## CKM matrix fits including constriants on New Physics

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I review the status of global fits to the CKM matrix within the framework of the Standard Model and also allowing for New Physics contributions in  $B - \overline{B}$  mixing. The driving force is coming from the large data sets collected by the *B*-factory experiments *BABAR* and Belle. Additional important inputs to the  $B_s$  sector are provided by the Tevatron experiments CDF and D0. In particular, when constraining New Physics in  $B - \overline{B}$  mixing in a model-independent analysis a nice interplay between the *B*-factories and the Tevatron experiments is observed.

### 1. Introduction

Within the Standard Model (SM) quark flavormixing is described by the 3 × 3 unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix [1]. The size of *CP* violation carried by the CKM matrix is proportional to a single parameter, the Jarlskog invariant  $J = \Im \left[ V_{ij} V_{kl} V_{il}^* V_{kj}^* \right]$  [2], where |J|/2 quantifies the area of the unitarity triangle (UT), defined by  $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0.$ 

The CKM matrix is parametrized by four independent real parameters. Inspired by the experimentally observed hierarchy of the CKM matrix, "Wolfenstein-type" parametrizations have been proposed in the literature (see e.g. Refs. [3, 4, 5]). Based on the improved Wolfenstein parametrization [4] the following set of CKM matrix parameters has been proposed [5] and is advertized by the PDG group [6]:  $\lambda = \frac{|V_{us}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}}, \ A\lambda^2 = \frac{|V_{cb}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}}, \ and \ \overline{\rho} + i \cdot \overline{\eta} = -V_{ud}V_{ub}^*/V_{cd}V_{cb}^*.$  This parametrization has several convenient properties: it is exact, it is unitary to all orders of  $\lambda$ , and it is phase-convention independent where  $(\overline{\rho},\overline{\eta})$  represent the coordinates of the apex of the rescaled UT  $((V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^*)/V_{cd}V_{cb}^* = 0).$ 

In many theoretical extensions of the SM sizeable New Physics (NP) effects are expected to contribute to the  $B_q - \bar{B}_q$  mixing amplitude where q = d, s (see e.g. [7]). Since  $\Gamma_{12}$  is dominated by longdistance physics one usually assumes that only  $M_{12}$ , which is dominated by intermediate top-quark contributions, is affected by NP. In this case and by assuming a  $3 \times 3$  unitary CKM matrix one can parameterize NP in mixing by two new parameters where two different parametrizations are typically in use:  $r_q^2 e^{2i\theta_q} = \frac{\langle \bar{B}_q^0 | M_{12}^{SM+NP} | B_q^0 \rangle}{\langle \bar{B}_q^0 | M_{12}^{SM} | \bar{B}_q^0 \rangle}$  [8], respectively,  $h_q e^{2i\sigma_q} = \frac{\langle \bar{B}_q^0 | M_{12}^{SM} | B_q^0 \rangle}{\langle \bar{B}_q^0 | M_{12}^{SM} | B_q^0 \rangle}$  [9, 10]. The SM values are  $r_q^2 = 1$  and  $2\theta_q = 0$ , respectively,  $h_q = 0$  and  $2\sigma_q = 0$ .

The case of  $K - \bar{K}$  mixing is more involved. The observable  $\epsilon_K$  obtains sizeable contributions from the charm quark box diagram while the short distance contribution to  $\Delta m_K$  is even dominated by interme-

Observable	Prediction in the presence of NP in mixing
$S(\psi K_S)$	$\sin\left(2\beta + 2\theta_d\right)$
$B \to \psi K^{0*}$	$\cos\left(2eta+2 heta_d ight)$
$\alpha$	$\pi-\gamma-eta- heta_d$
$\Delta m_q$	$\Delta m_q^{SM} \cdot r_q^2$
$\Delta\Gamma_s$	$-\Delta m_q^{SM} \left[ \Re \frac{\Gamma_{12}^{SM}}{M_{12}^{SM}} \cos 2\theta_q + \Im \frac{\Gamma_{12}^{SM}}{M_{12}^{SM}} \sin 2\theta_q \right]$
$A^q_{SL}$	$-\Re \frac{\Gamma_{12}^{\Gamma M}}{M_{12}^{SM}} \frac{\sin 2\theta_q}{r_q^2} + \Im \frac{\Gamma_{12}^{SM}}{M_{12}^{SM}} \frac{\cos 2\theta_q}{r_q^2}$

Table I Theoretical prediction of observables in the Bmeson system in the presence of NP in mixing.

diate charm quarks with a subdominant top quark contribution and a large, but hardly calculable long distance contribution from intermediate u quarks. If one assumes that NP does only modify the top quark amplitude the observable  $\Delta m_K$  provides only a weak constraint on the parameter  $r_K^2$ . In Ref. [10] the leading top-quark contribution in  $\epsilon_K$  has been modified by a term containing the parameters  $h_K$  and  $2\sigma_K$ . The UTfit collaboration simply parametrized any deviation from the SM in  $\epsilon_K$  by  $\epsilon_K^{exp} = C_{\epsilon_K} \cdot \epsilon_K^{SM}$  where  $C_{\epsilon_K} = \frac{\Im < \bar{K}^0 | H_{12}^{SM} + N^P | K^0 >}{\Im < \bar{K}^0 | H_{12}^{SM} | K^0 >}$  [11].

