Low Mass S-wave $K\pi$ and $\pi\pi$ Systems

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Knowledge of the details of the S-wave $K\pi$ and $\pi\pi$ systems limits the precision of measurements of heavy quark meson properties. This talk covers recent experimental developments in parametrizing and measuring these waves, and examines possibilities for the future.

I. INTRODUCTION

Hadrons were once regarded as a source of complication in the measurement of the dynamics of weak decays of heavy quark states. The realization that their interactions can be used to help in understanding the phases involved is quite a recent development. There are several examples. Interference between hadrons produced in decays of the D^0 and $\overline{D^0}$ mesons to the self-conjugate final state $K_s^0\pi^+\pi^-$ [31] has been used to measure D^0-$\overline{D^0}$ mixing parameters. [1, 2] It has also allowed the measurement of the CKM phase γ from the decays $B^+ \to D^0K^+$ and $B^+ \to \overline{D^0}K^+$. [3–6] Interference between S- and P-waves in the $K^+\pi^-$ systems produced in $B^0 \to J/\psi K^+\pi^-$ decays has been used to resolve the ambiguity in the sign of $\cos 2\beta$, where β is the CKM phase involved in these decays.

Central to all these analyses is the need to assign a partial wave composition at each point in the hadron phase space. Currently, models for the hadron interactions based on resonant composition are mostly used to do this (the "isobar model"). Sometimes a K-matrix description of the S-wave component is included. Uncertainties and ambiguities or questionable assumptions, especially with respect to the S-wave behaviour lead to significant systematic uncertainties in the results. As the precision of the measurements improves, it becomes a more pressing goal either to develop a good, model-independent strategy for describing the hadron amplitudes or, at least, a better understanding of how to describe the S-waves in the decays of heavy quark mesons.

It is also of intrinsic interest to study the low mass S-wave $K\pi$ and $\pi\pi$ systems to better understand the scalar states, labelled here as $\kappa(800)$ or $\sigma(500)$, that may exist [7]. Measurements of these amplitudes in this region are sparse. There is also merit, therefore, in pursuing the possibility of using heavy quark meson decays to add experimental data in these regions. This talk focusses on recent progress and future prospects on both fronts.

In the first section, information presently available on both $K\pi$ and $\pi\pi$ S-waves is briefly reviewed. The expected relationship between scattering amplitudes and those measured in decays of heavy quark mesons is then examined. The remainder of the talk discusses ways in which experimental observations, mostly of the available $K\pi$ data, are being studied.

II. S-WAVE SCATTERING DATA

Excellent experimental information comes from model-independent analyses of differential cross sections for reactions in which production of $K\pi$ or $\pi\pi$ systems is dominated by a pion exchange mechanism. The amplitudes so determined show clear evidence for the $K_0^*(1430)$ resonance in the iso-spin $I = 1/2$ $K\pi$ S-wave and for the $f_0(980)$ in the $I = 0$ $\pi\pi$ S-wave. No exotic states are found in the $I = 3/2$ $K\pi$ or $I = 2$ $\pi\pi$ waves. Data at the very low mass regions, where $\kappa(800)$ or $\sigma(500)$ poles may lurk far from the real axis, are relatively poor.

A. The $K\pi$ System

The SLAC E135 (LASS) experiment [8, 9] provides the best information on the $K^-\pi^+$ system. The data are shown in Fig. 1. Note that there are no data below 825 MeV/c^2 from this experiment, though some is available from an earlier era of experiments. The LASS collaboration determined that both $I = 1/2$ and $I = 3/2$ amplitudes $T(s)$ are unitary up to $K\eta$' threshold ($\sqrt{s} = 1454$ MeV/c^2)

$$T(s) = \sin\delta(s)e^{i\delta(s)} \tag{1}$$

where s is the squared invariant mass and $\delta(s)$ is the phase.

