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LOCAL QUARK-HADRON DUALITY IN ELECTRON SCATTERING

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Abstract

We present some recent developments in the study of quark-hadron duality in structure functions in the resonance region. To understand the workings of local duality we introduce the concept of truncated moments, which are used to describe the Q^2 dependence of specific resonance regions within a QCD framework.

1 Introduction

Deep inelastic lepton–nucleon scattering is one of the most effective tools to study the quark and gluon substructure of the nucleon. Since the early days of QCD, deep inelastic scattering data have been successfully analyzed within the framework of the operator product expansion (OPE). At large photon virtuality Q^2 and energy transfer ν, the structure functions can be interpreted in terms of universal parton (quark and gluon) distribution functions.

Much of the initial focus was on reaching higher Q^2 and exploring the small-x region, where $x = Q^2/2M\nu$ is the Bjorken scaling variable and M is the nucleon mass. However, recently attention has turned towards understanding the onset of scaling behavior in structure functions, and the dynamics of the transition from the region dominated by nucleon resonances at low hadron final state masses W. This has been motivated in part by the observation of the intriguing phenomenon of *Bloom-Gilman duality*, in which structure functions measured in the resonance region are found on on average to be approximately equivalent to the scaling function which describes the high-energy data [1, 2]. The equality between the resonance region (characterized by hadronic bound states) and the deep inelastic continuum (characterized by scattering from free quarks) is also referred to as *quark-hadron duality*.

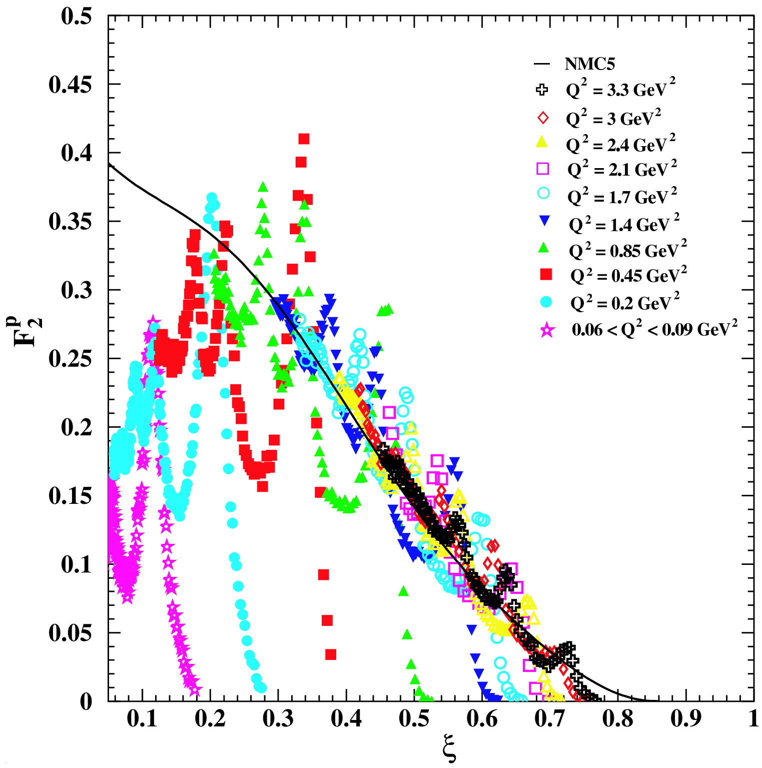

Figure 1: Proton F_2^p structure function data from Jefferson Lab and SLAC in the resonance region in the range $0.06 < Q^2 < 3.30$ GeV^2, as a function of the Nachtmann scaling variable ξ [2]. The solid curve is a fit to deep inelastic data at the same ξ but higher (W^2, Q^2), shown here at $Q^2 = 5$ GeV^2.

First observed by Bloom and Gilman in the late 1960s [1], the duality between resonance and scaling functions has been spectacularly confirmed by recent high-precision data from Jefferson Lab on unpolarized and polarized structure functions. These have enabled the global and local aspects of Bloom-Gilman duality to be quantified, including its flavor, spin and nuclear medium dependence (see Ref. [3] for a review). Data on the proton F_2^p structure functions are illustrated in Fig. 1 in the resonance region for various Q^2 values as a function of the Nachtmann scaling variable $\xi = 2x/(1 + \sqrt{1 + 4M^2x^2/Q^2})$ [4]. The data strongly suggest that duality works remarkably well for each of the prominent, low-lying resonances to rather low values of Q^2 (~ 1 GeV^2).