Often, the additional assumption is made that NP does not contribute to tree-mediated decays. However, on the non-perturbative level tree and penguin amplitudes can not be well separated. For this reason, it has been proposed to be more precise that decays proceeding through a Four Flavor Change obtain only SM contributions (SM4FFC) [5, 9]. According to these assumptions the following inputs used in the fit allowing for NP in mixing are considered to be free from NP contributions in their extractions from data: e.g.  $|V_{ud}|$ ,  $|V_{us}|$ ,  $|V_{ub}|$ ,  $|V_{cb}|$  and  $\gamma$ . Observables that are affected by NP in mixing are shown in Table I. It should be noted that  $\Im \frac{\Gamma_{12}^{SM}}{M_{12}^{SM}}$  is much smaller than  $\Re \frac{\Gamma_{12}^{SM}}{M_{12}^{SM}}$ , hence, terms containing  $\Im \frac{\Gamma_{12}^{SM}}{M_{12}^{SM}}$  are often neglected in the expressions given in Table I.

The first analysis making use of the early *B*-factory data  $(|V_{ub}|, |V_{cb}|, \Delta m_d, \sin(2\beta))$  with a special focus on the role of  $A_{SL}^d$  has been described in Ref. [12]. The

first complete model-independent analysis on data in the  $B_d$  sector using all relevant observables  $(|V_{ub}|, |V_{cb}|, \Delta m_d, \sin(2\beta), \cos(2\beta), \alpha, \gamma, A_{SL}^d)$  is discussed in Ref. [5] and could exclude a real CKM matrix even in the presence of NP in  $B_d$  mixing. A combined analysis for the Kaon and  $B_d$  sector with prospects in the  $B_s$  sector has been performed subsequently in Ref. [10] examining a Next-To-Minimal Flavour Violation (NMFV) scenario, and for the Kaon and  $B_d$ sector by the UTfit collaboration [11] with a focus on Minimal Flavour Violation (MVF). The advent of the first observation of  $B_s$  oscillations by CDF [13] triggered several analyses (see e.g. [14, 15, 16, 17, 18]) where not only  $\Delta m_s$  but also the role of  $A_{SL}$  in the  $B_d$  and  $B_s$  sector as well as  $\Delta \Gamma_s$  was discussed in several publications (see e.g. Refs. [16, 17, 18]).

#### 2. Inputs

In this section, the status of experimental measurements and theoretical input parameters as used in state-of-the-art CKM fits is summarized. The numerical values are given in Table II.

## 2.1. Inputs free from New Physics in mixing

Currently, the best determination of  $|V_{ud}|$  comes from superallowed  $\beta$ -decays where the uncertainty is dominated by the theoretical error, see e.g. Ref [19]. The matrix element  $|V_{us}|$  is determined from  $K_{e3}$  decays, from the ratio of rates between  $K \rightarrow \mu \nu$  and  $\pi \rightarrow$  $\mu\nu$ , from  $\tau$  decays, and from semileptonic hyperon decays. In his Moriond 2007 review talk, M. Jamin quotes an average value of  $|V_{us}| = 0.2240 \pm 0.0011 \, [20]$ . It should be noted that this average is dominated by the  $K_{e3}$  number where the quoted average has a significantly smaller theoretical uncertainty than quoted by others. This number is dominated by a recent and preliminary Lattice QCD (LQCD) calculation for the  $K \to \pi$  form factor  $f_+$  which has not evaluated all systematic uncertainties yet [21]. When using this  $|V_{us}|$  input the uncertainty on  $\lambda$  obtains similar contributions from  $|V_{ud}|$  and  $|V_{us}|$  resulting in a  $2\sigma$  deviation from the unitarity condition in the first family. When discarding this specific form factor calculation the weight of the  $K_{e3}$  number would get smaller in the  $|V_{us}|$  average but the size of the unitarity violation would remain more or less the same since the  $|V_{us}|$ values from the other methods give smaller results.

The matrix element  $|V_{cb}|$  is obtained from semileptonic decays  $B \to X_c \ell \nu$  where  $X_c$  is either a  $D^*$  meson (exclusive method) or a sum over all hadronic final states containing charm (inclusive method). The most precise value is currently provided by the inclusive method where the theoretical uncertainties have

been pushed below the 2 % level by determining nonperturbative parameters from moment measurements in  $B \to X_c \ell \nu$  and  $B \to X_s \gamma$ . The inclusive  $|V_{cb}|$  value used in this analysis is taken from Ref. [22]. The theoretical uncertainty on the exclusive  $|V_{cb}|$  determination in the calculation of the form factor value F at zero recoil is currently not competitive with the inclusive method. The central value is smaller than but compatible with the inclusive result given its large theoretical uncertainty.

A delicate input is  $|V_{ub}|$  for several reasons: It plays a crucial role in testing a NP phase in  $B_d - B_d$  mixing which can be detected by comparing the measured  $\sin 2\beta$  value with the predicted value without using the experimentally measured  $\sin 2\beta$  value. The predicted value is in particular sensitive to  $|V_{ub}|$  since the  $\sin 2\beta$  constraint (in the SM) is tangent to the  $|V_{ub}/V_{cb}|$  circle in the  $\overline{\rho}$ - $\overline{\eta}$  plane. However, the two methods to extract  $|V_{ub}|$ , the inclusive and the exclusive (using  $B \rightarrow \pi \ell \nu$ ) are suffering from significant theoretical uncertainties and, in addition, do not perfectly agree with each other. The exclusive numbers prefer values below  $4.0 \times 10^{-31}$ . For the fit input the exclusive numbers quoted by the Heavy Flavour Averaging group (HFAG) [24] I average them in a conservative way by keeping the smallest theoretical uncertainty as a common theoretical error:  $|V_{ub,excl}| = (3.60 \pm 0.10 \pm 0.50) \times 10^{-3}$ . The average of inclusive results quoted by HFAG using e.g. the Shape Function (SF) scheme [25] yields  $(4.52 \pm 0.19 \pm 0.27) \times 10^{-3}$  where the first uncertainty contains the statistical and experimental systematic uncertainty as well as the modelling errors for  $b \to u \ell \nu$ and  $b \rightarrow c \ell \nu$  transitions. Since several theoretical uncertainties are somehow guestimated (the HQE error on the  $m_b$  mass determined in moment fits, the matching scale in the BLNP calculation, subleading shape functions and weak annihilation) I add those linearily and use  $(4.52 \pm 0.23 \pm 0.44) \times 10^{-3}$  as an input where the first error is considered as a statistical one and the second as a theoretical one which will be scanned in the CKM fit  $^{2}$ . The inclusive and exclusive number

<sup>&</sup>lt;sup>1</sup> After the conference the HPQCD collaboration submitted an erratum for their former published Lattice QCD (LQCD) result for  $f_+$ . As a consequence, all LQCD values and the one from Light Cone Sum Rules (LCSR) prefer now values close to or even below  $3.5 \times 10^{-3}$  which leads to an even more pronounced discrepancy.