The $I = 3/2$ phase, assumed to be unitary, was measured by Estabrooks, *etal* [10] and is shown in Fig. 1(c) and (d). The LASS collaboration used this information to separate the $I = 1/2$ and $I = 3/2$ components of the amplitude seen in Fig. 1(a) and (b). The fit to their data, shown as the solid curves, are described by Eq. (1) with

$$\begin{aligned} I = 1/2: \quad \delta^{1/2}(s) &= \delta_R^{1/2}(s) + \delta_B^{1/2}(s) \\ \cot\delta_R^{1/2}(s) &= (s_0 - s)/(\sqrt{s}_0\Gamma_0) \\ q\cot\delta_B^{1/2}(s) &= 1/a_{1/2} + b_{1/2}q^2 \\ I = 3/2: \quad q\cot\delta^{3/2}(s) &= 1/a_{3/2} + b_{3/2}q^2 \end{aligned} \tag{2}$$

where q is the momentum of the K^- in the $K\pi$ center of mass. This parametrization is valid in the range

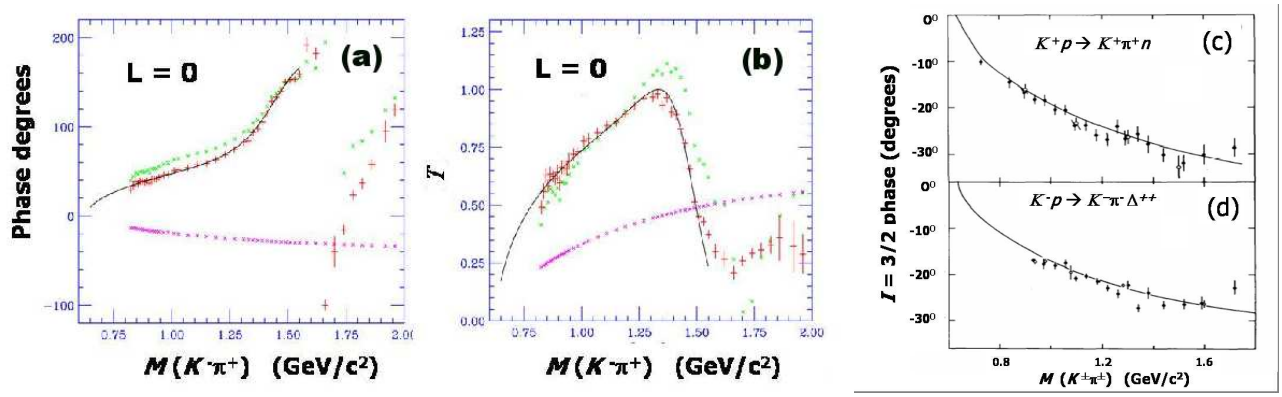

FIG. 1: S-wave $K^-\pi^+$ scattering amplitude extracted by the LASS collaboration from their data on the reaction $K^-p \to K^-\pi^+n$ at 11 GeV/c. The phase is shown in (a) and the magnitude in (b). The I-spin 1/2 component is represented by points on a solid curve and I-spin 3/2 is shown with no curve. $I = 3/2$ phases measured in data from Ref. [10] are plotted as a function of $K^\pm\pi^\pm$ invariant mass ofr (c) $K^+p \to K^+\pi^+n$ and (d) $K^-p \to K^-\pi^-\Delta^{++}$.

$825 < \sqrt{s} < 1454$ MeV/c^2, and includes one $I = 1/2$ resonance of mass $\sqrt{s_0} \sim 1435$ MeV/c^2 and width $\Gamma_0 \sim 275$ MeV/c^2. Non-resonant backgrounds in both waves are described by scattering lengths a and effective ranges b.

B. The $\pi\pi$ System

Phases for the $I = 0$ component, shown in Fig. 2(a), have been extracted in several analyses of $\pi^-p \to \pi^-\pi^+n$ data from Grayer, *etal* [11] It is clear that data below 600 MeV/c^2 in the region of the $\sigma(500)$ from these data are poor.

The $I = 2$ amplitude, assumed to be unitary up to $\rho\rho$ threshold (~ 1500 MeV/c^2), was derived from data on $\pi^+p \to \pi^+\pi^+n$ interactions at 12.5 GeV/c [12] and $\pi^+d \to \pi^-\pi^-pp_{\text{spectator}}$ at 9 GeV/c [13]. It was fit to the form shown in Fig. 2(b) [14]. For $I = 0$, the amplitude is unitary up to $K\bar{K}$ threshold where its elasticity drops suddenly. Slightly below this, the phase rises rapidly indicating the presence of the $f_0(980)$ resonance.

III. ROLE OF SCATTERING IN D DECAYS

In terms of our two goals, it would be useful if we could draw on these measurements to help in reducing ambiguities in models used in analyzing decays of heavy quark mesons. At the same time, we need to learn how to interpret such decays in learning more about the scattering amplitudes at small S values.

