Leptonic $D$ and $D_s$ Decays near $c\bar{c}$ Threshold

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We present recent results from the CLEO Collaboration on leptonic decay rates of $D$ and $D_s$ near $c\bar{c}$ production threshold. From these decay rates, we extract the decay constants, $f_{D^+} = (222.6 \pm 16.7^{+2.8}_{-3.4})$ MeV, $f_{D_s^+} = (274 \pm 10 \pm 5)$ MeV, and the ratio $f_{D^+}/f_{D_s^+} = 1.23 \pm 0.11 \pm 0.03$.

1. Introduction

Within the Standard Model, leptonic $D$ (or $B$) meson decays proceed via annihilation of the initial state quarks. The matrix element is described by the product of a hadronic current, a leptonic current, along with a $W$ propagator. The form of the latter two are well-known within the Standard Model, however, the hadronic matrix element, which represents the annihilation of the initial state heavy quark and light antiquark, depends on the details of the initial-state quark wave-functions, and is not calculable using standard techniques of perturbative QCD. This hadronic matrix element can be computed using either lattice QCD [1, 2, 3], or other techniques [4, 5, 6, 7, 8]. The matrix element is described by the product of a hadronic current, a leptonic current, along with a $W$ propagator. The form of the latter two are well-known within the Standard Model, however, the hadronic matrix element, which represents the annihilation of the initial state heavy quark and light antiquark, depends on the details of the initial-state quark wave-functions, and is not calculable using standard techniques of perturbative QCD. This hadronic matrix element can be computed using either lattice QCD [1, 2, 3], or other techniques [4, 5, 6, 7, 8].

The CLEO experiment, operating near $c\bar{c}$ threshold, is well positioned to measure these decay rates, and hence $f_{D^+}$ and $f_{D_s^+}$. Charge conjugate final states are implied throughout unless otherwise noted.

2. Measurement of $f_D^+$

To measure $f_D^+$ [9], we use 281 pb$^{-1}$ of data collected at the $\psi(3770)$ resonance. The proximity to the production threshold implies that the $\psi(3770)$ decays to $D\bar{D}$ with no additional particles. We exploit this clean final state, along with the hermiticity of the detector to reconstruct the neutrino from the missing momentum in the event. Specifically, we fully reconstruct a $D^-$ meson (the tag) in six hadronic final states, comprising $N_{tag} = 158, 354 \pm 496$ tags. To search for $D^+ \rightarrow \mu^+\nu$, we require a single extra charged particle with an energy deposition in the crystal calorimeter (CC), $E_{CC}^{\text{extra}} < 300$ MeV, and veto events with any additional photon candidates with energy larger than 250 MeV. From this subsample of events, we compute the square of the missing-mass ($MM^2$) recoiling against the $D^-\mu^+$ system. For $D^+ \rightarrow \mu^+\nu$, a peak at zero is obtained with a resolution of $\sigma(MM^2) \sim 0.025$ GeV$^2$. The $MM^2$ distribution is shown in Fig. 1 for data. The clear excess near zero is the $D^+ \rightarrow \mu^+\nu$ signal. Some $D^+ \rightarrow K_{S,L}\pi^+$ events pass the selection requirements and appear as a prominent, but well-separated peak near $MM^2 \simeq 0.25$ GeV$^2$.

Figure 1: Missing-mass squared distribution for $D^+ \rightarrow \mu^+\nu$ candidates. The peak near zero corresponds to signal events, and is expanded in the inset. The larger peak at $MM^2 \simeq 0.25$ GeV$^2$ corresponds to $D^+ \rightarrow K_{S,L}\pi^+$ events which pass the selection requirements.
The branching fraction is computed using:

$$B = \frac{N_{\text{cand}} - N_{\text{back}}}{N_{\text{tag}} \epsilon_{\text{CC}}},$$