<sup>&</sup>lt;sup>2</sup> According to M. Neubert the uncertainty due to the SF parameters determining the first and the second moment of the SF are underestimated in the HFAG 2006 average since the BLNP generator used to calculate partial rates is calculated on one-loop level. Hence the b-quark mass in the SF scheme has already an intrinsic uncertainty of O(60-70) MeV which is significantly larger than the one obtained from moment fits [22]. As a consequence, in future averages the uncertainty on  $|V_{ub,incl}|$  is expected to become larger.

are averaged again by keeping the smallest theoretical uncertainty as the common theoretical uncertainty (see Table II).

The UTfit group interprets all HFAG uncertainties in a statistical way and use the following inputs:  $|V_{ub,incl}^{UTfit}| = (4.49 \pm 0.33) \times 10^{-3}$  and  $|V_{ub,excl}^{UTfit}| =$  $(3.50 \pm 0.40) \times 10^{-3}$ . If all uncertainties were interpreted as coming from Gaussian distributions one obtained as an average  $|V_{ub}| = (4.09 \pm 0.25) \times 10^{-3}$ . If one followed the PDG error rescaling recipe one would obtain  $|V_{ub}| = (4.09 \pm 0.49) \times 10^{-3}$ ). As a result of the error treatment the discrepancy between the  $|V_{ub}|$ input and its prediction is much more pronounced in the UTfit analysis than in the analysis presented here.

The input for the UT angle  $\gamma$  is taken from a combined full frequentist analysis of the CKMfitter group using CP violating asymmetries in charged B decays to neutral  $D^{(*)}$  mesons plus charged  $K^{(*)}$  mesons as discussed in detail at this conference [26]. On the 68 % confidence level (CL) the result is  $(77\pm31)^{\circ}$ . This constraint differs significantly from the result obtained from the same data set by the UTfit group using a Bayesian approach ( $(82 \pm 20)^{\circ}$  at 95 % CL).

## 2.2. Inputs possibly affected by New Physics in mixing

The observable  $\epsilon_K$  has shifted by about 2.3 % (a 3.7  $\sigma$  effect) between the 2004 and 2006 edition of the PDG from  $(2.284 \pm 0.014) \times 10^{-3}$  [27] down to  $(2.232 \pm 0.007) \times 10^{-3}$  [6]. This shift is mainly caused by improved measurements of the branching fraction  $BF(K_L \to \pi^+\pi^-)$  performed by KTeV, KLOE and NA48 leading to a reduction of 5.5 % of the branching fraction values. The translation of  $\epsilon_K$  into a constraint on  $\overline{\rho}$  and  $\overline{\eta}$  suffers from sizeable uncertainties in the decay constant  $\hat{B}_K$  (see Table II, Ref. [28]) and, though of less importance, from uncertainties in the QCD corrections coming from  $\eta_{cc}$  [29, 30] and from the charm quark mass  $m_c(m_c)$  in the  $\overline{\text{MS}}$  scheme obtained from fits to energy and mass moments in  $B \to X_c \ell \nu, X_s \gamma$ decays [22]. As discussed in Ref. [22] an additional uncertainty of order 50 MeV should be assigned to this value of  $m_c(m_c)$  which I add linearly to the quoted uncertainty of 45 MeV from the Heavy Quark Expansion. Other uncertainties of less importance are coming from  $m_t$  [31], and the perturbative QCD corrections  $\eta_{tt}$  [29, 30] and  $\eta_{ct}$  [29, 30].

Within the SM the measurement of the S coefficient in the time-dependent CP asymmetry  $A_{CP} =$  $S \sin (\Delta m_d \cdot t) + C \cos (\Delta m_d \cdot t)$  in decays of neutral  $B_d$  mesons to final states  $(c\bar{c})K^0$  provides to a very good approximation a measurement of the parameter  $\sin 2\beta$ . The current uncertainty of 0.025 is still statistics dominated [24]. The difference between the measured S coefficient and  $\sin 2\beta$  has been estimated in Ref. [32] to be below the  $10^{-3}$  level. Less

