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ISOSPIN VIOLATING NUCLEON FORM FACTORS

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Abstract

A quantitative understanding of isospin violation is an increasingly important ingredient for the extraction of the nucleon's strange vector form factors. We present a theoretical analysis of the isospin violating form factors, both for single nucleons and for ^{4}He.

1 Isospin Violation and Nucleon Strangeness

The investigation of strangeness contributions to static properties of the nucleon is particularly interesting as it gives unambiguous access to low-energy manifestations of virtual or sea quark effects. Different strangeness currents of the form $\bar{s}\Gamma s$ test the strangeness component of different nucleon observables, such as mass ($\Gamma = 1$), spin ($\Gamma = \gamma_\mu\gamma_5$), or magnetic moment ($\Gamma = \gamma_\mu$). Here we are concerned with the magnetic moment only, or, more generally, with the nucleon form factors of the vector current.

The standard model provides two different flavor combinations of the three light quark contributions to the electric (G_E) and magnetic (G_M) form factors due to the electromagnetic and the weak vector currents,

$$G^{\gamma,p}_{E/M} = \frac{2}{3}G^u_{E/M} - \frac{1}{3}\big(G^d_{E/M} + G^s_{E/M}\big) , \tag{1}$$

$$G^{Z,p}_{E/M} = \Big(1 - \frac{8}{3}\sin^2\theta_W\Big)G^u_{E/M} - \Big(1 - \frac{4}{3}\sin^2\theta_W\Big)\big(G^d_{E/M} + G^s_{E/M}\big) , \tag{2}$$

so in order to obtain a full flavor decomposition of the vector current, one has to invoke isospin (or charge) symmetry in the form

$$G^{u,n}_{E/M} = G^{d,p}_{E/M} , \quad G^{d,n}_{E/M} = G^{u,p}_{E/M} , \tag{3}$$

and use the neutron electromagnetic form factors as the third input. If one relaxes this assumption and allows for isospin violation, however, the relation between weak vector form factors, electromagnetic form factors of proton and neutron, and strangeness is complicated by an additional term,

$$G_{E/M}^{Z,p} = \big(1 - 4\sin^2\theta_W\big) G_{E/M}^{\gamma,p} - G_{E/M}^{\gamma,n} - G_{E/M}^{s} - G_{E/M}^{u,d} \ , \qquad (4)$$

where $G_{E/M}^{u,d} = 2/3(G_{E/M}^{d,p} - G_{E/M}^{u,n}) - 1/3(G_{E/M}^{u,p} - G_{E/M}^{d,n})$. In other words, the isospin violating form factors $G_{E/M}^{u,d}$ generate "pseudo-strangeness", and in order to reliably extract strangeness effects, the former have to be calculated from theory.

2 Theory of Isospin Violating Form Factors

Chiral perturbation theory (ChPT) is ideally suited for an analysis of isospin violation. It is tailor-made to analyze the dependence of low-energy observables on quark masses, in particular on the light quark mass difference $m_u - m_d$, and the consistent inclusion of electromagnetic effects is also well-understood. As the isospin violating form factors can be calculated in SU(2) ChPT, they are not affected by convergence problems to the extent the strangeness form factors are (see Ref. [1] for a brief review on the latter).

Particular emphasis will be put on the analysis of the leading moments of the isospin violating form factors, magnetic moment as well as electric and magnetic radius terms,

$$G_E^{u,d}(t) = \rho_E^{u,d} t + \mathcal{O}(t^2) \ , \qquad G_M^{u,d}(t) = \kappa^{u,d} + \rho_M^{u,d} t + \mathcal{O}(t^2) \ . \qquad (5)$$

The two radius terms are unaffected by low-energy constants up to leading ($\rho_E^{u,d}$) and next-to-leading ($\rho_M^{u,d}$) order and can be expressed entirely in terms of the neutron-to-proton mass difference $\Delta m = m_n - m_p$ [2], with the result [3]

$$\rho_E^{u,d} = \frac{5\pi C}{6 M_\pi m_N} \ , \quad \rho_M^{u,d} = \frac{2C}{3M_\pi^2}\left\{1 - \frac{7\pi}{4}\frac{M_\pi}{m_N}\right\} \ , \quad C = \frac{g_A^2 m_N \Delta m}{16\pi^2 F_\pi^2} \ . \qquad (6)$$

It is remarkable that up to $\mathcal{O}(p^4)$ for $G_E^{u,d}$ and $\mathcal{O}(p^5)$ for $G_M^{u,d}$, no photon loops contribute, nor are there two-loop effects, nor does the pion mass difference $M_{\pi^+}^2 - M_{\pi^0}^2$ play a role.

