

Extraction of α_s and m_Q from Onia

N. Brambilla

Dipartimento di Fisica dell' Università di Milano and INFN, via Celoria 16, 20133 Milano, Italy

We briefly review how precise determinations of the strong coupling constant and of the heavy quark masses may be obtained from heavy quarkonium. Such determinations are competitive with heavy quark masses extraction from other systems and give an accurate value for the strong coupling constant at a relatively low energy scale. In particular we report about a recent determination of α_s from $\Gamma(\Upsilon(1S) \rightarrow \gamma X)/\Gamma(\Upsilon(1S) \rightarrow X)$ with CLEO data which includes color octet contributions and avoids model dependence in the extraction. The obtained value is $\alpha_s(M_{\Upsilon(1S)}) = 0.184_{-0.014}^{+0.015}$, which corresponds to $\alpha_s(M_Z) = 0.119_{-0.005}^{+0.006}$. Future prospects for more precise extractions of the heavy quark masses and α_s are discussed.

I. QCD AND THE ONIA

QCD is the theory of the strong interactions: we should be able to predict all the properties of hadrons starting from the QCD Lagrangian which is a function only of the coupling constant α_s and of the quark masses m . Therefore, from the theoretical predictions of physical observables and the corresponding experimental measurements, we should be able to extract the values of the coupling constant α_s and of the quark masses m . However, everything is complicated by the fact that QCD is a strongly coupled theory in the low energy region. At the scale Λ_{QCD} nonperturbative effects become dominant and α_s becomes large. The nonperturbative QCD dynamics originates the confinement of quarks that in turn is the reason for which the quark mass loses its most intuitive definition. Quarks are confined inside hadrons and thus we cannot directly measure their masses. The mass of the quark is a parameter defined in some renormalisation scheme at some renormalisation scale. Systems made by two heavy quarks, quarkonia in the following-, are characterized by a quark mass scale m_Q which is large, bigger than Λ_{QCD} . Then $\alpha_s(m_Q)$ is small and perturbative expansions may be performed at this scale. This introduces a great simplification and hints at a factorization between high and low energy contributions for quarkonia. For these systems, however, things are even more interesting [1]. They are nonrelativistic systems characterized by another small parameter, the heavy-quark velocity v , and by a hierarchy of energy scales: m_Q (hard), the relative momentum $p \sim m_Q v$ (soft), and the binding energy $E \sim m_Q v^2$ (ultrasoft). For energy scales close to Λ_{QCD} , perturbation theory breaks down and one has to rely on nonperturbative methods. Regardless of this, the nonrelativistic hierarchy $m_Q \gg m_Q v \gg m_Q v^2$ will persist also below the Λ_{QCD} threshold. While the hard scale is always larger than Λ_{QCD} , different situations may arise for the other two scales depending on the considered quarkonium system. The soft scale, proportional to the inverse radius r , may be a perturbative ($\gg \Lambda_{\text{QCD}}$) or a nonperturbative scale ($\sim \Lambda_{\text{QCD}}$) depending on the physical system. Finally, only for $t\bar{t}$

threshold states the ultrasoft scale may still be perturbative. Heavy quark-antiquark states probe confinement and nonperturbative physics [2] at different scales and are thus an ideal and to some extent unique laboratory where our understanding of nonperturbative QCD, its interplay with perturbative QCD and the behaviour of the perturbative bound state series may be tested and understood in a controlled framework. In particular in some regimes nonperturbative effects will appear in the form of local or nonlocal gluon condensates and will be suppressed in the computation of physical observables. In this framework quarkonia become very appropriate systems to be used for the study of the transition region from high to low energy, for information on the QCD vacuum structure and for precision determinations of the QCD parameters. Precisely this last point is the subject of this paper. In the next Sections we will discuss the systematic framework offered by Non Relativistic Effective Field Theories (NR EFT) [3] for the description of quarkonia and how one can take advantage of the accurate EFT calculations to make precise determinations of the QCD parameters. For some reviews of NR EFTs see [1, 3, 4, 5, 6]