Observable	Value and Uncertainties
$ V_{ud} $	$0.97377 \pm 0.00027$
$ V_{us} $	$0.2240 \pm 0.0011$ [20]
$ V_{cb} $	$0.0416 \pm 0.0007$ , see text
$ V_{ub} $	$(4.09 \pm 0.09 \pm 0.44) \times 10^{-3}$ , see text
$\gamma$	$B^{\pm} \to D^{(*)} K^{(*)\pm}$ , see text
$\alpha$	$B \to \pi \pi, \rho \rho, rho \pi$ , see text
$\sin 2\beta$	$0.678 \pm 0.025$ [24]
$\cos 2\beta$	see text
$\Delta m_d$	$(0.507 \pm 0.005) \text{ ps}^{-1} [46]$
$\Delta m_s$	$(17.77 \pm 0.12) \text{ ps}^{-1} [47]$
$A^d_{SL}$	$-0.0043 \pm 0.0046$ , see text
$A_{SL}$	$-0.0028 \pm 0.0013 \pm 0.0008$ [55]
$A^s_{SL}$	$0.0245 \pm 0.0196 [54]$
$\Delta\Gamma_s^{CP'}$	$(0.12 \pm 0.08) \text{ ps}^{-1}$
$\epsilon_K$	$(2.232 \pm 0.007) \times 10^{-3}$ [6]
$f_{B_s}$	$(268 \pm 17 \pm 20)$ MeV [28]
$B_s$	$1.29 \pm 0.05 \pm 0.08$ [28]
$f_{B_s}/f_{B_d}$	$1.20 \pm 0.02 \pm 0.05$ [28]
$B_s/B_d$	$1.00 \pm 0.02$ [48]
$\eta_B$	$0.551 \pm 0.007$ [49]
$m_t(m_t)$	$(163.8 \pm 2.0) \text{ GeV} [31]$
$\hat{B}_{K}$	$(0.78 \pm 0.02 \pm 0.09)$
$\eta_{tt}$	$0.5765 \pm 0.0065$ [29, 30]
$\eta_{ct}$	$0.47 \pm 0.04$ [29, 30]
$n_{cc}$	see text $[29, 30]$
$m_c(m_c)$	$(1.240 \pm 0.037 \pm 0.095) \text{ GeV} [22]$

Table II Input values and uncertainties used in the CKM fit.

stringent constraints on this difference are quoted in Refs. [33, 34, 35]. When interpreting the measured S coefficient as  $\sin(2\beta + 2\theta_d)$  the SM4FFC hypothesis does not rigourously apply. However, as pointed out in Ref. [36] the gluonic penguin is OZI suppressed and the Z-penguin is estimated to be small so that NP in decay is assumed to be negligible with respect to the leading tree amplitude. The effect from possible NP in  $K - \bar{K}$  mixing on  $\sin(2\beta + 2\theta_d)$  can be neglected as well due to the small value of  $\epsilon_K$ . The measurement on  $\sin(2\beta + 2\theta_d)$  results in four solutions on  $\beta + \theta_d$ . Two out of four solutions can be excluded by measuring the sign of  $\cos(2\beta + 2\theta_d)$ . For a recent review of BABAR and Belle measurements see Ref. [37]. The current experimental results from BABAR and Belle disfavour negative  $\cos(2\beta + 2\theta_d)$  values but it is considered to be difficult by the Heavy Flavour Averaging Group to average the different measurements, respectively, to determine a reliable confidence level as a function of  $\cos(2\beta + 2\theta_d)$  [24]. Hence, as a simplification, it is assumed here that  $\cos(2\beta + 2\theta_d) > 0$ .

The constraint on  $\alpha$  is obtained from timedependent and time-independent measurements in the decays  $B \to \pi\pi$ ,  $B \to \rho\rho$ , and  $B \to \rho\pi$ . The timedependent CP asymmetries measured in  $B \to \pi\pi$ and also in  $B \to \rho\rho$  provide information on the effective parameter  $\sin(2\alpha_{eff})$ . It is possible to translate this measurement into a constraint on  $\alpha$  exploiting isospin symmetry which allows to determine the difference  $\alpha - \alpha_{eff}$  from data [38]. Under the assumption of exact isospin symmetry the amplitudes  $A^{+-} \equiv A(B^0 \to \pi^+\pi^-), A^{00} \equiv A(B^0 \to \pi^0\pi^0),$ and  $A^{+0} \equiv A(B^+ \to \pi^+\pi^0)$  satisfy a triangular relationship:  $\sqrt{2}A^{+0} - \sqrt{2}A^{00} = A^{+-}$ . A corresponding relationship holds for the CP conjugated decays:  $\sqrt{2}\overline{A}^{+0} - \sqrt{2}\overline{A}^{00} = \overline{A}^{+-}$ .

The extraction of  $\alpha$  from the isospin analysis is independent of any possible NP contributions in the  $\Delta I = 1/2$  decay amplitude except for the singular point  $\alpha = 0$ . If there are no NP contributions in the  $\Delta I = 3/2$  decay amplitude the extraction provides  $\alpha = \pi - \gamma - \beta - \theta_d$ . As a consequence,  $\alpha$  is equivalent to  $\gamma$  if  $\beta + \theta_d$  is measured e.g. from  $B \to J/\psi K_S$  (with ambiguities).



Figure 1: Constraints on  $\alpha$  from the isospin analysis using  $B \to \pi\pi$  data.

The structure seen in the CL as a function of  $\alpha$  observed in in the isospin analysis (Figs. 1 and 3) can be easily understood. For  $B \to \pi\pi$  the eight solutions from the isospin analysis are clearly visible when using only the results for the *CP* asymmetries from *BABAR* while for Belle and the world averages [24] only four solutions are observed. This is due to the fact that in the latter case one of the two isospin triangles, the *B*-meson triangle, barely closes as illustrated in Fig. 2. For  $B \to \rho\rho$  there is only evidence so far for the decay  $B^0/\bar{B}^0 \to \rho^0\rho^0$  from a *BABAR* measurement but no



Figure 2: The two isospin triangles for the  $B \to \pi \pi$  system for the *B*-meson, respectively, the *B*-meson system using the world averages [24] for branching fractions and direct *CP* asymmetries.

CP asymmetry had been measured up to this point <sup>3</sup>. As a consequence, only a limit on  $\alpha - \alpha_{eff}$  can be extracted which explains the constant regions in the CL curve.

Concerning statistical issues in the isospin analysis there has been a recent debate in the literature.

<sup>&</sup>lt;sup>3</sup>At the Lepton-Photon conference 2007, *BABA*R has presented for the first time a time-dependent *CP* asymmetry measurement for  $B^0/\bar{B}^0 \to \rho^0 \rho^0$  though with still large uncertainties on the *S* and *C* coefficient.