In order to complete the chiral representation, we have to estimate the combination of low-energy constants entering $\kappa^{u,d}$. This is done by invoking resonance saturation: low-energy constants incorporate the effects of heavier states not included in the theory as explicit degrees of freedom. In our case, the relevant contributions are provided by vector mesons including $\rho - \omega$ mixing:

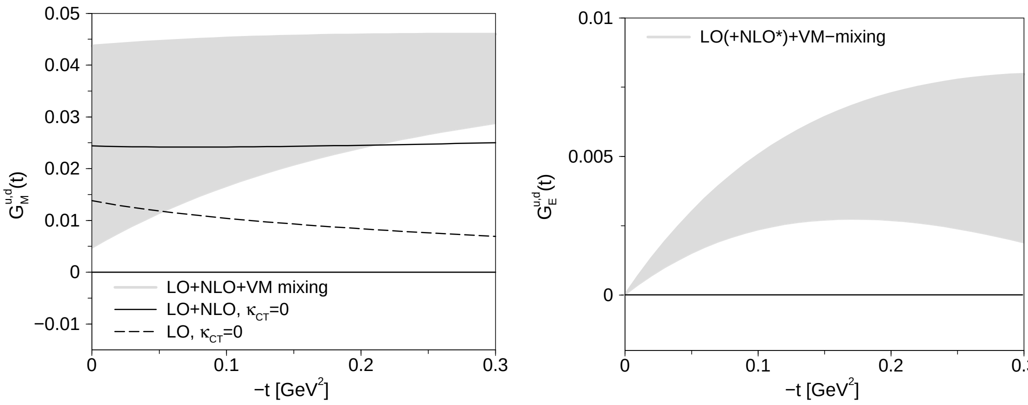

Figure 1: Isospin violating form factors. Figures taken from Ref. [3].

$$
\begin{aligned}
G_E^{u,d}(t)\Big|_{\rm mix} &= \frac{\Theta_{\rho\omega}\, t}{M_V(M_V^2-t)^2}\bigg[\Big(1+\frac{\kappa_\omega M_V^2}{4m_N^2}\Big)g_\omega F_\rho - \Big(1+\frac{\kappa_\rho M_V^2}{4m_N^2}\Big)g_\rho F_\omega\bigg]\ , \\
G_M^{u,d}(t)\Big|_{\rm mix} &= \frac{\Theta_{\rho\omega}}{M_V(M_V^2-t)^2}\Big[\big(t+\kappa_\omega M_V^2\big)g_\omega F_\rho - \big(t+\kappa_\rho M_V^2\big)g_\rho F_\omega\Big]\ , \qquad (7)
\end{aligned}
$$

where the necessary couplings can be extracted from experiment within certain errors; see Ref. [3] for details.

The total results for the isospin violating form factors are shown in Fig. 1. Isospin breaking remains at the percent level, the t-dependence of the form factors is rather moderate. We note that the symmetries of QCD do *not* dictate $\kappa^{u,d}$ to vanish, indeed we find $G_M^{u,d}(0) \neq 0$. Table 1 compares the specific linear combinations of $G_E^{u,d}$ and $G_M^{u,d}$ at $t \approx -0.1$ GeV^2 with the experimentally extracted values for strangeness form factors.

Table 1: Comparison of selected experimental measurements of strange form factors from SAMPLE [4], A4 [5], and HAPPEX [6] to the results of Ref. [3] for the isospin violating form factors.

experiment	electric/magnetic	G^s	$G^{u,d}$
SAMPLE	G_M	$0.37 \pm 0.20 \pm 0.26 \pm 0.07$	$0.02 \ldots 0.05$
A4	$G_E + 0.106\, G_M$	0.071 ± 0.036	$0.004 \ldots 0.010$
HAPPEX	$G_E + 0.080\, G_M$	$0.030 \pm 0.025 \pm 0.006 \pm 0.012$	$0.004 \ldots 0.009$

3 Isospin Mixing in Helium-4

Parity-violating electron scattering on ^{4}He gives clean access to the strange *electric* form factor G_E^s, as the $J^\pi = 0^+$ target does not allow for magnetic or axial vector transitions. However, in addition to effects of the isospin violating electric form factor, an $I = 1$ admixture in the ^{4}He wave function yields a contribution to the measured asymmetry A_{PV} [7],

$$A_{PV} = -\frac{G_\mu t}{4\pi\alpha\sqrt{2}}\Big\{4\sin^2\theta_W + \Gamma\Big\} , \quad \Gamma = -2\,\frac{F^{(1)}}{F^{(0)}} - \frac{2G_E^{\not{\chi}} - G_E^s}{(G_E^p + G_E^n)/2} , \qquad (8)$$

where $F^{(0/1)}$ are the nuclear form factors corresponding to isoscalar/isovector charge operators, and $G_E^{\not{\chi}} = (G_E^{u,p} - G_E^{d,n} - G_E^{d,p} + G_E^{u,n})/4$ is a different isospin breaking linear combination of single-nucleon form factors. The measured asymmetry $A_{PV} = \big[+6.40 \pm 0.23_{\rm stat} \pm 0.12_{\rm syst}\big] \times 10^{-6}$ at $t = -0.077$ GeV^2 [8] leads to $\Gamma = 0.010 \pm 0.038$. Single-nucleon isospin violation contributes 0.008 ± 0.003 to Γ, while isospin mixing in the ^{4}He wave function amounts to ≈ 0.003, leaving a mere strangeness contribution of $G_E^s = -0.001 \pm 0.016$.

4 Conclusions

The contributions of isospin violation are as yet smaller than some of the experimental uncertainties in extracting strange form factors, see Table 1, but in the case of the ^{4}He experiment, they are sufficient to explain the central value of the measured asymmetry (again within larger error bounds). Clearly, isospin breaking effects will become an essential ingredient for future precision extractions of strangeness matrix elements.

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