Measurement of the CKM angle $\phi_2(\alpha)$*

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We present recent measurements of the unitarity triangle angle ϕ_2 (α) using $B \to \pi\pi$, $B \to \rho\rho$, and $B \to \rho\pi$ decays. The measurements are based on data samples collected with the Belle and BaBar detectors at the KEKB and PEP-II e^+e^- colliders, respectively. We also report on a new measurement of a CP-violating asymmetry in $B^0 \to a_1^\pm \pi^\mp$ decay which will allow to constrain further the angle ϕ_2.

I. INTRODUCTION

The CP violation in the standard model (SM) can be described by the presence of a complex phase in the three-generation Cabibbo-Kobayashi-Maskawa [1] (CKM) quark-mixing matrix. Unitarity constraints on the matrix elements lead to six relations, one of the most interesting is $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$. This relation can be depicted as a triangle in the complex plane as shown in Fig 1. Checking unitarity of the CKM matrix implies measuring sides and angles of the triangle. This provides an important test of the SM. The phase angle ϕ_2 [2], defined as $\arg[-(V_{td}V_{tb}^*)/(V_{ud}V_{ub}^*)]$, represents the phase difference between V_{td} and -V_{ub}^*. It can be determined by measuring a time-dependent CP asymmetry in charmless $b \to u\overline{u}d$ decays such as $B^0 \to \pi^+\pi^-$, $\pi^+\pi^-\pi^0$, $\rho^+\rho^-$, and $a_1^\pm\rho^\mp$ [3]. The decay-rate asymmetry in these decays can arise due to the interference between the amplitudes of the direct decay of B and decay after $B\overline{B}$ mixing. The B decays are proceeded mainly through a tree and gluonic penguin loop diagrams as shown in Fig 2. The penguin loop amplitude is irrelevant to the ϕ_2 and contaminates the measurement. The penguin contribution can be constrained by using isospin relations or employing SU(3) flavor relations, which will be discussed later.

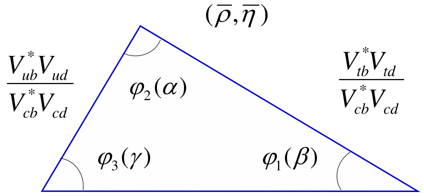

FIG. 1: The CKM unitarity triangle $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ in the complex $\overline{\rho} - \overline{\eta}$ plane.

The first analysis of time-dependent CP-violating asymmetries has been performed using $B^0 \to \pi^+\pi^-$ decays. This relatively clean channel provides hight-precision measurements but has a significant contribution from a penguin loop amplitude. This is indicated by observation of a large direct CP violation by Belle and by measurement of a relatively large branching fraction of $B^0 \to \pi^0\pi^0$ decay. The measurements of CP asymmetries in $B^0 \to \pi^+\pi^-$ are followed by that in $B^0 \to \rho^+\rho^-$ decays. The contamination from a $b \to u$ penguin amplitude is measured to be much smaller here. However, a large width of ρ mesons makes reconstruction of $B^0 \to \rho^+\rho^-$ decays a challenging task. The extraction of ϕ_2 from measurements in $B^0 \to \pi^+\pi^-$ and $B^0 \to \rho^+\rho^-$ decays can be performed using an isospin analysis which allows one to constrain the contribution from the penguin amplitude generally with an eight-fold ambiguity. The latest measurements in $B^0 \to \pi^+\pi^-$ and $B^0 \to \rho^+\rho^-$ decays are discussed in Section III and IV.

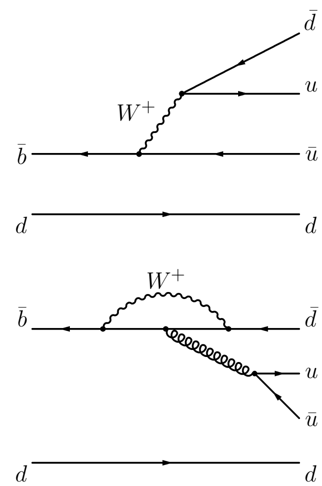

FIG. 2: Tree-level (top) and gluonic penguin (bottom) diagrams for the decays $B^0 \to \pi^+\pi^-$, $\rho^+\rho^-$, $\rho^\pm\pi^\mp$, $a_1^\pm\pi^\mp$.

Snyder and Quinn [4] showed that the angle ϕ_2 can be determined without discrete ambiguities using a time-dependent Dalitz plot (DP) analysis of $B^0 \to \pi^+\pi^-\pi^0$ decays. The DP analysis allows one to measure the complex amplitudes of $\pi^+\pi^-\pi^0$ decays which are related to ϕ_2 via an isospin relation. Previously a quasi-two-body analysis of $B^0 \to \rho^+\pi^-$ decays has been used to constrain ϕ_2 using this decay mode. Recent results on the DP analyses are presented in Section V.