Figure 3: Constraints on  $\alpha$  from the isospin analysis using  $B \rightarrow \rho \rho$  data.

In Ref. [39] it has been shown that the result of the isospin analysis in  $B \to \pi\pi$  and  $B \to \rho\rho$  shows a clear prior dependence when being performed in the framework of Bayesian statistics. In a reply, the UTfit collaboration [40] argued that the result of the isospin analysis for  $B \to \pi\pi$  when considering 95 % probability intervals shows only a weak prior dependence. Moreover, it was advertized that additional information on top of the isospin analysis should be used, namely, that SM QCD amplitudes can not exceed a certain size in order to avoid SU(3) flavour symmetry breaking effects of more than 100 %. With this additional input the constraint on  $\alpha$  are getting stronger and the prior dependence becomes weaker.

While there is indeed no large difference between credibility intervals with a probability content of 95 %in the  $B \rightarrow \pi \pi$  analysis for the specific data set considered by the UTfit collaboration the result of a Bayesian analysis is the full a-posteriori probability density function (PDF) often used as input in subsequent analyses and not just intervals of a specific probability content as pointed out in Ref. [41]. In addition, significant differences are observed for e.g. 95 % probability intervals when considering the  $B \to \rho \rho$  isospin analysis. While there is no objection to use additional theory input on top of the isospin analysis which improves the constraints on  $\alpha$  as discussed e.g. in detail in Ref. [5] this kind of analysis is not equivalent to the isospin analysis and does not preserve the exact degeneracy as expected from the remaining symmetries of the problem [41].

The UT angle  $\alpha$  can also be determined from a  $B \rightarrow \rho \pi$  Dalitz plot analysis which has the principal advantage that no ambiguities are present although,

with small statistics, it is possible that mirror solutions may occur. The BABAR collaboration has performed such an analysis in Ref. [42] based on a sample of  $375 \times 10^{-6} B\bar{B}$  events. A similar analysis but also taking into account twobody final state  $B \to \rho \pi$ branching fractions has been shown by Belle [43] based on a sample of  $449 \times 10^{-6} B\bar{B}$  events. The constraint on  $\alpha$  for the Dalitz plot results from BABAR and Belle as well as for their combination are shown in Fig. 4. The combined constraint shows the particular feature that the occuring mirror solutions have relatively small CL values compared to the preferred solution around 120°. The preferred solution itself deviates from the SM prediction for  $\alpha$  by about  $2\sigma$ . The fact that the combination differs significantly from a naive average of both experimental CL curves is due to the fact that all the Dalitz plot observables, the Uand I coefficients, are averaged (and not just  $\alpha$ ) including their correlations which is crucial for a correct average. The combined analysis of the UTfit collab-



Figure 4: Constraints on  $\alpha$  from the  $B \to \rho \pi$  Dalitz plot analyses.

oration at the time of the FPCP07 conference, also taking into account the correlation between the Uand I coefficients, finds a significantly different constraint. One part of the discrepancy might stem from a slightly different input data set. The inputs used by the UTfit collaboration for their Winter 2007 analysis stemmed from analyses presented at the ICHEP06 conference: The BABAR analysis used was based on a slightly smaller data set  $(347 \times 10^6 B\bar{B} \text{ pairs})$  [44] while for the Belle analysis the ICHEP06 result based on the same statistics, however, with slightly different systematic uncertainties was taken [45]. The Summer 2007 UTfit analysis relies on the same inputs as the analysis presented here and, in contrast to the Winter 2007 analysis, only one preferred region around 110° is observed (see Fig. 5) demonstrating the sensitivity of the  $B \rightarrow \rho \pi$  analysis to small changes in the inputs. Nonetheless, the global constraint from the UT-fit group shows still significant differences compared to the Frequentist analysis of the CKMfitter group.



Figure 5: Constraints on  $\alpha$  from the  $B \rightarrow \rho \pi$  Dalitz plot analyses as performed by the UTfit collaboration in the Summer 2007 analysis.

The combined constraint on  $\alpha$  from  $B \to \pi\pi$ ,  $B \to \rho\rho$  and  $B \to \rho\pi$  is shown in Fig. 6. Two preferred regions are visible around 90° and 115° due to the fact that the preferred solution from  $B \to \rho\pi$  is disfavoured by the  $B \to \pi\pi$  and  $B \to \rho\rho$ constraints while the preferred combined  $B \to \pi\pi$ and  $B \to \rho\rho$  region around 90° coincides with one of the disfavoured  $B \to \rho\pi$  solutions. The SM prediction for  $\alpha$  lies just in between these two regions.

The oscillation frequency in the  $B_d$  sector  $\Delta m_d$  is measured with O(1 %) precision mainly due to the *B*-factory data [46]. Since 2006  $\Delta m_s$  is also known with good precision thanks to the observation [13] of and improved measurement [47] of  $B_s$  oscillations by CDF. The translation of the measured value for  $\Delta m_q$  into constraints on CKM parameters  $|V_{tq}V_{tb}^*|^2$ (q = d, s) suffer from significant uncertainties on  $f_{B_d}\sqrt{B_d}$ , respectively,  $f_{B_s}\sqrt{B_s}$ . The input values for these hadronic parameters can be calculated in LQCD. For  $f_{B_s}$ ,  $B_s$ , and the ratio  $f_{B_s}/f_{B_d}$ , the central values and uncertainties are used as quoted in a recent review by Tantalo where the first uncertainty reflects a statistical error and the second the range of various LQCD results [28]. For the ratio  $B_s/B_d$ Ref. [48] is used since no corresponding value and uncertainty has been provided in Ref. [28]. The value and uncertainty for the perturbative QCD correction  $\eta_B$  is taken from Ref. [49].