(2)

where $N_{\text{cand}} = 50$ is the number of signal candidates in the region $|MM^2| < 0.050 \text{ GeV}^2$, $N_{\text{back}} = 2.81 \pm 0.30 \pm 0.27$ is the expected number of background events, $N_{\text{tag}} = 158,354 \pm 496$ is the number of fully-reconstructed $D^-$ tags, $\epsilon_{\mu} = 69.4\%$ is the efficiency for reconstructing and identifying the muon, and $\epsilon_{\text{CC}} = 96.1\%$ is the fraction of events that do not have any additional photon candidates with energy larger than 250 MeV. An additional correction of $(1.5 \pm 0.4 \pm 0.5)\%$ is applied to account for the higher efficiency for reconstructing a $D^-$ tag in $D^+ \rightarrow \mu^+\nu$ events than in generic hadronic events.

The resulting branching fraction is

$$B(D^+ \rightarrow \mu^+\nu) = (4.40 \pm 0.66^{+0.99}_{-0.12}) \times 10^{-4}. \quad (3)$$

Using Eq. (2) we determine the decay constant to be:

$$f_{D^+} = (222.6 \pm 16.7^{+2.8}_{-3.4}) \text{ MeV.} \quad (4)$$

3. Measurement of $f_{D_s^+}$

The measurements of $f_{D_s^+}$ at CLEO require higher energy running in order to produce the $D_s\bar{D_s}$ pair. A scan of the energy region from 3970 to 4260 MeV was performed, and it was determined that the optimal energy for $D_s$ physics was 4170 MeV [13], where $D_s\bar{D_s}$ is dominant, e.g., $\sigma(D_s\bar{D_s}) = (916 \pm 50)$ pb and $\sigma(D_s\bar{D_s}) = (35 \pm 19)$ pb. A slight complication with using $D_s\bar{D_s}$ is the additional ($\sim 150$ MeV) photon(s) from the $D_s^\ast$ decay. Two independent analyses have been carried out. The first analysis is similar to the $D^+ \rightarrow \mu^+\nu$ measurement described previously, where, in addition to measuring $B(D_s^+ \rightarrow \mu^+\nu)$, we also measure $B(D_s^+ \rightarrow \tau^+\nu)$, where, $\tau^+ \rightarrow \pi^+\nu\bar{\nu}$. In the second analysis, we measure $B(D_s^+ \rightarrow \tau^+\nu)$, $\tau^+ \rightarrow e^+\nu\bar{\nu}$.

3.1. Measurement of $B(D_s^+ \rightarrow (\mu^+, \tau^+)\nu)$ using Missing Mass

We use 314 pb$^{-1}$ of data collected at $E_{cm} = 4170$ MeV for this analysis. We search for final states consistent with either $D_s^+ \rightarrow \mu^+\nu$ or $D_s^+ \rightarrow \tau^+\nu$. The branching fraction is obtained from:

$$B = \frac{N_{\text{cand}} - N_{\text{back}}}{N_{\text{tag}} \epsilon},$$

(5)

where $N_{\text{tag}}$ is the number of reconstructed $D_s\bar{D_s}$ events and $\epsilon$ is the efficiency for reconstruction and identification of the $\mu^+$ for $D_s^+ \rightarrow \mu^+\nu$, or the $\pi^+$ for $D_s^+ \rightarrow \tau^+\nu$, $\tau^+ \rightarrow \pi^+\nu\bar{\nu}$. We therefore absorb the full reconstruction of the $D_s^\ast$ into the denominator, and do not rely on Monte Carlo simulation for the efficiency of the $\sim$100 MeV photon.

To determine $N_{\text{tag}}^*$, we first fully reconstruct a hadronic $D_s^-$ tag in eight tag modes, from which we obtain $31,302 \pm 472$ $D_s^-$ tags. To identify $D_s^+,\bar{D_s}$ events, we combine a $D_s^-$ tag with any additional photon candidate in the event and form the missing-mass squared $(MM^2)$ recoiling against the $\gamma D_s^+$ system, $MM^2 = (E_{cm} - E_{D_s} - E_\gamma)^2 - (\vec{p}_{D_s} - \vec{p}_\gamma)^2$. This quantity peaks at $M_{D_s^\ast}^2$, regardless of whether the photon came from the $D_s^\ast$ (the tag) or from the $D_s$. The distribution of $MM^2$ is shown in Fig. 2 for all eight tag modes combined. A fit to this distribution yields $18645 \pm 426 \pm 1081$ $D_s^\ast\bar{D_s}$ events within $\pm 2.5$ standard deviations of $M(D_s)$.

