MENU 2007
11th International Conference
on Meson-Nucleon Physics and
the Structure of the Nucleon
September10-14, 2007
IKP, Forschungzentrum Jülich, Germany

EFFECTIVE QUANTUM FIELD THEORIES

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Abstract

After a short introduction to effective field theory, I concentrated in my talk on a recent application of the method: previous analyses of K_{e4} data neglected an important isospin breaking effect, generated by the pion mass difference and by $\pi^0 - \eta$ mixing. Once it is taken into account, the previous discrepancy between NA48/2 data on K_{e4} decays and the prediction of $\pi\pi$ scattering lengths in the framework of chiral perturbation theory disappears.

1 Introduction

In my talk, I provided a short introduction to effective quantum field theories. It is not necessary to provide here yet another introduction to the method, because there are many excellent reviews available on the market. A selected list of recent work is given in the bibliography [1], see also the contribution of Bijnens to this conference [2].

However, there is a point which is worth elaborating here. Chiral perturbation theory (ChPT) [3], combined with Roy equations, allows one to make very precise predictions for the values of the threshold parameters in elastic $\pi\pi$ scattering [4] – see Colangelo's contribution at KAON07 for a status report [5]. Several experiments allow one to confront these predictions with experimental data: i) $K^+ \to \pi^+\pi^- e^+\nu_e$ decays [6,7], ii) the pionium lifetime, measured by the DIRAC collaboration [8], and iii) the cusp effect in $K \to 3\pi$ decays, investigated by the NA48/2 collaboration [9–11].

The experiments performed by the NA48/2 collaboration have generated an impressive data basis, as a result of which the matrix elements of K_{e4} and

$K \to 3\pi$ decays can be determined with an unprecedented accuracy [7,9,11]. The interpretation of these measurements was the main topic of my talk. In particular, I pointed out that the theoretical predictions and the measurements are performed in two different settings: the predictions concern pure QCD, in the isospin symmetry limit $m_u = m_d$, with photons absent. To be more precise, the convention is to choose the quark masses and the renormalization group invariant scale of QCD such that the pion and the kaon masses coincide with the values of the charged ones, and the pion decay constant is $F_\pi = 92.4$ MeV. [I do not specify the masses of the heavy quarks, because in the present context, their precise values do not matter.] I refer to this framework as a *paradise world.*

On the other hand, experiments are all carried out in the presence of isospin breaking effects, generated by real and virtual photons, and by the mass difference of the up and down quarks: this is the *real world*, described by the Standard Model. We are thus faced with the problem to find the relation between quantities measured in the real world, where isospin breaking effects are always present, and the predictions made in the paradise world. I discuss the relevant points here for the case of K_{e4} decays. See also Ref. [12], and section 6 in the recent review Ref. [13].

2 K_{e4} decays

2.1 General

In the NA48/2 experiment, the general purpose Monte Carlo program PHOTOS [14] is used to calculate electromagnetic corrections. In addition, the Sommerfeld factor is applied, to account for the Coulomb interaction between charged particles [15].

In my talk, I pointed out that in these prescriptions to perform radiative corrections, one specific mechanism is not included. Namely, the kaon may decay first into a neutral pion pair, that then annihilates into two charged pions, or first decay into a charged pion pair, that then re–scatters. In the real world, the neutral pion mass is smaller than the charged one by about 4.6 MeV[1]. As a result of this, the two contributions to the decay matrix element have a different holomorphic structure: the neutral (charged) pion loop generates a branch point at $s_\pi = 4M_{\pi^0}^2$ (at $s = 4M_\pi^2$), and the *phase* of the relevant form factor is affected with a cusp, and does not vanish at the threshold $s = 4M_\pi^2$.

[1] Throughout the text, I use the symbols M_π (M_{π^0}) for the charged (neutral) pion mass.

2.2 Partial wave expansion: isospin symmetry limit

The matrix element for $K^+(p) \to \pi^+(p_1)\pi^-(p_2)e^+(p_e)\nu_e(p_\nu)$ is

$$T = \frac{G_F}{\sqrt{2}} V_{us}^\star \overline{u}(p_\nu)\gamma^\mu(1-\gamma_5)v(p_e)(V_\mu - A_\mu), \tag{1}$$

where the last factor denotes hadronic matrix elements of the strangeness changing (vector and axial vector) currents,

$$V_\mu - A_\mu \quad = \quad \langle \pi^+(p_1)\pi^-(p_2)\,\mathrm{out} \mid (\bar{s}\gamma_\mu u - \bar{s}\gamma_\mu\gamma_5 u) \mid K^+(p)\rangle \,. \tag{2}$$