Figure 6: The combined constraint on  $\alpha$  from  $B \to \pi \pi$ ,  $B \to \rho \rho$  and  $B \to \rho \pi$ .

CP violation in  $B_d$  mixing  $(|q/p| \neq 1)$  can be measured using the untagged dilepton rate asymmetry <sup>4</sup>

$$A_{SL}^{d} = \frac{N_{\ell^+\ell^+} - N_{\ell^-\ell^-}}{N_{\ell^+\ell^+} + N_{\ell^-\ell^-}} = 2(1 - |q/p|).$$
(1)

Theoretical calculations for  $A_{SL}^d = \Im \frac{\Gamma_{12}}{M_{12}}$  at Next-To-Leading Order (NLO) are available [50, 51, 52] with the most recent one giving a SM prediction of  $A_{SL}^d =$  $(-4.8^{+1.0}_{-1.2}) \times 10^{-4}$  [52]. As a consequence, a measurement of  $A_{SL}^d$  is not useful for a precise determination of CKM parameters but very helpful to constrain possible NP contributions to neutral *B*-meson mixing. A weighted average of different results from BABAR Belle and CLEO results in  $-0.0043 \pm 0.0046$  [53] corresponding to a  $-0.8 \sigma$  deviation from the SM prediction. The SM prediction for  $A_{SL}^s$  in the  $B_s$  sector is at least one order of magnitude smaller than the one for  $A_{SL}^d$  [52]. Compared to the SM prediction the first direct measurement from D0 has a quite large uncertainty:  $A_{SL}^s = 0.0245 \pm 0.0196$  [54]. D0 has also measured an inclusive dimuon asymmetry of  $A_{SL} = -0.0028 \pm 0.0013 \pm 0.0008$  [55] which is a mixture between the asymmetries  $A_{SL}^{d}$  and  $A_{SL}^{s}$ :  $A_{SL} = (0.582 \pm 0.030) A_{SL}^{d} + (0.418 \pm 0.047) A_{SL}^{s}$  [52]. The measured value corresponds to a  $-1.6 \sigma$  deviation from the SM prediction.

 $<sup>^4</sup>$  With a tagged time-dependent decay asymmetry one measures the ratio  $\frac{1-|q/p|^4}{1+|q/p|^4}.$ 

A measurement of the lifetime difference in the  $B_s$  sector provides information on the mixing phase  $2\phi_s = -2\beta_s + 2\theta_s$  where the SM prediction  $\beta_s \approx \lambda^2 \eta = (0.945^{+0.201}_{-0.069})^{\circ}$  (95 % *CL*) is very close to zero with a small uncertainty. In a recent analysis, the D0 experiment has measured the untagged time-dependent decay rates for  $B_s \rightarrow \psi \phi$  with an angular analysis which allows to disentangle the *CP*-even and *CP*-odd final states in this particular vector-vector final state [56]. The result of the analysis is  $\Delta\Gamma_s = (0.17 \pm 0.08(stat) \pm 0.02(sys))$  ps<sup>-1</sup> and  $\phi_s = -0.79 \pm 0.56(stat)^{+0.01}_{-0.14}(sys)^5$ . The analysis measures effectively the product  $\Delta\Gamma_s^{CP'} \equiv \Delta\Gamma_s \cdot \cos \phi_s = \Delta\Gamma_s^{SM} \cdot \cos^2 \phi_s$ . Taking into account the correlation between the measured  $\Delta\Gamma_s$  and  $\phi_s$  results one obtains  $\Delta\Gamma_s^{CP'} = (0.12 \pm 0.08)$  ps<sup>-1</sup> which is used in the NP fit as presented in Sec. 5.

#### 3. The Standard Model CKM fit

In this section the results of the Standard Model CKM fit are summarized as obtained from all inputs quoted in Sec. 2 except the ones for  $A_{SL}^d$ ,  $A_{SL}^s$ ,  $A_{SL}$ , and from  $\Delta\Gamma_s^{CP'}$ . The numerical values of the fit results are given in Table III. The constraints on  $\overline{\rho}$  and  $\overline{\eta}$  are visualized in Fig. 7. As a comparison, the Winter



<sup>&</sup>lt;sup>5</sup>The result contains a four-fold ambiguity which can be reduced to a two-fold ambiguity by fixing the strong phase between the CP-even and CP-odd amplitude as it has been done in Ref. [52]. Out of the two remaining solutions the one being in good agreement with the SM is retained for the further analysis. A different strategy is pursued in Ref.[57].

