D meson spectroscopy

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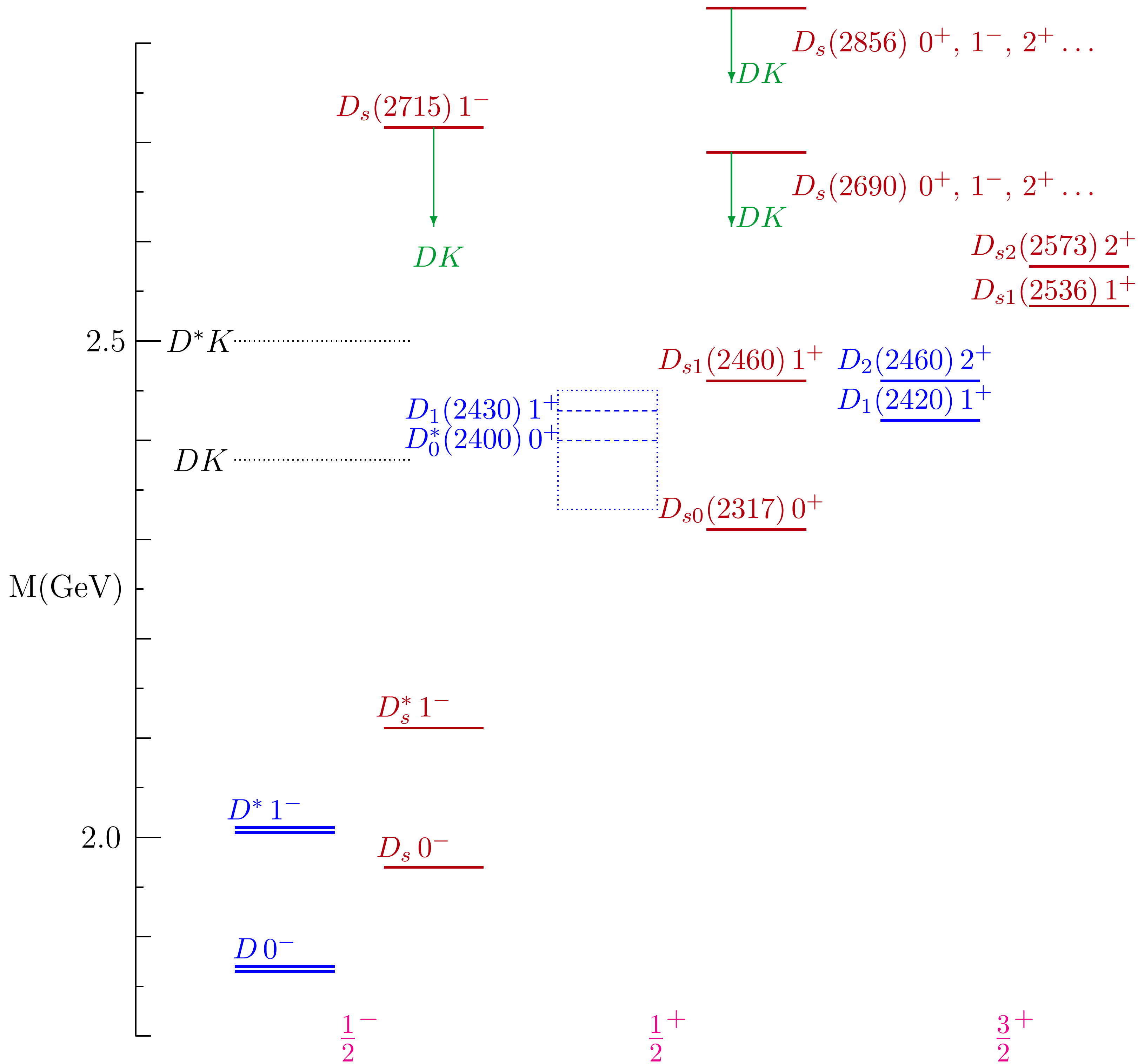


