# Factorization Approaches to $B$ Meson Decays 

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#### Abstract

We compare the theoretical frameworks and the phenomenological applications of the factorization approaches to exclusive $B$ meson decays, which include QCD-improved factorization, perturbative QCD, and soft-collinear effective theory. Recent progress on two-body nonleptonic $B$ meson decays made in these approaches are reviewed.


## 1. Introduction

"Factorizations" in the naive factorization assumption and in factorization theorem have very different meanings. The former refers to the factorization of a process into subprocesses. For example, a $B$ meson decay amplitude $A\left(B \rightarrow M_{1} M_{2}\right)$ is written, in the factorization assumption, as the product [1],

$$
\begin{equation*}
A\left(B \rightarrow M_{1} M_{2}\right) \propto f_{M_{2}} F^{B M_{1}} \tag{1}
\end{equation*}
$$

where the meson decay constant $f_{M_{2}}$ arises from the production of the meson $M_{2}$ from the vacuum, and the form factor $F^{B M_{1}}$ is associated with the $B \rightarrow$ $M_{1}$ transition. The latter refers to the factorization of perturbative and nonperturbative dynamics in a QCD process. According to factorization theorem, the above amplitude is expressed as

$$
\begin{equation*}
A\left(B \rightarrow M_{1} M_{2}\right) \propto \phi_{B} \otimes H \otimes \phi_{M_{1}} \otimes \phi_{M_{2}} \tag{2}
\end{equation*}
$$

where $\otimes$ denotes the convolution over parton kinematic variables, the hard kernel $H$ absorbs perturbative dynamics, and the $B\left(M_{1}, M_{2}\right)$ meson distribution amplitude $\phi_{B}\left(\phi_{M 1}, \phi_{M 2}\right)$ absorbs nonperturbative dynamics in the $B \rightarrow M_{1} M_{2}$ decay. A piece of contribution to $B$ meson decays is factorizable, if it respects either Eq. (11) in the sense of the factorization assumption, or Eq. (2) in the sense of factorization theorem. Below we shall use the terms "factorizable" and "nonfactorizable" without specifying which sense it refers to.

## 2. Collinear vs. $k_{T}$ Factorization

Both collinear and $k_{T}$ factorizations are the fundamental tools of perturbative QCD, where $k_{T}$ denotes parton transverse momenta. We first explain these two types of theorems by considering the simplest scattering process $\pi\left(P_{1}\right) \gamma^{*} \rightarrow \gamma\left(P_{2}, \epsilon\right)$ as an example. The momentum $P_{1}$ of the pion and the momentum $P_{2}$ of the out-going on-shell photon are chosen as

$$
\begin{equation*}
P_{1}=\left(P_{1}^{+}, 0, \mathbf{0}_{T}\right), \quad P_{2}=\left(0, P_{2}^{-}, \mathbf{0}_{T}\right) \tag{3}
\end{equation*}
$$

The leading-order ( LO ) quark diagram, in which the anti-quark $\bar{q}$ carries the on-shell fractional momentum $k=\left(x P_{1}^{+}, 0, \mathbf{0}_{T}\right)$ and the internal quark carries $P_{2}-k$, leads to the amplitude,

$$
\begin{align*}
G^{(0)}\left(x, Q^{2}\right) & =\frac{\operatorname{tr}\left[k\left(\not P_{2}-k\right) \gamma_{\mu} \not P_{1} \gamma_{5}\right]}{\left(P_{2}-k\right)^{2}} \\
& =-\frac{\operatorname{tr}\left[k P_{2} \gamma_{\mu} \not P_{1} \gamma_{5}\right]}{x Q^{2}} \tag{4}
\end{align*}
$$

with the leading spin structure $P_{1} \gamma_{5}$ of the pion and the momentum transfer squared $Q^{2} \equiv 2 P_{1} \cdot P_{2}$. We have suppressed other constant factors, such as the electric charge, the color number, and the pion decay constant, which are irrelevant in the following discussion.

