K-matrix
and
Dalitz plot analysis

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Outline

- **Introduction:**
  - Dalitz plot and K-matrix formalism …the **issue**
- **Analysis:**
  - Implementation of K-matrix formalism in D-decays
  - Examples from FOCUS
    - **$D^+ \rightarrow \pi^+ \pi^- \pi^+$** (2003) 1500 evts
    - **$D+ \rightarrow K^- \pi^+ \pi^+$** (2007) **53000 evts**
- **Results and Conclusions**
  - What we have learnt so far
  - How we should proceed …
    - ***prospects for the future***

Dalitz plot: the revenge

- **SPIRES** search for " **title Dalitz and date after 1999**"

new millennium

137 entries

after 2005

42 entries

- **Experiments:**

FOCUS, E791, CLEO (-c) , BaBar-Belle, BES

- ➡ **From D to B decays**
- ➡ **From decay dynamics to CPV to New Physics**

$B \to \rho\pi$ **α angle**

$B \to D^{(*)}K^{(*)}$ **γ angle**

The issue

- **to go from** $B \to \pi\pi\pi$ **to** $B \to \rho\pi$

  means selecting and filtering the desired states among the possible contributions, e.g. $\sigma\pi$, $f_0(980)\pi$, $\sigma\rho$, $\sigma\sigma$, $\rho\pi\pi$...

- **a model for D^0 decay is needed**
  - **$(K\pi)\pi$, $K(\pi\pi)$**

…and a question

- **In the era of precision measurements**
  - **How to deal with the underlying strong dynamics effects?**
    - The $\pi\pi$, $K\pi$ **S–wave** are characterized by broad, overlapping states: **unitarity** is **not** explicitly guaranteed by a simple **sum of Breit -Wigner (BW)** functions
    - **Independently of the nature of** σ,κ (genuine resonances or strong dynamics structures), they are **not simple BW's**
    - **f_0(980)** is a **Flatté**-like function, coupling to KK and $\pi\pi$

.. a possible answer

a ***bridge*** of knowledge and terminology

- Many problems are already well known in nuclear and intermediate energy physics

↓

K-matrix

  - **A cultural bridge towards the high energy community**
  - **A common jargon**
- An effort has been made in FOCUS to apply it to the Heavy Flavor sector …..
  - interesting for future B-studies

What is K-matrix?

E.P.Wigner, Phys. Rev. 70 (1946) 15

S.U. Chung et al. Ann. Physik 4 (1995) 404

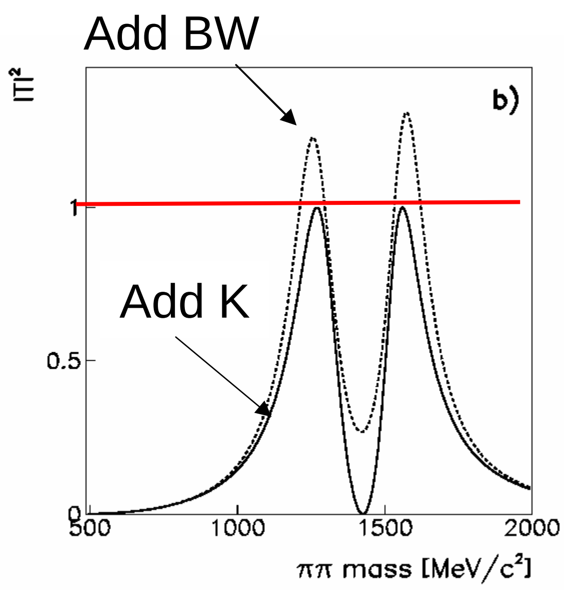
- It follows from S-matrix and, because of S-matrix unitarity, it is real

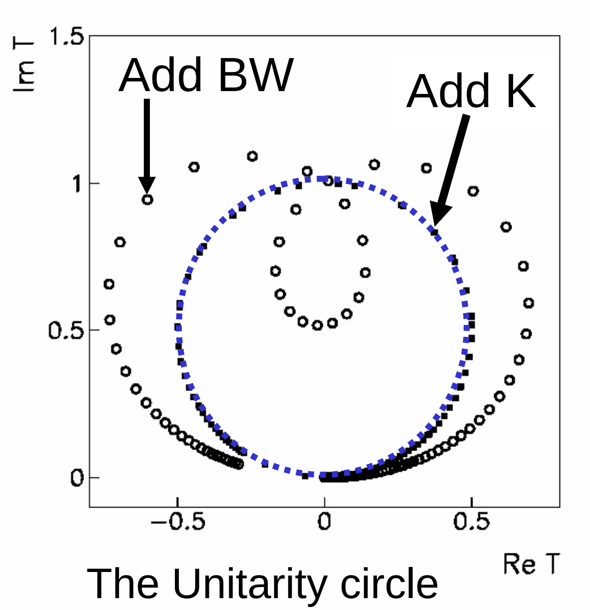
$$S = I + 2i\rho^{1/2} T \rho^{1/2}$$

$$K^{-1} = T^{-1} + i\rho \qquad T = (I - iK \cdot \rho)^{-1} K$$

- Viceversa, any real K-matrix would generate an unitary S-matrix
- This is the real advantage of the K-matrix approach:
  - **It (heavily) simplifies the formalization of any scattering problem since the unitarity of S is automatically respected.**

- For a single-pole problem, far away from any threshold, a K-matrix amplitude reduces to the standard BW formula
    - **The two descriptions are equivalent**
- In all the other cases, the BW representation is no longer valid
    - **The most severe problem is that it does not respect unitarity**





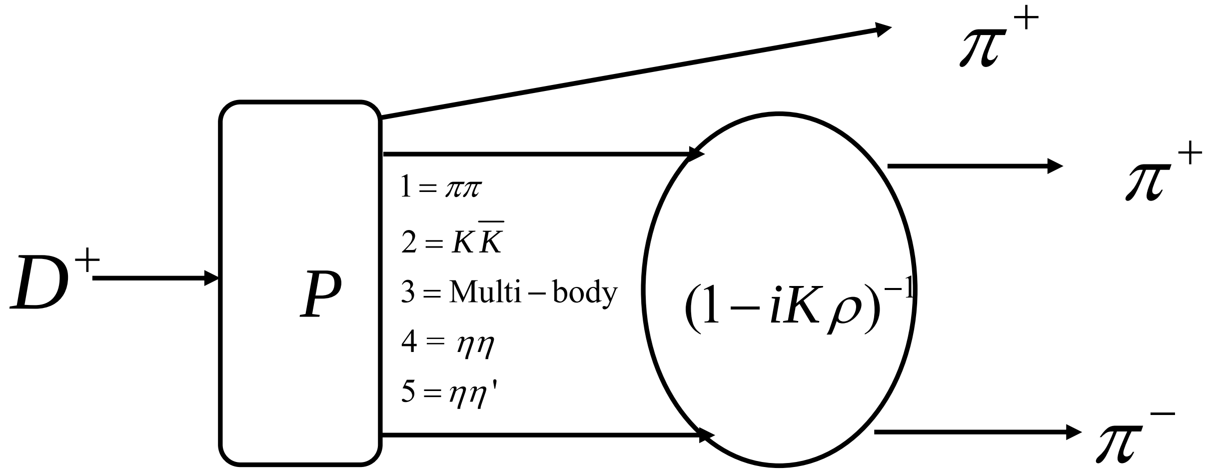
Adding BWs *a la* "traditional Isobar Model"

- Breaks Unitarity
- Heavily modify the phase motion!