Consider the decay $D \to (AB)C$ in which a two hadron system $f = AB$ and another system C are produced. A simple assumption about the decay amplitude $F(s)$ for such a process is that it can be factorized into short and long-range effects.

$$F_f(s) = T_{fk}(s)Q_k(s) \tag{3}$$

Here $Q_k(s)$ describes the short-range decay of D to C and an intermediate hadron system k, and $T_{kf}(s)$ describes the subsequent re-scattering within the system k to produce the final state f. We take $Q_k(s)$ to encode not only the relatively real ratio of decay modes for the weak decay of the D that would imply an s-dependence of its magnitude, but also any other short-range effects that may occur in the Ck system which might also impart an s-dependence of its phase.

If s is below the threshold where AB scattering becomes inelastic ($K\eta'$ for L = even waves when $AB = K\pi$ and K^+K^- when $AB = \pi\pi$ systems, for example), then $T_{kf}(s)$ is simply equal to the AB elastic scattering amplitude. If the phase of $Q_k(s)$ is independent of s, then the phase of $F(s)$ will have the same s-dependence as $T(s)$, *i.e.* that observed in elastic scattering. This is a statement of the (more rigorously derived) Watson theorem [15]. If partial wave expansions of $F(s)$ and $T(s)$ are made, then this condition must hold for each partial wave. It must hold, in particular, for the S-waves.

The Watson theorem could provide a very useful constraint, therefore, in the analysis of heavy quark states, so its range of applicability is of great importance. The considerations above lead us to expect the following:

- If C is a di-lepton ($\ell\nu$, for example) then Ck scattering would be unlikely and the phase of Q_k would have little, if any, s-dependence.

- The same might also be true if C were a massive hadron (such as a J/ψ) with a small interaction radius.

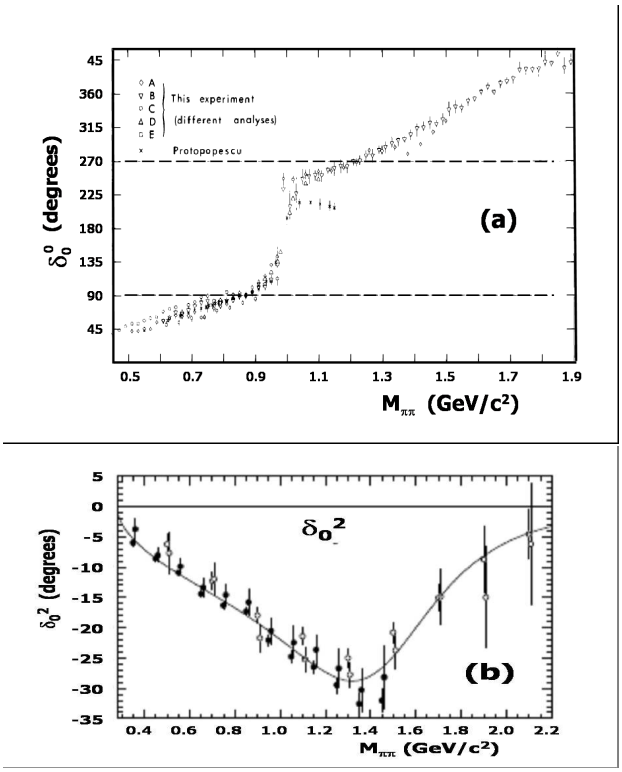

FIG. 2: (a) $I = 0$ phase of the S-wave $\pi\pi$ scattering amplitude extracted from data for the reactions $\pi^- p \to \pi^-\pi^+ n$ from Ref. [11]. The phase in degrees is plotted as a function of $\pi\pi$ invariant mass M. (b) The $I = 2$ phase shifts plotted as a function of $\pi\pi$ invariant mass M. The $I = 2$ amplitude is assumed to be unitary up to 1500 MeV/c and is obtained from data for the reactions $\pi^+ p \to \pi^+\pi^+ n$ (12.5 GeV/c) [12] and $\pi^+ d \to \pi^-\pi^- pp_{\text{spectator}}$ (9 GeV/c) [13].

- In cases where C is a hadron with mass comparable to A or B, significant scattering in the Ck system Would be likely. This could lead to a dependence of both the magnitude and the phase of $Q_k(s)$ upon s.

- The likelihood of Ck scattering probably increases at shorter range.

We should expect, therefore, to find any deviations from the Watson theorem to be most pronounced when the mass of C is comparable with that of A or B, and at small s in the AB system. We also note that, the Watson theorem applies to every partial wave, and it makes little sense to impose it only on the S-wave alone.