$D_s(2856)\ 0^+,\ 1^-,\ 2^+\ldots$
DK
$D_s(2715)\ 1^-$
$D_s(2690)\ 0^+,\ 1^-,\ 2^+\ldots$
DK
DK
$D_{s2}(2573)\ 2^+$
$D_{s1}(2536)\ 1^+$
2.5
D^*K
$D_{s1}(2460)\ 1^+$
$D_2(2460)\ 2^+$
$D_1(2430)\ 1^+$
$D_1(2420)\ 1^+$
$D_0^*(2400)\ 0^+$
DK
$D_{s0}(2317)\ 0^+$
M(GeV)
$D_s^*\ 1^-$
$D^*\ 1^-$
2.0
$D_s\ 0^-$
$D\ 0^-$
$\frac{1}{2}^-$
$\frac{1}{2}^+$
$\frac{3}{2}^+$

General considerations.

- HQ limit (m_c considered large).

A meson X = slow c + light 'stuff'. The 'light stuff' = light (anti)quark + gluons.

Pauli:

$$M_X = m_c + \overline{\Lambda}_X + \frac{1}{2m_c} \langle X | c^\dagger (\vec{\pi} \cdot \vec{\sigma})^2 c | X \rangle + O\left(\frac{1}{m_c^2}\right), \qquad \vec{\pi} = \vec{p} - \vec{A}$$

$\overline{\Lambda}_X$ depends on the light quark (mass), $m_s - m_{u,d}$, and of the state of the 'light stuff': $\vec{J} = \vec{L} + \vec{s}$, $P = (-1)^{L+1}$,

$$J^P = \frac{1}{2}^-, \frac{1}{2}^+, \frac{3}{2}^+, \ldots$$

The spin of the c, $s_c = 1/2$ combines with J to form pairs of mesons

$$(0^-, 1^-), \quad (0^+, 1^+), \quad (1^+, 2^+), \ldots$$

The mass splitting within each pair is $O(1/m_c)$.

$$(\vec{\pi} \cdot \vec{\sigma})^2 = \vec{\pi}^2 - (\vec{\sigma} \cdot \vec{B})$$

$\vec{B}$ - chromomagnetic field. $\langle X | \vec{B} | X \rangle = \kappa \vec{J} \Rightarrow$ splitting within the doublets: $-2\,\kappa (\vec{J} \cdot \vec{s}_c)$.
E.g. $D^* - D$ mass splitting: $\mu_g^2 = \langle D | c^\dagger (\vec{\sigma} \cdot \vec{B}) c | D \rangle$, then $\langle D^* | c^\dagger (\vec{\sigma} \cdot \vec{B}) c | D^* \rangle = -\mu_g^2/3$:

$$\mu_g^2 = \frac{3}{4} (M_{D^*}^2 - M_D^2) \approx 0.41\,\mathrm{GeV}^2$$

Compare with $\mu_g^2 \approx 0.36\,\mathrm{GeV}^2$ from $B^* - B$ splitting. (Looks OK.)

$\mu_\pi^2 = \langle D|c^\dagger \vec{\pi}^2 c|D\rangle$ is not directly known. Estimates from QCD sum rules, lattice(?), direct measurements from kinematics in $B \to X_c \ell \nu$ give $\mu_\pi^2 \approx (0.4 \div 0.5)\,\mathrm{GeV}^2$. Inequality following from positivity of $(\vec{\pi}\cdot\vec{\sigma})^2$: for any state X

$$\langle X|c^\dagger \vec{\pi}^2 c|X\rangle \geq \langle X|c^\dagger (\vec{\sigma}\cdot\vec{B}) c|X\rangle$$

$\Rightarrow \mu_\pi^2 \geq \mu_g^2$.