From Scattering to Production

- Thanks to I.J.R. Aitchison (Nucl. Phys. A189 (1972) 514), the K-matrix approach can be extended to production processes
- In technical language,
  - From $$T = (I - iK \cdot \rho)^{-1} K$$
  - To $$F = (I - iK \cdot \rho)^{-1} P$$
- The P-vector describes the coupling at the production with each channel involved in the process
  - **In our case the production is the D decay**

First FOCUS study: $D^+, D_s^+ \to \pi^+\pi^-\pi^+$



$$F = (I - iK \cdot \rho)^{-1} P$$

Describes coupling of resonances to D

Comes from scattering data

Beside restoring the proper dynamical features of the resonances, K-matrix allows for the inclusion of all the knowledge coming from scattering experiments: **enormous amount of results and science!**

$\pi\pi$ S-wave scattering parametrization

"K-matrix analysis of the 00++-wave in the mass region below 1900 MeV"

V.V Anisovich and A.V.Sarantsev Eur.Phys.J.A16 (2003) 229

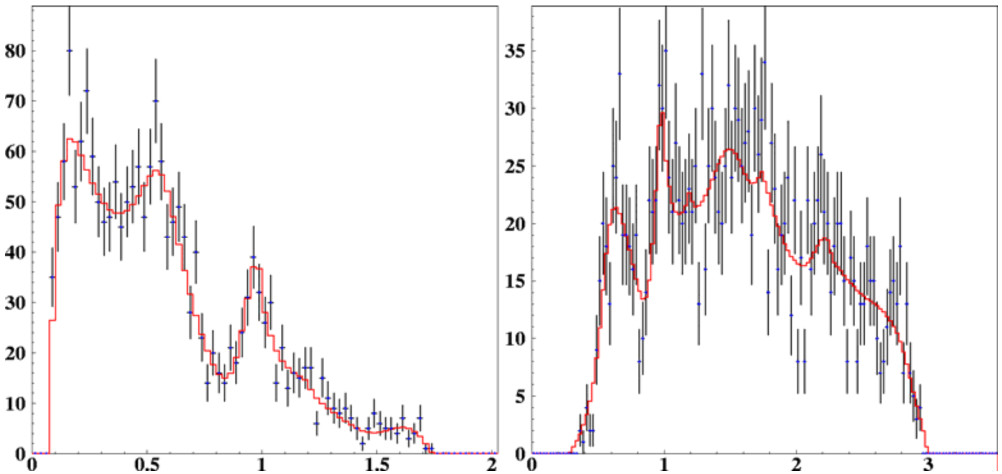
- A global fit to (all) the available data has been performed

*	**GAMS**	$\pi p \rightarrow \pi^0\pi^0 n, \eta\eta n, \eta\eta' n$, $\lvert t\rvert<0.2$ (GeV/c^2)	
*	**GAMS**	$\pi p \rightarrow \pi^0\pi^0 n$, $0.30<\lvert t\rvert<1.0$ (GeV/c^2)	
*	**BNL**	$\pi p^- \rightarrow K\bar{K}n$	
*	**CERN-Munich**	$\pi^+\pi^- \rightarrow \pi^+\pi^-$	
*	**Crystal Barrel**	$p\bar{p} \rightarrow \pi^0\pi^0\pi^0, \pi^0\pi^0\eta, \pi^0\eta\eta$	At rest, from liquid H_2
*	**Crystal Barrel**	$p\bar{p} \rightarrow \pi^0\pi^0\pi^0, \pi^0\pi^0\eta$	At rest, from gaseous H_2
*	**Crystal Barrel**	$p\bar{p} \rightarrow \pi^+\pi^-\pi^0, K^+K^-\pi^0, K_sK_s\pi^0, K^+K_s\pi^-$	At rest, from liquid H_2
*	**Crystal Barrel**	$n\bar{p} \rightarrow \pi^0\pi^0\pi^-, \pi^-\pi^-\pi^+, K_sK^-\pi^0, K_sK_s\pi^-$	At rest, from liquid D_2
*	**E852**	$\pi^- p \rightarrow \pi^0\pi^0 n$, $0<\lvert t\rvert<1.5$ (GeV/c^2)	

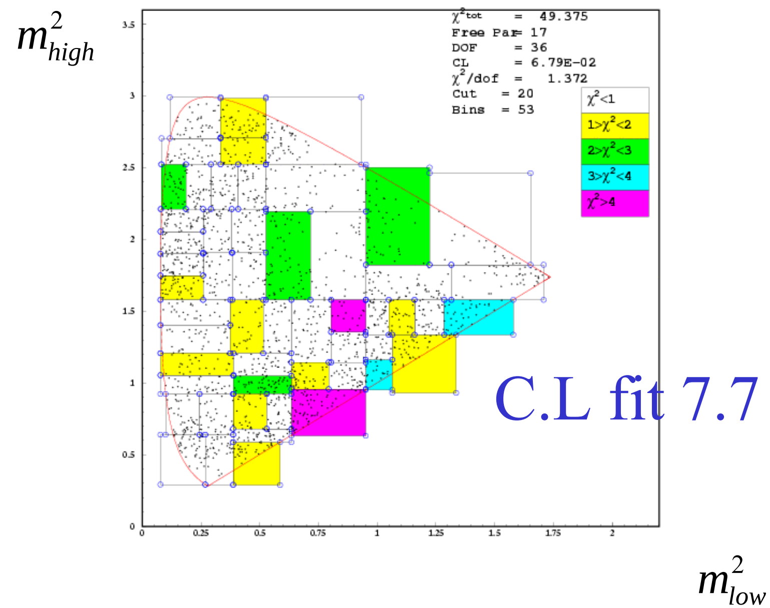
- It provided the K-matrix input to our three-pion D analysis