II. EFFECTIVE FIELD THEORIES

It is possible to take advantage from the existence of a hierarchy of scales in quarkonia to introduce NR EFTs, which are simpler but equivalent to QCD. A hierarchy of EFTs may be constructed by systematically integrating out modes associated to high energy scales not relevant for the quarkonium system. Such integration is made in a matching procedure that enforces the complete equivalence between QCD and the EFT at a given order of the expansion in v ($v^2 \sim 0.1$ for $b\bar{b}$, $v^2 \sim 0.3$ for $c\bar{c}$, $v \sim 0.1$ for $t\bar{t}$). The EFT realizes a factorization at the Lagrangian level between the high energy contributions carried by matching coefficients and the low energy contributions carried by the dynamical degrees of freedom. The Poincaré symmetry remains intact in a nonlinear realization at the level of the NR EFT imposing exact relations among

the EFT matching coefficients [7, 10].

By integrating out the hard modes one obtains Non-relativistic QCD [8, 9, 10]. NRQCD is making explicit at the Lagrangian level the expansions in mv/m and mv^2/m . It is similar to HQET, but with a different power counting. It also accounts for contact interactions between quarks and antiquark pairs (e.g. in decay processes) and hence has a wider set of operators. In NRQCD soft and ultrasoft scales are left dynamical and their mixing may complicate calculations, power counting and the consideration of the nonperturbative effects. In the last few years the problem of systematically treating the remaining dynamical scales in an EFT framework has been addressed by several groups [11, 12, 13] and has now reached a good level of understanding. Therefore one can go down one step further and integrate out also the soft scale in a matching procedure to the lowest energy and simplest EFT that can be introduced for quarkonia, where only ultrasoft degrees of freedom remain dynamical. We call such EFT potential NonRelativistic QCD (pNRQCD) [12, 13] (an alternative EFT is in [14]). pNRQCD is making explicit at the Lagrangian level the expansion in mv^2/mv . This EFT is close to a Schrödinger-like description of the bound state and hence as simple. The bulk of the interaction is carried by potential-like terms, but non-potential interactions, associated with the propagation of low-energy degrees of freedom ($Q\bar{Q}$ colour singlets, $Q\bar{Q}$ colour octets and low energy gluons), are generally present. They start to contribute at NLO in the multipole expansion of the gluon fields and are typically related to nonperturbative effects [13, 16].

In this EFT frame, it is important to establish when Λ_{QCD} sets in, i.e. when we have to resort to non-perturbative methods. For low-lying resonances, it is reasonable to assume $mv^2 \gtrsim \Lambda_{\text{QCD}}$. The system is weakly coupled and we may rely on perturbation theory, for instance, to calculate the potential. In this case, we deal with weak coupling pNRQCD. The theoretical challenge here is performing higher-order calculations and the goal is precision physics. This is the case that we will consider in this paper.

A. The QCD potential and the Static Energy

The masses may be extracted from a calculation of the energy levels and to obtain the energy levels we need the potential. The $Q\bar{Q}$ potential is a Wilson coefficient of pNRQCD [17] obtained by integrating out all degrees of freedom but the ultrasoft ones. It is given by a series of contributions in an expansion in the inverse of the mass of the quark. If the quarkonium system is small, the soft scale is perturbative and the potentials can be entirely calculated in perturbation theory [3]. As matching coefficients the potentials undergo renormalization, develop

a scale dependence and satisfy renormalization group equations, which eventually allow to resum potentially large logarithms [15]. The static singlet potential (the contribution at zero order in the mass expansion) is known at three loops apart from the constant term [17, 18, 25]. The first log related to ultrasoft effects arises at three loops. Such logarithm contribution at N³LO and the single logarithm contribution at N⁴LO may be extracted respectively from a one-loop and two-loop calculation in the EFT and have been calculated in [17, 19]. The static energy given by the sum of a constant, the static potential and the ultrasoft corrections is free from renormalon ambiguities. By comparing it (at the NNLL) with lattice calculations one sees that the QCD perturbative series converges very nicely to and agrees with the lattice result [26] in the short range and that no nonperturbative linear (“stringy”) contribution to the static potential exist [3, 20]. This is an example of how precise calculations may be performed in this framework. Once the renormalon contribution has been cancelled, in this case between the static potential and the pole mass [13, 21, 22], we are left with a well behaved perturbative series and we can unambiguously define power corrections. It is possible to make predictions of physical quantities (in this case the $Q\bar{Q}$ static energy) at high order in the perturbative expansion and with a small error (including nonperturbative corrections which are suppressed in the power counting) and to make a connection with the lattice results. It is remarkable that the dependence on the lattice spacing can be predicted in perturbation theory.

