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Hadron structure from lattice QCD

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Abstract

Some elements and current developments of lattice QCD are reviewed, with special emphasis on hadron structure observables. In principle, high precision experimental and lattice data provide nowadays a very detailed picture of the internal structure of hadrons. However, to relate both, a very good controle of perturbative QCD is needed in many cases. Finally chiral perturbation theory is extremely helpful to boost the precision of lattice calculations. The mutual need and benefit of all four elements: experiment, lattice QCD, perturbative QCD and chiral perturbation theory is the main topic of this review.

1 Elements of lattice QCD

All information on hadron structure is, in principle, contained in the generating functional of QCD

$$Z[J^a_\mu, \overline{\eta}^i, \eta^i] = \int \mathcal{D}[A^{a\mu}, \overline{\psi}^i, \psi^i] \exp\left(\mathrm{i}\int d^4x \left[\mathcal{L}_{\mathrm{QCD}} - J^a_\mu A^a_\mu - \overline{\psi}^i \eta^i - \overline{\eta}^i \psi^i\right]\right) \tag{1}$$

The tasks to be performed are:
i.) to classify this information in a suitable manner, namely in terms of specific correlators
ii.) to calculate these from Eq.(1), which is done numerically by lattice QCD after analytic continuation to Euclidean space-time,
iii.) and to relate them to experimental observables, which is done by Operator Product Expansion and perturbative QCD.

i.) Let us illustrate the physical content of specific correlators with a few typical examples:

$\left\langle P(p)\middle|\overline{q}(x)\gamma_\mu D_{\mu_1}...D_{\mu_n}q(x)\middle|P(p)\right\rangle$: Here D_{μ_j} is the gauge invariant derivative. For a suitable symmetrization of the Lorentz indices these operators are of twist-2 and thus have a simple probabilistic interpretation in terms of moments of the momentum distribution of a quark species q in a hadron P, e.g. a proton, $\int_0^1 dx x^n (q(x) \pm \overline{q}(x))$. For mixed symmetry combinations these correlators describe, e.g., specific correlations between the quark and gluon fields. (One should remember that the commutator $\left[D_{\mu_i}, D_{\mu_j}\right]$ gives the gluon field strength tensor.)

$\left\langle P(p)\middle|\overline{q}(x)\Gamma_\mu q(x)\overline{q}'(x)\Gamma'_\mu q'(x)\middle|P(p)\right\rangle$:
This quantity describes diquark correlations within a proton.

$\left\langle P(p')\middle|\overline{q}(x)\gamma_\mu q(x)\middle|P(p)\right\rangle$: This quantity provides the form factors of a proton.

$\left\langle 0\middle|\overline{d}(-z)\not{z}[-z,z]u(z)\middle|\rho^+(p,s)\right\rangle$: This expression provides information on the ρ distribution amplitude. While distribution functions correspond to the squares of a wave functions, integrated over $k_\perp$, distribution amplitudes correspond to the wave functions themselves, integrated over $k_\perp$.

$\left\langle 0\middle|\overline{u}(z)u(z)\middle|0\right\rangle$: Is the best known of an infinite tower of vacuum condensates, which characterize the QCD-vacuum itself, just as any other 'hadronic' state.

Recently this collection of correlators was substantially enlarged, namely by various non-local correlators. Correlators, which are non-local along the light cone coordinate

$$\left\langle P(p,s)\middle|\overline{q}(-z^-,\mathbf{z}_\perp=\mathbf{0})\gamma_\mu[-z,z]q(z,\mathbf{z}_\perp=\mathbf{0}))\middle|P(p',s')\right\rangle \tag{2}$$

parametrize the Generalized Parton Distributions (GPDs), see [1] and references therein, which provide a comprehensive description of hadron structure, allowing to combine formerly unrelated information in a set of analytic functions, namely the GPDs. Finally correlators which are also non-local in the transverse direction

$$\left\langle P(p,s)\middle|\overline{q}(-z^-,\mathbf{0})\gamma_\mu[-z,\mathbf{0};z,\mathbf{z}_\perp]q(z,\mathbf{z}_\perp)\middle|P(p',s')\right\rangle \tag{3}$$

are intimately connected to various, surprisingly large spin-phenomena, as observed e.g. by the HERMES experiment, see [2]. (They actually challenge

our current theoretical understanding of perturbative QCD.)
Obviously each of these (and many other) correlators would deserve a chapter on its own, but this would far exceed the scope of this small review.
To summarize: **The task is to calculate for each hadron the matrix elements of many different quark-gluon-operators**.

ii.) Obviously, this is an easy task to do, if one knows the exact many-particle hadron wave function. The latter is provided by lattice QCD, however, only in Euclidean space-time.

