

Charmonium from Lattice QCD

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Charmonium is an attractive system for the application of lattice QCD methods. While the sub-threshold spectrum has been considered in some detail in previous works, it is only very recently that excited and higher-spin states and further properties such as radiative transitions and two-photon decays have come to be calculated. I report on this recent progress with reference to work done at Jefferson Lab.

1. Introduction

Between 3 and 3.7 GeV a number of states exist which are believed to be the bound states of a charm quark and an anti-charm quark and whose widths are narrow owing to their being below the threshold to decay to a pair of open charm mesons coupled with the suppression of annihilation channels at this high mass scale. Because the hadronic contributions to their widths are so small, radiative transitions between them constitute considerable branching fractions, and the rates of these transitions have been measured with some accuracy by a number of experiments (Yao et al. [2006]). Additionally the $C = +$ states can decay to a pair of photons - this process when time-reversed can serve as a production mechanism (two-photon fusion) at e^+e^- machines.

Rates for these radiative processes have been computed in various varieties of quark-model, and are typically fairly successful when one sets parameters using the experimental spectrum, however corrections beyond approximations like non-relativistic dynamics are often uncontrolled in these models.

In the current century the charmonium picture has filled out considerably and new mysteries have arisen owing to the high statistics and new production methods made possible by CLEO-c and the B -factories. The remaining expected sub-threshold states, η'_c , h_c , have been observed, as have radiative transitions from the $\psi(3770)$ down to the χ_{cJ} . The above-threshold spectrum is rapidly being mapped (Swanson [2006]), with some states living up to the expectations of potential models (Uehara et al. [2006]) and others coming as something of a surprise (Choi et al. [2003]). The increasingly complete set of exclusive data in e^+e^- looks set to allow determination of the vector spectrum with some confidence.

In a series of recent works (Dudek et al. [2007, 2006], Dudek and Edwards [2006]), members of the Jefferson Lab lattice group have investigated the possibility of computing excited spectral and radiative quantities using lattice QCD. These initial studies have been carried out on quenched lattices with rather promising results. In the sections that follow I will briefly describe the work done.

2. Excited and higher spin states

The mass spectrum of a field theory considered in Euclidean space-time can be extracted from the time-dependence of a two-point correlation function,

$$C_{ij}(t) = \sum_{\vec{x}} \langle \mathcal{O}_i(\vec{x}, t) \mathcal{O}_j(\vec{0}, 0) \rangle,$$

where $\mathcal{O}_{i,j}$ are operators that have the right quantum numbers to produce a particular state from the vacuum which are constructed from the fundamental fields of theory. For example in QCD we might try to study the pseudoscalar spectrum by considering an operator $\bar{\psi}\gamma^5\psi$. The correlator receives contributions from all states in the theory with the appropriate quantum numbers,

$$C_{ij}(t) = \sum_{\alpha} \frac{Z_i^{\alpha*} Z_j^{\alpha}}{2m_{\alpha}} \exp -m_{\alpha}t. \quad (1)$$

(In a quantum mechanical bound state model we might think of radial excitations being labeled by α). In practice extracting anything other than the ground state mass from fits to the time-dependence of a single correlator is difficult and often unstable. This is particularly troublesome in a system like charmonium where there are significant approximate degeneracies, e.g. the $\psi(3686)$ and $\psi(3770)$. These degeneracy problems are made worse on a cubic lattice where states are labeled not by a continuum spin, but by an irreducible representation of the cubic group. The continuum spin content of these various irreps is shown in Table I. This indicates that, for example, components of a 3^{--} state would appear in the same correlators as 1^{--} states. Since from potential models we expect there to be a $3^{--}({}^3D_3)$ roughly degenerate with the $1^{--}({}^3D_1)$ $\psi(3770)$, we anticipate that there should be three roughly degenerate excited states above the ground state in a T_1^{--} correlator. Extracting this from a fit to a single correlator is not practical.