The appearance of *global* duality in structure functions (*i.e.*, the weak Q^2 dependence of moments at low Q^2) implies strong cancellations between nucleon resonances, resulting in the dominance of the leading twist contribution to the moments. Within the OPE the existence of duality is understood in terms of the suppression of higher twist ($\propto 1/Q^{2n}$ with $n \geq 1$) contributions to moments of structure functions [5, 6]. However, while the OPE can be used to describe the global aspects of duality in terms of moments, the similarity of the resonance and scaling functions in individual resonance regions, over restricted regions of x (or W) — termed *local* duality — is difficult to understand from QCD.

To see how such cancellations may take place, simple models have been considered recently [7–9], in which the resonance transitions can be evaluated exactly and the degree to which duality holds quantified. Although this can yield clues as to how local duality can arise in nature, the connection with QCD is still not very clear. In this talk, we describe a new approach to local Bloom-Gilman duality, in terms of *truncated* moments of structure functions, which allow one to follow the Q^2 evolution of specific resonances, or resonance regions, explicitly [10, 12–17]. With the evolution equations obtained from perturbative QCD, the analysis in terms of truncated moments offers the first direct connection between local duality and QCD.

2 Duality and the OPE

Before the advent of QCD, quark-hadron duality in structure functions was interpreted in the context of finite-energy sum rules, in analogy with the s- and t-channel duality observed in hadron-hadron scattering. In QCD, this duality is understood in the language of the operator product expansion, in which moments of structure functions are organized in powers of $1/Q^2$. For the F_2 structure function, for example, one has for the n-th moment:

$$M_2^{(n)}(Q^2) = \int_0^1 dx \; x^{n-2} \; F_2(x, Q^2) \tag{1}$$

$$= \sum_{\tau=2}^{\infty} \frac{A_\tau^{(n)}(\alpha_s(Q^2))}{Q^{\tau-2}} \; , \tag{2}$$

where $A_\tau^{(n)}$ are the matrix elements of operators with twist $\leq \tau$ (where twist is defined as the mass dimension minus the spin, n, of the operator).

The leading term in Eq. (2) is associated with free quark scattering, and is responsible for the scaling of structure functions, while the $1/Q^{\tau-2}$ terms involve nonperturbative, long-distance interactions between quarks and gluons. The weak Q^2 dependence of the low moments of the structure function is then interpreted as indicating that the non-leading, $1/Q^2$-suppressed, interaction terms do not play a major role even at low Q^2.

An important consequence of duality is that the strict distinction between the resonance and deep inelastic regions becomes entirely artificial. To illustrate this, consider that at $Q^2 = 1$ GeV^2 around 2/3 of the total cross section comes from the resonance region, $W < 2$ GeV [6]. However, the resonances and the deep inelastic continuum conspire to produce only about a 10% higher-twist correction to the lowest moment of the scaling F_2 structure function at the same Q^2. Even though each resonance is built up from a multitude of twists, when combined the resonances interfere in such a way that they resemble the leading-twist component [8].

This by itself is quite a remarkable observation. But how can it be made useful in practice? If the degree to which duality holds, or the extent to which duality is violated, is understood, then the resonance data, when properly averaged, can be used to extract information on the leading-twist (scaling) parts of structure functions. Furthermore, if the inclusive–exclusive connection via local duality is taken seriously, one can relate structure functions measured in the resonance region to electromagnetic transition form factors [1, 11].

3 Truncated Moments

While "global duality" — the duality for structure function moments — can be analyzed in terms of the OPE, a simple interpretation of "local duality" — the x dependence of the functions themselves, or for integrals over restricted regions of x — is elusive. Attempts have been made the understand the emergence of a scaling function from resonances within QCD-inspired models of the nucleon, which have shed some light on the possible dynamics behind the emergence of local duality [7–9].