$B^0 \to a_1^\pm\pi^\mp$ is another channel which allows us to extract the angle ϕ_2. Recently BaBar has measured

*University of Cincinnati preprint number UCHEP-07-07.

a CP-violating asymmetry in this decays. However the extraction of ϕ_2 has not yet been performed for this channel. It is expected to be done using a flavor SU(3) symmetry. The measurements in $B^0 \to a_1^\pm \pi^\mp$ decays are described in Section VI.

II. ANALYSES OVERVIEW

Analyses of $B \to \pi\pi$, $B \to \rho\pi$, $B \to \rho\rho$, and $B \to a_1\pi$ decays have several common features.

Charged pions are identified using information from time-of-flight counters and the central tracking chambers. Most analyses also use information from aerogel threshold cherenkov counters (Belle) and from a DIRC detector (BaBar). B decays are selected by combining charged pions originating from the interaction region, and adding π^0's in the decays which involve $\rho^\pm$ mesons.

B decays are identified using two kinematic variables: the beam-energy-constrained mass $M_{\rm bc} \equiv \sqrt{E^2_{\rm beam} - p^2_B}$ and energy difference $\Delta E \equiv E_B - E_{\rm beam}$, where $E_{\rm beam}$ is the beam energy, and E_B and p_B are the energy and momentum of the reconstructed B candidate, all evaluated in the center-of-mass (CM) frame.

Flavor of the B meson accompanying the signal B is identified using a tagging algorithm which identifies its decay products (mainly leptons and kaons). The tagging algorithm provides the flavor of the tagged meson and a tagging quality.

The dominant background originates from $e^+e^- \to q\bar{q}$ $(q = u, d, s, c)$ continuum events. The separation of continuum background from the signal events can be done using the event topology: $q\bar{q}$ events tend to be more jet-like while $B\overline{B}$ events are more spherical in the CM frame.

The Belle analyses use event-shape variables, specifically, 16 modified Fox-Wolfram moments [5] combined into a Fisher discriminant [6]. We form signal and background likelihood functions $\mathcal{L}_s$ and $\mathcal{L}_{BG}$ by multiplying a probability density function (PDF) for the Fisher discriminant by a PDF for $\cos\theta_B$, where θ_B is the polar angle in the CM frame between the B direction and the beam axis. The PDFs for signal and $q\bar{q}$ are obtained from Monte Carlo (MC) simulations and the data sideband, respectively. We calculate the ratio $\mathcal{R} = \mathcal{L}_s/(\mathcal{L}_s + \mathcal{L}_{BG})$. The continuum background is reduced by setting a threshold on $\mathcal{R}$. In $B^0 \to \rho^+\rho^-$ analysis PDFs for $\mathcal{R}$ are included into a likelihood function. In some analyses we also set a threshold on the absolute value of the cosine of the angle between a thrust axis of the reconstructed signal B meson and that of the rest of the event, $\cos_{\rm th}$.

In the BaBar analyses the $q\bar{q}$ background is usually removed by directly applying thresholds on $\cos_{\rm th}$ and on the second-to-zeroth Fox-Wolfram momentum R_2. Further discrimination is achieved by using an artificial neural network which includes event shape variables. The neural net is trained using MC simulated events and off-peak data. The output of the net, $\mathcal{N}$ is included as a PDF into the fitting likelihoods.

The decay time difference Δt between the two B mesons can be determined by measuring the distance between the decay vertices of these mesons. Since the B^0 and $\overline{B}^0$ are produced approximately at rest in the $\Upsilon(4S)$ CM system, $\Delta t \simeq \Delta z/\beta\gamma c$, where c is the speed of light and $\beta\gamma$ is a Lorentz boost of the $\Upsilon(4S)$ mesons and equals 0.425 (0.559) for Belle (BaBar). The decay vertices of the signal and tag-side B mesons are reconstructed by fitting charged tracks that have hits in the silicon vertex detector using an interaction point constraint.

III. $B^0 \to \pi^+\pi^-$

The time-dependent rate for $B \to \pi^+\pi^-$ decays tagged with $B^0(Q = +1)$ and $\overline{B}^0$ $(Q = -1)$ mesons is given by

$$\mathcal{P}_{\pi\pi}(\Delta t) = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}}\{1 + Q[\mathcal{A}\cos(\Delta m \Delta t) + \mathcal{S}\sin(\Delta m \Delta t)]\}, \quad (1)$$

where τ_{B^0} is the B^0 lifetime, Δm is the mass difference between the two B^0 mass eigenstates, Δt is the proper-time difference between the two B decays in the event, and $\mathcal{A}$ [7] and $\mathcal{S}$ are CP asymmetry coefficients which are to be obtained from a fit to the data. If the decay amplitude is dominated by a tree diagram, $\mathcal{S} = \sin(2\phi_2)$ and $\mathcal{A} = 0$. The presence of an amplitude with a different weak phase (such as from a gluonic penguin diagram) gives rise to direct CP violation and shifts $\mathcal{S}$ from $\sin(2\phi_2)$:

$$\mathcal{S}_{\rm meas} = \sqrt{1 - \mathcal{A}^2_{\rm meas}}\sin 2\phi_2^{\rm eff}, \quad (2)$$

where $\mathcal{A}, \mathcal{S}_{\rm meas}$ are measured coefficients, $\phi_2^{\rm eff} = \phi_2 + \delta\phi_2$, and $\delta\phi_2$ is the phase shift.