Given this one might consider more reliable ways to extract the excited state spectrum. A variational method utilizing a large basis of operators satisfies this need. Its major advantage is that it utilizes the

Λ	d_Λ	J
A_1	1	0, 4, 6, ...
A_2	1	3, 6, 7, ...
E	2	2, 4, 5, ...
T_1	3	1, 3, 4, ...
T_2	3	2, 3, 4, ...

Table I The table shows the single-valued irreducible representations Λ of the cubic group O , together with their dimensions d_Λ and continuum spin content J . Additional superscripts are employed to denote charge conjugation C and parity P .

orthogonality of states in a space of operators - while states might be degenerate and hence hard to separate on the basis of mass, they remain orthogonal and hence easier to separate on the basis of their state vectors.

In Dudek et al. [2007], an operator basis was constructed based upon operators that in the continuum would have the structure of fermion bilinears with a number of symmetric covariant derivatives

$$\mathcal{O}_{\mu\nu\rho\dots} = \bar{\psi}(x)\Gamma_\mu \overleftrightarrow{D}_\nu \overleftrightarrow{D}_\rho \dots \psi(x).$$

Including up to two derivatives, these operators give access to almost all continuum J^{PC} with $J \leq 3$. Suitable linear combinations of these operators can be constructed that transform as the irreducible representations of Table I. These are related to the operators used in Liao and Manke [2002].

Once a matrix of correlators, $C_{ij}(t)$, has been computed (for a given irrep), the mass spectrum follows from solution of a generalized eigenvalue problem that can be shown to be the quantum mechanical variational solution. We solve

$$C(t)v_\alpha = \lambda_\alpha(t)C(t_0)v_\alpha, \quad (2)$$

for the eigenvalues $\lambda_\alpha(t)$ which are related to state masses, and for the eigenvectors v_α which are related to the overlap of our operators onto the mass eigenstates, the Z_i^α in eqn (1).

We computed correlators on quenched anisotropic lattices with $a_s \sim 0.1$ fm and $a_t^{-1} \sim 6$ GeV. Full details can be found in Dudek et al. [2007].

We show in figure 1 the mass spectrum extracted for negative parity and charge conjugation. In the T_1 representation we see precisely the level structure we expected, namely a ground state and three closely spaced excited states above. Looking at the states in the other irreps we see that one possible continuum spin assignment of the states in the first excited “band” would be to have two spin-1 states, one spin-2 state and one spin-3 state¹. We gain a good deal of

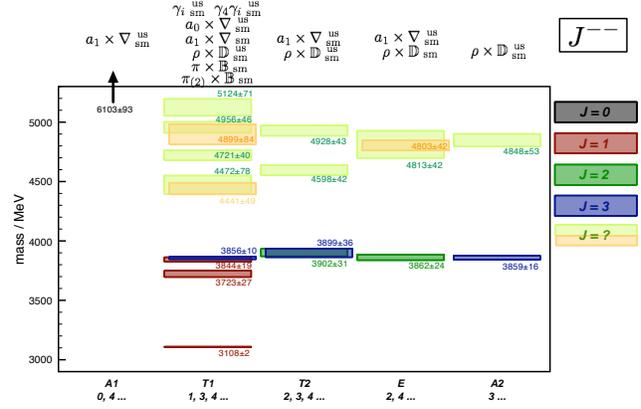


Figure 1: Extracted mass spectrum for $PC = --$ listed by lattice irreducible representation. Operator labels listed in Dudek et al. [2007]