The trivial collinear factorization of Eq. (4) reads,

$$
\begin{align*}
G^{(0)}\left(x, Q^{2}\right) & =\int d x^{\prime} \phi^{(0)}\left(x ; x^{\prime}\right) H^{(0)}\left(x^{\prime}, Q^{2}\right) \\
\phi^{(0)}\left(x ; x^{\prime}\right) & =\delta\left(x-x^{\prime}\right) \\
H^{(0)}\left(x, Q^{2}\right) & =-\frac{\operatorname{tr}\left[k P_{2} \gamma_{\mu} P_{1} \gamma^{5}\right]}{x Q^{2}} \tag{5}
\end{align*}
$$

The zeroth-order distribution amplitude $\phi^{(0)}$ is proportional to $\delta\left(x-x^{\prime}\right)$, implying that the parton entering the LO hard kernel $H^{(0)}$ carries the same momentum as the parton entering the distribution amplitude does. The trivial $k_{T}$ factorization of Eq. (4) reads 2],

$$
\begin{align*}
G^{(0)}\left(x, Q^{2}\right)= & \int d x^{\prime} d^{2} k_{T}^{\prime} \Phi^{(0)}\left(x ; x^{\prime}, k_{T}^{\prime}\right) \\
& \times H^{(0)}\left(x^{\prime}, Q^{2}, k_{T}^{\prime}\right) \\
\Phi^{(0)}\left(x ; x^{\prime}, k_{T}^{\prime}\right)= & \delta\left(x-x^{\prime}\right) \delta\left(\mathbf{k}_{T}^{\prime}\right) \\
H^{(0)}\left(x, Q^{2}, k_{T}\right)= & -\frac{\operatorname{tr}\left[k P_{2} \gamma_{\mu} \not P_{1} \gamma^{5}\right]}{x Q^{2}+k_{T}^{2}} \tag{6}
\end{align*}
$$

Because of the zeroth-order wave function $\Phi^{(0)} \propto$ $\delta\left(\mathbf{k}_{T}^{\prime}\right), H^{(0)}$ does not depend on the parton transverse momentum actually.

The $O\left(\alpha_{s}\right)$ quark diagrams for Eq. (4) from full QCD are displayed in Fig. 11 in which the upper line represents the $q$ quark. The collinear factorization of


Figure 1: $O\left(\alpha_{s}\right)$ quark diagrams for $\pi \gamma^{*} \rightarrow \gamma$ with $\times$ representing the virtual photon vertex.
these radiative corrections is given by

$$
\begin{align*}
G^{(1)}\left(x, Q^{2}\right)= & \int d x^{\prime} \phi^{(1)}\left(x ; x^{\prime}\right) H^{(0)}\left(x^{\prime}, Q^{2}\right) \\
& +H^{(1)}\left(x^{\prime}, Q^{2}\right) \tag{7}
\end{align*}
$$

where the first-order distribution amplitude $\phi^{(1)}$ is defined by the effective diagrams in Fig. [2 [3]. Expressions from Figs. 2( c ), 2(e), and 2(f) are proportional to $\delta\left(x-x^{\prime}-l^{+} / P_{1}^{+}\right)$, where $l$ is the loop momentum carried by the collinear gluon. The $\delta$-function indicates that the exchange of the collinear gluon modifies the momentum fraction of the parton entering $H^{(0)}$ from $x$ to $x-l^{+} / P_{1}^{+}$. The $k_{T}$ factorization of Fig. 1 leads to 2]

$$
\begin{align*}
G^{(1)}\left(x, Q^{2}\right)= & \int d x^{\prime} d^{2} k_{T}^{\prime} \Phi^{(1)}\left(x ; x^{\prime}, k_{T}^{\prime}\right)  \tag{8}\\
& \times H^{(0)}\left(x^{\prime}, Q^{2}, k_{T}^{\prime}\right)+H^{(1)}\left(x, Q^{2}\right),
\end{align*}
$$

where the first-order wave function $\Phi^{(1)}$ from Figs. $2(\mathrm{c}), 2(\mathrm{e})$, and $2(\mathrm{f})$ is proportional to $\delta\left(x-x^{\prime}-\right.$ $\left.l^{+} / P_{1}^{+}\right) \delta\left(\mathbf{k}_{T}^{\prime}+\mathbf{l}_{T}\right)$. In this case the collinear gluon exchange modifies both the longitudinal and transverse parton momenta flowing into the hard kernel.