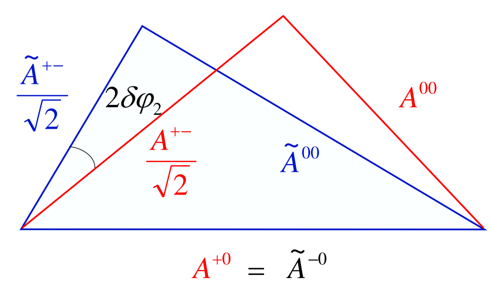

FIG. 3: The isospin triangles for $B \to \pi\pi$ decays. The amplitudes $\widetilde{A}$ are defined as $\widetilde{A} = e^{i2\phi_3}\bar{A}$.

The Belle analysis [8] is based on a data sample consisting of 535 million $B\overline{B}$ pairs. The analysis is organized in two steps. We first determine the yields of signal and background components using an unbinned extended maximum likelihood (ML) fit to M_{bc}, ΔE, and the kaon identification probability $x_{\pm}$ for the positively and negatively charged tracks. The fit yields 1464 ± 65 $\pi^+\pi^-$ candidates. We subsequently perform a fit to the Δt distribution for the CP parameters $\mathcal{A}$ and $\mathcal{S}$. The fit to 16831 events yields $\mathcal{A}_{\pi^+\pi^-} = 0.55 \pm 0.08(stat) \pm 0.05(syst)$ and $\mathcal{S}_{\pi^+\pi^-} = -0.61 \pm 0.10(stat) \pm 0.04(syst)$.

The BaBar measured the CP-violating parameters using a sample of 383 million $B\overline{B}$ events [9]. The CP-violating parameters are obtained from an unbinned extended ML fit to 309540 events. The likelihood function contains 117 parameters which are varied in the fit. In order to enrich the data sample with the signal events, the event selection requirements are lowered in the BaBar analysis and additional PDF's for background discriminating variables (six in all) are included in the likelihood function. The fit results are $\mathcal{C}_{\pi^+\pi^-} = -0.21 \pm 0.09(stat) \pm 0.02(syst)$ and $\mathcal{S}_{\pi^+\pi^-} = -0.60 \pm 0.11(stat) \pm 0.03(syst)$.

Both Belle and BaBar measurements indicate a large mixing-induces CP-violation with a significance greater than 5.3σ and 5.1σ, respectively, for any values of $\mathcal{A}_{\pi^+\pi^-}$. Belle also observed large direct CP violation. The case of no direct CP violation, $\mathcal{A}_{\pi^+\pi^-} = 0$, is ruled out with a significance of 5.5σ. The difference between $\mathcal{A}, \mathcal{S}$ measurements of Belle and BaBar, as estimated by Heavy Flavor Averaging Group (HFAG) [10] group, constitutes about 2.1σ.

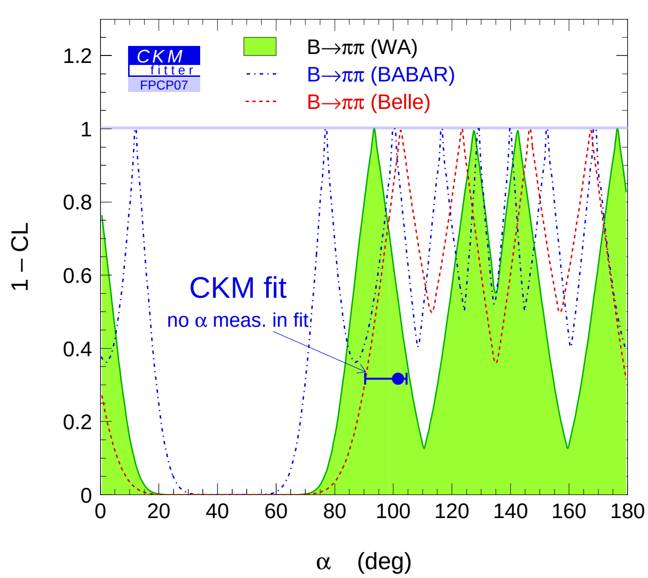

FIG. 4: 1 - C.L. vs $\phi_2(\alpha)$ obtained from the isospin analysis of $B \to \pi\pi$ decays by the CKMfitter group. The dashed curve represents Belle measurements only; the dot-dashed curve represents BaBar measurements and the hatched region is a combined constraint from Belle and BaBar.