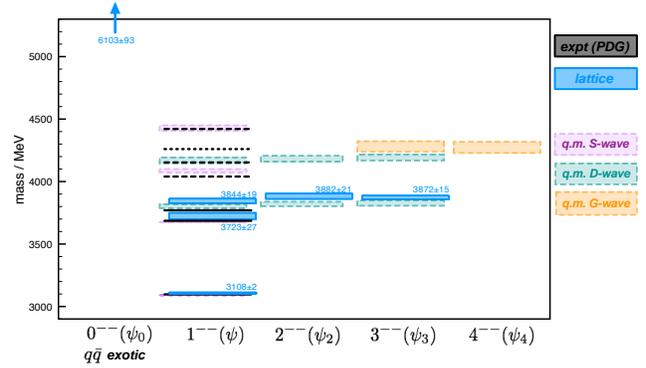


Figure 2: Extracted mass spectrum for $PC = --$ listed by assigned continuum spin.

support for this hypothesis from studying the eigenvectors extracted from eqn (2). Consider the lattice irrep projections of the “ $a_1 \times \nabla$ ” operator:

$$\begin{aligned} \mathcal{O}_{T_2}^i &= |\epsilon^{ijk}| \bar{\psi}(x) \gamma_5 \gamma_j \overleftrightarrow{D}_k \psi(x) \\ \mathcal{O}_E^i &= Q^{ijk} \bar{\psi}(x) \gamma_5 \gamma_j \overleftrightarrow{D}_k \psi(x), \end{aligned}$$

where $|\epsilon^{ijk}|$, Q^{ijk} are Clebsch-Gordan coefficients for the lattice cubic group. In the continuum we know the form that the overlap of these operators onto a spin-2 state takes, so that

$$\begin{aligned} \langle 0 | \mathcal{O}_{T_2}^i | 2^{--}(\vec{p}, r) \rangle &= Z |\epsilon^{ijk}| \epsilon_{jk}(\vec{p}, r) \\ \langle 0 | \mathcal{O}_E^i | 2^{--}(\vec{p}, r) \rangle &= Z Q^{ijk} \epsilon_{jk}(\vec{p}, r), \end{aligned}$$

in both the three-dimensional T_2 and the two-dimensional E corresponds to the five spin projections of a spin-2 meson

¹In the continuum the appearance of a e.g. spin-2 state

where Z is common to both. Z can be extracted from the eigenvectors and if it is found to be close in value in the T_2 and E cases then we conclude that it is likely that we have a spin-2 state.

We apply this eigenvector inspection method whenever possible and where the result is conclusive we assign the continuum spin shown by the color coding in figure 1. For the states above the first excited band this method gave inconclusive answers and for this reason we do not try to assign a continuum spin.

This first calculation was performed only at one (quenched) lattice spacing and consequently our results are not extrapolated to the continuum. Nevertheless we present our results for continuum spin assigned states in figure 2 along with experimental state masses taken from the PDG (Yao et al. [2006]) and potential model masses taken from Barnes et al. [2005].

It is clear that we are in agreement with the gross structure predicted by potential models, and in particular we appear to have successfully extracted something like the $\psi(3686)/\psi(3770)$ system. We believe that this has not been achieved before in a lattice calculation. Extracted state masses appear to be systematically high with respect to potential models and experiment - our suspicion is that this is due to some combination of computation at finite lattice spacing and the quenched approximation² - this hypothesis can be tested with further calculation now that this method has been demonstrated.

Other PC combinations were also considered. In figure 3 we show our results for J^{++} . It is clear that again we are observing masses systematically higher than the potential model states. That we miss the spin-4 state near 4 GeV may be related to the fact that our operators, which have a maximum of two spatial derivatives do not have any overlap with spin-4 mesons in the continuum limit. This could be remedied by enlarging the operator basis.

With $PC = +-$, as well as spin-singlets with odd J , one also has the possibility of exotic quantum numbers, i.e. those not accessible to a $q\bar{q}$ Fock state. In a quenched heavy-quark calculation these can only arise through non-trivial gluonic excitation giving rise to states usually described as ‘‘hybrids’’. Our extracted mass spectrum is shown in figure 4 where exotic states with 0^{+-} , 2^{+-} quantum numbers appear above 4.5 GeV.