In the following, I concentrate on the matrix element of the axial vector current, because it carries information on the $\pi\pi$ final state interactions and, in particular, on the $\pi\pi$ phases. One decomposes A_μ into Lorentz scalars,

$$A_\mu = -i\frac{1}{M_K}\left[(p_1+p_2)_\mu F + (p_1-p_2)_\mu G + (p_e+p_\nu)_\mu K\right] . \tag{3}$$

The form factors F, G, K are holomorphic functions of the three variables

$$s_\pi = (p_1+p_2)^2 \,,\; t = (p_1-p)^2 \,,\; u = (p_2-p)^2 \;. \tag{4}$$

Sometimes, it is useful to use instead

$$s_\pi = (p_1+p_2)^2 \,,\; s_\ell = (p_e+p_\nu)^2 \,,\; \cos\theta_\pi \,, \tag{5}$$

where θ_π is the angle of the π^+ in the CM system of the two charged pions, with respect to the dipion line of flight in the rest system of the kaon [7,16]. In the isospin symmetry limit, one identifies the $\pi\pi$ phases in the matrix element in a standard manner, by performing a partial wave expansion, and using unitarity and analyticity, although, in the present case, this is a slightly intricate endeavor [17]. It is useful to introduce a particular combination of form factors,

$$F_1 = F + \frac{(M_K^2 - s_\pi - s_\ell)\sigma}{\lambda(M_K^2, s_\pi, s_\ell)^{1/2}} \cos\theta_\pi G \,. \tag{6}$$

Here, $\sigma = \sqrt{1-4M_\pi^2/s_\pi}$, and $\lambda(x,y,z)$ is the triangle function. The form factor F_1 has a simple partial wave expansion,

$$F_1 = f(s_\pi, s_\ell) + \sum_{k\geq 1} P_k(\cos\theta_\pi) f_k(s_\pi, s_\ell) \,. \tag{7}$$

In the low–energy region $s_\pi \leq 16M_\pi^2$, f_k carry the $\pi\pi$ phases [17] in the pertinent isospin channel. In the following, I consider the lowest partial

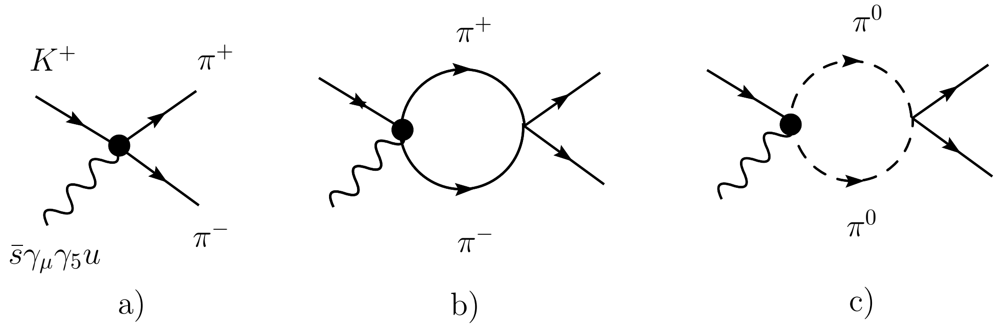

Figure 1: Some of the graphs that contribute to the matrix element of the axial current at tree and one–loop order. The filled vertex indicates that the axial current also couples to a single kaon line. That graph contributes to the form factor K. There are many additional graphs at one–loop order, not displayed in the figure.

wave $f(s_\pi, s_\ell)$. In the interval $4M_\pi^2 \leq s_\pi \leq 16M_\pi^2$, its phase coincides with the isospin zero S-wave phase δ_0^0 in elastic $\pi\pi$ scattering,

$$f_+ = e^{2i\delta_0^0} f_-,\; f_\pm = f(s_\pi \pm i\epsilon, s_\ell)\,. \tag{8}$$

It is instructive to calculate the form factors in chiral perturbation theory and to verify that F_1 indeed has the behaviour just discussed. For this, it is sufficient to consider the effective Lagrangian

$$\mathcal{L}_2 = \frac{F_0^2}{4}\langle D_\mu U D^\mu U^\dagger + 2B_0\mathcal{M}(U + U^\dagger)\rangle\,, \tag{9}$$

where the covariant derivative $D_\mu U$ contains the external vector and axial vector currents, and $\mathcal{M} = \mathrm{diag}(\hat{m}, \hat{m}, m_s)$. Some of the graphs that contribute at tree level and at one loop are displayed in Figure 1. The result is [18]

$$f(s_\pi, s_\ell) = \frac{M_K}{\sqrt{2}F_0}\left\{1 + \Delta(s_\pi) + H(s_\pi, s_\ell) + O(p^4)\right\}\,, \tag{10}$$

with

$$\begin{aligned} \Delta(s_\pi) &= \frac{1}{2F_0^2}(2s_\pi - M_\pi^2)\bar{J}(s_\pi)\,, \\ 16\pi^2\bar{J}(s_\pi) &= \sigma\left(\ln\frac{1-\sigma}{1+\sigma} + i\pi\right) + 2\,, \quad s_\pi \geq 4M_\pi^2\,. \end{aligned} \tag{11}$$