$D^+ \rightarrow \pi^+ \pi^- \pi^+$ K-matrix fit results

PLB 585 (2004) 200



Low mass projection High mass projection

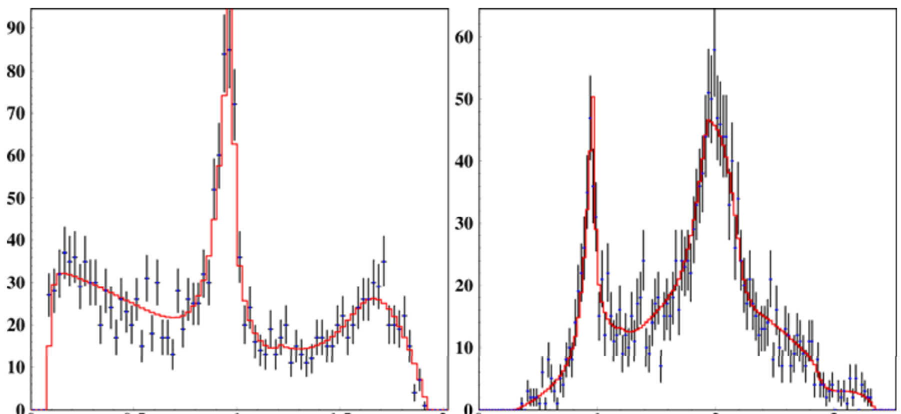


C.L fit 7.7 %

decay channel	fit fractions (%)	phase (deg)
$(S\text{-wave})\pi^+$	$56.00 \pm 3.24 \pm 2.08$	0(fixed)
$f_2(1275)\pi^+$	$11.74 \pm 1.90 \pm 0.23$	$-47.5 \pm 18.7 \pm 11.7$
$\rho^0(770)\pi^+$	$30.82 \pm 3.14 \pm 2.29$	$-139.4 \pm 16.5 \pm 9.9$

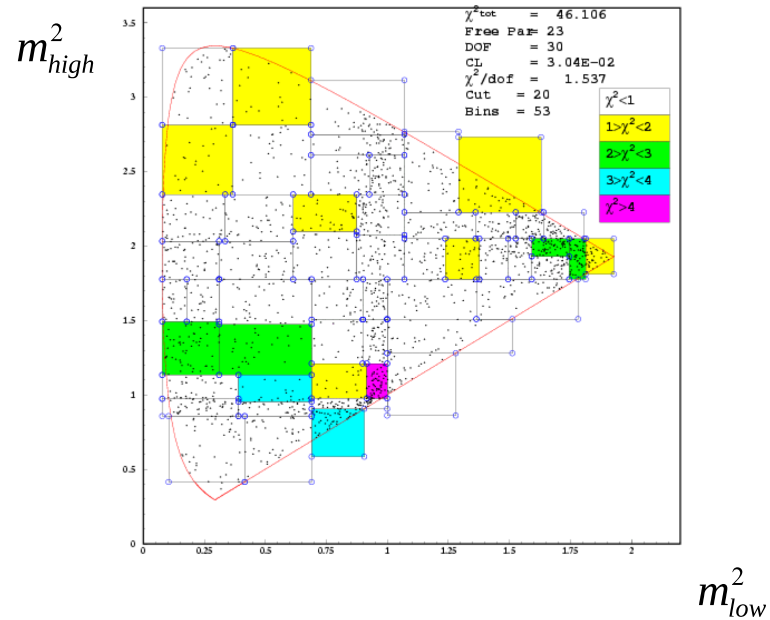
Reasonable fit with <u>no retuning</u> of the A&S K-matrix.
No new ingredient (resonance) required not present in the scattering!

$D_s^+ \rightarrow \pi^+ \pi^- \pi^+$ K-matrix fit results



Low mass projection | High mass projection

decay channel	fit fractions (%)	phase (deg)
$(S\text{-wave})\pi^+$	$87.04 \pm 5.60 \pm 4.17$	0(fixed)
$f_2(1275)\pi^+$	$9.74 \pm 4.49 \pm 2.63$	$168.0 \pm 18.7 \pm 2.5$
$\rho^0(1450)\pi^+$	$6.56 \pm 3.43 \pm 3.31$	$234.9 \pm 19.5 \pm 13.3$



C.L fit 3 %

Yield D^+ =1527 ± 51 evts

Yield D_s =1475 ± 50 evts

FOCUS E831

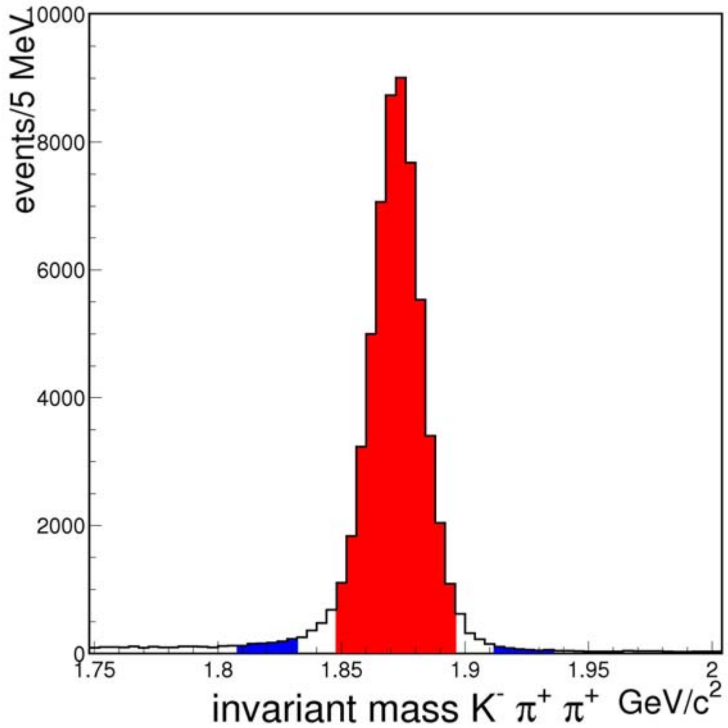
The high statistics test

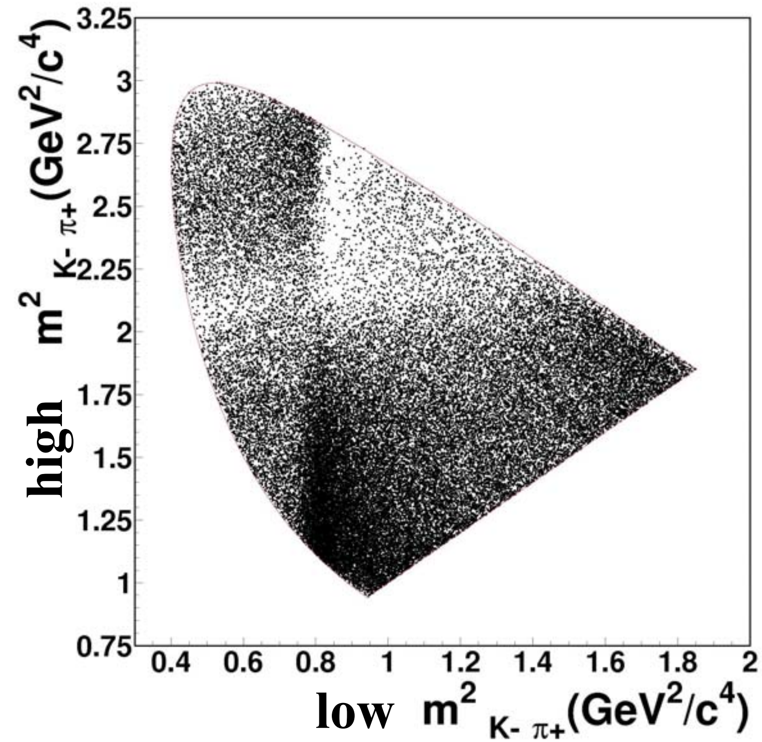
- Three-pion analysis suggested:
  - two-body dominance
  - consistency with scattering data