B. The QCD perturbative series of the $Q\bar{Q}$ energies and the nonperturbative contributions

In weak coupling pNRQCD the soft scale is perturbative and the potentials are purely perturbative objects. Nonperturbative effects enter energy levels and decay calculations in the form of local or nonlocal electric and magnetic condensates [23, 24]. We still lack a precise and systematic knowledge of such non-perturbative purely glue dependent objects. It would be important to have for them lattice determinations or data extraction (see e.g. [27]) The leading electric and magnetic nonlocal correlators may be related to the gluelump masses [13] and to some existing lattice (quenched) determinations [3, 28].

However, since the nonperturbative contributions are suppressed in the power counting it is possible to obtain good determinations of the masses of the lowest quarkonium resonances with purely perturbative calculations in the cases in which the perturbative series is convergent (after that the appropriate subtractions of renormalons have been performed) and large logarithms are resummed. In this framework power corrections are unambiguously defined.

III. m_c AND m_b EXTRACTION

The lowest heavy quarkonium states are suitable systems to obtain a precise determination of the mass of the heavy quarks b and c . Perturbative determinations of the $\Upsilon(1S)$ and J/ψ masses have been used to extract the b and c masses. These determinations are competitive with those coming from different systems and different approaches (for the b mass see e.g. [35]).

Determinations of the quark masses from the perturbative calculation of Υ and J/ψ $1S$ masses differ for the order of the perturbative calculation considered, for the order of the resummation of the logarithms in v and the way in which nonperturbative corrections are taken into account. Higher order terms and the residual scale dependence of the result give the theoretical error on the mass. The main uncertainty in these determinations comes from nonperturbative contributions (local and nonlocal gluon condensates) together with possible effects due to subleading renormalons. We report some example of such determinations in Tab. 1.

reference	order	$\overline{m}_b(\overline{m}_b)$ (GeV)
[36]	NNNLO*	$4.210 \pm 0.090 \pm 0.025$
[30]	NNLO +charm	$4.190 \pm 0.020 \pm 0.025$
[38]	NNLO	4.24 ± 0.10
[37]	NNNLO*	4.346 ± 0.070
[39]	NNNLO*	4.20 ± 0.04
[40]	NNNLO*	4.241 ± 0.070
[41]	NNLL*	4.19 ± 0.06
reference	order	$\overline{m}_c(\overline{m}_c)$ (GeV)
[30]	NNLO	1.24 ± 0.020
[38]	NNLO	1.19 ± 0.11

TABLE I: Different recent determinations of $\overline{m}_b(\overline{m}_b)$ and $\overline{m}_c(\overline{m}_c)$ in the $\overline{\text{MS}}$ scheme from the bottomonium and the charmonium systems. The displayed results use either a direct calculation of the lowest energy level in perturbation theory or non-relativistic sum rules. The * indicates that the theoretical input is only partially complete at that order. For the detailed discussion about how the error has been computed see the original references, for a review see [3].

Once the quark masses have been obtained, the renormalon subtraction and the same calculational approach have been exploited also to obtain the energy levels of the lowest resonances. In [29] a prediction of the B_c mass has been obtained. The NNLO calculation with finite charm mass effects [30] predicts a mass of 6307(17) MeV that well matches the CDF measurement [31] and the lattice determination [32]. The same procedure seems to work at NNLO even for higher states (inside the theory errors that grow) [30]. Including logs resummation at NLL, it

is possible to obtain a prediction for the mass of $\eta_b = 9421 \pm 11(\text{th})_{-8}^{+9}(\delta\alpha_s)$ MeV (where the second error comes from the uncertainty in α_s) and for the B_c hyperfine separation $\Delta = 65 \pm 24_{-16}^{+19}$ MeV [33]. A NLO calculation reproduces in part the $1P$ fine splitting [34].