$D^* - D$ mass splittings in more detail.

$$\Delta_u = M(D^{*0}) - M(D^0) = 142.12 \pm 0.07\,\mathrm{MeV}$$

$$\Delta_d = M(D^{*+}) - M(D^+) = 140.64 \pm 0.10\,\mathrm{MeV}$$

$$\Delta_s = M(D_s^*) - M(D_s) = 143.8 \pm 0.4\,\mathrm{MeV}$$

What do these numbers tell us?

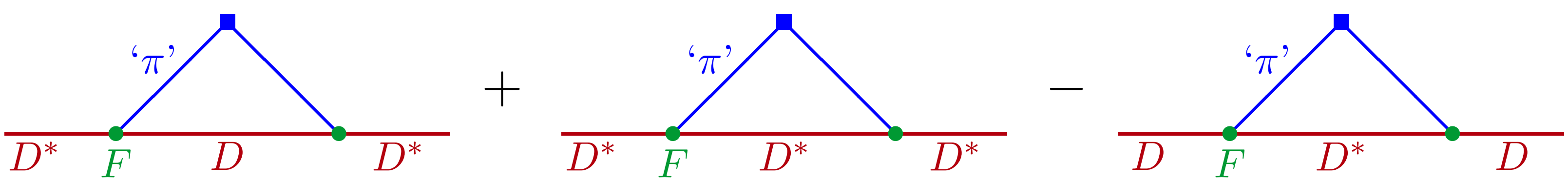
E.M. effect $\sim m_s$ effect (?)

• Trying to understand the m_s effect

Chiral logarithm estimate. Neglect $m_{u,d}$, i.e. $m_\pi^2 \to 0$.

$$\Delta_s - \Delta_d = \Big(\langle D_s^* | m_K^2 \bar{K} K + \frac{1}{2} m_\eta^2 \eta^2 | D_s^* \rangle - \langle D_s | m_K^2 \bar{K} K + \frac{1}{2} m_\eta^2 \eta^2 | D_s) \rangle \Big)$$

$$- \Big(\langle D_d^* | m_K^2 \bar{K} K + \frac{1}{2} m_\eta^2 \eta^2 | D_d^* \rangle - (\langle D_d | m_K^2 \bar{K} K + \frac{1}{2} m_\eta^2 \eta^2 | D_d) \rangle \Big)$$



■ $= (m_K^2 + m_\eta^2/2) \times 1$, $F : D^{*+} \to D^0 \pi^+$ vertex (in n.r. normalization), $\Gamma(D^{*+} \to D^0 \pi^+) = |F|^2 p_\pi^3 / 6\pi$ $\Rightarrow |F| \approx (1/220)\,\mathrm{MeV}$.

$$\Delta_s - \Delta_d \approx (m_K^2 + m_\eta^2/2) \frac{\Delta_d \, |F|^2}{6\pi^2} \ln \frac{\Lambda}{\mu} \approx (20\,\mathrm{MeV}) \ln \frac{\Lambda}{\mu}$$

Compare with exp. $3.2 \pm 0.4\,\mathrm{MeV}$. Clearly an s_c dependent $\pi D^{(*)}$ non-pole scattering amplitude is required.

- s quark effect in μ_π^2:

$$(D_s - D^+) - (B_s - B^0) = \left\{ \left[\mu_\pi^2(s) - \mu_\pi^2(d) \right] - \left[\mu_g^2(s) - \mu_g^2(d) \right] \right\} \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right)$$

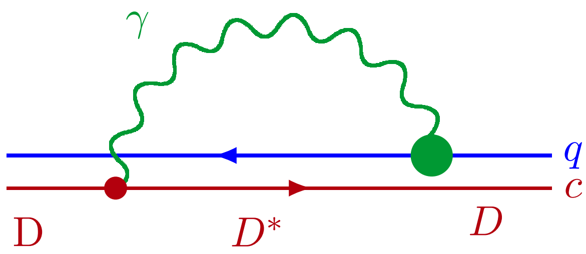
PDG07: $(D_s - D^+) = 98.9 \pm 0.4\,\mathrm{MeV}$, $(B_s - B^0) = 86.6 \pm 0.8\,\mathrm{MeV} \Rightarrow$
$(D_s - D^+) - (B_s - B^0) = 12.3 \pm 0.9\,\mathrm{MeV}$. (Notice: $\sim 10\,\mathrm{MeV}$ s quark effect.)
$[\mu_g^2(s) - \mu_g^2(d)]/(2m_c) = (3/4)\,(\Delta_s - \Delta_d) = 2.4 \pm 0.3\,\mathrm{MeV}$. $\Rightarrow$