Figure 2: Square of the missing mass recoiling against a $\gamma D_s^+$ candidates.

To search for $D_s^+ \rightarrow \mu^+\nu$ and $D_s^+ \rightarrow \tau^+\nu$, $\tau^+ \rightarrow \pi^+\nu\bar{\nu}$, we require a single additional charged particle and no additional photon candidates with energy in excess of 250 MeV. The signatures for $D_s^+ \rightarrow \mu^+\nu$ and $D_s^+ \rightarrow \tau^+\nu$, $\tau^+ \rightarrow \pi^+\nu\bar{\nu}$ are similar in that they both have a $D_s$ tag and a single high momentum charged particle. In addition to the difference in the energy depositions of muons and pions, the two-body versus three-body decay implies significantly different missing mass $(MM^2)$ distributions. To suppress backgrounds with neutrals, we veto events which have an energy deposition (excluding the tag) in the $CC$ exceeding 250 MeV. The two-body leptonic decay form a $MM^2$ distribution that peaks near zero with a resolution of $\sim$0.025 GeV$^2$. The three-body leptonic decay covers a broad $MM^2$ region, which peaks near 0.1
where we use the Standard Model ratio for R: 

\[ R = \frac{\Gamma(D_s^+ \to \tau^+ \nu)}{\Gamma(D_s^+ \to \mu^+ \nu)} = \left( \frac{m_{\tau^+}}{m_{\mu^+}} \right)^2 \frac{1 - \frac{m_{\tau^+}^2}{m_{\mu^+}^2}}{1 - \frac{m_{\tau^+}^2}{m_{D_s^+}^2}} \approx 9.72. \]

We thus find:

\[ B(D_s^+ \to \mu^+ \nu) = (0.594 \pm 0.066 \pm 0.031)\%, \]

where the 5.2% systematic error is dominated by the 5% uncertainty on \( N_{\text{tag}} \).

We also compute \( B(D_s^+ \to \tau^+ \nu, \tau^+ \to \pi^+ \nu \bar{\nu}) \) using cases (i)-\( \tau \) and (ii)-\( \tau \). For these two cases, we find yields of 31 and 25 events, and expected backgrounds of 3.5\( ^{+1.7}_{-1.1} \) and 5.1\( ^{+1.6}_{-1.0} \) events, respectively. The fraction of \( D_s^+ \to \tau^+ \nu, \tau^+ \to \pi^+ \nu \bar{\nu} \) events in the respective \( MM^2 \) regions are 32% and 45%. We thus find:

\[ B(D_s^+ \to \tau^+ \nu, \tau^+ \to \pi^+ \nu \bar{\nu}) = (8.0 \pm 1.3 \pm 0.4)\%. \]

With the measured branching fractions, \( B(D_s^+ \to \mu^+ \nu) \) and \( B(D_s^+ \to \tau^+ \nu, \tau^+ \to \pi^+ \nu \bar{\nu}) \), we measure the ratio of partial widths, \( R = 13.4 \pm 2.6 \pm 0.2 \) (defined in Eq. 8), which is consistent with the Standard Model value of 9.72.

We may improve on the precision of \( B(D_s^+ \to \mu^+ \nu) \) by combining the \( D_s^+ \to \mu^+ \nu \) and \( D_s^+ \to \tau^+ \nu, \tau^+ \to \pi^+ \nu \bar{\nu} \).
π⁺νBAR candidates. We can still use Eq. 7 except ε′ and ε′′ increase from 91.4% and 7.9% to 96.2% and 45.2%, respectively. We thus find an effective branching fraction:

$$B^{\text{eff}}(D_s^+ → μ^+ν) = (0.638 ± 0.059 ± 0.033)\%.$$  \hspace{1cm} (11)

Again, the dominant systematic uncertainty (5%) is on the number of $D_s^+$ tags.