The basic idea is that by analytic continuation of the time-coordinate $t \to -\mathrm{i}t_E$ the exponential weight in Eq.(1) becomes positive definite. (Note that $\mathcal{L} \to -\mathcal{H}_E$ and thus $\mathrm{i}S \to -S_E$.) If one then places some source, i.e. a combination of quarks and possibly gluons with the desired quantum numbers, on the lattice, Euclidean time evolution projects out the desired exact (in principle) hadron wave function. For the proton, on can choose, e.g., as source ($C = i\gamma^2\gamma^4 = C^{-1}$ is the charge conjugation matrix):

$$\hat{B}_\alpha(t,\vec{p}) = \sum_{\vec{x}} e^{\mathrm{i}\vec{p}\cdot\vec{x}} \epsilon_{ijk} \hat{u}^i_\alpha(x)\ \hat{u}^j_\beta(x)(C^{-1}\gamma_5)_{\beta\gamma}\hat{d}^k_\gamma(x) \tag{4}$$

and then gets

$$\begin{aligned} |B\rangle \quad &\sim \quad c_0|N\rangle \ + \ c_1|N'\rangle \ + \ c_2|N\pi\rangle \ + \ ... \\ &\Rightarrow \quad c_0 e^{-E_N t_E}|N\rangle \ + \ c_1 e^{-E_{N'} t_E}|N'\rangle \ + \ c_2 e^{-E_{N\pi} t_E}|N\pi\rangle \ + \ ... \end{aligned} \tag{5}$$

For sufficiently large t_E this sum will obviously converge to the lowest energy state. Over the years rich experience was gained in the development of sources which maximize from the very beginning the overlap with a specific physical hadron state. For a (quenched) show-case example see Fig.1. (Note that to the right one sees the slope of the lowest nucleon resonance with negative parity.)

Once the propagation in imaginary time has projected the original source onto the physical wave function on can calculate physical correlators from

$$\frac{\tilde{\Gamma}_{\alpha\beta}\langle B_\beta(t,\vec{p})\mathcal{O}\overline{B}_\alpha(0,\vec{p})\rangle}{\Gamma_{\alpha\beta}\langle B_\beta(t,\vec{p})\overline{B}_\alpha(0,\vec{p})\rangle} \tag{6}$$

This procedure allows to calculate many of the quantities of interest, however, it is limited to those operators which contain only coordinate differences which are oriented either spatially or along the light cone direction in which the hadron is Lorentz-contracted (typically called z^-). Otherwise the