Here, M_π (F_0) denotes the pion mass (pion decay constant), at leading order in the chiral expansion. The quantity $H(s_\pi, s_\ell)$ is real in the interval of elastic $\pi\pi$ scattering. It is now seen that f indeed has the property Eq. (8) at this order in the low–energy expansion, with

$$\delta_0^0 = \frac{(2s_\pi - M_\pi^2)}{32\pi F_0^2}\sigma \,. \tag{12}$$

This is the phase of the isospin zero S-wave, in tree approximation.

2.3 Partial wave expansion: the real world

In reality, experiments are not carried out in the paradise world of the previous subsection: we have not included so far photons, nor did we consider isospin breaking effects generated by different up and down quark masses. Here, I investigate these effects in several steps [19]:

i) I assume that the manner in which real and virtual photons are treated in the analysis of the NA48/2 experiment (PHOTOS + Sommerfeld factor) is a decent approximation to the effects generated by soft photons.

ii) This procedure misses the effects generated by the pion and kaon mass differences, and by the quark mass difference $m_d - m_u$. These must therefore be taken into account separately.

iii) ChPT is the appropriate tool to evaluate these contributions.

iv) I assume that PHOTOS+Sommerfeld factor + mass effects provide a good approximation to the full isospin breaking contributions.

Remark: One may envisage a more ambitious procedure [20], by working out the relevant matrix elements in the framework of ChPT including photons and leptons [21], and then constructing a new event generator, to be used in the analysis of K_{e4} decays. [A one–loop calculation was already performed in Ref. [22]. It needs to be checked, and brought into a form which is suitable for the present purpose.] Eventually, such an analysis might lead to an improved algorithm, but I consider it a long term project. End of remark.

According to iii), we simply need to perform a ChPT calculation of the effects generated by the mass differences. This is rather easy to achieve at one–loop order: one adapts the quark mass matrix, $\mathcal{M} \to \mathrm{diag}(m_u, m_d, m_s)$, and enlarges the Lagrangian $\mathcal{L}_2$ [23],

$$\mathcal{L}_2 \to \mathcal{L}_2 + C\langle QUQU^\dagger\rangle \,,\; Q = \frac{e}{3}\mathrm{diag}(2,-1,-1)\,, \tag{13}$$

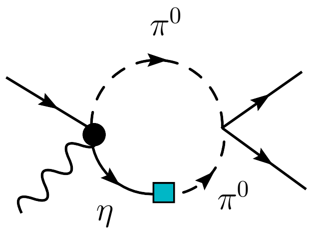

Figure 2: The contribution from $\pi^0 - \eta$ mixing, at leading order in $m_d - m_u$.The filled square denotes the vertex from $\pi^0 - \eta$ mixing. Otherwise, the notation is the same as in Figure 1.

where C is a new low–energy constant, that breaks the isospin symmetry of the meson masses: $M_\pi \neq M_{\pi^0}, M_K \neq M_{K^0}$.

The effect of the replacement Eq. (13) is twofold [19]: first, as just mentioned, the meson masses split. As a result of this, the loop contributions in Fig. 1b),c) have a different threshold, and the phase of the form factor f generates a cusp. Second, in addition to the graphs displayed in Figure 1, there is a new contribution shown in Figure 2: the kaon interacts with the axial current to generate a $\pi^0\eta$ intermediate state. Because $m_u \neq m_d$, the η can transform back into a neutral pion, that then re–scatters with the second neutral pion into a charged pion pair.

Working out the relevant diagrams, one finds [24, 25] that the phase Eq. (12) becomes in the elastic region

$$\delta_0^0 \to \delta = \frac{1}{32\pi F_0^2}\left\{(4\Delta_\pi + s_\pi)\sigma + (s_\pi - M_{\pi^0}^2)\left(1 + \frac{3}{2R}\right)\sigma_0\right\}\,, \tag{14}$$

with

$$\Delta_\pi = M_\pi^2 - M_{\pi^0}^2\,, \quad \sigma_0 = \sqrt{1 - 4M_{\pi^0}^2/s_\pi}\,, \quad R = \frac{m_s - \hat{m}}{m_d - m_u}\,. \tag{15}$$

The one–loop expressions for the form factors F, G given in Refs. [22] contain the effects considered here, up to terms of order $\alpha_{QED}(m_d - m_u)$.