Recently it was found that local quark-hadron duality can be studied within a perturbative QCD context in terms of *truncated* moments of structure functions, which are integrals of structure functions over restricted intervals of x (or W) [10,12–17]. For the F_2 structure function, the n-th truncated moment for the interval $x_{\rm min} \le x \le x_{\rm max}$ is defined as:

$$\mathcal{M}^{(n)}(x_{\rm min}, x_{\rm max}, Q^2) \;\; = \;\; \int_{x_{\rm min}}^{x_{\rm max}} dx\; x^{n-2} F_2(x, Q^2)\ . \tag{3}$$

In the limit $x_{\rm min} \to 0$ and $x_{\rm max} \to 1$, one recovers the usual (full) moment of Eq. (1), $\mathcal{M}^{(n)}(0, 1, Q^2) \to M_2^{(n)}(Q^2)$.

The remarkable feature of the truncated moments is that they obey Q^2 evolution equations which are similar to the DGLAP evolution equations for parton distribution functions. In particular, the evolution equation for the

n-th truncated moment can be written as [10, 15]:

$$\frac{d\mathcal{M}_n}{dt}(x_{\min}, x_{\max}, Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\left(P'_n \otimes \mathcal{M}_n\right)(x_{\min}, x_{\max}, Q^2) , \quad (4)$$

where $\otimes$ denotes convolution, and P'_n is defined in terms of the QCD splitting function P as:

$$P'_n\left(z, \alpha_s(Q^2)\right) = z^n P\left(z, \alpha_s(Q^2)\right) . \quad (5)$$

The truncated moments therefore satisfy DGLAP-like evolution with a modified splitting function $P \to P'_n$.

The truncated moments can be used to determine the extent to which nucleon structure function data in specific regions in x (or W) are dominated by leading twists [10]. This is done by constructing empirical truncated moments and evolving then to a different Q^2 using the evolution equations in Eq. (4). Deviations of the evolved moments from the experimental data at the new Q^2 then reveal any higher twist contributions in the original data.

Psaker *et al.* [10] have analyzed recent data on the proton F_2 structure function from Jefferson Lab, from which moments were constructed at a range of Q^2 values, from $Q^2 \sim 1$ GeV^2 up to 9 GeV^2. The data at low Q^2 contain significant contributions arising from kinematical effects associated with finite values of $Q^2/\nu^2 = 4M^2x^2/Q^2$. These so-called "target mass corrections" are formally related to twist-two operators, and hence contain no additional information on nonperturbative multi-parton correlations. In the literature there are well known approaches for how to remove the target mass effects, and here this is done by applying the standard TMC prescription from Ref. [18] (see also Ref. [19] for a recent review).

To determine the extent to which the F_2 data at low Q^2 are dominated by leading twist, one assumes that the data at the highest Q^2 value available, namely $Q^2 = 9$ GeV^2, contain only twist-2 contributions. After evolving down to $Q^2 = 1$ GeV^2 and applying the target mass corrections, the truncated moments are compared with the actual data at 1 GeV^2. Preliminary results indicate the presence of higher twists in the data at $Q^2 = 1$ GeV^2. To quantify the higher twist content of the truncated moments in various resonance regions, several intervals in W are considered: $W^2_{\rm th} \leq W^2 \leq 1.9$ GeV^2, corresponding to the $\Delta(1232)$ (or first) resonance region (where $W_{\rm th} = M + m_\pi$ is the inelastic threshold); $1.9 \leq W^2 \leq 2.5$ GeV^2 for the $S_{11}(1535)$ (or second) resonance region; and $2.5 \leq W^2 \leq 3.1$ GeV^2 for the $F_{15}(1680)$ (or third) resonance region.