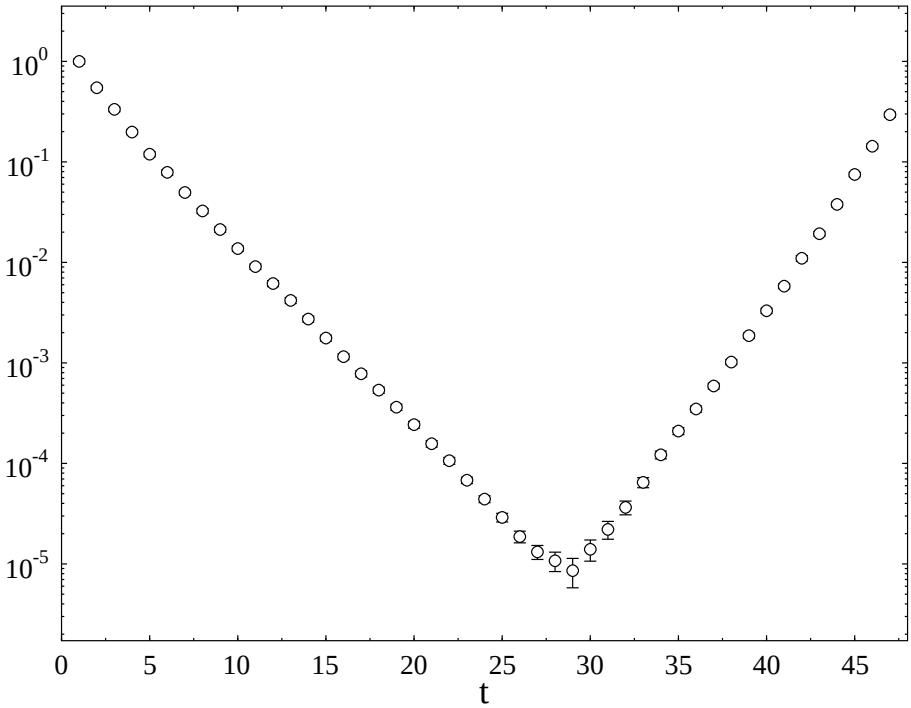

Figure 1: A show-case example for a clean exponential slope (nucleon, quenched Wilson fermions, QCDSF).

operator itself is affected by the analytic continuation to Euclidean times. Probably for many cases the resulting effects can be corrected for by making use of the analytic properties of the continuum expressions, but such efforts are, to the best of my knowledge, still in their infancy.

ii.) Linking lattice results and experiment is far more difficult than usually appreciated. It is only simple for very few cases, especially hadron masses, as these are direct experimental observables. However, reproducing known hadron spectra has only limited impact beyond fixing the quark masses. Basically it demonstrates that QCD is correct and that the technical aspects of lattice QCD are well understood, which nobody seriously doubts anyway. The real strength of lattice QCD is that it provides detailed direct information on the quark-gluon structure of hadrons. As the latter is not directly observable in experiment, it is crucial for the whole program of 'Hadron structure from lattice QCD' that one succeeds to link the correlators calculated on the lattice to experimental observables with help of perturbative QCD in the continuum. The latter has made tremendous progress in recent years.

Presently NLO calculations are the standard and NNLO calculations and beyond start to become common. Furthermore, resummation techniques have been developed which allow to improve very significantly the phenomenological agreement and to reduce the estimated systematic uncertainties. Unfortunately, this is not the place to give justice to these developments. Let me stress, however, one point which is specific to the lattice: lattice propagators differ from their continuum form. They contain periodic functions, as known from Brillouin zones from solid state lattices. Therefore, also the radiative corrections and thus the renormalization effects differ. The problem is aggravated by the fact that the discrete hypercubic symmetry group is much smaller than the continuous Lorentz group, such that on the lattice operators mix, further complicating the renormalization issues. Finally, lattice results correspond to all order calculations in perturbation theory and can, therefore, only be compared meaningfully to fixed order continuum results to the extent that the latter converge.
The bottom line of this section is that **the control of all theoretical uncertainties is today the main theme of modern hadron theory, both on and off the lattice**.

2 The choice of lattice actions

This question which discretization of Eq.(1), i.e. which lattice action is the best choice is often debated with close to religious fervor. As the computer resources absorbed by lattice-QCD add up world-wide to many millions of Euros per year, this is not just a question of academic interest, but a major issue for all universities and laboratories involved. In the following I will not argue for our position in this debate, but instead I will show examples from different groups without any assessment of the different actions used. I would like, however, to caution the reader that typically the unknown systematic uncertainties are largest for those choices for which the purely statistically uncertainties cited by the different groups are smallest. Otherwise the choice of the 'best' action would obviously not require any debate. One might, in fact, conclude from the fact that many highly experienced groups favor so many different choices, that presently no choice is clearly superior to any other, and that a mixture of strategies is in fact the wisest decision.
Some of the presently used fermion actions are:
Chiral fermions: Overlap fermions are the only realization with exact chiral symmetry presently used. For all other actions chiral symmetry is more or less heavily violated and only recovered in the limit of vanishing lattice constant $a \to 0$. Note that in practice the range of a reachable by numerical