The $MM^2$ distribution for all selected $D_s^+ → μ^+ν$ and $D_s^+ → τ^+ν$, $τ^+ → π^+νBAR$ candidates is shown in Fig. 4 [13]. Overlayed is a curve that represents the expected shape, normalized to the number of events in the $MM^2$ region below 0.2 GeV². We find good agreement between the shape in data and expectations.

$$\frac{f_{D_s^+}}{f_{D^+}} = 1.23 ± 0.11 ± 0.04. \hspace{1cm} (13)$$

4. Measurement of $D_s^+ → τ^+ν, \, τ^+ → e^+νBAR$

In the second measurement of $B(D_s^+ → τ^+ν)$, we use 298 pb⁻¹ of data collected at $E_{cm} = 4170$ MeV. We utilize the decay $τ^+ → e^+νBAR$, where we benefit from the large value of $B(τ^+ → e^+νBAR) \sim 18\%$, and the excellent electron identification capabilities of the CLEO-c detector. We fully reconstruct the three hadronic decay channels: $D_s^- → φπ^−, K^{*0}K^−$ and $K_S^0K^-$. Charged hadrons are identified using standard selection criteria [12], and the intermediate resonances, $φ \rightarrow K^+K^−, K^{*0} \rightarrow K^−π^+$, and $K_S^0 \rightarrow π^+π^−$, are required to have an invariant mass within ±10 MeV, ±75 MeV and ±12 MeV of their known values [12]. Signal candidates are required to be reconstructed invariant mass, $M(D_s)$ within ±20 MeV of the known $D_s$ mass ($m_{D_s}$). We also define sideband regions, $35 < |M(D_s) - m_{D_s}| < 55$ MeV, to study the combinatorial background. The invariant mass distributions of the three $D_s^−$ tag channels are shown in Fig. 5.

![Figure 4: Square of the missing mass recoiling against γDs;μ or π+ candidates. The curve is the expected shape from simulation, normalized to the number of events with MM2 < 0.2 GeV.](image)

![Figure 5: Invariant mass distributions of Ds− candidates from data. The points are data, the solid line is a fit, and the dashed line is the background.](image)

To ensure we have $D_s^−,\overline{D_s^+}$, we compute the mass recoiling against the reconstructed $D_s^+$, and require it to be within ±55 MeV of the $D_s$ mass [13]. We then select the subset of events with a single additional charged track with $p > 200$ MeV that has opposite charge to the $D_s$ tag and is consistent with being a positron. The discriminating variable we use to identify $D_s^+ → τ^+ν, \, τ^+ → e^+νBAR$ is $E_{\text{extra}}$, the total energy remaining in the calorimeter after all showers associated with the tag and the positron are removed. In signal events, the only additional particles beyond the $D_s$ tag and the positron are the two neutrinos and either a photon from $D_s^+ → γD_s$, or a π⁰ from $D_s^+ → π⁰D_s$. Kinematically, these photons populate the energy regions from 114-170 MeV (for $γD_s^-$) and 39-117 MeV (from $π⁰D_s^+$).

The distribution of $E_{\text{extra}}$ in data is shown in Fig. 6. The large excess at low values of $E_{\text{extra}}$. 