The angle ϕ_2 can be extracted using an isospin relations [11]. The $SU(2)$ isospin symmetry allows one to relate the amplitudes A^{+-}, A^{+0}, and A^{00} of the $B \to \pi^+\pi^-, \pi^+\pi^0, \pi^0\pi^0$ decays and corresponding amplitudes $\bar{A}$ for the charge-conjugated processes as follows

$$\frac{1}{\sqrt{2}}A^{+-} + A^{00} = A^{+0}$$
$$\frac{1}{\sqrt{2}}\bar{A}^{+-} + \bar{A}^{00} = \bar{A}^{-0}. \quad (3)$$

These relations lead to two triangles in the complex plane as shown in Fig 3. The angle between these triangles is the phase difference $2\delta\phi_2$. There are six unknown parameters (five amplitudes, and the angle ϕ_2) and six observables: the branching fractions for $B \to \pi^+\pi^-$, $\pi^+\pi^0$, and $\pi^0\pi^0$, the CP parameters $\mathcal{A}_{\pi^+\pi^-}$ and $\mathcal{S}_{\pi^+\pi^-}$ and the time integrated asymmetry$\mathcal{A}_{\pi^0\pi^0}$ for $B^0 \to \pi^0\pi^0$ decays. The angle ϕ_2 can be determined with an eight-fold ambiguity corresponding to four possible orientations of the isospin triangles and the two fold ambiguity from solving Eq. 2. The confidence level (C.L.) for ϕ_2 obtained by the CKMfitter group [12] is presented in Fig. 4. The curves on this plot correspond to the Belle (BaBar) measurements only and to the combined Belle and BaBar constraint. The combined constraint is obtained using averaged measurements of the branching fractions and asymmetries. The eight peaks in the 1-C.L. distribution for BaBar (dot-dashed curve) correspond to the eight-fold ambiguity. However the Belle constraint contains only 4 peaks. This can be explained by the large value of the measured asymmetry $\mathcal{A}_{\pi^+\pi^-} = 0$; one of the isospin triangle becomes flat, an apex of the triangle lies on the base. This leads to merging of the two solutions and to the four-fold ambiguity on ϕ_2. The combined Belle and BaBar constraint on ϕ_2 consistent with the standard model is $93.5^{+12.1}_{-10.0}$ at 68% C.L.

IV. $B^0 \to \rho^+\rho^-$

As it was discussed in previous section, $B \to \pi\pi$ decays constrain ϕ_2 with a four-fold ambiguity. In order to choose a single solution and further constrain the angle ϕ_2 one can exploit other decay channels. In particular, ϕ_2 can be determined by measuring CP-violating parameters in $B^0 \to \rho^+\rho^-$ decays. $B^0 \to \rho^+\rho^-$ channel have several advantages comparing with $B^0 \to \pi^+\pi^-$:

- The branching fraction for $B^0 \to \rho^+\rho^-$ decays is about 4.4 times larger than that for $B^0 \to \pi^+\pi^-$.
- The size of the penguin amplitude in $B^0 \to \rho^+\rho^-$ decays is constrained to be small with respect to the leading tree diagram by the small branching fraction of $B^0 \to \rho^0\rho^0$ [13].

However there are certain complications in the measurement of ϕ_2:

- The relatively large width of ρ mesons ($\sim$ 150MeV) leads to a substantial combinatorial background in $\rho^+\rho^-$ decays.

- $\rho^+\rho^-$ is a vector-vector final state. The CP-violating parameters receive contributions from a longitudinally polarized state (CP-even) and two transversely polarized states (an admixture of CP-even and CP-odd states). However, recent measurements of the polarization fraction by Belle [14] and BaBar [15] show that the longitudinal polarization fraction is near unity ($f_L = 0.968 \pm 0.023$ [10]).

- The isospin analysis in $B \to \rho\rho$ decays can be complicated by a possible contribution of the isospin I = 1 amplitude which might appear due to the finite width of ρ mesons. Similar to $B^0 \to \pi^+\pi^-$ decays, electroweak penguin amplitudes can contribute to the $B^0 \to \rho^+\rho^-$ decays. However both these contributions are expected to be small [16].

Belle measured CP-violating parameters using 535 million $B\overline{B}$ pairs. Similar to $B^0 \to \pi^+\pi^-$ analysis, the measurements are done in two steps: we first obtain the yields of signal an background components using an unbinned extended ML fit to the three-dimensional $(M_{\rm bc}, \Delta E, \mathcal{R})$ distribution. A fit to 176843 events yields $N_{\rho\rho+\rho\pi\pi} = 576 \pm 53$. We subsequently perform a fit to the Δt distribution to determine the CP parameters $\mathcal{A}$ and $\mathcal{S}$. The time dependent decay rate for $B \to \rho^+\rho^-$ decays is given by Eq. 1. The likelihood function includes the following event categories: signal and $\rho\pi\pi$ non-resonant decays, signal decays that have at least one π meson incorrectly identified (referred to as SCF events), continuum background ($q\bar{q}$), $b \to c$ background, and charmless ($b \to u$) background. The fit results are $\mathcal{A} = 0.16 \pm 0.21 \pm 0.08$ and $\mathcal{S} = 0.19 \pm 0.30 \pm 0.08$.