simulations is very limited. Therefore, if the a-dependencies are not well described by simple linear fits, one faces a most severe problem.
Approximately chiral fermions: The best known example are domain wall fermions, which introduce a fifth space dimension. In the limit that the extension of this fifth lattice direction goes to infinity, chiral symmetry becomes exact. More severe approximations are involved in the construction of the 'Chirally Improved'- and 'Perfect Action' - fermions, which on the other hand are computationally substantially less expensive.
Fermion actions with remnants of chiral symmetry: Staggered fermions are certainly the most controversial choice. For them the continuum limit $a \to 0$ and the chiral limit $m_\pi \to 0$ do not commute, instead of the pion one gets 5 varieties of particles with different masses, quadratically divergent contributions to propagators (for finite a), etc. Therefore, the validity of this approach depends crucially on the availability and control of the corresponding 'Staggered Chiral Perturbation Theory' which so far is only given for the meson, but not the baryon sector. On the other hand, however, staggered fermion simulations seem to provide the phenomenologically most impressive results.
Twisted-mass fermions have also pions of different masses, due to the addition of an additional isospin dependent term (vanishing in the continuum limit). This construction allows, however, to reach very small quark masses and the authors argue quite convincingly that the isospin violating effects are small in the baryon sector and anyway quite well under control.
Fermion actions without chiral symmetry: Some of the most used actions are variants of Wilson fermions, for which chiral symmetry is broken explicitly. They are in general very well understood and do not invokes any fundamental problems. There exist extremely efficient algorithms for Wilson and the results are in general phenomenologically satisfactory, with the exception of a few disturbing puzzles.

3 Some recent results

For hadron spectroscopy the quark masses are basically free parameters, which are determined by the optimal fit of the physical hadron masses. In view of the discussion above, quark masses in the continuum depend on the renormalizaton/factorization scheme and the chosen scale(s). By general convention one cites today typically the values in the $\overline{MS}$-scheme and at the scale $\mu = 2$ GeV. Fig. 2 gives a recent compilation from [3] for the strange quark mass. Here $N_f = 2$ and $N_f = 2 + 1$ refers to the number of dynamical

quarks. For $N_f = 2$ only up and down quark-antiquark fluctuations are taken into account, for $N_f = 2+1$ also strange ones. Note that the results are generally smaller than the values used by many model-builders.

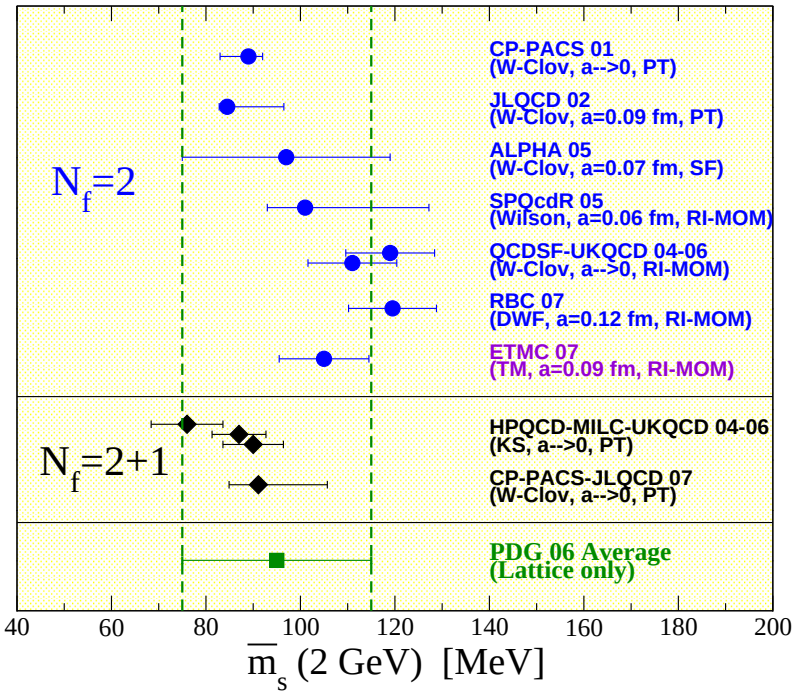

Figure 2: A recent compilation of lattice results for the strange quark mass.

Technically the next difficult observables are two-point functions, which provide coupling constants as well as moments of parton distribution amplitudes. The ρ coupling constants to the vector and tensor quark currents are defined e.g. by

$$\langle\, 0 \,|\, \overline{q}_2(0)\gamma^\mu q_1(0) \,|\, \rho(p;\lambda)\,\rangle \;=\; f_\rho\, m_\rho\, \varepsilon^\mu_\lambda \tag{7}$$

$$\langle\, 0 \,|\, \overline{q}_2(0)\sigma^{\mu\nu} q_1(0) \,|\, \rho(p;\lambda)\,\rangle \;=\; i f^T_\rho(\mu)\, \left(\varepsilon^\mu_\lambda p^\nu - \varepsilon^\nu_\lambda p^\mu\right)\,, \tag{8}$$

Fig.3 shows recent result obtained by the UKQCD+RBC collaboration with domain wall fermions (hence the residual mass m_{res}, which is associated with the fifth dimension and has to be extrapolated to zero.)