The results for the $n = 2$ moment generally indicate deviations from leading twist behavior at the level of $\lesssim 20\%$ for all $Q^2 \geq 0.75$ GeV^2, with

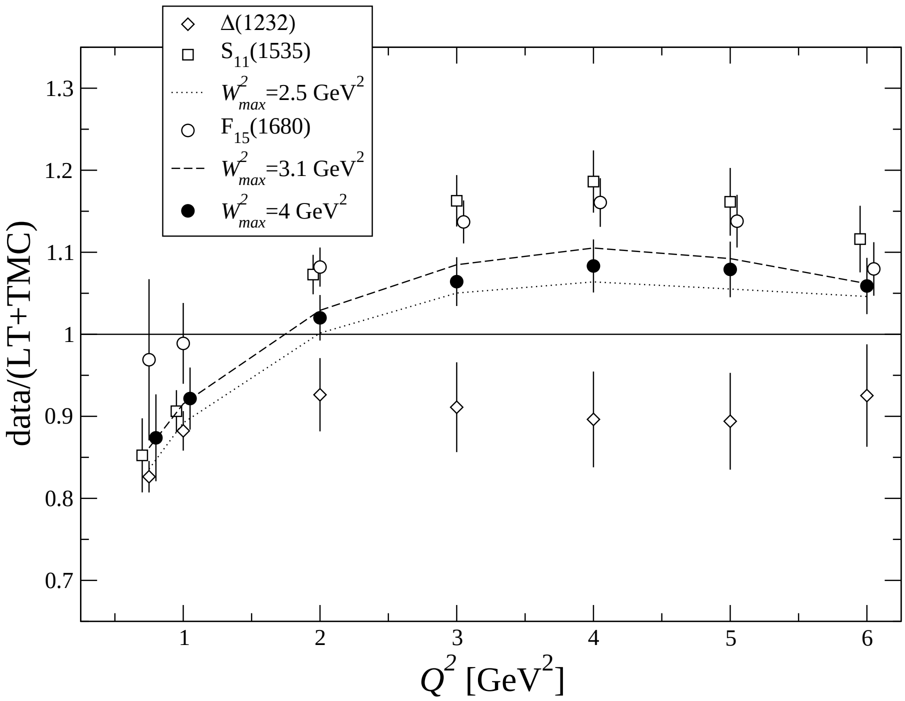

Figure 2: Q^2 dependence of the ratio of truncated moments $\mathcal{M}_2$ calculated from the data and from leading twist evolution from $Q^2 = 9$ GeV^2 (including target mass corrections), for various intervals in W. (Some of the points are offset slightly for clarity.) From Psaker *et al.* [10].

significant Q^2 dependence for $Q^2 \lesssim 3$ GeV^2, decreasing at larger Q^2. In the Δ region (diamonds), the higher twist contributions are approximately -10% of the total. For the S_{11} region (squares), on the other hand, the higher twists constitute $\lesssim +15\%$ of the total moment (except at $Q^2 \leq 1$ GeV^2, where the higher twists change sign). Combined, the higher twist contribution from first two resonance regions (dotted curve) is $\approx 5\%$ at $Q^2 = 4$ GeV^2. The higher twist content of the F_{15} region (open circles) appears to be similar to the S_{11} within errors, and the first three resonance regions combined (dashed curve) contribute $\lesssim 10\%$ for $Q^2 > 1$ GeV^2. Integrating up to $W^2_{\rm max} = 4$ GeV^2, the data on the $n = 2$ truncated moment are found to be leading twist dominated at the level of $90 - 95\%$ for $Q^2 > 1$ GeV^2.

The overall magnitude of the higher twists for the higher moments is qualitatively similar to the $n = 2$ moments, however, the Q^2 values at which they start decreasing in importance are larger. At low Q^2 values the higher twist contributions are also relatively larger for higher moments: at $Q^2 = 1$ GeV^2, for example, the magnitude of the higher twist component of the $W^2 < 4$ GeV^2 region increases from $\sim 15\%$ for the $n = 2$ moment, to $\sim 25\%$ for $n = 4$, and $> 40\%$ for $n = 6$. This behavior can be understood from the relatively greater role played by the nucleon resonances and the large-x region, which is emphasized more by the higher moments.

The relatively small size of the higher twists at scales ~ 1 GeV^2 is consistent with the qualitative observations made in earlier data analyses about the approximate validity of Bloom-Gilman duality [2]. The truncated moment analysis is able to for the first time quantify the degree to which this duality holds as a function of Q^2. The fact that duality works better (*i.e.* higher twists are smaller) when more resonances are included has also been borne out in quark model studies [7–9] (see also Ref. [3] and references therein).