A compilation of values of the b and c mass has been presented by the Quarkonium Working Group in Chapter 6 of [1] and is reported in Figures 1 and 2. The mass determinations presented in such Figures include (relativistic and nonrelativistic) sum rule results, lattice QCD results, semileptonic B decays as well as $\Upsilon(1S)$ and J/ψ $1S$ determinations. One can see that the determinations from quarkonium are competitive with respect to determinations coming from other systems (heavy-light, B decays). The original works to which the results in such Figures refer are explicitly given and discussed in [1]. We refer to [1] also for an extended review of the different mass schemes, the different heavy quark mass extractions approaches and the renormalon subtraction.

From these determinations the QWG reported the following values for the $\overline{\text{MS}}$ masses:

$$\begin{aligned}\overline{m}_b(\overline{m}_b) &= 4.22 \pm 0.05 \text{ GeV} \\ \overline{m}_c(\overline{m}_c) &= 1.28 \pm 0.05 \text{ GeV}.\end{aligned}$$

which are displayed by the darker gray area in Figures 1 and 2 and the following ranges:

$$\begin{aligned}\overline{m}_b(\overline{m}_b) &= 4.12 - 4.32 \text{ GeV} \\ \overline{m}_c(\overline{m}_c) &= 1.18 - 1.38 \text{ GeV}\end{aligned}$$

corresponding to the lighter gray area in Figures 1 and 2. For the details of the calculation of these averages and ranges see [1].

We see that the QWG values for the b and c mass attribute to them an error of 1% and 4% respectively. This is a smaller error than the one given in the PDG [42].

More recent and more accurate mass determinations (from lattice unquenched calculation $\overline{m}_b(\overline{m}_b) = 4.4 \pm 0.030$ GeV [44]; from semileptonic B decays, $\overline{m}_c(\overline{m}_c) = 1.224 \pm 0.017 \pm 0.054$ GeV [43]; from low momentum sum rules $\overline{m}_b(\overline{m}_b) = 4.164 \pm 0.025$ GeV $\overline{m}_c(\overline{m}_c) = 1.286 \pm 0.013$ GeV [45], and a new preliminary calculation of the mass of the b in the the potential subtracted scheme with unquenched lattice Fermilab action [46]) would call for a new critical analysis and discussion of such extractions and errors and an updated mass compilation.

A. m_t from $t\bar{t}$ bar systems

In [41, 48] the total cross section for top quark pair production close to threshold in e^+e^- annihilation is