$$f_{D_s^+} = 274 ± 13 ± 7 \text{ MeV} \hspace{1cm} (12)$$

Combining this with our previous result for $f_{D^+} = (222.6 ± 16.7 ± 3.4) \text{ MeV}$ we determine the ratio:
is the $D_s^+ \to \pi^+ \nu$, $\tau^+ \to e^+ \nu \bar{\nu}$ signal. The broad background which peaks near 1 GeV is predominantly semi-leptonic decays, such as $D_s^+ \to \phi e^+ \nu$, $\eta e^+ \nu$, $\eta^' e^+ \nu$ and $K^0 e^+ \nu$. The Cabibbo-suppressed decay, $K_L^0 e^+ \nu$, produces a small peaking component in the signal region. The shape of this background is taken from Monte Carlo simulation, and is normalized to our measured rate for $D^+ \to K^0_S e^+ \nu$ of $B(D^+ \to K^0_S e^+ \nu) = (0.27 \pm 0.10)\%$. We choose the signal region as $E_{\text{extra}} < 400$ MeV, which is chosen based on optimizing the signal significance. The expected non-peaking background in the signal region is estimated by scaling the number of data events with $E_{\text{extra}} > 600$ MeV by the MC ratio of events in the sideband ($E_{\text{extra}} > 600$ MeV) to signal region ($E_{\text{extra}}^\text{MC} < 400$ MeV). The yields of $D_s^-$ tags and $D_s^+ \to \tau^+ \nu$, $\tau^+ \to e^+ \nu \bar{\nu}$ signal events are shown in Table I. The scale factor, $s$ shown in Table I, is a correction to account for slight differences in the expected number of background events in the signal and sideband regions. Using the efficiency to reconstruct the final state, $D_s^+ \to \tau^+ \nu$, $\tau^+ \to e^+ \nu \bar{\nu}$ of $(71.4 \pm 0.4)\%$ and the $B(\tau^+ \to e^+ \nu \bar{\nu}) = (17.84 \pm 0.05)\%$, we find:

$$B(D_s^+ \to \tau \nu) = (6.24 \pm 0.71 \pm 0.36)\%.$$ (14)

The 5.8\% systematic uncertainty is dominated by the 4.3\% contribution from the simulation of $K_L^0$ showering in the calorimeter.

Using Eq. 1, we find $f_{D_s^+} = (275 \pm 16 \pm 8)$ MeV. When this result is combined with the result in Eq. 12 we obtain:

$$f_{D_s^+} = 274 \pm 10 \pm 5 \text{ MeV}$$ (15)

5. Summary

We have presented measurements of the branching fractions $B(D^+ \to \mu^+ \nu$, $D_s^+ \to \mu^+ \nu$ and $D_s^+ \to \tau^+ \nu$, $\tau^+ \to \pi^+ \nu \bar{\nu}$ with the CLEO-c detector. The results are the most precise measurements of these leptonic decay rates to date. Using Eq 1 we extract the decay constants:

$$f_{D^+} = (222.6 \pm 16.7^{+2.8}_{-3.4}) \text{ MeV}.$$ (16)

$$f_{D_s^+} = (274 \pm 10 \pm 5) \text{ MeV}$$ (17)

$$f_{D_s^+}/f_{D^+} = 1.23 \pm 0.11 \pm 0.03$$ (18)

Our measurement of $f_{D_s^+}$ is consistent with and significantly more precise than the recent measurement by BaBar [14]. The only other measurement of $f_{D^+}$ was reported by BES based on 1 signal candidate. Recent lattice QCD predictions [15, 16] of both $f_{D^+}$ and $f_{D_s^+}$ are typically $\sim 10\%$ lower than our measurements, whereas the ratio of $f_{D_s^+}/f_{D^+}$ is in good agreement with our measurement.

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References


Table I Summary of $D_{s}^{-} \rightarrow \phi \pi^{-}$, $D_{s}^{-} \rightarrow K^{-} K^{*0}$, $D_{s}^{-} \rightarrow K^{-} K_{S}^{0}$ tagged events (yield, background from sidebands, sidebands scale factor ($s$), and sideband-subtracted yield), and $D_{s}^{+} \rightarrow \tau^{+} \nu$, $\tau^{+} \rightarrow e^{+} \nu \bar{\nu}$ events (yield, background from $D_{s}^{-}$ sidebands, background from $D_{s}^{+}$ semileptonic decays, and sideband-subtracted yield).

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