The result is

$$\frac{f^T_\rho}{f_\rho} \;=\; 0.681(20) \tag{9}$$

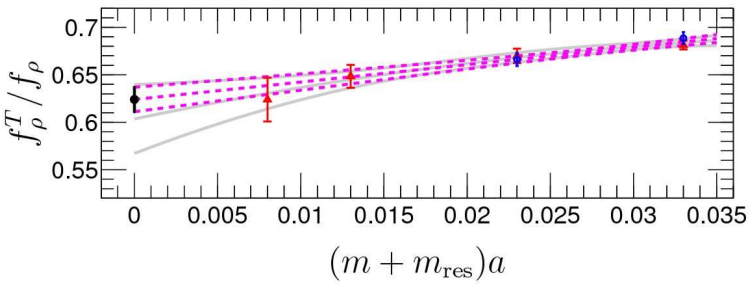

Figure 3: Chiral extrapolations for f_ρ^T/f_ρ for a UKQCD/RBC simulation with domain-wall fermions. The broken red lines represent a linear fit to the mass behavior and the solid grey lines a quadratic fit. Taken from [4]

which is somewhat smaller than earlier results from the BGR collaboration, 0.742 ± 0.014, which used quenched Chirally Improved fermions, see [5].

The calculation of many different decay constants for heavy quark systems is the main emphasis of the Fermilab/MILC collaboration, which uses staggered quarks. These results are important for the analysis of BABAR and BELLE results. Instead of discussing these in detail we show in Fig.4 another result of this collaboration, namely the D to K decay formfactor. Note that the lattice paper cited here actually antedated the experimental one, see [6].

Distribution amplitudes are the $k_\perp$ integrals of the exact wave functions, e.g., for pseudoscalar hadrons one defines

$$\phi_\Pi(x,\mu^2) = Z_2(\mu^2) \int^{|k_\perp|<\mu} d^2k_\perp \, \phi_{\Pi,BS}(x,k_\perp). \tag{10}$$

when $\phi_{\Pi,BS}$ is the Bethe-Salpeter wave function of the pseudoscalar. $\phi_\Pi(x,\mu^2)$ contains as valuable information as distribution functions, which are given by the $k_\perp$ integrals of the squares of the wave functions. Phenomenologically, distribution amplitudes play a major role in the description of exclusive reactions. With the definitions

$$\begin{aligned} \langle 0|\overline{q}(-z)\gamma_\mu\gamma_5[-z,z]u(z)|\Pi^+(p)\rangle &= i f_\Pi p_\mu \int_{-1}^{1} d\xi \, e^{-i\xi p\cdot z}\phi_\Pi(\xi,\mu^2) \\ \phi_\Pi(\xi,\mu^2) = \frac{3}{4} \quad (1-\xi^2) &\left(1+\sum_{n=1}^{\infty} a_n^\Pi(\mu^2) C_n^{3/2}(\xi)\right). \end{aligned} \tag{11}$$

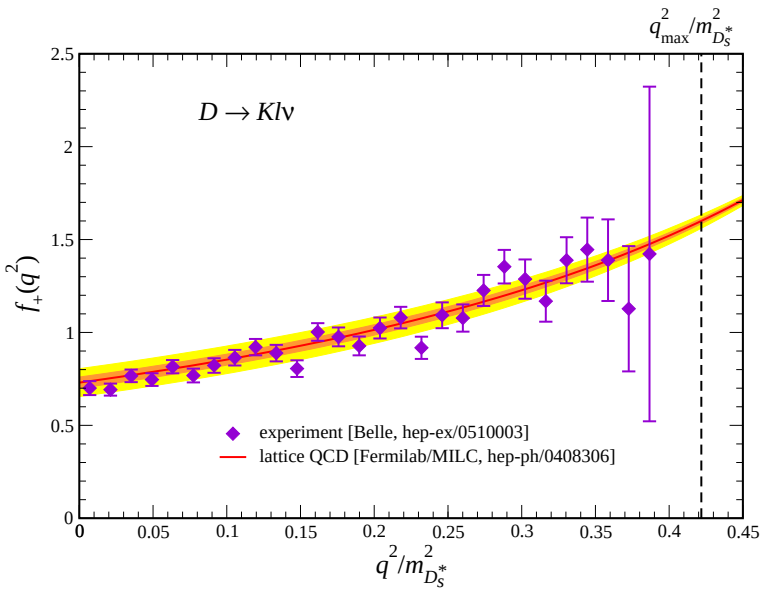

Figure 4: Form factor $f_+(q^2)$ for $D \to Kl\nu$ vs. $q^2/m^2_{D_s^*}$ compared with Belle data.