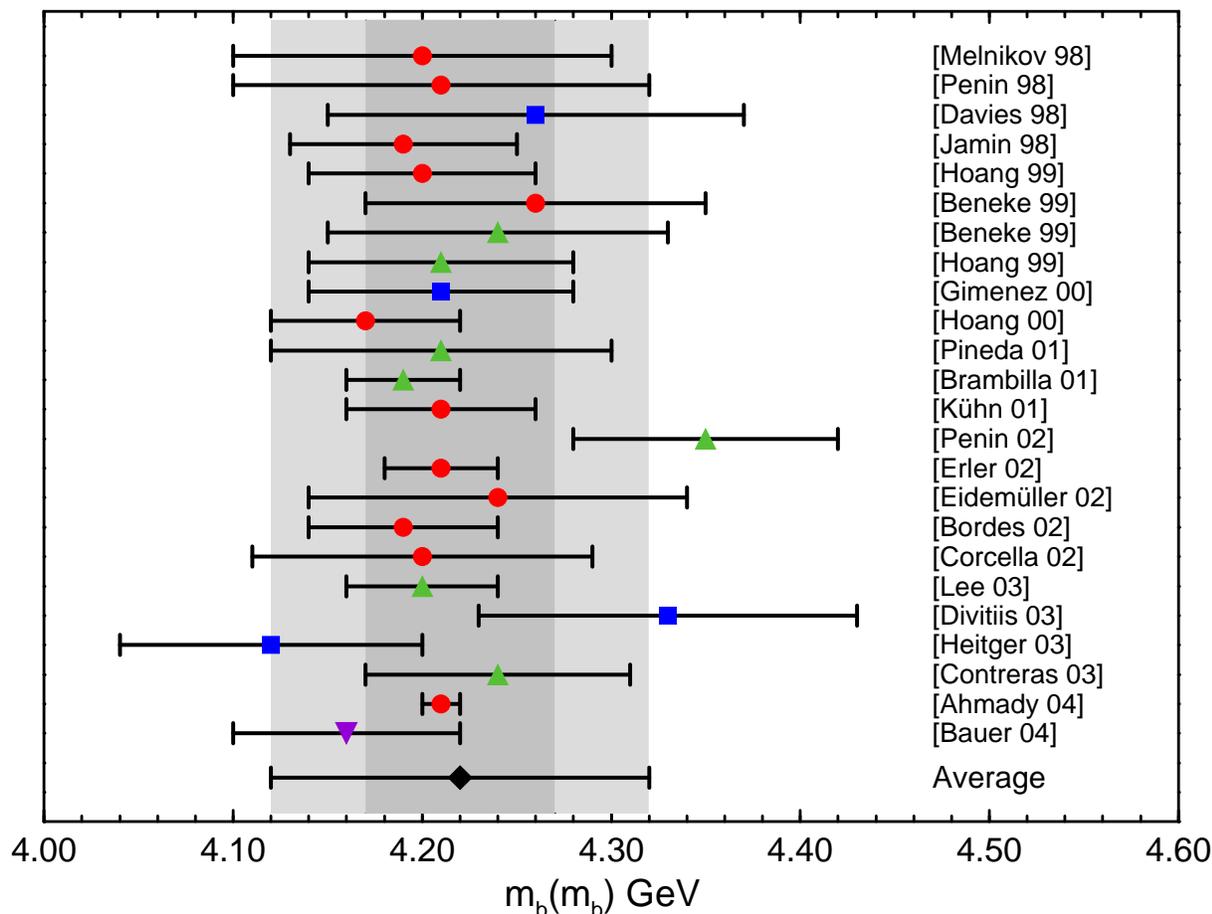


FIG. 1: Collection of recent bottom quark mass determinations. The circles represent sum rule results, the triangles Up-silon 1S determinations, the squares lattice QCD results and the upside down triangle a determination from semileptonic B decays. The full diamond gives the QWG global average for $\overline{m}_b(\overline{m}_b)$. The darker and lighter shaded areas represent the QWG error estimates corresponding to a 1σ error and a range respectively. This Table is taken from Chapter 6, pag. 360 of [1]. For a detailed discussion and the explicit references to the original works see [1].

investigated at NNLL in the weakly coupled EFT. The summation of the large logarithms in the ratio of the energy scales significantly reduces the scale dependence. Studies like these will make feasible a precise extractions of the strong coupling, the top mass and the top width at a future ILC. The present theoretical uncertainties for top mass extraction at the ILC is about 100 MeV [3, 47].

IV. α_s EXTRACTION FROM QUARKONIA

The summary of values of $\alpha_s(M_Z)$ from various processes as reported by the PDG 2006 [42] is given in Fig. 3. We see that the value of α_s as determined from quarkonium is considerably smaller than the other determinations. The effect is seen also in Fig. 4 where the values of $\alpha_s(\mu)$ are reported at the values of μ where they are measured. The determination of α_s from Υ decays is one of the few ones at a relatively low

energy with a relatively small error. It follows from theory calculations of ratio of hadronic and leptonic Υ decays [57] and use of sum rules for the Υ system [58, 59], the smaller error being obtained in the first case. Here we will report about a determination for α_s from the Υ decays [62] that has recently solved this inconsistency.

Heavy quarkonium leptonic and non-leptonic inclusive decay rates have historically provided ways to extract α_s and served as additional confirmation of the validity of QCD. Ratios of these quantities are very sensitive to α_s if the data are sufficiently precise. In particular, today the inclusive decay widths of J/ψ , $\psi(2S)$ and $\Upsilon(1S)$ are known with a few percent error, the ones of $\Upsilon(2S)$, $\Upsilon(3S)$ with a 10% error and most of the other inclusive decays are known with an error of 15-20%. In the last few years the error on charmonium P-wave inclusive decays have been reduced to half [42]. On the theory side NRQCD [9] and pNRQCD [49] have provided powerful factorization for-