- It was important (mandatory) to **test** the formalism **@ high statistics**
  - the $D^+ \to K^- \pi^+ \pi^+$ channel, i.e. my latest nightmare

The $D^+ \rightarrow K^- \pi^+ \pi^+$ decay

53653 evts…another story!





e-Print: **arXiv:0705.2248** [hep-ex] (to appear in Phys. Lett. B)

The Kπ S-wave scattering parametrization

(Mike Pennington)

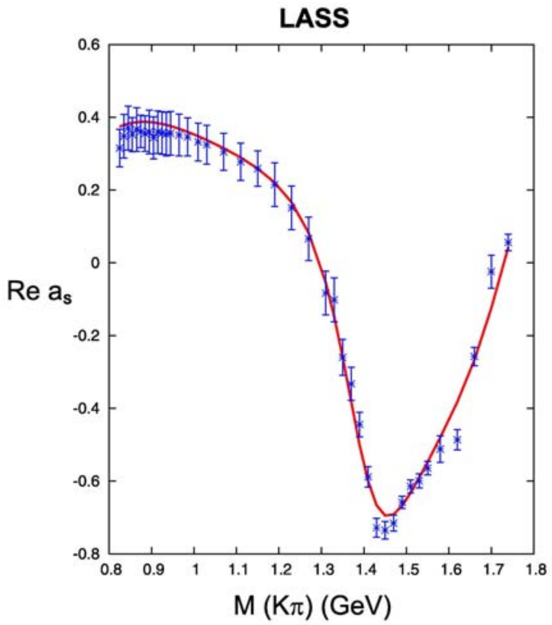
- two isospin states (I=1/2 and I=3/2) ⟺ two K-matrices
  - fit S-wave $K^- \pi^+ \to K^- \pi^+$ LASS data above 825 MeV

Nucl. Phys,.B 296 (1988) 493

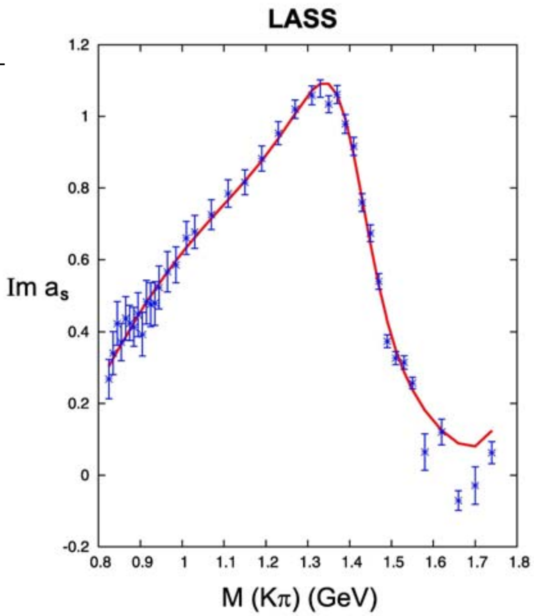
and $K^- \pi^- \to K^- \pi^-$ scattering from Estabrooks *et al*

Nucl. Phys,.B 133 (1978) 490

- extrapolate down to Kπ threshold according to dispersive analysis consistent with ChPT (Buttiker et al, Eur.Phys.J C33 (2004) 409).



$K^- \pi^+ \to K^- \pi^+$



I=1/2 K-matrix

1 pole -2 channels (Kπ -Kη')

$$K_{11} = \left(\frac{s - s_{01/2}}{s_{norm}}\right)\left(\frac{g_1 \cdot g_1}{s_1 - s} + C_{110} + C_{111}\tilde{s} + C_{112}\tilde{s}^2\right)$$

$$K_{22} = \left(\frac{s - s_{01/2}}{s_{norm}}\right)\left(\frac{g_2 \cdot g_2}{s_1 - s} + C_{220} + C_{221}\tilde{s} + C_{222}\tilde{s}^2\right)$$

$$K_{12} = \left(\frac{s - s_{01/2}}{s_{norm}}\right)\left(\frac{g_1 \cdot g_2}{s_1 - s} + C_{120} + C_{121}\tilde{s} + C_{122}\tilde{s}^2\right)$$

$\mathbf{g_1}$,$\mathbf{g_2}$: real couplings of the $\mathbf{s_1}$ pole to the first and second channel

$\mathbf{s_{01/2}}$ =0.23 GeV^2 is the Adler zero position in the I=1/2 ChPT elastic scattering amplitude

$s = m^2(K\pi)$

$s_{norm} = m^2_K + m^2_\pi$

$\tilde{s} = s/s_{norm} - 1$

Values of parameters for the $I = 1/2$ K-matrix

Pole (GeV^2)	Coupling (GeV)	C_{11i}	C_{12i}	C_{22i}
$s_1 = 1.7919$				
	$g_1 = 0.31072$			
	$g_2 = -0.02323$			
		$C_{110} = 0.79299$	$C_{120} = 0.15040$	$C_{220} = 0.17054$
		$C_{111} = -0.15099$	$C_{121} = -0.038266$	$C_{221} = -0.0219$
		$C_{112} = 0.00811$	$C_{122} = 0.0022596$	$C_{222} = 0.00085655$

S-matrix pole : E = M-iΓ/2= 1.408 –i0.110 GeV

I=3/2 K-matrix

1 channel scalar function

$$K_{3/2} = \left(\frac{s - s_{03/2}}{s_{norm}} \right) \left(D_{110} + D_{111}\tilde{s} + D_{112}\tilde{s}^2 \right)$$

$\mathbf{s_{03/2}}$ =0.27 GeV^2 is the Adler zero position in the I=3/2 ChPT elastic scattering amplitude