$$\mu_\pi^2(s) - \mu_\pi^2(d) \approx 0.040\,\mathrm{GeV}^2$$

About 10% effect in μ_π^2.

- Trying to understand the EM difference $\Delta_u - \Delta_d$

$$\Delta_u - \Delta_d = \frac{4}{3}\,\frac{Q_c}{2m_c}\,\left[\langle D^0|c^\dagger(\vec{\sigma}\cdot\vec{B})c|D^0\rangle - \langle D^+|c^\dagger(\vec{\sigma}\cdot\vec{B})c|D^+\rangle\right]$$



Only D^* intermediate state contributes in the HQ limit. Define $\mu_{u,d}$:

$$\langle D^{0,+}|\vec{j}_i^{\,e.m.}(q)|D^{*0,+}(\vec{a})\rangle = e\,\mu_{u,d}(q^2)\,\epsilon_{ijk}a_j q_k$$

So that $\Gamma(D^{*0}\to D\gamma) = 4\alpha\,|\mu_u(0)|^2\,\omega_\gamma^3/3$ ($\approx 26\,\mathrm{KeV}$). $|\mu_u(0)|\approx 1.0\,\mathrm{Gev}^{-1}$, $|\mu_d(0)|\approx 0.25\,\mathrm{Gev}^{-1}$ ($\Gamma(D^{*+}\to D+\gamma)\approx 1.6\,\mathrm{KeV}$).

$$\Delta_u - \Delta_d = \frac{4}{3}\,\frac{4\pi\,\alpha\,Q_c}{2m_c}\int[\mu_u(-q^2)-\mu_d(-q^2)]\,\frac{d^3q}{(2\pi)^3}$$

The exp. value $\Delta_u - \Delta_d \approx 1.5\,\mathrm{MeV}$ requires $\int[\mu_u(-q^2)-\mu_d(-q^2)]\,q^2\,dq \approx 1.3\,\mathrm{GeV}^2$. For a simple form factor $\mu(-Q^2) = \mu(0)\,M^4/(Q^2+M^2)^2$ this requires $M\approx 1\,\mathrm{GeV}$. Possibly OK (?)

Surprises of the $\frac{1}{2}^+$ states.

$D_{s0}(2317)$ and $D_{s1}(2460)$ are much like D_s and D_s^*. Even too much like.
Indeed, $M(D_{s1}) - M(D_{s0}) = 141.6 \pm 1.2\,\mathrm{MeV}$, i.e. practically coincides with
$M(D_s^*) - M(D_s) = 143.8 \pm 0.4\,\mathrm{MeV}$.

Bardeen, Eichten, Hill '03: The $\frac{1}{2}^+$ states are the parity doubles of $\frac{1}{2}^-$.