BaBar analysis [15] is based on a data sample of 347 million $B\overline{B}$ pairs. The signal yield, f_L, and CP-violating parameters are obtained simultaneously from an unbinned extended ML fit to 33902 events. The background discriminating variables are m_{ES}, ΔE, Δt, $m_{\pi^\pm\pi^0}$, $\cos\theta_\pm$, and $\mathcal{N}$. PDFs for these variables are included in the likelihood function while the event selection requirements are lowered. The fit results are $N_{\rho\rho} = 615 \pm 57$, $f_L = 0.977 \pm 0.024^{+0.015}_{-0.013}$, $\mathcal{C} = -0.07 \pm 0.15 \pm 0.06$ and $\mathcal{S} = -0.19 \pm 0.21^{+0.05}_{-0.07}$. The CP-violating parameters measured by Belle are consistent with those obtained by BaBar. The values of $\mathcal{A}_{\rho^+\rho^-}$ and $\mathcal{S}_{\rho^+\rho^-}$ are also consistent with no CP violation ($\mathcal{A} = \mathcal{S} = 0$).

The angle ϕ_2 can be constrained using an isospin analysis similar to that used in $B \to \pi\pi$ decays. The six underlying parameters are: five decay amplitudes for $B \to \rho\rho$ and the angle ϕ_2. The observables are the branching fractions for $B \to \rho^+\rho^-$, $\rho^+\rho^0$ [10], and $\rho^0\rho^0$ [13]; the CP parameters $\mathcal{A}$ and $\mathcal{S}$; and the parameter $\mathcal{A}_{\rho^0\rho^0}$ for $B \to \rho^0\rho^0$ decays. The last parameter is not yet measured, but nevertheless one can constrain ϕ_2. We apply the isospin relations to the decay amplitudes corresponding to the longitudinal polarization (CP-even state). The branching fractions are multiplied by the corresponding longitudinal polarization fractions. We neglect possible contributions from electroweak penguins and $I = 1$ amplitudes [16] and possible interference between signal and non-resonant components. The resulting function 1−C.L. obtained by the CKMfitter group is shown in Fig. 5. The curves on this plot correspond to the Belle (BaBar) measurements only and to a combined constraint. The combined constraint is obtained using averaged measurements of the branching fractions and asymmetries. The distribution has more than one peak due to ambiguities that arise when solving for ϕ_2. The "flat-top" regions in Fig. 5 arise because $\mathcal{A}_{\rho^0\rho^0}$ is not measured. The absence of the flat-top regions in Belle measurements can be explained by the fact that the isospin triangle is not closed; the relatively large measured branching fraction of $B^\pm \to \rho^\pm\rho^0$ decay leads to the squashed triangle. This results in only a two-fold ambiguity on ϕ_2. The solution consistent with the standard model is $72.5° < \phi_2 < 111.5°$ at 68% C.L.

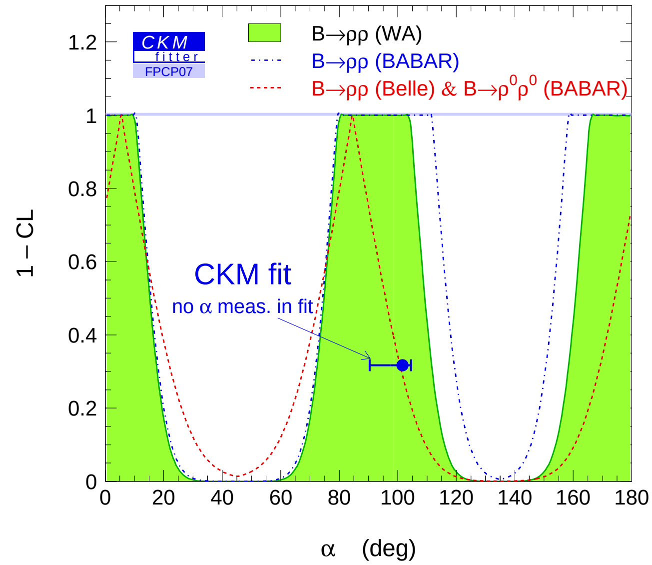

FIG. 5: 1 - C.L. vs $\phi_2(\alpha)$ obtained from the isospin analysis of $B \to \rho\rho$ decays by the CKMfitter group. The dashed curve represents Belle measurements only; the dot-dashed curve represents BaBar measurements and the hatched region is a combined constraint from Belle and BaBar.

Recently, an alternative model-dependent approach to extract ϕ_2 using flavor $SU(3)$ symmetry has been proposed [17]. This method could potentially give more stringent constraints on ϕ_2.