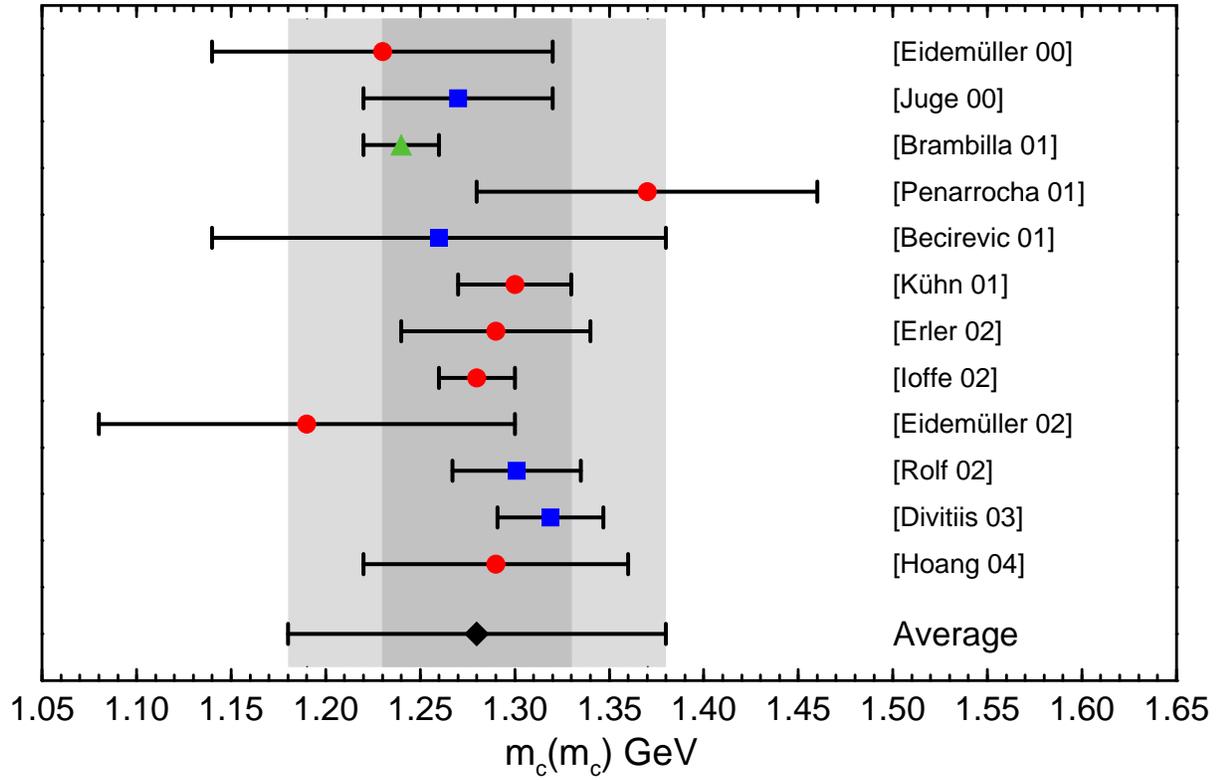


FIG. 2: Collection of recent charm quark mass determinations. The circles represent sum rule results, the triangles J/ψ 1S determinations and the squares quenched lattice QCD results. The full diamond gives the QWG global average for $\overline{m}_c(\overline{m}_c)$. The darker and lighter shaded areas represent the QWG error estimates corresponding to a 1σ error and a range respectively. This Table is taken from Chapter 6, pag. 363 of [1]. For a detailed discussion and the explicit references to the original works see [1].

mulas for the inclusive decays.

S and P wave quarkonium inclusive decays are today known in the NRQCD factorization up to order v^7 in the relativistic expansion [9, 50, 64] and at different orders in the perturbative expansion of the matching coefficients (see e.g. [65] for a review). In pNRQCD the nonperturbative matrix elements of the four quark operators on the quarkonium states can be further decomposed in the product of quarkonium wave functions (or derivatives of quarkonium wave functions) in the origin and glue dependent operators, with a substantial reduction in the number of nonperturbative (and unknown) contributions [49]. A lattice calculation of such nonlocal gluonic correlators is however still missing.