the tasks reduces to the calculation of matrix elements of the operators

$$\mathcal{O}_{\{\mu_0 \ldots \mu_n\}} = \overline{q}\gamma_{\{\mu_0}\gamma_5 \overleftrightarrow{D}_{\mu_1} \ldots \overleftrightarrow{D}_{\mu_n\}} u \tag{12}$$

which give directly the ξ^n-moments of the distribution amplitude and thus allow to fix the expansion coefficients a_n^Π. Fig. 5 shows QCDSF results [7] for the pion, obtained with improved $N_f = 2$ Wilson fermions.

The final result is $[a_2^\pi(\mu^2 = 4\ \mathrm{GeV}^2) = 0.201(114)$ in rough agreement with the results from analyses of QCD sumrules, B-decays and transition formfactors, which give $a_2^\pi(4\ \mathrm{GeV}^2) = 0.17 \pm 0.15$

All of the examples given so far are only appetizers for the richness of present day lattice results. To finish this overview let us discuss one specific more complex example. Transverse asymmetries, observed in e.g. by HERMES [2], can be related by perturbative QCD to the so-called Boer-Mulders functions, which in turn are related to the spatial distribution of quarks in the transverse plain (perpendicular to its momentum direction), which in turn is parametrized by GPDs, moments of which can be calculated on the lattice. Some recent QCDSF results are shown in Fig.6. For details see [8,9]. These results were obtained with dynamical improved Wilson fermions.

Another example for the power of the GPD framework is given in Fig.7, where recent LHPC results for the spin and orbital angular momentum struc-

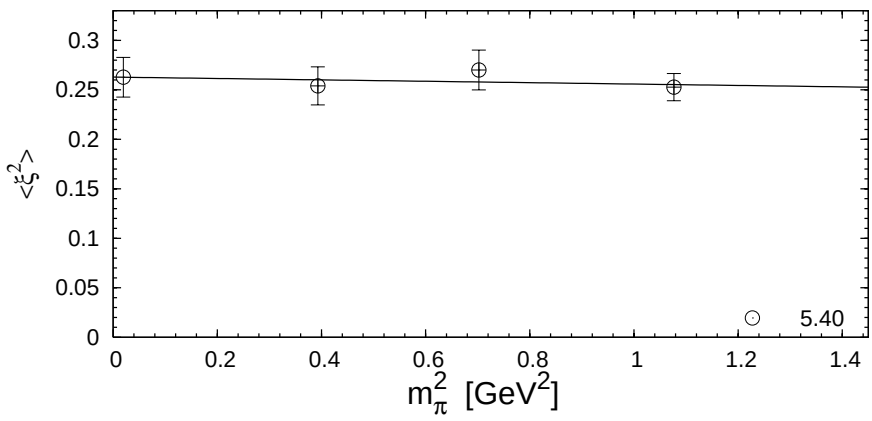

Figure 5: Chiral extrapolation of $\langle \xi^2 \rangle_\pi$ for $\beta = 5.40$ for $\mathcal{O}^a_{412}$ in the $\overline{\text{MS}}$ scheme at $\mu^2 = 4\,\text{GeV}^2$

ture of the nucleon are shown. These results were obtained with a mixed action, combining a staggered fermion sea with domain wall valence quarks, see [10] .

4 Conclusion

Lattice QCD, perturbative QCD, chiral perturbation theory (a topic which was barely addressed but is also of crucial importance) and experiment have reached in the last years such an accuracy that finally detailed questions regarding the quark gluon structure of hadrons can be precisely answered with controlled theoretical uncertainties. A few examples were given, to illustrate the wealth of information to be expected from lattice calculations in the near future. All of this promisees to change the nature of hadron physics as QCD-motivated models will be more and more replaced by full-fledged QCD calculations.