With $PC = -+$, the odd- J states are exotic. Our extracted mass spectrum listed by lattice irrep is shown in figure 5. This case demonstrates the difficulty in continuum spin assignment; the set of five

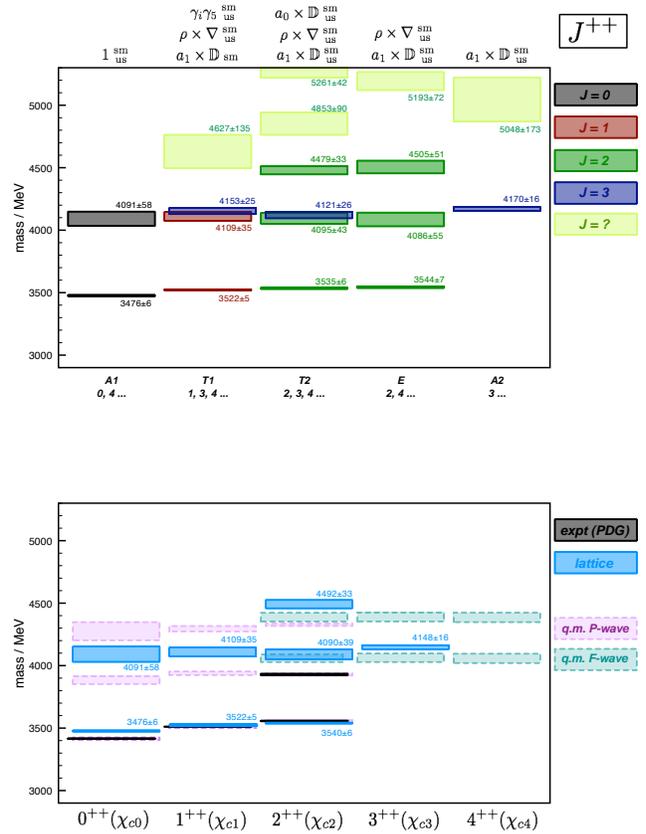


Figure 3: Extracted mass spectrum for $PC = ++$ listed by lattice irreducible representation and continuum spin assigned states.

levels near 4.3 GeV could, on the basis of their mass degeneracy, be interpreted either as a single 0^{-+} and a single (non-exotic) 4^{-+} or as two 0^{-+} states, an exotic 1^{-+} and a 2^{-+} . In previous cases we used the eigenvector inspection method to break these ambiguities, but unfortunately here the method produces inconclusive results. We display the two possible spectra in figure 6 where we note that the potential model does have a 4^{-+} state in this mass range.

It is worth pointing out that previous studies of the 1^{-+} state in charmonia have not taken into account the spin ambiguity and hence they may have in fact reported the mass of a non-exotic 4^{-+} state. It is clear that further study with more operators and higher statistics is needed in order to make a definitive statement.

We believe that we have demonstrated the power of using a variational solution in a large, carefully constructed operator basis to extract excited states in lattice QCD. Of course there remain numerous issues to deal with, including the effect of multiparticle ($D\bar{D}$) states when one relaxes the quenched approximation, but given that they too are orthogonal states we should be well-equipped with the method outlined

²In particular the problem of scale setting when one has an incorrect running of the coupling. In Dudek et al. [2007] the effect of finite box size was tested and was found not to be the source of the level raising effect.

3. Radiative transitions

Sub- $D\bar{D}$ -threshold charmonia have very narrow widths such that radiative transitions between them constitute considerable branching fractions, these have been measured by a range of experiments and their relative magnitudes give us clues to internal structure.