I consider the result Eq. (14) to be very interesting, for the following reasons. First, due to the presence of the phase space factor σ_0, the phase δ does not vanish at the threshold $s_\pi = 4M_\pi^2$. This unexpected behaviour of the phase is the cusp effect already experienced in $K \to 3\pi$ decays, with the role of neutral and charged pions interchanged. Second, the difference $\delta - \delta_0^0$ is positive for s_π above the threshold, and even *increases* at large s_π,

$$\delta - \delta_0^0 \;\; = \;\; \frac{3s_\pi}{64\pi F_0^2}\frac{1}{R} + O(1)\,, \quad s_\pi/M_\pi^2 \gg 1\,. \tag{16}$$

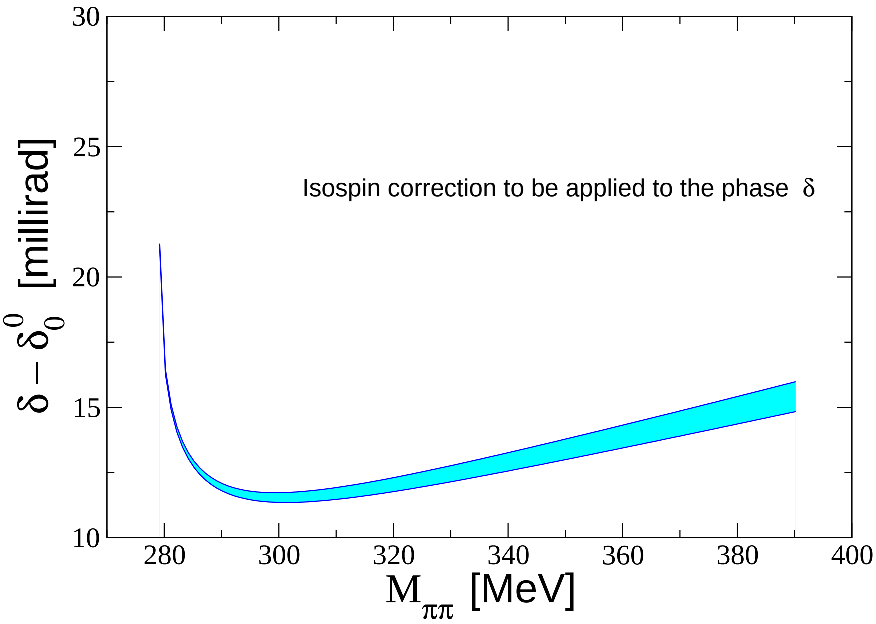

Figure 3: The isospin breaking correction that must be subtracted from the phase δ measured in K_{e4} decays. The width of the band reflects the uncertainty in the ratio R.

We now come to the main point. According to point iv) above, it is the phase δ that is measured in K_{e4} decays (up to contributions from higher orders in the chiral expansion). Therefore, before comparing the phase so determined with ChPT predictions, one has to subtract from the measured phase the (positive) difference $\delta - \delta_0^0$, because $\delta_0^0 = \delta - (\delta - \delta_0^0)$. In Figure 3 we display this difference in the relevant decay region, for $R = 37 \pm 4$[2]. The width of the band reflects the uncertainty in R. [Two–loop contributions are modest in the analogous case of the scalar form factor of the pion [25].] It is seen that the isospin correction is quite substantial – well above the uncertainties quoted for the measured phase [7]. [In Ref. [26], the cusp in K_{e4} decays was investigated as well. The expressions presented there do not agree with Eq. (14), because these authors do not take into account derivative couplings of the $\pi\pi$ amplitude, as is dictated by chiral symmetry.]

[2]This value for R should be considered as preliminary – it was used in my talk for illustrative purposes. A more refined estimate will be provided in Ref. [25]. Of course, the conclusions to be drawn from the isospin breaking effects considered here will not change.

Colangelo has performed fits to K_{e4} data, with and without isospin breaking corrections applied. It turns out that the former discrepancy [27] of the NA48/2 data with the prediction [4] disappears, once isospin breaking effects are taken into account in the manner just described, see Refs. [5, 7]. Colangelo also shows that the former agreement between the chiral prediction and the E865 data [6] becomes marginal. This is an issue that should be understood, because it is independent of the special effects considered here. On the other hand, since the NA48/2 data are so precise, they will dominate the E865 result, in any case.

Acknowledgments

It is a pleasure to thank the organizers for the invitation to give this talk, and for the excellent organization of this exciting conference. I thank M. Bissegger, G. Colangelo, A. Fuhrer, B. Kubis, H. Leutwyler and A. Rusetsky for a most enjoyable collaboration, B. Bloch–Devaux for explaining to me with patience details of the data analysis of K_{e4} decays, and Z. Was for discussions on PHOTOS.

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