5 Acknowledgements

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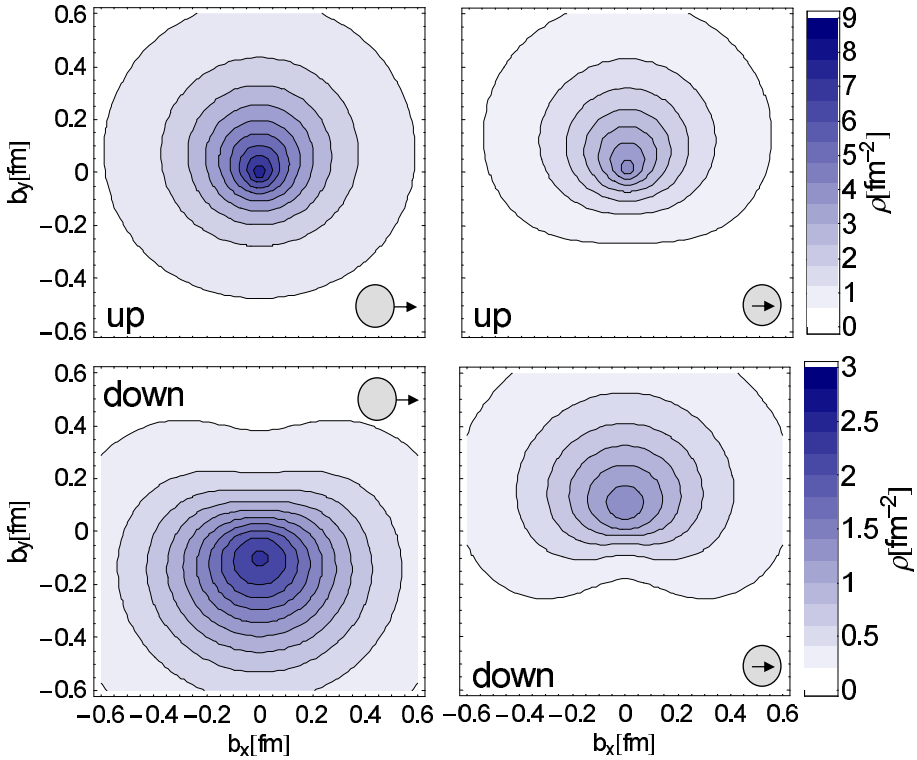

Figure 6: Transverse distribution of up and down quarks in a nucleon (integrated over the longitudinal momentum fraction x) for different relative orientations of quark (inner arrow) and nucleon (outer arrow) spins. The nucleon is moving towards the observer

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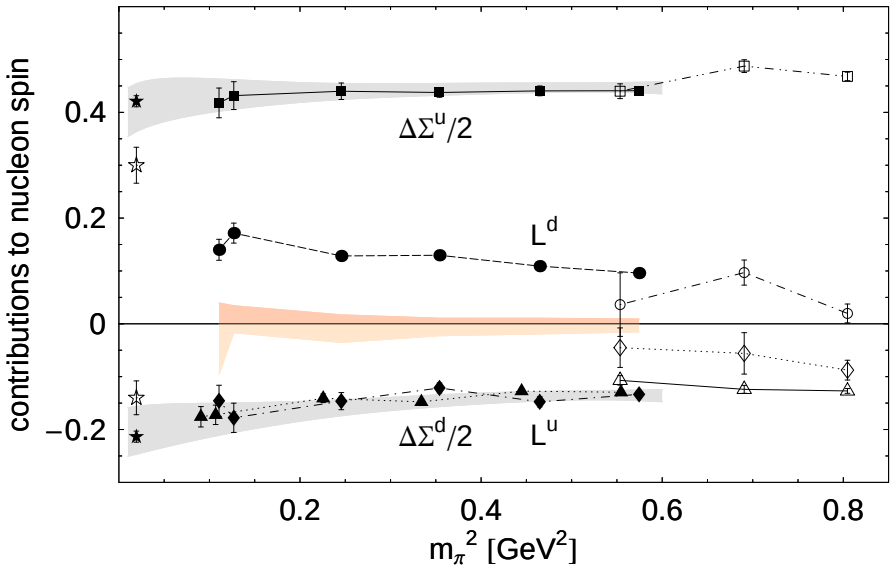

Figure 7: LHPC results for the nucleon spin structure. Filled stars: HERMES result

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