- Would be degenerate if the chiral symmetry was realized linearly. Naively: at $m_q \to 0$ the L and R chiral components q_L and q_R obey the same QCD equations, so that $c\bar{q}_L$ and $c\bar{q}_R$ would be degenerate.
- Spontaneous breaking (nonlinear realization) of the chiral symmetry: $(c\bar{q}_L) \pm (c\bar{q}_R) = J^\pm$ are split.
- $J^+ \to J^- + (\pi, K, \eta)$ in S wave with the coupling $\Delta M / f_\pi$. (Goldberger-Treiman)
- Non-strange 0^+ and 1^+ decay very strongly $D(0^+, 1^+) \to D(0^-, 1^-) + \pi$, $\Gamma \sim 300\,MeV$. Thus these are not readily identifiable.
- Strange $D_s(0^+, 1^+)$ are below the threshold for the allowed decay $D_s(0^-, 1^-) + K \Rightarrow$ narrow, decaying through isospin breaking $D_{s0}(2317) \to D_s\pi^0$, $D_{s1}(2460) \to D_s^*\pi^0$, $D_s\gamma, D_s\pi\pi$
- Predict in B mesons, applying overall shift $351 \pm 35\,\mathrm{MeV}$ to B_s and B_s^*: $B_{s0}(5718 \pm 35)$ and $B_{s1}(5765 \pm 35)$. 35 Mev is the BEH estimate of $O(1/m_c)$ pre-HQ correction.

Comments on the parity doubling scheme

- As soon as the chiral symmetry is spontaneosly broken, the Goldstone coupling is $G_A\,\Delta M/f_\pi$ with $G_A \neq 1$. (Apriori G_A can be anything.)
  Axial current:
  $$\langle +|A_\mu|-\rangle = G(q^2)(p_+ + p_-)^\nu \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)$$
  $$= G(q^2)\left[(p_+ + p_-)_\mu - q_\mu \frac{M_+^2 - M_-^2}{q^2}\right]$$
  Linear realization (Wigner-Weyl):
  $$M_+ = M_-, \qquad G_A = G(0) = 1$$
  Nonlinear (Nambu-Goldstone):
  $$g_\pi = G_A(M_+^2 - M_-^2)/f_\pi$$
  No constraints on G_A. The axial charge $Q_A = \int d^3x A_0(x)$ vanishes.
  Indeed,
  $$\langle +|A_0|-\rangle = G(q^2)\,2M_-\,\frac{\vec{q}^2}{\vec{q}^2 - (\Delta M)^2}$$

- Not clear why the $1^+ - 0^+$ mass splitting should be the same as $1^- - 0^-$, once the overall masses of $\frac{1}{2}^+$ and $\frac{1}{2}^-$ are split. Generally can be different if $G_A \neq 1$.

- The prediction for $B_{s0,s1}$ is not specific for the sheme. In fact this is general HQ.

Indeed, according to the HQ mass formula

$$(B_{s0} - B_s) - (D_{s0} - D_s) = -\left\{\left[\mu_\pi^2(J^+) - \mu_\pi^2(J^-)\right] - \left[\mu_g^2(J^+) - \mu_g^2(J^-)\right]\right\}\left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)$$

experimentally $\mu_g^2(J^+) = \mu_g^2(J^-) \approx 0.36 \div 0.40\,\mathrm{GeV}^2$. Furthermore, $\mu_\pi^2 \geq \mu_g^2 \Rightarrow \mu_\pi^2(sJ^+) \geq 0.36\,\mathrm{GeV}^2$ and $0.40\,\mathrm{GeV}^2 \leq \mu_\pi^2(sJ^-) \leq 0.54\,\mathrm{GeV}^2$. Hence

$$(B_{s0} - B_s) - (D_{s0} - D_s) < \left[\mu_\pi^2(J^-) - \mu_g^2(J^-)\right]\left(\frac{1}{2m_c} - \frac{1}{2m_b}\right) < 42\,\mathrm{MeV}$$

$\Rightarrow M(B_{s0}) < 5760\,\mathrm{MeV}$ and $M(B_{s1}) < 5808\,\mathrm{MeV}$.

Both are below the corresponding $B^{(*)}K$ threshold (5773 and 5818 MeV). Should be narrow.

Hard to undestand $D_{s1} - D_{s0} \approx 142\,\text{MeV} \gg D_{s2}(2573) - D_{s1}(2536) \approx 38\,\text{MeV}$.