These transition widths can be computed using lattice QCD by considering three-point correlators of the type

$$C_{i\mu j}(t) \sim \langle \mathcal{O}_i(\vec{x}, t_f) \mathcal{V}_\mu(\vec{y}, t) \mathcal{O}_j(\vec{0}, 0) \rangle,$$

where $\mathcal{O}_{i,j}$ are operators having overlap with meson states and \mathcal{V}_μ is a lattice representation of the vector current. For example we could extract the $J/\psi \rightarrow \eta_c \gamma$ matrix element from the large Euclidean times value of the correlator

$$C_{\mu\nu}(t) = \sum_{\vec{x}} e^{-i\vec{p}_f \cdot \vec{x}} e^{i\vec{q} \cdot \vec{y}} \times \langle [\bar{\psi} \gamma_5 \psi](\vec{x}, t_f) [\bar{\psi} \gamma_\mu \psi](\vec{y}, t) [\bar{\psi} \gamma_\nu \psi](\vec{0}, 0) \rangle.$$

Correlators of this type, using only point-like operators were evaluated (details can be found in Dudek et al. [2006]) and transition form-factors extracted for a set of transitions between J^{PC} ground states.

The $J/\psi \rightarrow \eta_c \gamma$ transition form-factor shown in figure 7 is the most statistically precise signal, but it suffers from a large systematic issue related to quenching. It is well known that the experimental hyperfine splitting in charmonium is not reproduced well by studies utilizing the quenched approximation. As such we have an ambiguity when computing the phase space that is required to scale a matrix-element to a width (or vice-versa) - should we use the experimental value or the value extracted from the spectrum portion of our lattice calculation? In figure 7 we show the experimental width³ scaled to a matrix-element by both possibilities and the lattice data fitted with an exponential in photon virtuality, Q^2 , used to extrapolate back to $Q^2 = 0$.

A transition with reasonable statistical precision and a very small phase-space ambiguity is the electric dipole transition $\chi_{c0} \rightarrow J/\psi \gamma$. Our results are shown in figure 8 where the fit uses a form motivated by the quark model. Note the points at slightly timelike Q^2 are not included in the fit - the agreement with the extrapolated curve then lends support to the fitting form used.

Results for other transitions can be found in Dudek et al. [2006] as can comparison of the lattice results to

³We note that there is ongoing work at CLEO to confirm the single measurement from Crystal Ball

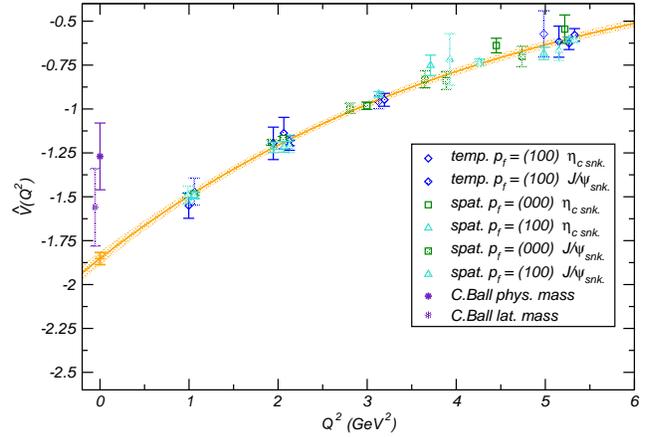


Figure 7: Transition form factor for $J/\psi \rightarrow \eta_c \gamma$.

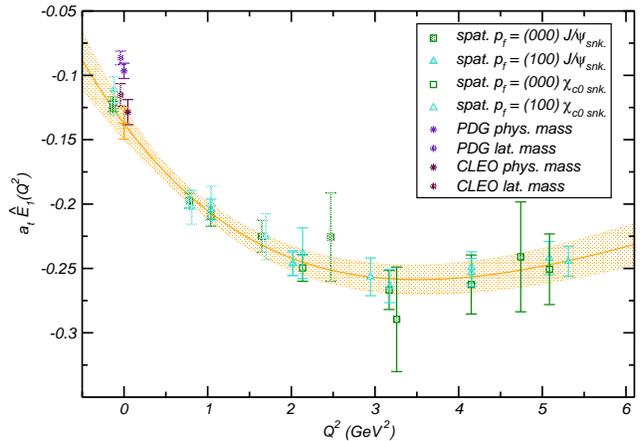


Figure 8: Electric dipole transition form factor for $\chi_{c0} \rightarrow J/\psi \gamma$. Experimental data from PDG(2005) and CLEO(Adam et al. [2005])

potential model expectations. Work is currently underway combining the excited state technology of the first section with the radiative transition technology to make it possible to study transitions involving excited and high-spin states. This would include experimentally measured transitions like $\psi(3686) \rightarrow \chi_{cJ} \gamma$.