IV. $K\pi$ PRODUCTION FROM HEAVY QUARK MESON DECAYS

A. Decays with Leptons or Massive Hadrons

In a study of semi-leptonic decays of D mesons

$$D^0 \;\to\; K^-\pi^+\ell\nu. \tag{4}$$

the FOCUS collaboration [16] observed asymmetry in the distribution of $\cos\theta$, where θ is the angle between K^- and the $\ell\nu$ system in the $K^-\pi^+$ rest frame. The asymmetry results from interference between S- and P-waves, and is proportional to the cosine of the relative phase γ between them. As seen in Fig. 3 this follows the behaviour observed in the LASS data quite closely as the $K^-\pi^+$ invariant mass moves through the $K^*(890)$ region. This conforms to the first of our expectations.

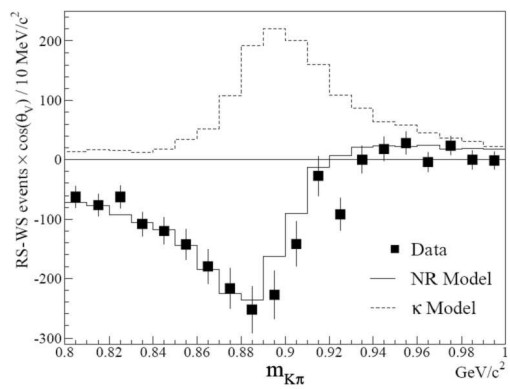

FIG. 3: Asymmetry in cosine of $K^-\pi^+$ helicity angle in $D^0 \to K^-\pi^+\ell\nu$ data from the FOCUS collaboration [16] as a function of invariant mass for the $K^-\pi^+$ system. The behaviour observed in the LASS data is indicated by the solid curve.

The BABAR collaboration observed [17] similar behaviour in the asymmetry of the helicity distribution for $K^+\pi^-$ produced in

$$B^0 \;\to\; J/\psi K^+\pi^- \tag{5}$$

decays. Fig. 4 shows the two solutions for the S-P phase difference γ in the $K^*(890)$ invariant mass region. The relative phase is dominated by the rapid variation in the P-wave due to the $K^*(890)$ and only one solution is physically meaningful. It is seen in the figure that this matches the LASS phase variation (though shifted by $+\pi$ radians) very well.

This conforms to the second of our expectations, but it is not obvious why there is an overall phase shift of $+\pi$ radians.

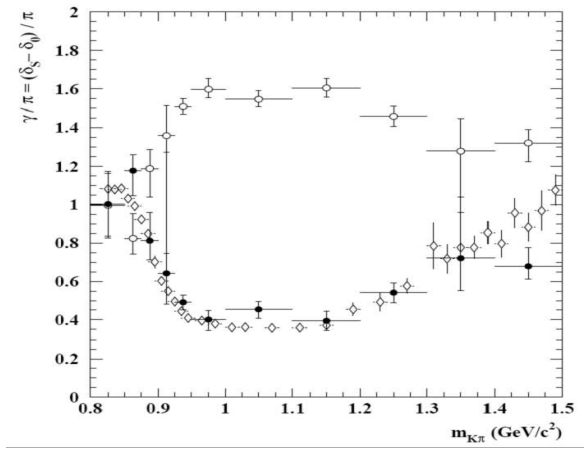

FIG. 4: Two solutions (open and closed circles with error bars) for the relative phase $\gamma = \delta_S - \delta_P$ between S- and P-wave amplitudes of $K^+\pi^-$ systems produced in $B^0 \to J/\psi K^+\pi^+$ decays from the BABAR collaboration [17]. Values for γ are plotted as a function of $K^+\pi^-$ invariant mass. The solid circles are for the only physical solution and follow the LASS data points, plotted as triangles, closely.