$s = m^2(K\pi)$

$s_{norm} = m^2_K + m^2_\pi$

$\tilde{s} = s/s_{norm} - 1$

$$D_{110} = -0.22147$$

$$D_{111} = 0.026637$$

$$D_{112} = -0.00092057$$

P and F-vectors

- P-vectors

$$(P_{1/2})_{1=K\pi} = \frac{\beta g_1 e^{i\theta}}{s_1 - s} + (c_{10} + c_{11}\hat{s} + c_{12}\hat{s}^2)e^{i\gamma_1}$$

$$(P_{1/2})_{2=K\eta'} = \frac{\beta g_2 e^{i\theta}}{s_1 - s} + (c_{20} + c_{21}\hat{s} + c_{22}\hat{s}^2)e^{i\gamma_2}$$

$$P_{3/2} = (c_{30} + c_{31}\hat{s} + c_{32}\hat{s}^2)e^{i\gamma_3}$$

$$\hat{s} = s - s_c$$

$$s_c = 2 \text{ GeV}^2$$

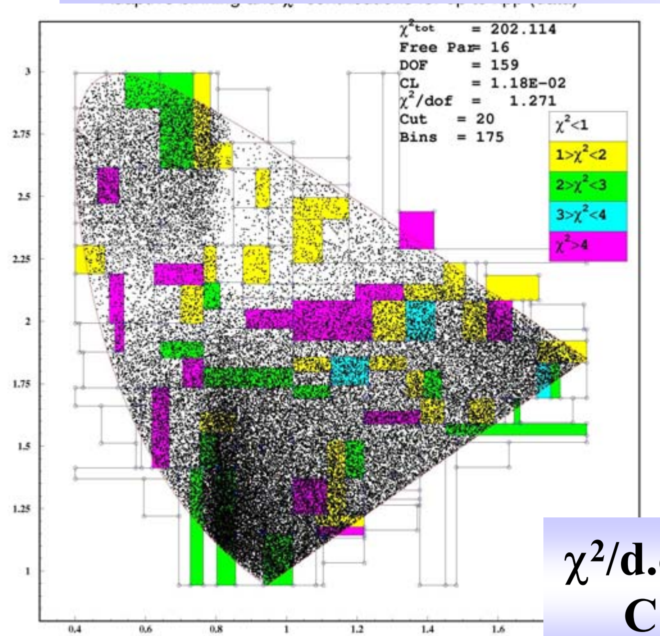
…and F-vectors

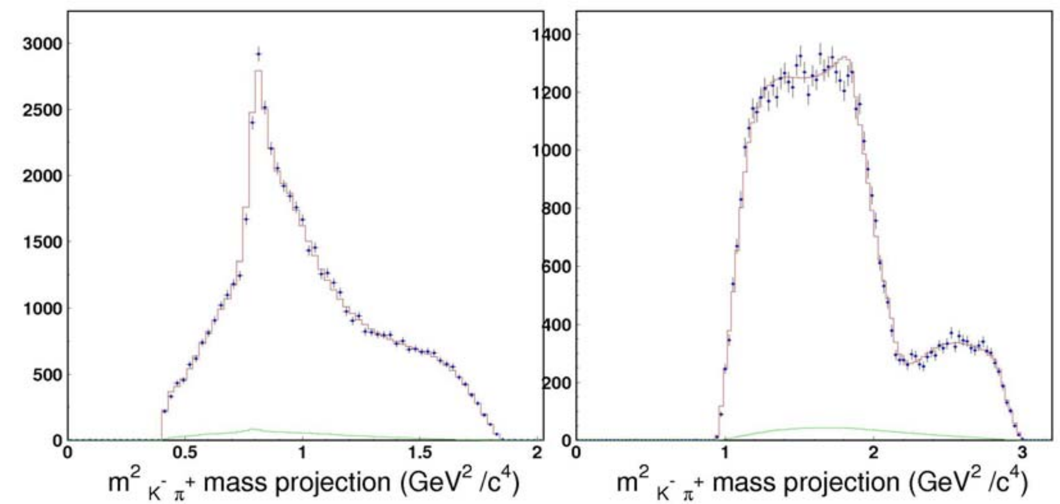
$$F_{3/2} = (I - iK_{3/2}\rho)^{-1} P_{3/2}$$

$$(F_{1/2})_{1=K\pi} = (I - iK_{1/2}\rho)^{-1}_{1j} (P_{1/2})_j$$

β, θ, c_{ij} , γ_i are the free parameters, all the others are fixed to scattering

…finally ready to fit $D^+ \to K^- \pi^+ \pi^+$





χ^2/d.o.f =1.27
C.L 1.2 %

total D decay amplitude

$$M = (F_{1/2})_1(s) + F_{3/2}(s) + \sum_n a_n e^{i\delta_n} A_n$$

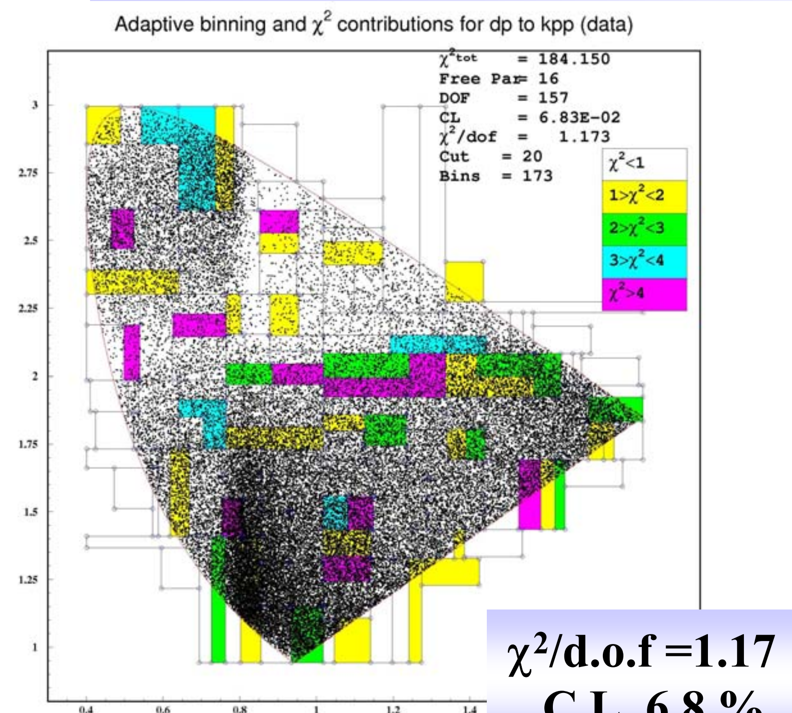
BW-like for J>0 states

coefficient	*phase (deg)*
$\beta = 3.389 \pm 0.152 \pm 0.002 \pm 0.068$	$\theta = 286 \pm 4 \pm 0.3 \pm 3.0$
$c_{10} = 1.655 \pm 0.156 \pm 0.010 \pm 0.101$	$\gamma_1 = 304 \pm 6 \pm 0.4 \pm 5.8$
$c_{11} = 0.780 \pm 0.096 \pm 0.003 \pm 0.090$	
$c_{12} = -0.954 \pm 0.058 \pm 0.0015 \pm 0.025$	
$c_{20} = 17.182 \pm 1.036 \pm 0.023 \pm 0.362$	$\gamma_2 = 126 \pm 3 \pm 0.1 \pm 1.2$
$c_{30} = 0.734 \pm 0.080 \pm 0.005 \pm 0.030$	$\gamma_3 = 211 \pm 10 \pm 0.7 \pm 7.8$
Total S-wave fit fraction $= 83.23 \pm 1.50 \pm 0.04 \pm 0.07$ %	
Isospin 1/2 fraction $= 207.25 \pm 25.45 \pm 1.81 \pm 12.23$ %	
Isospin 3/2 fraction $= 40.50 \pm 9.63 \pm 0.55 \pm 3.15$ %	