Parameter	Value and Uncertainties (95 $\%$ CL)
$\lambda_{fit}$	$0.2258\substack{+0.0016\\-0.0017}$
$A_{fit}$	$0.817\substack{+0.030\\-0.028}$
$\overline{ ho}_{fit}$	[0.108, 0.243]
$\overline{\eta}_{fit}$	[0.288, 0.375]
$J_{fit}$	$(2.74^{+0.63}_{-0.22}) \times 10^{-5}$
$\left V_{ud}^{fit}\right $	$0.97419 \pm 0.0037$
$\left V_{us}^{fit}\right $	$0.2257 \pm 0.0016$
$ V_{us}^{pred} $	$0.2275 \pm 0.0011$
$\left V_{cb}^{fit}\right $	$(41.7 \pm 1.3) \ 10^{-3}$
$ V_{ub}^{fit} $	$(3.62^{+0.25}_{-0.16}) \times 10^{-3}$
$ V_{ub}^{pred} $	$(3.54^{+0.18}_{-0.16}) \times 10^{-3}$
$\left V_{cd}^{fit}\right $	$0.2255 \pm 0.0016$
$ V_{cs}^{fit} $	$0.97334 \pm 0.00037$
$ V_{td}^{fit} $	$(8.73^{+0.43}_{-1.14}) \times 10^{-3}$
$ V_{ts}^{fit} $	$(40.9 \pm 1.3) \ 10^{-3}$
$ V_{tb}^{fit} $	$0.999124\substack{+0.000053\\-0.000055}$
$\beta_{fit}$	$(21.5^{+2.1}_{-1.3})^{\circ}$
$\beta_{pred}$	$(26.8^{+2.9}_{-6.2})^{\circ}$
$\alpha_{fit}$	$[84.8^{\circ}, 108.5^{\circ}]$
$\alpha_{pred}$	$[85.4^{\circ}, 107.1^{\circ}]$
$\gamma_{fit}$	$[50.7^{\circ}, 73.1^{\circ}]$
$\gamma_{pred}$	$[50.5^{\circ}, 72.9^{\circ}]$
$\Delta m_d^{pred}$	$(0.42^{+0.33}_{-0.12}) \text{ ps}^{-1}$
$\Delta m_s^{pred}$	$(23.4^{+6.4}_{-8.2}) \text{ ps}^{-1}$
$\beta_s^{pred}$	$(0.9455^{+0.201}_{-0.069})^{\circ}$
$\epsilon_{K}^{pred}$	$(2.05^{+1.40}_{-0.71}) \times 10^{-3}$

Table III Selected numerical results from the CKM fit within the framework of the SM using the inputs as described in the text.

Figure 7: The constraints on  $\overline{\rho}$  and  $\overline{\eta}$  from the individual constraints and by combining all individual constraints. Coloured regions indicate CL's of at least 95 %. The allowed region at 95 % CL is shown in yellow and inscribed by the red contour. The combined fit also includes the input from the branching fraction measurements of  $B \to \tau \nu_{\tau}$  as described in Sec. 4. This individual constraint is not shown in order to guarantee readibility of the plot.

2007 analysis of the UTfit group obtains for  $\overline{\rho}$  and  $\overline{\eta}$  the following 95 % credibility intervals: [0.107, 0.222], respectively, [0.307, 0.373].

## 4. Constraints from $B^+ \rightarrow \tau^+ \nu_{\tau}$

The decay  $B^+ \to \tau^+ \nu_{\tau}$  is interesting for two reasons. First, if measured with good precision it provides a stringent constraint on the product  $|V_{ub}| \cdot f_B$ where  $f_B$  is the decay constant of the charged B meson. Hence, combining  $BF(B^+ \to \tau^+ \nu_{\tau})$  with the measurement of  $\Delta m_d$  allows to remove the dependency on  $f_B$  (assuming isospin symmetry  $f_B = f_{B_d}$ ) when translating these measurements into constraints on  $\overline{\rho}$  and  $\overline{\eta}$ .

Second, although the decay is mediated at leading order by a tree amplitude this process is sensitive to NP through a charged Higgs boson exchange. In a Two-Higgs-Doublet model of type II the prediction of the branching fraction [58] is given by

$$BF(B^+ \to \tau^+ \nu_\tau) =$$

$$BF(B^+ \to \tau^+ \nu_\tau)_{SM} \left( 1 - \tan^2 \beta \frac{m_B^2}{m_{H^+}^2} \right) \qquad (2)$$

with the charged Higgs mass  $m_{H^+}$  and the ratio of the two Higgs vacuum expectation values  $\tan \beta$ , and the SM prediction

$$BF(B^+ \to \tau^+ \nu_\tau)_{SM} =$$

$$\frac{G_F^2 m_B}{8\pi} m_\tau^2 \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B =$$

$$(0.96^{+0.38}_{-0.20}) \times 10^{-4} (95 \% CL) \qquad (3)$$

using  $|V_{ub}^{fit}| = (3.62^{+0.25}_{-0.16}) \times 10^{-3}$  from the CKM fit result as shown in Table III and  $f_B = f_{B_d} = (226 \pm 15 \pm 26)$  MeV following from the inputs provided in Sec. II.

At this conference the BABAR collaboration has presented new results for the branching fraction  $BF(B^+ \to \tau^+ \nu_{\tau})$  [59]. The combined result of analyses where the other B meson is tagged by either reconstrucing a hadronic decay or a  $B \to D^{(*)}\ell\nu$  decay is  $BF(B^+ \to \tau^+\nu_{\tau}) = (1.20^{+0.40+0.29}_{-0.38-0.30} \pm 0.22) \times 10^{-4}$  corresponding to a 2.6  $\sigma$  access [59]. Belle had already reported a 3.5  $\sigma$  evidence with hadronic tags only and measured  $BF(B^+ \to \tau^+ \nu_{\tau}) = (1.79^{+0.56+0.46}_{-0.49-0.51}) \times$  $10^{-4}$  [60]. Fig. 8 shows the constraint from  $BF(B^+ \rightarrow$  $\tau^+ \nu_{\tau}$ ) in blue and from  $\Delta m_d$  in yellow. The green region represents the 95 % CL region when both inputs are combined illustrating the correlation between  $BF(B^+ \to \tau^+ \nu_{\tau})$  and  $\Delta m_d$  due to the decay constant  $f_B$ . At present, the precision of  $BF(B^+ \rightarrow \tau^+ \nu_{\tau})$ is not sufficient to compete with the combined SM CKM fit. In Fig. 9 are shown the constraints from  $BF(B^+ \to \tau^+ \nu_{\tau})$  on tan  $\beta$  and  $m_{H^+}$  in a Two-Higgs-Doublet model of type II using the values for  $f_B = f_{B_d}$ and  $|V_{ub}|$  as quoted in Sec. 2.