Thanks to the EFTs factorization between high energy contributions, calculable in QCD perturbation theory, and low energy nonperturbative contributions, it is possible to consider appropriate ratios of inclusive decays at some order of the expansion in α_s and in v . In particular, the ratio $\Gamma(H \rightarrow \gamma gg)/\Gamma(H \rightarrow ggg)$ (H being a quarkonium state) appears particularly promising for the extraction of α_s [51, 52, 63]. since both the wave function at the origin and the relativistic corrections cancel out. However, the first mea-

surements of J/ψ and Υ inclusive radiative decays delivered a photon spectrum not compatible with the early QCD predictions. Inside the EFT approach it was understood that colour octet contributions, ignored in the early calculations, become very important in the upper end-point region of the spectrum [53]. By considering such octet contributions, using pNRQCD to calculate them and Soft Collinear Effective Theory (SCET) to resum end-points singularities, a good description of the photon spectrum has been achieved recently, at least for the $\Upsilon(1S)$ state [54]. These recent theoretical advances combined with new and more precise data from CLEO on $\Upsilon(1S)$ radiative decay [55], has made the ratio $R_\gamma \equiv \Gamma(\Upsilon(1S) \rightarrow \gamma X)/\Gamma(\Upsilon(1S) \rightarrow X)$, (X being hadrons) particularly suitable for the α_s extraction at the bottom mass scale. For the perturbative calculation of the matching coefficients appearing in such ratio see [9, 56]. Colour octet contributions also affect the ratio R_γ and are parametrically of the same order of the relativistic corrections. They have so far either been ignored [55] or estimated to be small [57] in the available extractions of α_s from this ratio. In [62], recent determinations of the $\Upsilon(1S)$ colour octet matrix elements both on the lattice [60] and in the contin-

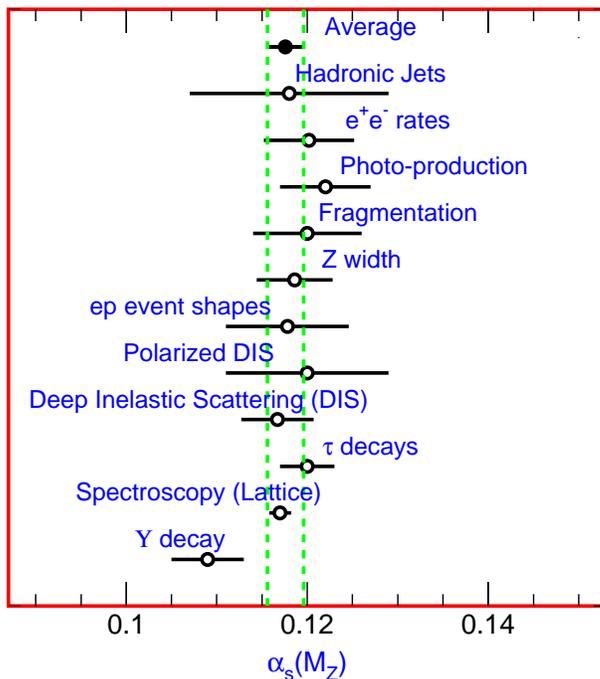


FIG. 3: Summary of values of $\alpha_s(M_Z)$ from various processes, taken from the PDG [42]. The value shown indicate the process and the measured value of α_s extrapolated to $\mu = M_Z$. The error shown is the total error including theoretical uncertainties. The PDG average coming from these measurements and quoted in the text is also shown. Notice that the value of α_s extracted from Υ decays is considerably lower than all the other determinations.

uum [61] have been taken into account. This, together with the good theoretical description [54] of the photon spectrum [55], allows for a consistent extraction of $\alpha_s(M_{\Upsilon(1S)})$ at NLO. The final result obtained in [62] is

$$\alpha_s(M_{\Upsilon(1S)}) = 0.184^{+0.014}_{-0.013}, \quad (1)$$

which corresponds to

$$\alpha_s(M_Z) = 0.119^{+0.006}_{-0.005}, \quad (2)$$

very close to the central value of the PDG [42] with competitive errors. The key ingredients to get these numbers have been the precise CLEO data [55], the use of a QCD calculation to extrapolate the photon spectrum at low z , and accurate estimates of the colour octet matrix elements, which have been possible thanks to recent lattice and continuum calculations. At present, the main uncertainty in the extraction of α_s comes from the systematic uncertainties in R_γ^{exp} .