Figure 2: $O\left(\alpha_{s}\right)$ effective diagrams for the pion wave function.

It is observed that $H^{(0)}$ in Eq. (8) depends on $k_{T}^{\prime}$ nontrivially in the first-order $k_{T}$ factorization. Being
convoluted with $\Phi^{(0)}$, the partons entering the next-to-leading (NLO) hard kernel $H^{(1)}$ are still on-shell. To acquire a nontrivial $k_{T}$ dependence, $H^{(1)}$ must be convoluted with the higher-order wave functions $\Phi^{(i)}$, $i \geq 1$ : the gluon exchanges in $\Phi^{(i)}$ render the incoming partons of $H^{(1)}$, ie., the incoming partons of the quark diagrams $G^{(1)}$ and the effective diagrams $\Phi^{(1)}$ off-shell by $k_{T}^{2}$ [4]. We thus derive $H^{(1)}\left(x, Q^{2}, k_{T}\right)$ according to the formula,

$$
\begin{align*}
& H^{(1)}\left(x, Q^{2}, k_{T}\right)=G^{(1)}\left(x, Q^{2}, k_{T}\right)  \tag{9}\\
& -\int d x^{\prime} d^{2} k_{T}^{\prime} \Phi^{(1)}\left(x, k_{T} ; x^{\prime}, k_{T}^{\prime}\right) H^{(0)}\left(x^{\prime}, Q^{2}, k_{T}^{\prime}\right)
\end{align*}
$$

with the $\bar{q}$ quark carrying the momentum $k=$ $\left(x P_{1}^{+}, 0, \mathbf{k}_{T}\right)$. This is the way to obtain a $k_{T^{-}}$ dependent hard kernel without breaking gauge invariance, since the gauge dependences cancel between $G^{(1)}$ and $\Phi^{(1)}$. A physical quantity is wtitten as a convolution of a hard kernel with model wave functions, which are determined by methods beyond a perturbation theory, such as lattice QCD and QCD sum rules, or extracted from experimental data. A gauge-invariant hard kernel then leads to gauge-invariant predictions from $k_{T}$ factorization.

## 3. $B$ Decays in QCDF, SCET $_{\phi}$, and PQCD

Factorization theorems have been applied to exclusive $B$ meson decays, and different approaches have been developed. Below we compare the frameworks of perturbatiove QCD (PQCD) [5, 6, 7], QCD-improved factorization (QCDF) [8], and soft-collinear effective theory (SCET) [9, 10]. The $B \rightarrow \pi$ transition form factor $F^{B \pi}$ involved in the semileptonic decay $B \rightarrow$ $\pi l \nu$ is expressed, in collinear factorization, as

$$
\begin{equation*}
F^{B \pi}=\int d x_{1} d x_{2} \phi_{B}\left(x_{1}\right) H\left(x_{1}, x_{2}\right) \phi_{\pi}\left(x_{2}\right) \tag{10}
\end{equation*}
$$

with the LO hard kernel $H^{(0)} \propto\left(1+2 x_{2}\right) /\left(x_{1} x_{2}^{2}\right)$. The parton momentum fractions $x_{1}$ and $x_{2}$ are carried by the spectator quarks on the $B$ meson and pion sides, respectively. Obviously, the above integral is logarithmically divergent for the asymptotic model $\phi_{\pi} \propto x(1-x)$ 11].