S-wave fraction
83 ±1.5 %

component	*fit fraction (%)*	*phase δ_j (deg)*	*coefficient*
$K^*(892)\pi^+$	13.61 ± 0.98	0 (fixed)	1 (fixed)
	$\pm 0.01 \pm 0.30$		
$K^*(1680)\pi^+$	1.90 ± 0.63	1 ± 7	0.373 ± 0.067
	$\pm 0.009 \pm 0.43$	$\pm 0.1 \pm 6$	$\pm 0.009 \pm 0.047$
$K_2^*(1430)\pi^+$	0.39 ± 0.09	296 ± 7	0.169 ± 0.017
	$\pm 0.004 \pm 0.05$	$\pm 0.3 \pm 1$	$\pm 0.010 \pm 0.012$
$K^*(1410)\pi^+$	0.48 ± 0.21	293 ± 17	0.188 ± 0.041
	$\pm 0.012 \pm 0.17$	$\pm 0.4 \pm 7$	$\pm 0.002 \pm 0.030$

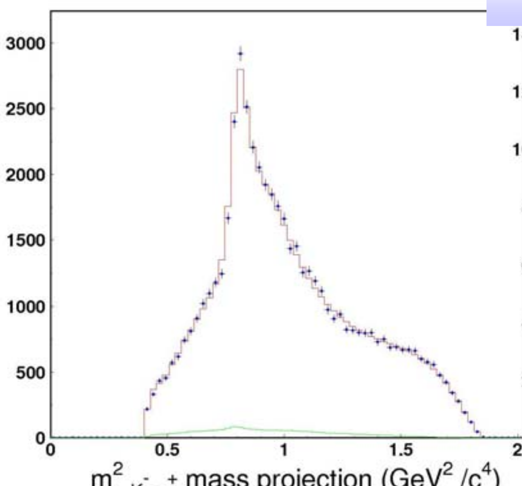
Comparison with the isobar fit

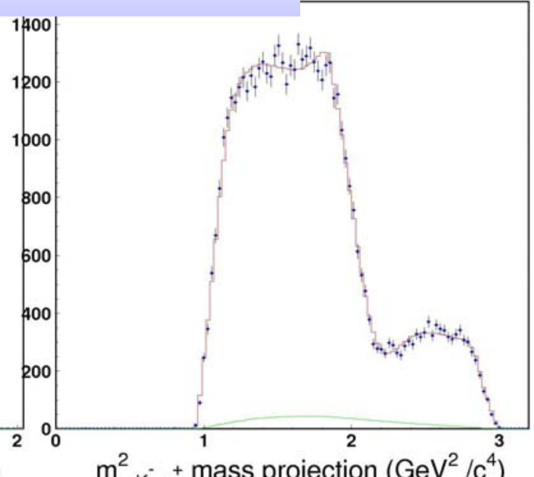


- serves as the standard for fit quality
- requires two "ad hoc" scalars states with free masses and widths (BW) with no reference to how these states appear in other Kπ interactions (an effective data description)

χ^2/d.o.f =1.17
C.L 6.8 %

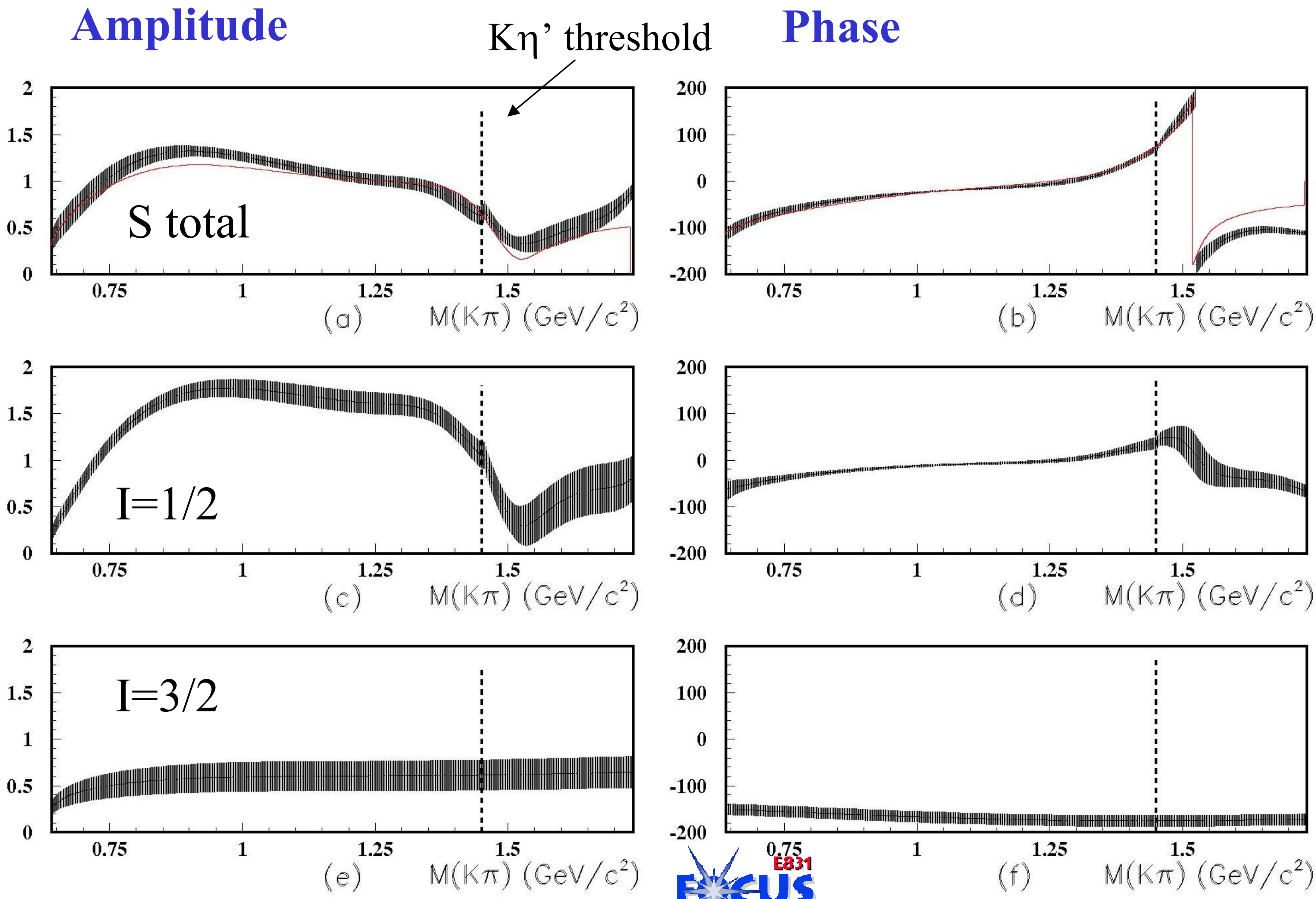
k	**m=856±17** **Γ=464±28**	$K_0^*(1430)$	**m=1461±4** **Γ=177±8**