The impact of this determination of α_s , if included in the world average, will be to increase it.

V. FUTURE PROSPECTS FOR MASS AND α_s EXTRACTIONS

The mass and α_s determinations from quarkonium that we have presented are already competitive with

the results obtained from other physical systems.

In the near future $\alpha_s(m_c)$ may be extracted from the R_γ ratio for the J/ψ provided that a new measurement of the inclusive photon spectrum for radiative J/ψ decays will be performed at BESIII [66]. In a similar way, the discovery and the measurement of the η_b mass with a few MeV accuracy will provide a determination of $\alpha_s(M_Z)$ with 3 per mille error from the hyperfine separation calculated at NLL [33].

For an improved determination of α_s from the lattice calculations of the quarkonium spectrum, we need a nonperturbative unquenched determination of $\Lambda_{\overline{\text{MS}}}$ and results on the spectrum obtained with different formulations of sea quarks, besides staggered quarks. Also the improvement in the lattice extraction of the masses would require an improved accuracy in the conversion from the bare lattice mass to the $\overline{\text{MS}}$ mass. In particular the two loop matching in such conversion would be needed for the Fermilab and the NRQCD actions. A nonperturbative matching would also be desirable.

For what concerns the mass extraction from the $\Upsilon(1S)$ and J/ψ masses in perturbation theory at present, as it has been discussed, the major theoretical error comes from our ignorance of the ultra-soft nonperturbative corrections. A lattice calculation of the nonperturbative chromoelectric correlator together with its matching from lattice to $\overline{\text{MS}}$ scheme

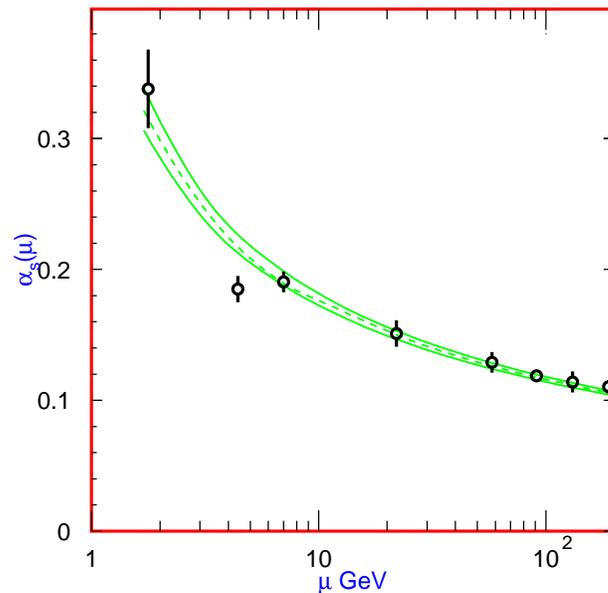


FIG. 4: Summary of values of $\alpha_s(\mu)$ at the values of μ where they have been measured, taken from the PDG [42]. The line shows the central value and the $\pm 1\sigma$ limits of the PDG average. The data are in increasing order of μ : τ width, Υ decays, deep inelastic scattering, e^+e^- event shapes at 22 GeV from the JADE data, shapes at TRISTAN at 58 GeV, Z width, and e^+e^- event shapes at 135 and 189 GeV. Notice how the determination from the Υ decays is the only one outside the band.

is needed.

Further improvements in the mass determinations from nonrelativistic sum rule would require the full NNLL calculation; a complete NNNLO computation would also be useful to have a better control on the theoretical uncertainties. For low momentum sum rules, improved determinations of the R measurements around bottomonium and charmonium region would be crucial.

We conclude noticing that, within the EFT approach and the factorization scheme, precision calculations in quarkonium may be applied to all the physical observables of the lowest resonances, spectra and decays included. To this respect it is particularly interesting the example of the calculation of M1 transitions for the lowest quarkonia resonances. In this case

the Poincarè invariance of the EFT imposes exact relations among matching coefficients that set to zero the nonperturbative corrections at order v^2 . At this order the M1 transitions may be exactly calculated in perturbation theory [67].

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