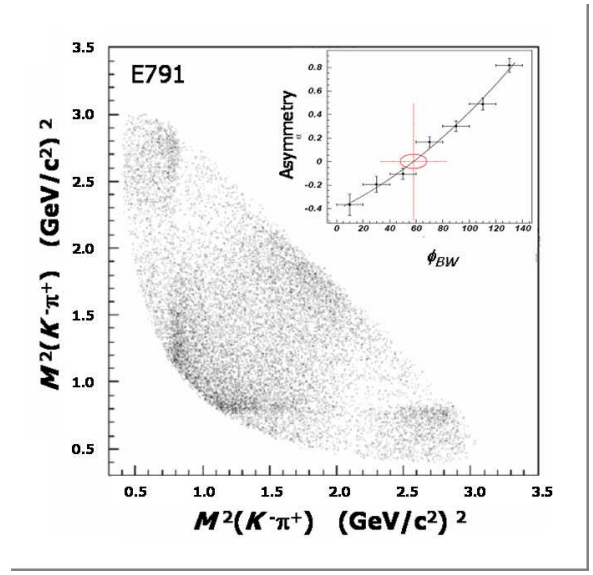

FIG. 5: Dalitz plot for $D^+ \to K^-\pi^+\pi^+$ decays. Data are from E791 [18] and show distributions of squared invariant mass for one $K^-\pi^+$ combination plotted against the other (symmetrized). The inset shows the asymmetry in the cosine of the $K^-\pi^+$ helicity angle (between K^- and the other π^+ in the $K^-\pi^+$ rest frame) as a function of ϕ_{BW}, the $K^*(890)$ Breit-Wigner phase.

B. Decays to Light Hadrons

The most detailed experimental information comes from studies of $D^+ \to K^-\pi^+\pi^+$ decays. These Cabibbo favoured decays are known to contain a large S-wave component.

A study of $\sim 15,000$ such decays by the E791 collaboration [18] provides an illustration. The E791 Dalitz plot is shown in Fig. 5 where significant S-P interference is evident from the asymmetry of the $K^*(890)$ bands. This is plotted in Fig. 5 vs. the Breit-Wigner phase $\phi_{BW} = \tan^{-1} M_0\Gamma(M_0)/M_0^2 - s)$, where M_0 is the mass and $\Gamma(\sqrt{s})$ the invariant mass-dependent width of the $K^*(890)$. The asymmetry is zero when $\phi_{BW} = 56°$, almost 80° below the phase found in the LASS data [8]. This shows that the phase of $Q_{K\pi}(s)$ at this invariant mass is $\sim -80°$ and is entered in Table II.

1. Isobar Model Fits

In the earliest analyses of these decays, a model with interfering resonances like the $K^*(890)$, the $L = 0$ $K_0^*(1430)$ and a constant non-resonant "NR" 3-body amplitude could account for the voids and asymmtries observed in the Dalitz plot. With their larger sample, the E791 isobar model analysis showed that additional structure in the S-wave was required to achieve an acceptable fit, and the addition of a $\kappa(800)$ Breit-Wigner isobar, which interfered destructively with the NR term, worked well.

The "isobar model" description of the $L = 0, 1$ and 2 wave amplitudes F_L in the $K^-\pi^+$ systems for this fit can be summarized as:

$$F_0(s) = c_{00} + \alpha_{10} BW_{K_0^*(1430)}(s) + \alpha_{20} BW_{\kappa(800)}(s) \quad (6)$$

$$F_1(s) = \alpha_{11} BW_{K_1^*(890)}(s) + \alpha_{21} BW_{K_0^*(1688)}(s) \quad (7)$$

$$F_2(s) = \alpha_{21} BW_{K_2^*(140)} \quad (8)$$

where the $BW(s)$ are relativistic Breit-Wigner functions with s-dependent widths, and the α_{iL} are complex coefficients determined in the fit. The overall phase was defined by setting $\alpha_{11} = 1.0$, and α_{00} was the NR term.

Two further isobar model analyses of this decay mode were recently made, one by FOCUS [19] and the other by CLEOc [20]. Each used samples ~ 3.5 times larger. The conclusions, and estimates of the resonant fractions [32], of both were in good general agreement with E791.

Parameters for κ and $K_0^*(1430)$ S-wave Breit Wigner isobars are compared for the three experiments in Table IV B 1. The $K_0^*(1430)$ parameters in this model disagree significantly with those obtained by the LASS experiment or with the World average [21]. There is general agreement that this description of the S-wave, described by Eq. (6), with two broad, Breit Wigner resonances, one of which is also near threshold, is theoretically problematic and could

account for this discrepancy. It would be virtually certain that this amplitude would have an s-dependent phase that would differ from the Watson theorem expectation.

TABLE I: Breit Wigner Parameters for the $K^-\pi^+$ S-wave Isobar States. All quantities are in MeV/c^2.