V. $B^0 \to \rho^\pm \pi^\mp$

An alternative way to measure the angle ϕ_2 is to perform a time-dependent Dalitz plot analysis in $B^0 \to \pi^+\pi^-\pi^0$ decays. It was pointed out by Snyder and Quinn [4] that the Dalitz analysis allows one to determine ϕ_2 without discrete ambiguities. The time-dependent rate for $B^0 \to \pi^+\pi^-\pi^0$ decays is given by

$$\begin{aligned}\mathcal{P}(\Delta t, ds_+, ds_-) \quad &\sim e^{-|\Delta t|/\tau_{B^0}} \{ \left(|A_{3\pi}|^2 + |\overline{A}_{3\pi}|^2\right) \\ &-Q \times [\left(|A_{3\pi}|^2 - |\overline{A}_{3\pi}|^2\right) \cos(\Delta m \Delta t) \\ &-2\mathrm{Im}\left[\frac{q}{p} A^*_{3\pi} \overline{A}_{3\pi}\right] \sin(\Delta m \Delta t)]\} \ , \quad (4)\end{aligned}$$

where $Q = +1(-1)$ corresponds to $B^0(\overline{B}^0)$ tags, the parameters q and p are the mass eigenstates of neutral B mesons with the mass difference Δm, and the τ_{B0} is the B^0 meson average lifetime. The amplitudes $A_{3\pi}(\overline{A_{3\pi}})$ of the $B^0(\overline{B}^0)$ decays depend on the Dalitz plot variables $s_{+,-} = (p_{+,-} + p_0)^2$ and $s_0 = (p_+ + p_-)^2$, where $p_{+,-,0}$ are the four-momenta of the $\pi^{+,-,0}$. The amplitudes can be factorized as follows

$$\begin{aligned}A_{3\pi}(s_+, s_-) &= \sum_{\kappa=(+,-,0)} T^\kappa_{J=1} F^\kappa(s_\kappa) A^\kappa \ , \\ \frac{q}{p}\overline{A}_{3\pi}(s_+, s_-) &= \sum_{\kappa=(+,-,0)} T^\kappa_{J=1} F^\kappa(s_\kappa) \overline{A}^\kappa \ , \quad (5)\end{aligned}$$

where $T^\kappa_{J=1}$, $F^\kappa(s_\kappa)$, and $A^\kappa(\overline{A}^\kappa)$ are helicity distributions, lineshapes, and complex amplitudes corresponding to $B^0(\overline{B}^0) \to \rho^+\pi^-, \rho^-\pi^+, \rho^0\pi^0$ decays for $\kappa = +, -, 0$. The lineshapes $F^\kappa(s)$ is modeled as a sum of the $\rho(770)$ resonance and its radial excitations $\rho(1450)$, and $\rho(1700)$:

$$F^\kappa(s) = B_{\rho(770)} + \beta B_{\rho(1450)} + \gamma B_{\rho(1700)}, \quad (6)$$

where B are Breit-Wigner functions and β and γ are the relative complex amplitudes of the two resonances. Inserting Eq. (5) and Eq. (6) to Eq. (4) one obtains 26 free parameters which are to be determined from a fit to the data.

The Belle analysis [18] is based on a data sample of 449 million $B\overline{B}$ pairs. We first obtain the signal yield from an unbinned extended ML fit to the $M_{\rm bc} - \Delta E$ and Dalitz plot distribution. The fit yields 971 ± 42 $B^0 \to \pi^+\pi^-\pi^0$ events. The lineshape parameters β and γ are determined from a time-integrated Dalitz plot fit with a larger Dalitz plot acceptance. The 26 coefficients are subsequently determined from a Dalitz-Δt fit to the 2824 events in a small signal region. The measured parameters also allow to extract a ratio $\mathcal{B}(B^0 \to \rho^0\pi^0)/\mathcal{B}(B^0 \to \rho^+\pi^-)$ which is measured to be $0.133 \pm 0.022 \pm 0.023$. This value is consistent with a Belle previous measurement of the branching fraction of $B^0 \to \rho^0\pi^0$ decays using a quasi-two-body approach [19], $\mathcal{B}(B^0 \to \rho^0\pi^0)^{\rm Belle}/\mathcal{B}(B^0 \to \rho^+\pi^-)^{\rm WA} = 0.130^{+0.049}_{-0.046}$.

BaBar performed analysis using 375 million $B\overline{B}$ pairs [20]. The 26 coefficients and event yields are obtained simultaneously from an unbinned extended ML fit. The likelihood function contains in total 68 parameters which are varied in the fit. The fit yields $N_{3\pi} = 2067 \pm 68$ candidates.

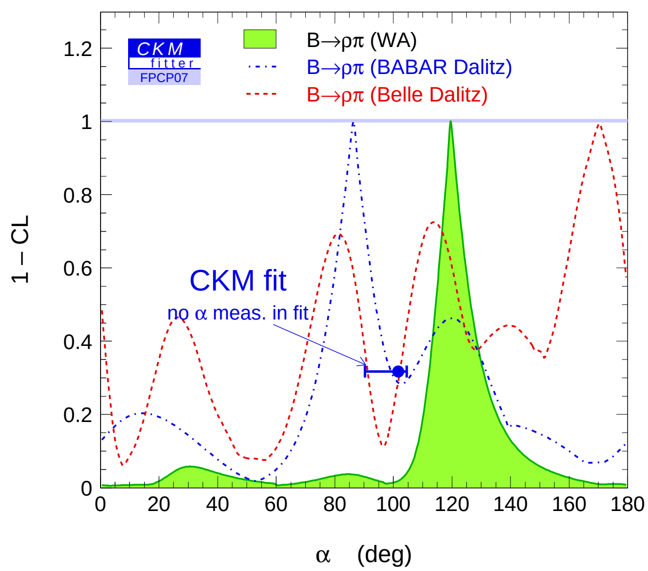

FIG. 6: 1 - C.L. vs $\phi_2(\alpha)$ obtained from the Dalitz analysis of $B \to \rho\pi$ decays by the CKMfitter group. The dashed curve represents Belle measurements only; the dot-dashed curve represents BaBar measurements and the hatched region is a combined constraint from Belle and BaBar.