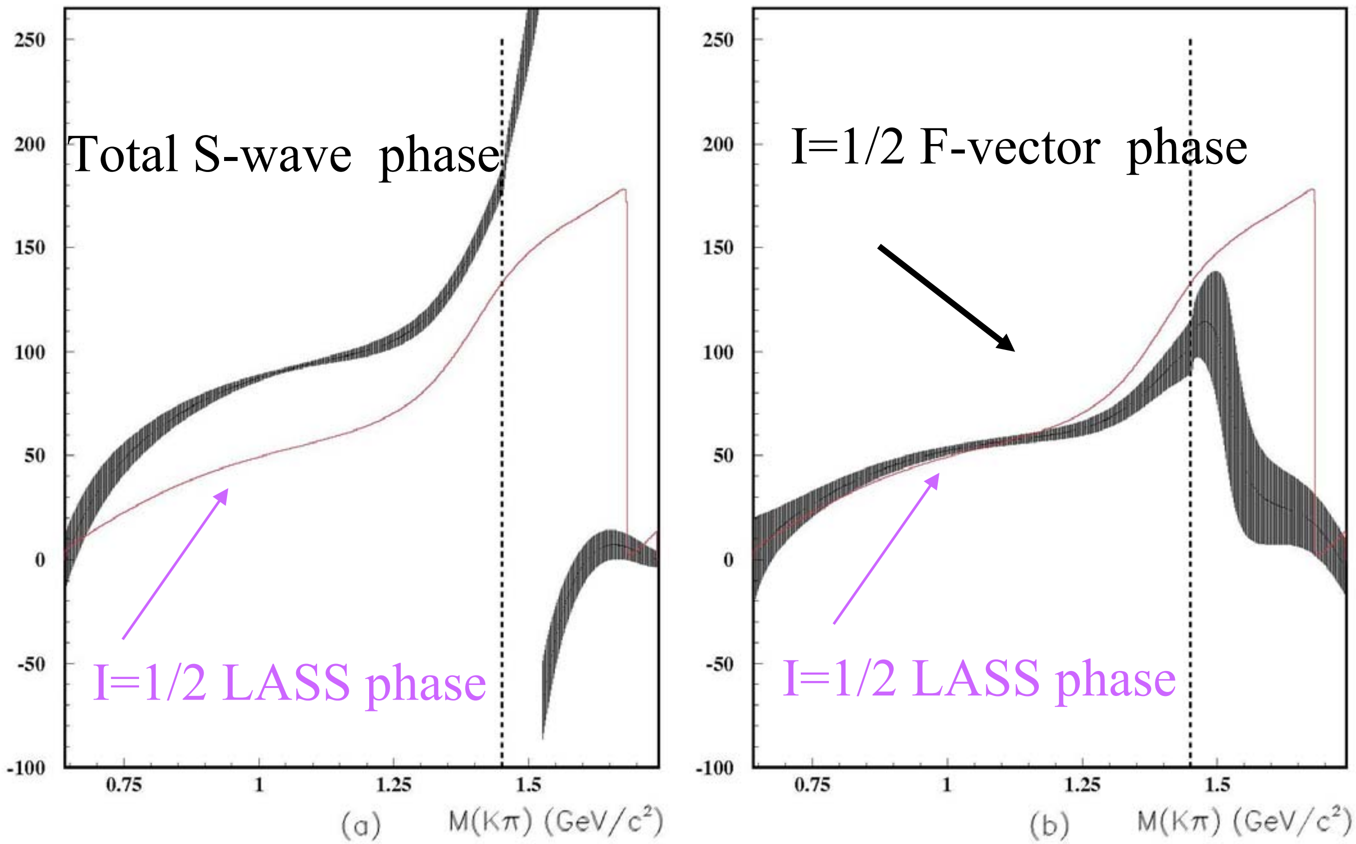


- Isobar and K-matrix fits show
  - same "hot spots" in the adaptive binning scheme
  - good agreement in vector-tensor fit parameters

What else can we infer from F-vectors?



Phase comparison



Kη' threshold

Results (I)

- The hypothesis of the **two -body dominance** is **consistent** with the **high statistics $D^+ \rightarrow K^-\pi^+\pi^+$**
- The **first determination** in **D decays** of the **I=1/2 and I=3/2 for the S-wave Kπ** system has been performed
- Our results show **close consistency with Kπ scattering data**, and consequently, with **Watson's theorem predictions for two-body Kπ interactions** in the low Kπ mass region where elastic processes dominate.

Results (II)

- Our K-matrix representation fits along the real energy axis inputs on scattering data and ChPT in close agreement with those used by Descotes-Genon and Moussallam (Eur. Phys. J C48 (2006) 553) that locate *k* with

mass (653 ± 15) MeV/c^2

and

width (557 ± 24) MeV/c^2

different from isobar fit parameters !

- Whatever ***k* is revealed by our data**, it is the **same** as that found in **scattering data**

Results (III)

- Our K-matrix description gives a fit quality globally good.
- However it deteriorates at higher $K\pi$ mass
  - Two channels: $K\pi$ and $K\eta'$:
  - **Reliable info on the former, poor constraints on the latter**

- **Improvements**: using a number of **D-decay chains** with **$K\pi$ final state interactions** and inputting all these in one **combined analysis** in which **several inelastic** channels are included in the **K-matrix formalism**.

for the future!