		E791	Focus	CLEO c	PDG
κ	$M_\circ$	$797 \pm 19 \pm 42$	883 ± 13	805 ± 11	672 ± 40
	$\Gamma_\circ$	$410 \pm 43 \pm 85$	355 ± 13	453 ± 21	550 ± 34
$K^*_\circ$	$M_\circ$	$1459 \pm 7 \pm 12$	1461 ± 4	1461 ± 3	1414 ± 6
	$\Gamma_\circ$	$175 \pm 12 \pm 12$	177 ± 8	169 ± 5	290 ± 21

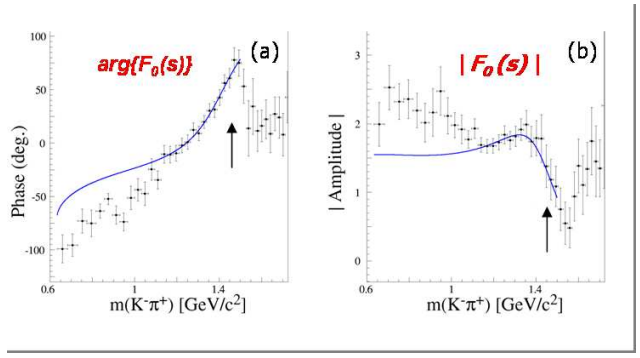

FIG. 6: (a) Phase and (b) magnitude of the decay amplitude $F_0(s)$ at 38 discrete values of s (squared invariant mass of the $K^-\pi^+$ systems from $D^0 \to K^-\pi^+\pi^+$ decays taken from Ref. [22]. The LASS model in Eq. (2), shifted by $-75°$ and scaled to the region where $s > 1.2$ (GeV/c^2)^2 is shown as the blue, continuous curve.

2. *Model-Independent Measurement*

A test of the Watson theorem requires a measurement of the phase of $F(s)$ at several values of s in a model-independent way. The first attempt to do this for the S-wave for $K^-\pi^+$ produced in this decay mode was made by the E791 collaboration [22].

They replaced the analytical function describing the S-wave in Eq. (6) by a set of 38 complex values at discrete values for s, using a spline interpolation for other values. The P- and D-waves were paramerized, as before, by the form in Eqs. (7) and (8) and the coefficients α_{iL} were allowed to float. A fit was then made to determine the best values (magnitude and phase) for each of the 38 S-wave points. The result, shown in Fig. 6, is compared with the LASS model for the $I = 1/2$ $K^-\pi^+$ system in Eq. (2). Agreement is good in the $K^*_0(1430)$ region (above ~1100 MeV/c^2) after a shift in phase of 75° and an arbitrary scale factor are applied to the LASS amplitude. A very significant discrepancy is, however, seen for lower values of invariant mass.

This might conform to our 4th expectation, but there are two problems. First, though $I = 1/2$ $K^-\pi^+$ production probably dominates, $I = 3/2$ production in these decays cannot be excluded. Second, the isobar model form in Eqs. (7) and (8) for the P- and D-waves, upon which this result depends, is questionable. The P-wave contains more than one Breit-Wigner, and both waves are assumed to be dominated by resonant behaviour. More importantly, neither wave is likely to follow the Watson theorem, so a test of the S-wave alone cannot be conclusive.

The CLEO collaboration [20] has attempted to overcome the latter difficulty. Using their high purity sample of $\sim 60,000$ events, they proceeded in the same way as E791, interpolating the S-wave between discrete values of s, while parametrizing the P- and D-waves as above. Their results were very similar. They then fixed the S-wave and, using a similar procedure, fit the $P-$wave parametrized in the same way. Then they repeated this for the D-wave. In each step, only one wave was allowed to float with the others fixed.

This procedure can converge only by simultaneously floating all waves at once, and this was not done. Also, the phases have to be defined in some part of the phase space (as little as possible) in order for this to work.

It seems that a truly model-independent measurement of the S-wave decay amplitudes in these hadron decays is desirable, but has yet to be made, and will probably need to await the much larger data samples of the B factories, BES or the Panda experiment.

3. ***I***-*spin Analysis by the FOCUS Collaboration*

Using a sample of about 50,000 events with 96% purity, the FOCUS collaboration used the K-matrix method to separate $I = 1/2$ and $I = 3/2$ production for this decay [19].

The P- and D-waves were described as in Eqs. (7) and (8), however, the S-wave amplitude $F_0(s)$ in Eq. (6) was replaced by the sum of two terms, one for each I-spin. Each was described by

$$F_f(s) = (I - i\rho K(s))^{-1}_{fk} P_k(s) \quad (9)$$

$$T_{kf}(s) = (I - i\rho K(s))^{-1}_{ki} K_{if}(s) \quad (10)$$

$$Q_f(s) = K^{-1}_{fk}(s) P_k(s). \quad (11)$$

Here, the amplitudes F, T, P and Q were introduced for each I-spin and their indices K and f labelled intermediate and final states ($1 = K\pi$, $2{=}\eta' K$). The production vector $P_k(s)$ describing the couplings of the D decay to the two intermediate states was de-

scribed by parameters obtained from the fit, and ρ_{kf} was the phase space matrix for the two channels.