4. Two-photon decays

At first sight it is not clear how one would go about evaluating the matrix element for the process $\eta_c \rightarrow \gamma\gamma$ in lattice QCD. In the previous section we outlined how to extract the matrix element for a radiative transition between two QCD eigenstates from a three-point function evaluated at large Euclidean times. This issue here is that the photon is not an eigenstate of QCD - taking a vector interpolating field to large Euclidean time would not yield a photon state, but instead the lightest QCD vector eigenstate (the J/ψ in this case).

However, all is not lost, for while the photon is not a QCD eigenstate, it can be constructed from a linear superposition of QCD eigenstates. The precise field-theoretic mechanism for this is the LSZ reduction. The connection in Euclidean space-time, for a different physical process, is made in Ji and Jung [2001] and for the process in question an outline appears in Dudek and Edwards [2006]. The end result is that the following relationship connects the matrix element of interest to a Euclidean three-point function computable on the lattice: $\langle \eta_c(p) | \gamma(q_1, \lambda_1) \gamma(q_2, \lambda_2) \rangle \sim$

$$e^2 \epsilon_\mu(q_1, \lambda_1) \epsilon_\nu(q_2, \lambda_2) \int dt_i e^{-\omega_1(t_i - t)} \times \left\langle \int d^3 \vec{x} e^{-i\vec{p}\cdot\vec{x}} \mathcal{O}(\vec{x}, t_f) \int d^3 \vec{y} e^{i\vec{q}_2\cdot\vec{y}} \mathcal{V}^\nu(\vec{y}, t) \mathcal{V}^\mu(\vec{0}, t_i) \right\rangle \quad (3)$$

The difference with respect to the radiative transitions between hadrons considered above is that an integral over the Euclidean time position of a vector source is now involved.

The details of the lattice computation of this object can be found in Dudek and Edwards [2006], here we mention only that an isotropic lattice was used. In figure 9(a) we display the integrand of equation 3, having computed with an operator $\bar{\psi}\gamma_5\psi$ fixed at $t_f = 37$, a conserved vector current insertion at $t = 4, 16, 32$ and a vector interpolating field at all possible source positions, $t_i = 0 \rightarrow 37$. It is clear that provided one is not too close to the Dirichlet wall or to the sink position, one can capture the entire integral by summing timeslices. In figure 9(b) the results of summing timeslices to compute the integral for all possible insertion positions and a number of Q^2 are shown - clear plateaus are visible at intermediate times indicating dominance of the η_c over the possible excited states.

Given the confidence that the integral can be captured on a lattice of this temporal length, one can use a much faster method to compute the transition form-factor that places the sum over timeslices into a ‘‘sequential source’’, reducing the computation time by a factor of $\mathcal{O}(L_t)$. Results using this method are shown in figure 10 along with PDG values and results inferred from Uehara et al. [2007].