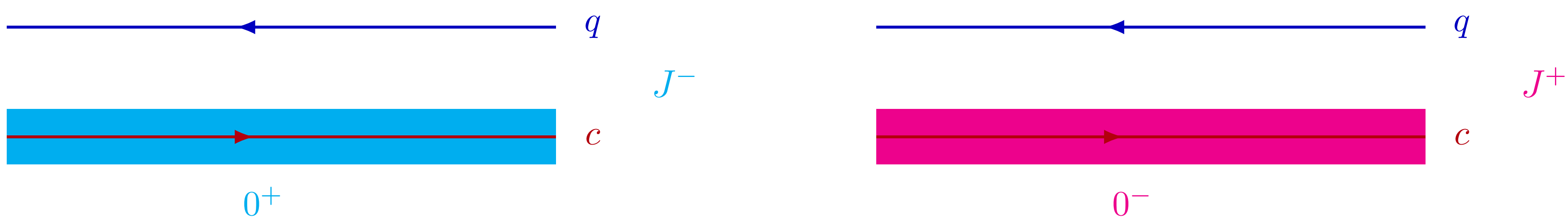
Nonrelativistic picture: consider $\vec{j} = \vec{L} + \vec{s}_q$, chromomagn field $\vec{B} = g_L \vec{L} + g_s \vec{s}$. ($\vec{J} = \vec{j} + \vec{s}_c$). Ratio of the splittings in $P_{1/2}$ and $P_{3/2}$:

$$\frac{\Delta m_{1/2}}{\Delta m_{3/2}} = \frac{(g_L + g_s)/2 + 5(g_L - g_s)/6}{g_L + g_s + (g_L - g_s)/3}$$

In order to reproduce 142/38 one needs $g_s/g_L = -1.3$ (wrong sign!?)

- Alternative (exotic) explanation(?)

$D_{s0}(2317)$ and $D_{s1}(2460)$ are in a sense not $\frac{1}{2}^+$ states of the light quark, but rather $\frac{1}{2}^-$ states essentially the same as D_s and D_s^*. The parity comes from a 'coating' of the heavy quark at shorter distances $\lesssim 1/(600\,\text{MeV})$ by a nonperturbative 0^- gluon field. (Possible short-distance nonperturbative gluon field phenomena in $0^\pm$ channels were discussed by NSVZ in early 80's.)



The true $\frac{1}{2}^+$ states are above the DK (D^*K) threshold and are broad.

- Arguments

  - The same mass splitting: $D_{s1} - D_{s0} = D_s^* - D_s$. No need to tweak g_s vs. g_L.
  - $D_{s0} \to D_s \pi^0$ goes as well by π^0 emission from the glue (anomaly) as by the $\eta - \pi$ mixing.
  - Large μ_π^2: $|\mu_\pi| = (0.6 \div 0.7)\,\mathrm{GeV}$
  - Analogs in c baryons (?): $\Lambda_c(\frac{1}{2}^-) - \Lambda_c(\frac{1}{2}^+) = 309\,\mathrm{MeV}$, $\Xi_c(\frac{1}{2}^-) - \Xi_c(\frac{1}{2}^+) = 320\,\mathrm{MeV}$
  - Possibly no such effect in heavy quarkonia, since the size of charmonium and bottomonium is $\lesssim$ the size of the 'coating'
  - A convincing prediction is needed
  - An estimate of $(D_{s1} \to D_s \gamma)/(D_{s1} \to D_s^* \pi\pi)$ (?)

• Higher D states ?

$D_s(2690)$, $D_s(2715)$, $D_s(2860)$

Radial excitations of the known ? $D_s^* \to D_s(2715)$, $\Delta M \approx 600\,\mathrm{MeV}$,
$D_{s0}(2317) \to D_s(2860)$, $\Delta M \approx 545\,\mathrm{MeV}$, $D_s(2690)$: $\frac{3}{2}^- \Rightarrow J^P = 1^-$ (why broad?).

Conclusions.

• Ground-state mesons, D, D^*, D_s, D_s^* look OK.

• Excited states: more questions than answers ...