The measured amplitudes A^κ and $\overline{A}^\kappa$ can be related to the angle ϕ_2 using an isospin relation [21] for neutral B decays only

$$e^{2i\phi_2} = \frac{\overline{A}^+ + \overline{A}^- + 2\overline{A}^0}{A^+ + A^- + 2A^0}. \quad (7)$$

This relation allows one to determine ϕ_2 without discrete ambiguities in the limit of high statistics. The confidence level for ϕ_2 obtained by the CKMfitter group is presented in Fig. 6. The curves on this plot correspond to the Belle (BaBar) measurements only and to a global constraint on ϕ_2 obtained by combining Belle and BaBar measurements. The preferred region for the combined constraint is around 120°.

It is worth mentioning that ϕ_2 can be further constrained by including branching fractions $\mathcal{B}(B^0 \to$

$\rho^{\pm}\pi^{\mp}$), $\mathcal{B}(B^+ \to \rho^+\pi^0)$, $\mathcal{B}(B^+ \to \rho^0\pi^+)$ and asymmetries for $\mathcal{A}(B^+ \to \rho^+\pi^0)$ and $\mathcal{A}(B^+ \to \rho^0\pi^+)$ decays [10] into the isospin analysis (a pentagon isospin analysis). As an example, 1 - C.L. curves on ϕ_2 obtained by Belle from the Dalitz analysis and from the full Dalitz and pentagon combined analysis are presented in Fig. 7. Combined Belle and BaBar Dalitz and pentagon constrain is being recently prepared by the CKMfitter group.

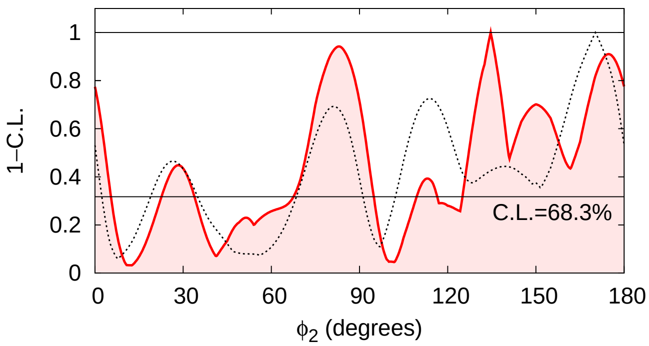

FIG. 7: 1 - C.L. vs $\phi_2(\alpha)$ obtained by Belle. The dotted curve represents the Dalitz analysis and the solid curve represents the combined Dalitz and an isospin pentagon analysis.

VI. $B^0 \to a_1^{\pm}\pi^{\mp}$

Another channel which allows to measure ϕ_2 is $B^0 \to a_1^{\pm}\pi^{\mp}$. As the final state $a_1^{\pm}\pi^{\mp}$ is not a CP eigenstate, one has to consider four decay modes with different charge and flavor combinations: $B^0 \to a_1^{\pm}\pi^{\mp}$ and $\overline{B}^0 \to a_1^{\pm}\pi^{\mp}$. The decay rates can be written as [22]

$$\mathcal{P}_{a_1^{\pm}\pi^{\mp}}(\Delta t) = (1 \pm \mathcal{A}_{CP}^{a_1\pi})\frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}}\{1 - Q \times [(C_{a_1\pi} \pm \Delta C_{a_1\pi})\cos(\Delta m_d \Delta t) - (S_{a_1\pi} \pm \Delta S_{a_1\pi})\sin(\Delta m_d \Delta t)]\}, \quad (8)$$

where $\Delta t = t_{\rho\pi} - t_{tag}$ is the proper-time difference between the fully reconstructed and the associated B decay, and $Q = +1(-1)$ corresponds to $B^0(\overline{B}^0)$ tags. The parameters $S_{a_1\pi}$ and $C_{a_1\pi}$ are associated with mixing-induced CP violation (related to ϕ_2) and flavor-dependent direct CP violation, respectively. The parameters $\Delta S_{a_1\pi}$ and $\Delta C_{a_1\pi}$ are CP-conserving. $\Delta C_{a_1\pi}$ describes the asymmetry between the rates $\Gamma(B^0 \to a_1^+\pi^-) + \Gamma(\overline{B}^0 \to a_1^-\pi^+)$ and $\Gamma(B^0 \to a_1^-\pi^+) + \Gamma(\overline{B}^0 \to a_1^+\pi^-)$. $\Delta S_{a_1\pi}$ depends in addition on difference in strong phases between the amplitudes contributing to $B \to a_1\pi$ decays.