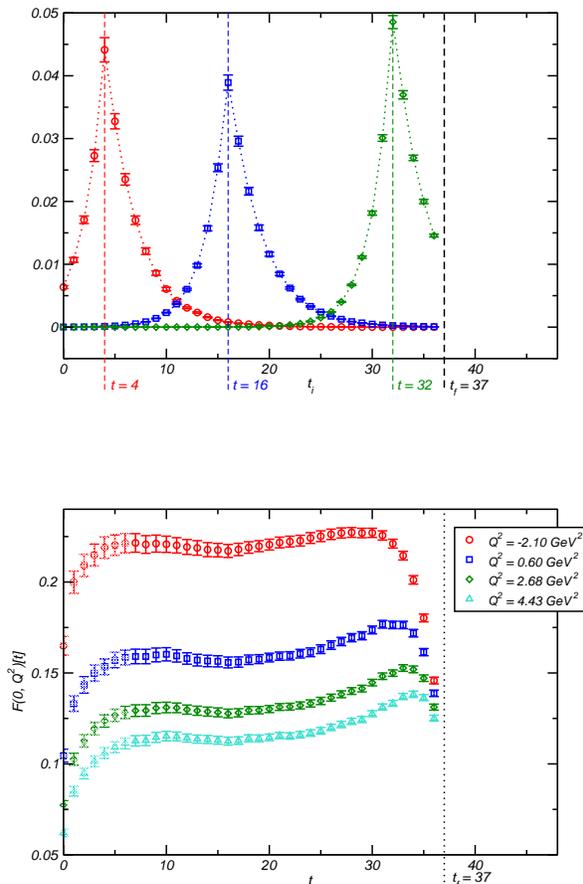


Figure 9: (a) Integrand in equation 3 at three values of vector current insertion time ($t = 4, 16, 32$) with pseudoscalar sequential source at sink position $t_f = 37$. (b) Pseudoscalar two-photon form-factor as a function of time slice, t , from equation 3. First six time slices ghosted out due to the Dirichlet wall truncating the integral.

Of course here the errors displayed on the lattice data are statistical only and must be augmented by an uncertainty due to scaling from our fixed lattice spacing to the continuum and one related to the lack of light-quark loops within the quenched approximation. This is the first demonstration of this method, such controlled studies will doubtless follow now that efficacy has been demonstrated.

5. Summary

Several new techniques have emerged that much expand the range of charmonium quantities that can be considered in lattice QCD. Initial studies with quenched lattices are clearly systematics dominated, but this can be expected to be improved in the near

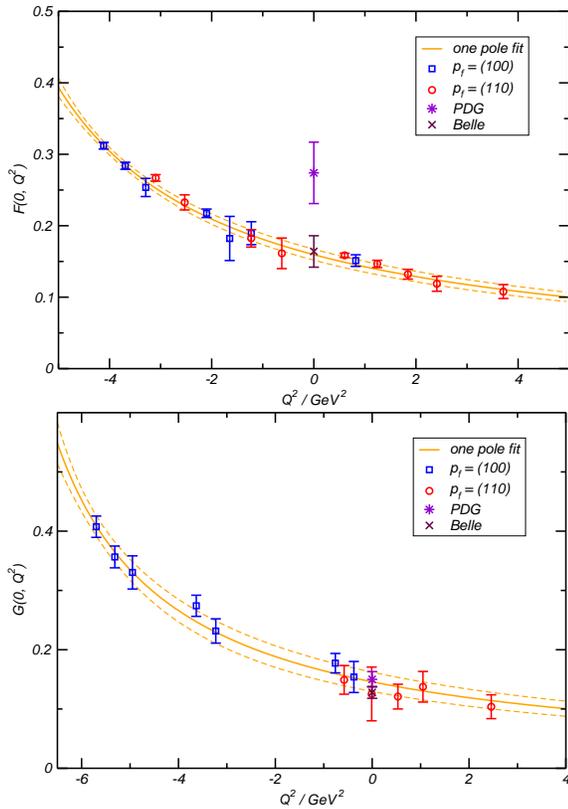


Figure 10: (a) $\eta_c \rightarrow \gamma\gamma^*$ amplitude. (b) $\chi_{c0} \rightarrow \gamma\gamma^*$ amplitude. Fits are one-pole forms as described in Dudek and Edwards [2006].

future by use of dynamical lattices, in particular the anisotropic dynamical lattices being generated under USQCD at Jefferson Lab. These same methods applied to the light quark sector will provide invaluable information for future meson spectroscopy projects like GlueX.

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