4 Conclusion

Structure functions at low Q^2 provide fertile ground for studying the transition from perturbative to nonperturbative QCD dynamics. The dramatic empirical confirmation of Bloom-Gilman duality in inclusive electron–nucleon scattering has stimulated considerable interest in understanding the dynamical origins of duality and of low Q^2 physics in general.

We have outlined here how the operator product expansion can be used to understand the qualitative features of global duality in terms of moments of structure functions. The phenomenon of quark-hadron duality appears to go further beyond this, however, and a non-trivial connection is found to exist between resonance and scaling functions for individual resonance regions. Understanding the emergence of this *local* duality from first principles is considerably more challenging, and to date the only attempts to do so have been within QCD-inspired models of the nucleon — which at best can only serve to give us clues as to the underlying dynamics.

In this talk we have described a new approach to local duality in terms of *truncated* moments of structure functions. The crucial feature of these moments is that they obey Q^2 evolution equations similar to the DGLAP equations obeyed by parton distribution functions. The same techniques for evolving the parton distributions can therefore be used for the truncated moments.

We find that at a scale of $Q^2 = 1$ GeV^2 the Δ resonance region contains about -10% higher twist contributions to the total $n = 2$ moment. The higher twists in the first and second resonance regions are positive, each contributing $\lesssim +15\%$ higher twist to the moment. Combined, the entire nucleon resonance region up to $W^2_{\rm max} = 4$ GeV^2 contains about $5 - 10\%$ higher twist for $Q^2 > 1$ GeV^2. Similar behavior is found also for the $n = 4$ and $n = 6$ truncated moment ratios, where the relatively greater role played by the resonances due to the large-x enhancement means larger higher twists at the same Q^2.

These results represent the first quantitative determination of higher twists in individual resonance regions within a QCD framework. This work thus opens the way to further study of local duality in other structure functions, such as the longitudinal structure function F_L or spin-dependent structure functions.

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References

[1] E. D. Bloom and F. J. Gilman, *Phys. Rev. Lett.* **25**, 1140 (1970).

[2] I. Niculescu *et al.*, *Phys. Rev. Lett.* **85**, 1182, 1186 (2000).

[3] W. Melnitchouk, R. Ent and C. Keppel, *Phys. Rept.* **406**, 127 (2005).

[4] O. Nachtmann, *Nucl. Phys.* **B 63**, 237 (1973).

[5] A. De Rujula, H. Georgi and H. D. Politzer, *Phys. Rev.* **D 15**, 2495 (1977).

[6] X. D. Ji and P. Unrau, *Phys. Rev.* **D 52**, 72 (1995).

[7] F. E. Close and N. Isgur, *Phys. Lett.* **B 509**, 81 (2001).

[8] N. Isgur, S. Jeschonnek, W. Melnitchouk and J. W. Van Orden, *Phys. Rev.* **D 64**, 054005 (2001).

[9] F. E. Close and W. Melnitchouk, *Phys. Rev.* **C 68**, 035210 (2003).

[10] A. Psaker, W. Melnitchouk, M. E. Christy and C. E. Keppel, in preparation.

[11] W. Melnitchouk, *Phys. Rev. Lett.* **86**, 35 (2001).

[12] S. Forte and L. Magnea, *Phys. Lett.* **B 448**, 295 (1999).

[13] S. Forte, L. Magnea, A. Piccione and G. Ridolfi, *Nucl. Phys.* **B 594**, 46 (2001).

[14] A. Piccione, *Phys. Lett.* **B 518**, 207 (2001).

[15] D. Kotlorz and A. Kotlorz, *Acta Phys. Polon.* **B 36**, 3023 (2005).

[16] D. Kotlorz and A. Kotlorz, *Eur. Phys. J.* **C 48**, 457 (2006).

[17] D. Kotlorz and A. Kotlorz, *Phys. Lett.* **B 644**, 284 (2007).

[18] H. Georgi and H. D. Politzer, *Phys. Rev.* **D 14**, 1829 (1976).

[19] I. Schienbein *et al.*, arXiv:0709.1775 [hep-ph], to appear in *J. Phys. G.*