The parameters C, ΔC, S, and ΔS have recently been measured by BaBar collaboration using a data sample of 384 million $B\overline{B}$ pairs [23]. The likelihood function includes the following components: signal, generic $B\overline{B}$ background, continuum $q\bar{q}$ background, $B^0 \to a_2^{\pm}(1320)\pi^{\mp}$, and non-resonant $\rho\pi\pi$. The fitting observables are $m_{\rm ES}$, ΔE, a Fisher discriminant F for continuum suppression, m_{a_1}, Δt and an angle between the flight direction of the bachelor pion from B meson and normal to the plane of the 3π resonance calculated in the 3π rest frame. An unbinned extended ML fit to 29300 events yields 608 ± 53 signal events and the following parameters: $C_{a_1\pi} = -0.10 \pm 0.15 \pm 0.09$, $\Delta C_{a_1\pi} = 0.26 \pm 0.15 \pm 0.07$, $S_{a_1\pi} = 0.37 \pm 0.21 \pm 0.07$, $\Delta S_{a_1\pi} = -0.14 \pm 0.21 \pm 0.06$, and $\mathcal{A}_{CP}^{a_1\pi} = -0.07 \pm 0.07 \pm 0.02$. These measurement indicate no direct and mixing-induced CP violation in $B^0 \to a_1^{\pm}\pi^{\mp}$ decays. As shown in [22], the effective angle $\phi_2^{\rm eff}$ can be calculated as follows

$$\phi_2^{\rm eff} = \frac{1}{4}[\arcsin(\frac{S_{a_1\pi} + \Delta S_{a_1\pi}}{\sqrt{1 - (S_{a_1\pi} + \Delta S_{a_1\pi})^2}} + \arcsin(\frac{S_{a_1\pi} - \Delta S_{a_1\pi}}{\sqrt{1 - (S_{a_1\pi} - \Delta S_{a_1\pi})^2}}]. \quad (9)$$

$\phi_2^{\rm eff} = \phi_2 + \delta\phi_2$ is measured to be $78.6° \pm 7.3°$. The extraction of ϕ_2 can be performed using an SU(3) flavor symmetry [22]. However it can not be done at the moment as branching fractions for SU(3)-related decays have not yet been measured.

VII. SUMMARY

We have discussed the measurements of the CKM phase angle ϕ_2 using $B \to \pi\pi$, $\rho\rho$, $\rho\pi$, and $a_1^{\pm}\pi^{\mp}$ decays. The combined constraint for the first three channels is presented in Fig. 8. There are two preferred regions around 88° and 115°. This can be explained by the fact that the ϕ_2 preferred region obtained from $B \to \rho\pi$ Dalitz analysis doesn't agree well with that obtained from the isospin analysis of $B \to \pi\pi$ and $B \to \rho\rho$ decays. The angle ϕ_2 can be constrained as $(114.5^{+4.4}_{-8.3})°$ at 68% C.L. (solution around the main peak) and $80.0° < \phi_2 < 122.7°$ at 90% C.L. As it can be seen, there is no 'stringent' constrain at 90% C.L. with the current data. More data are awaited to improve our knowledge on ϕ_2.

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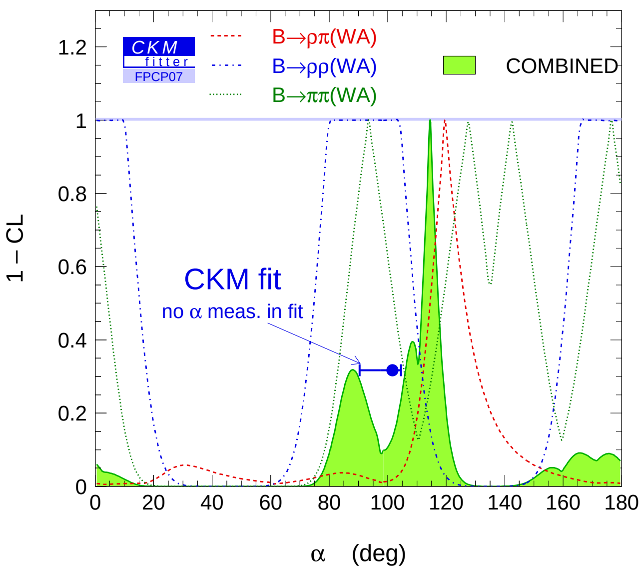

FIG. 8: World average constraints on $\phi_2(\alpha)$ obtained from an isospin analyses of $B \to \pi\pi$ (dotted curve) and $B \to \rho\rho$ (dot-dashed curve) decays, and from Dalitz analysis of $B \to \rho\pi$ decays (dashed curve). The hatched region is a combined constraint for these three channels.

[2] Angles ϕ_1, ϕ_2, and ϕ_3 in Belle are referred to as α, β, and γ in BaBar.
[3] Charge-conjugate modes are included throughout this paper unless noted otherwise.
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