# 5. Constraints on New Physics in $B - \overline{B}$ mixing

When performing a combined fit without using  $\epsilon_K$ as an input and allowing for NP in  $B_d$  and  $B_s$  mixing one obtains the constraints on the NP parameters  $r_q^2$  and  $2\theta_q$  as shown in Figs. 10 and 11. For both



Figure 8: Constraints on  $\overline{\rho}$  and  $\overline{\eta}$  from the world averages of  $BF(B^+ \to \tau^+ \nu_{\tau})$  (blue) and  $\Delta m_d$  (yellow) and when both are combined (green).



Figure 9: Constraints from  $BF(B^+ \to \tau^+ \nu_{\tau})$  on  $\tan \beta$ and  $m_{H^+}$  in a Two-Higgs-Doublet model of type II using the values for  $f_B = f_{B_d}$  and  $|V_{ub}|$  as quoted in Sec. 2.

neutral *B*-meson systems the SM point shows decent CL with the current data although the best fit values in the  $B_d$  case prefer small negative  $2\theta_d$  values mainly caused by the inputs from  $|V_{ub}|$  and  $\sin(2\beta)$ . Also  $r_d^2$  values smaller than 1 are preferred due to the slight discrepancy between the  $\alpha$  constraint and the  $\alpha$  SM prediction. The constraints on  $r_d^2$  and  $2\theta_d$ profit from the inputs for  $A_{SL}^d$  and  $A_{SL}$  highlighting the importance of the D0 measurement and the nice

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Figure 10: Constraints on the NP parameters  $r_d^2$  and  $2\theta_d$  from a combined fit allowing for NP in  $B_d$  and  $B_s$  mixing using all inputs listed in Table 2.



Figure 11: Constraints on the NP parameters  $r_s^2$  and  $2\theta_s$  from a combined fit allowing for NP in  $B_d$  and  $B_s$  mixing using all inputs listed in Table 2.

interplay between these inputs from the *B*-factories on one hand and the Tevatron on the other hand. The preferred large negative values for both,  $A_{SL}^d$  and  $A_{SL}$ , disfavour large positive  $2\theta_d$  values and at the same time  $r_d^2$  values smaller than 1. This is best illustrated in Fig. 12 where all inputs but  $A_{SL}^d$  and  $A_{SL}$ are used. In this case a second allowed region appears which is inconsistent with the SM point. The double-peak structure observed is caused by the cor-



Figure 12: Constraints on the NP parameters  $r_d^2$  and  $2\theta_d$  from a combined fit allowing for NP in  $B_d$  and  $B_s$  mixing using all inputs except  $A_{SL}^d$  and  $A_{SL}$ .

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a possible significant deviation from  $2\theta_d = 0$  in the future depends significantly on the central value and and the prospects for the uncertainty on  $\sin(2\beta)$  and, more importantly, on  $|V_{ub}|$ . To reduce the allowed size for  $r_d^2$  requires improved precision on  $\gamma$  and/or  $\alpha$ , but even more important is an improvement for (LQCD) calculations of  $f_{B_d}\sqrt{B_d}$ .

The allowed range in  $r_s^2$  is significantly smaller than the one for  $r_d^2$  thanks to the more precise value for  $f_{B_s}\sqrt{B_s}$  compared to  $f_{B_d}\sqrt{B_d}$ . Currently, there is no exclusion better than at 90 % CL for the NP phase  $2\theta_s$ . The fact that values for  $2\theta_s$  around  $\pm 90^\circ$  are disfavoured are due to the fact that the measured value for  $\Delta \Gamma_s^{CP'}$  is larger than the SM prediction while NP ( $\propto \cos^2 2\theta_s$ ) can only lower the measured value with respect to the SM prediction. With improved measurements of  $\Delta \Gamma_s^{CP'}$  and/or  $\Delta \Gamma_s$  from Tevatron and eventually from LHCb the SM value  $2\theta_s = 0$  might be excluded. However, the best sensitivity to  $2\theta_s$  will only come from a tagged time-dependent angular analysis of the decay  $B_s \rightarrow \psi \phi$  on a high statistics sample eventually carried out by LHCb and possibly also by ATLAS and CMS.

The preferred value in  $r_s^2$  is smaller than 1 although the difference in CL with respect to  $r_s^2 = 1$  is small. The current situation with preferred values for both,  $r_d^2$  and  $r_s^2$ , equal but smaller than 1 is consistent with a Minimal Flavour Violation scenario.

Constraints in  $h_d$  and  $2\sigma_d$ , respectively,  $h_s$  and  $2\sigma_s$  corresponding to Figs. 10 and 11. are shown in Figs. 13 and 14.

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responding structure in the  $\alpha$  input. To pin down



Figure 13: Constraints on the NP parameters  $h_d$  and  $2\sigma_d$  from a combined fit allowing for NP in  $B_d$  and  $B_s$  mixing using all inputs listed in Table 2.



Figure 14: Constraints on the NP parameters  $h_s$  and  $2\sigma_s$  from a combined fit allowing for NP in  $B_d$  and  $B_s$  mixing using all inputs listed in Table 2.

Similar in line, the constraint on  $h_K$  and  $2\sigma_K$  from a combined analysis in the  $K-\bar{K}$ -,  $B_d-\bar{B}_d$ - and  $B_s-\bar{B}_s$ system as presented in Ref. [10] shows that all  $2\sigma_K$ values are allowed. Except for  $2\sigma_K$  around 70° and 155° the parameter  $2\sigma_K$  is constrained to be smaller than  $\approx 0.55$  at 95 % CL. According to the conclusions of Ref. [16] the current constraints on the  $K - \bar{K}$ -,  $B_d - \bar{B}_d$ - and  $B_s - \bar{B}_s$ -system are still compatible with a NMFV scenario since the sufficiently large  $h_q$  values (q = K, d, s) are still allowed <sup>6</sup>.

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