Real values for all elements of the K matrices (2×2 for $I = 1/2$ and 1×1 for $I = 3/2$) were determined by a fit to the LASS measurements of T.

The fit required a very large contribution from the $I = 3/2$ S-wave (40.50%) interfering destructively with $I = 1/2$ (207.25%) giving a total S-wave (83.23%).

The phases of the resulting decay amplitudes, $F(s)$, are shown in Fig. 7, with the K-matrix fit to the LASS elastic scattering data for $I = 1/2$ superimposed. The phase of the total amplitude $F(s)$, similar to that found by E791 and by CLEOc, differs significantly from the LASS $I = 1/2$ data. The $I = 1/2$ component is shown alone in Fig. 7(b) and shows better agreement.

The parametrization of the production vector was chosen specifically to allow an s-dependence in its phase, thereby allowing a deviation from the Watson theorem. Some deviations are, indeed, evident in Fig. 7(b), as the invariant mass approaches the $K_0^*(1430)$ region. It is possible that this is due to the $K_0^*(1430)$ pole and the onset of effects from the $K\eta'$ channel. However, another interpretation may be possible. The phase shown in this figure is shifted by an arbitrary amount to achieve good agreement at the lower invariant masses. Were a different shift in phase applied, agreement would, in fact, be good at the high invariant mass end and poor at low mass, consistent with our expectation number 4.

This analysis attempted to make a valid comparison between scattering and decay. It appears premature, however, to conclude that the Watson theorem works for these hadronic decays especially because the same is not required for the P- nor D-waves in this analysis.

V. $\pi\pi$ PRODUCTION FROM HEAVY QUARK MESON DECAYS

Decays of $D^+ \to \pi^-\pi^+\pi^+$ result in S-wave enriched $\pi^+\pi^-$ systems. Unfortunately, these decays are Cabibbo suppressed. Another decay rich in S-wave content is $D^0 \to K^0\pi^+\pi^-$. This is partly Cabibbo favoured and partly doubly suppressed, so is somewhat complex with many resonances contributing. Consequently, model-independent analyses of these decays have yet to be attempted.

A. $D^+ \to \pi^-\pi^+\pi^+$ Decays

The largest sample yet studied comes from the CLEO experiment [23]. It consists of only about $4,000$ events with a purity of $\sim 95\%$. It has been used to test a variety of models.

An isobar model fit to the Dalitz plot confirms, as does a similar analysis by FOCUS [24], the E791 collaboration conclusion [25] that structure in the low mass $\pi^+\pi^-$ system is well described by a scalar $\sigma(500)$ Breit Wigner destructively interfering with a constant NR amplitude. The fit obtained was of marginal quality.

CLEO tried several variations on the isobar model. They included an $I = 2$ $\pi^+\pi^+$ S-wave contribution, slightly improving the fit quality. They also tried variations of $\pi\pi$ S-wave isobar model. In one, as suggested in Ref. [26], they replaced the σ Breit-Wigner by a simple pole

$$1/(m_0^2 - s - im_0\Gamma) \;\to\; 1/(s_0 - s) \qquad (12)$$

where $s_0 = (0.47 - 0.22i)$. Other S-wave models used were an amplitude based on the linear sigma model [27] and an amplitude derived by N.Achamov described in the CLEO paper Ref. [23]. All models provided fits to the data slightly more acceptable than the isobar model, but no clear distinction was obvious.

B. $D^0 \to K^0\pi^+\pi^-$ Decays

Isobar model fits to large samples have been made by BABAR [4] and Belle [5] collaborations. They each require two σ states in the $\pi^+\pi$ S-wave, one similar to that found in isobar fits to the previous channel by E791, FOCUS and CLEOc, and the other at a mass of ~ 1.0 GeV/c^2. Both collaborations have also used K-matrix parametrizations of the S-wave $\pi^+\pi^-$ amplitude, obtaining slightly better fits. These fits involve no assumptions about σ states, and they do enforce the Watson theorem in this system. However, the other waves are those defined by the isobar model, with no such constraint. None of the fits so far published have acceptable quality.

