbf{b} + \frac{r_1}{2}) |\varphi(\mathbf{r}_{\perp}, z)|^2 d^2b d^2r_1 dz$$

$$- \frac{4k}{\pi} \int t_{pN}(\mathbf{b} + \frac{r_1}{2}) t_{nN}(\mathbf{b} - \frac{r_1}{2}) t_{pN}^C(\mathbf{b} + \frac{r_1}{2}) |\varphi(\mathbf{r}_{\perp}, z)|^2 d^2b d^2r_1 dz,$$

where $t_{nN}^{(C)}(\mathbf{p}_{nN}) = e^{i\chi_{nN}(\mathbf{p}_{nN}) - \frac{1}{2i}}$, $F_{nN(pN)}^{(C)}(0) = \frac{k}{\pi} \int t_{nN(pN)}^{(C)}(\xi) d^2\xi = \frac{m_p}{m_n(p)} f_{n(n(p))}^{(C)}(0)$, $F_{ppN}^C(0) = \frac{k}{\pi} \int t_{pN}^C(\xi) d^2\xi$, $\mathbf{r}_{\perp}$ is the $\mathbf{r}$ component perpendicular to the momentum of incident deuteron, $f_{n(n)}^{(C)}(0)$ is the nuclear and the Coulomb amplitude of the proton (neutron)-carbon zero-angle elastic coherent scattering. So spin dependent part of forward scattering amplitude $d_1$ is determined be difference of forward scattering amplitudes for deuterons with $m = \pm 1$ and $m = 0$. In accordance with [1], [2] spin dichroism is determined by $Im(d_1)$. At scattering of deuterons on light nucleus the characteristic radius of the deuteron is large comparing with the radius of nucleus. For this reason for estimation of effects, when integrating, we can suppose that the functions $t_{pN}$ and $t_{nN}$ act on $\varphi$ as $\delta$-function. Then

$$Im(d_1) = -\frac{3}{2\pi} \text{Re} \left\{ F_{nN}(0) \int t_{pN}^C(\xi) \left\{ \frac{1}{\sqrt{2}} \frac{u(r)W(r)}{r^2} - \frac{1}{4} \frac{W(r)^2}{r^2} \right\} \frac{\xi^2 - 2z^2}{r^2} d^2\xi dz \right\} + \frac{6}{k} \text{Re} \left\{ F_{pN}(0)F_{nN}(0) \right\} G - \frac{12}{k} \text{Im} \left\{ F_{ppN}^C(0)F_{nN}(0) \right\} G,$$

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\[ G = \int_0^\infty \left\{ \frac{1}{\sqrt{2}} \frac{u(r)W(r)}{r^2} - \frac{1}{4} \frac{W(r)^2}{r^2} \right\} dr, \quad r^2 = \xi^2 + z^2, \quad u(r) \text{ and } W(r) \]

are the deuteron radial wave functions corresponding to the S-wave and to the D-wave. Now we can evaluate the deuteron spin dichroism. According to [1], [2] spin dichroism \( A \) and tensor polarization can be written as

\[ p_{zz} \approx -\frac{4}{3} A, \quad p_{xx} = p_{yy} \approx \frac{2}{3} A, \]

where \( A = I_0 - I_{\pm 1} I_0 + I_{\pm 1} = N_a z^2 M_r (\sigma_{\pm 1} - \sigma_0) = \frac{2\pi N_a z k M_r \text{Im}(d_1)}{\xi}, \)

\( I_0 \) and \( I_{\pm 1} \) are the occupation numbers for deuterons in spin state \( m = 0 \) and \( m = \pm 1 \) after the target, \( z \) is thickness of target in g/cm\(^2\), \( N_a \) is Avogadro number, \( M_r \) is molar mass for targets matter, \( \sigma_{\pm 1} \) and \( \sigma_0 \) are the deuteron total cross-section of scattering for spin state \( m = \pm 1 \) and \( m = 0 \) respectively.

For estimation of nucleon-carbon strong interaction in (4) lets consider optical Woods-Saxon potential for 5.25 MeV nucleons \( V_{nN}(r) = V_{pN}(r) = -52.9 - 0.9i \exp(2(r - 3.045)). \) For Coulomb p-C interaction in (4) we consider Coulomb screening potential. For calculation of parameter \( G \) the deuterons wave functions from [6] was applied. Obtained value \( G \) is about 0.05.

In (4) the first item is describe contribution of interference of nuclear n-C and Coulomb p-C interactions (lets denote that as NC), the second item is describe contribution of interference of nuclear p-C and n-C interactions (NN) and the third item is describe contribution of interference of nuclear p-C, n-C and Coulomb p-C interactions (NNC). Dependencies on energy of contributions of every items to \( \sigma_{\pm 1} - \sigma_0 \) are shown on Fig.1 a).

So for carbon target with \( z = 0.1 g/cm^2 \) and for energy conditions of experiment (6-13 MeV) [2] dichroism is about 0.01. On the Fig.1 b) is shown dependence of averaged effective difference of total cross-section \( \sigma_{\pm 1} - \sigma_0 \) on averaged deuteron energy inside carbon targets obtained in experiments.

There are some reasons that can essentially increase birefringence effect. The first is interaction of nucleon with carbon. On the Fig.1 c), d) are shown the estimated total cross-section, calculated by Woods-Saxon potential and eikonal approximation in comparison with experimental total cross-section. Interaction of nucleon with carbon has a lot of resonances in energy region of carried out experiment. Experimental cross-section for some energy interval in 2-2.5 times more than estimated that can result in increasing of effects up to 4-6.25 times for that energy interval. At the second, parameter G is very sensitive to deuterons wave functions at small distances. At the third, the increasing of weight of D-state (in [6] it is 4.85%) is increase birefringence effects. According to Fig.1 a) Coulomb scattering play very important role in birefringence value and behavior. Position of peak, caused by Coulomb interaction is sensitive to Coulomb potential so it can be shifted for realistic interaction. Fig.1 a) and Fig.1 d) give qualitative explanation of experimental results on Fig.1 b): sign of dichroism, strong dependence on energy, non-monotone and non-linear dependence of dichroism on target thickness.
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Figure 1: a) Dependencies on deuteron energy of contributions of items NC, NN, NNC and their sum to $\sigma_{\pm 1} - \sigma_0$; b) dependence of averaged effective difference of total cross-section $\sigma_{\pm 1} - \sigma_0$ on averaged deuteron energy inside carbon targets obtained in experiments; c) and d) dependencies on nucleon energy of total nucleon-carbon cross-section calculated by eikonal approximation and from experimental data.

References