Conclusions

- Dalitz plot analysis is teaching us much about hadronic decays. It will definitely keep us company over the next few years
- Some complications have already emerged
  - expecially in the charm field

  others (unexpected) will only become clearer when we delve deeper into the beauty sector
  - **B_s will be a new chapter** (PLB645 (2007) 201: $B_s \to K\pi\pi$, $B_s \to KK\pi$)
- There will be work for both theorists and experimentalists
  - **Synergy invaluable!**

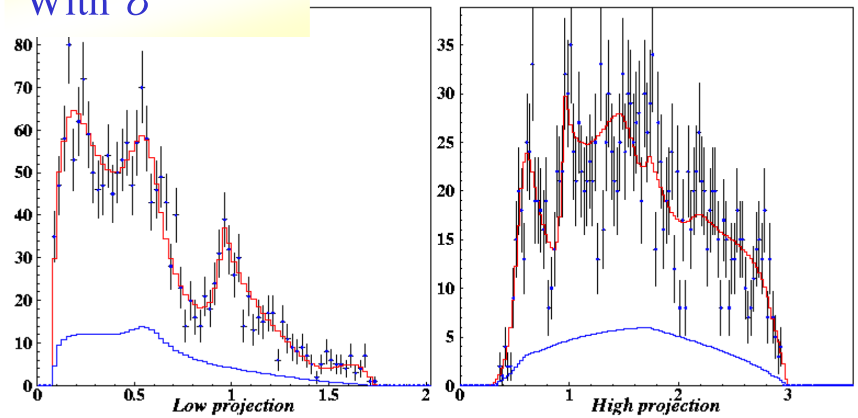
The are no shortcuts toward ambitious and high-precision studies and NP search

Back-up slides

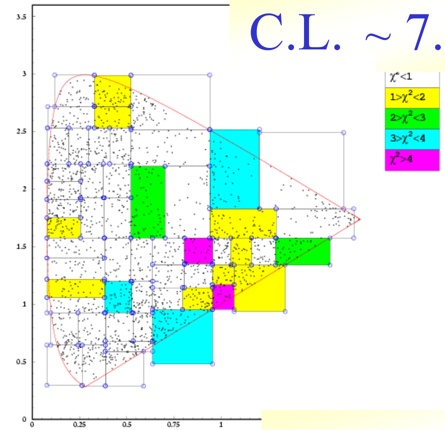
Isobar analysis of $D^+ \to \pi^+\pi^+\pi$ would instead require a new scalar meson: σ

$m = 442.6 \pm 27.0$ *MeV/c*
$\Gamma = 340.4 \pm 65.5$ *MeV/c*

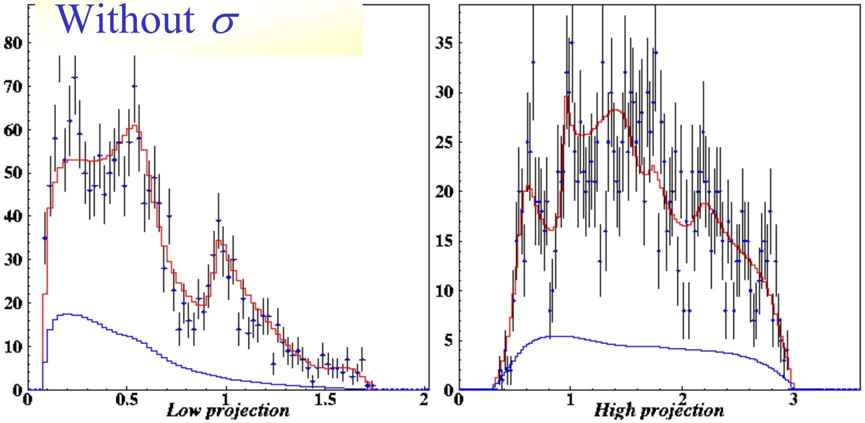
With σ



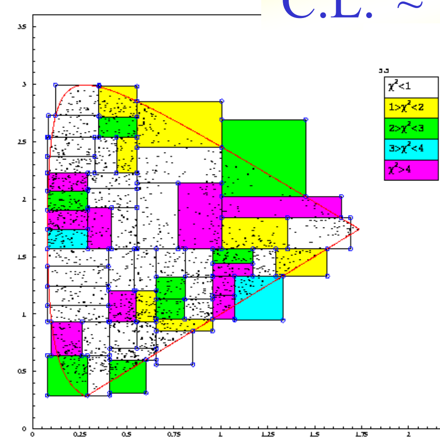
C.L. ~ 7.5%



Without σ



C.L. ~ 10^{-6}



Total D decay amplitude

$$M = (F_{1/2})_1(s) + F_{3/2}(s) + \sum_n a_n e^{i\delta_n} A_n$$

for J >0

$$A_n = F_D F_r \times |p_1|^J |p_3|^J P_J \cos(\theta_{13}^r) \times \frac{1}{m_r^2 - m_{12}^2 - i m_r \Gamma_r}$$

$F = 1$	**Spin 0**	$P_J = 1$
$F = (1 + R^2 p^2)^{-1/2}$	**Spin 1**	$P_J = (-2\overrightarrow{p_3} \cdot \overrightarrow{p_1})$
$F = (9 + 3R^2 p^2 + 3R^4 p^4)^{-1/2}$	**Spin 2**	$P_J = 2(p_3 p_1)^2 (3\cos^2 \vartheta_{13} - 1)$

Isobar fit parameters

Table 2

Fit fractions, phases, and coefficients from the isobar fit to the FOCUS $D^+ \to K^-\pi^+\pi^+$ data. The first error is statistic, the second error is systematic from the experiment, and the third error is systematic induced by model input parameters for higher resonances

Channel	Fit fraction (%)	Phase δ_j (deg)	Coefficient
non-resonant	29.7 ± 4.5	325 ± 4	1.47 ± 0.11
	$\pm 1.5 \pm 2.1$ (see text)	$\pm 2 \pm 1.2$	$\pm 0.06 \pm 0.06$
$K^*(892)\pi^+$	13.7 ± 0.9	0 (fixed)	1 (fixed)
	$\pm 0.6 \pm 0.3$		
$K^*(1410)\pi^+$	0.2 ± 0.1	350 ± 34	0.12 ± 0.03
	$\pm 0.1 \pm 0.04$	$\pm 17 \pm 15$	$\pm 0.003 \pm 0.01$
$K^*(1680)\pi^+$	1.8 ± 0.4	3 ± 7	0.36 ± 0.04
	$\pm 0.2 \pm 0.3$	$\pm 4 \pm 8$	$\pm 0.02 \pm 0.03$
$K_2^*(1430)\pi^+$	0.4 ± 0.05	319 ± 8	0.17 ± 0.01
	$\pm 0.04 \pm 0.03$	$\pm 2 \pm 2$	$\pm 0.01 \pm 0.01$
$K_0^*(1430)\pi^+$	17.5 ± 1.5	36 ± 5	1.13 ± 0.05
	$\pm 0.8 \pm 0.4$	$\pm 2 \pm 1.2$	$\pm 0.01 \pm 0.02$
$\kappa\pi^+$	22.4 ± 3.7	199 ± 6	1.28 ± 0.10
	$\pm 1.2 \pm 1.5$ (see text)	$\pm 1 \pm 5$	$\pm 0.015 \pm 0.04$
	Mass (MeV/c^2)	Width (MeV/c^2)	
$K_0^*(1430)$	$1461 \pm 4 \pm 2 \pm 0.5$	$177 \pm 8 \pm 3 \pm 1.5$	
κ	$856 \pm 17 \pm 5 \pm 12$	$464 \pm 28 \pm 6 \pm 21$	