Further progress in this system may come from the larger data samples form the B factories, or from the next generation of charm factories.

C. Other Channels

A number of B and D decays in which $K\pi$ systems are produced have now been published. A comparison between the general characteristics of the S-wave amplitudes observed and those of the LASS scattering data are summarized in Table II [9]. The two examples above are included and represent the best examples of the validity of the Watson theorem. Hopefully, an underlying pattern for these characteristics will eventually emerge, provided that such decays are studied with this goal in mind. Any pattern is certainly not yet evident.

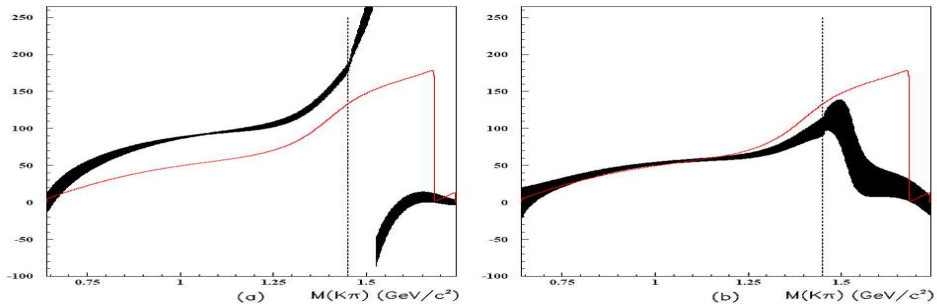

FIG. 7: Decay amplitudes for $K^-\pi^+$ systems from $D^+ \to K^-\pi^+\pi^+$ decays from Ref. [19] for a(a) the total S-wave and (b) the $I = 1/2$ contribution. The black bands correspond to one standard deviation limits about the central fit in each of the amplitudes. Scattering amplitudes from a K-matrix fit to the LASS data [8] are shown as red continuous lines.

TABLE II: Qualitative comparison between $K\pi$ S-wave decay amplitude $F_0(s)$ and the LASS Model in a variety of cases studied. Three characteristics are compared. The S-P relative phase at the $K^*(890)$ mass. The observed value minus that observed in LASS data, $\Delta\phi_{SP}$, is tabulated to the nearest 15°). The general shape of $|F_0(s)|$ for s values both below and above $s_0 = 1.0$ GeV/c^2 are also tabulated.

Decay Process	ϕ°_{SP} **approx**	$\|F_0\|$ $s < s_0$	$\|F_0\|$ $s > s_0$
$B^0 \to J/\psi K^+\pi^-$ [17]	+180	Poorly defined	Similar to LASS
$B^+ \to K^+\pi^-\pi^+$ [28]	0	Unknown	Similar to LASS
$B^+ \to K^+\pi^-\rho$ [29]	+180	Unknown	Unknown
$D^0 \to K^-K^+\pi^0$ [30]	−90	Similar to LASS	Similar to LASS
$D^+ \to K^-\pi^+\pi^+$ [22]	−75	Very different	Similar to LASS
$D_s^+ \to K^-K^+\pi^+$	−90	Similar to LASS	Similar to LASS
$D^+ \to K^-\pi^+\ell\nu$ [16]	0	Similar to LASS	Similar to LASS

VI. CONCLUSIONS

The most reliable data on S-wave amplitudes are still those from LASS or CERN-Munich data on elastic scattering. Data at the lowest energies, however, are somewhat sparse. It would surely help in the understanding of the pole structure relevant to the firm establishment of existence or otherwise of light scalar $\kappa(800)$ or $\sigma(500)$ states to improve on this situation. It is difficult, at present, to see how hadronic decays of D or B mesons will help since I-spin considerations, and uncertainties in the D form-factor, make such measurements difficult. It appears, therefore, that such information is most likely to come from high statistics studies of D semi-leptonic decays, or decays of B mesons to $J/\psi K^+\pi^-$ (or $J/\psi\pi^+\pi^-$) from BABAR or Belle or the new facilities in BES or Darmstadt.

Progress in understanding ways to parametrize the S-wave decay amplitudes in Dalitz plot analyses is slow, but the body of information is growing. The prize could be less systematic uncertainty resulting from model dependence of such analyses in measurements of important phenomena such as D^0-$\overline{D^0}$ mixing or of the CKM angle γ.

A positive trend is that more techniques beyond the isobar model, with its known problems and strong model-dependence, are being developed.

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[31] Charge conjugate states are implied unless specified otherwise throughout this talk
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