An end-point singularity implies that exclusive $B$ meson decays are dominated by soft dynamics. That is, a heavy-to-light form factor is not calculable in collinear factorization, and $F^{B \pi}$ should be treated as a soft object [8]. This is the basis of QCDF, and subleading corrections are added systematically [12]. The above treatment has been further elucidated in the framework of SCET [13]: only the 1 term in $H^{(0)}$ contains the end-point singularity, which leads to an $O(\Lambda)$, ie., soft object $f^{\mathrm{NF}}$. The $2 x_{2}$ term does not, leading to an $O\left(\sqrt{m_{B} \Lambda}\right)$ object $f^{\mathrm{F}}$ with the $B$ meson
mass $m_{B}$. Therefore, at leading power in $1 / m_{B}$, the $B \rightarrow \pi$ form factor can be split into the nonfactorizable and factorizable components,

$$
\begin{equation*}
F^{B \pi}=f^{\mathrm{NF}}+f^{\mathrm{F}} \tag{11}
\end{equation*}
$$

which have different power counting in the strong coupling constant $\alpha_{s}$ : the former is of $O\left(\alpha_{s}^{0}\right)$ and the latter of $O\left(\alpha_{s}\right)$. The values of $f^{\mathrm{NF}}$ and $f^{\mathrm{F}}$ have been determined from a fit to the $B \rightarrow \pi \pi$ data [14].

The formulation of the $B \rightarrow \pi$ transition in $k_{T}$ factorization theorem is different. When the parton transverse momenta are included, $f^{\mathrm{NF}}$ does not develop an end-point singularity, and both $f^{\mathrm{NF}}$ and $f^{\mathrm{F}}$ are factorizable. Hence, they are of the same order in $\alpha_{s}$, and can be combined into a single term, giving [5, 15],

$$
\begin{align*}
F^{B \pi} & =\int d x_{1} d x_{2} d^{2} k_{1 T} d^{2} k_{2 T} \Phi_{B}\left(x_{1}, k_{1 T}\right) \\
& \times H\left(x_{1}, x_{2}, k_{1 T}, k_{2 T}\right) \Phi_{\pi}\left(x_{2}, k_{2 T}\right) \tag{12}
\end{align*}
$$

The end-point singularity is smeared into the large logarithm $\ln ^{2}\left(m_{B} / k_{T}\right)$, and absorbed into the pion wave function $\Phi_{\pi}$. Resumming this logarithm to all orders in the conjugate $b$ space [16, 17], we derived the Sudakov factor $S\left(m_{B}, b\right)$, which describes the parton distribution in $b$. Since $f^{\mathrm{NF}}$ has been included, the large-energy symmetry [12] is respected in PQCD. Recently, it was proposed that the end-point singularity is attributed to a double counting of soft degrees of freedom in collinear factorization [18]. The zero-bin subtraction removes the double counting, and leads to a modified SCET formalism for $f^{\mathrm{NF}}$, labelled by $\mathrm{SCET}_{\phi}$ hereafter. The power counting of $\mathrm{SCET}_{\phi}$ in both $1 / m_{B}$ and $\alpha_{s}$ is then consistent with that of PQCD. The regularization of the end-point singularity introduces the logarithms $\ln \mu_{ \pm}$in $\mathrm{SCET}_{\phi}$ [18], whose treatment has not yet been explained.

When applying the above factorization approaches to two-body nonleptonic $B$ meson decays, further difference appears in the treatment of annihilation amplitudes. The $O\left(\alpha_{s} m_{0} / m_{B}\right)$ annihilation amplitudes from the scalar penguin operators are divergent in collinear factorization, where $m_{0}$ is the chiral enhancement scale. Because of the end-point singularity, an annihilation amplitude has been parameterized as

$$
\begin{equation*}
\alpha_{s} \ln \frac{m_{B}}{\Lambda}\left(1+\rho_{A} e^{i \delta_{A}}\right) \tag{13}
\end{equation*}
$$

in QCDF [8], where $\Lambda$ is a hadronic scale and the free parameter $\rho_{A}$ is postulated to vary in the range $0 \leq \rho_{A} \leq 1$. It is not clear what mechanism generates the strong phase $\delta_{A}$. With the similar zerobin subtraction, an annihilation amplitude is factorizable in $\mathrm{SCET}_{\varphi}$, but found to be almost real [19]. A strong phase can only be generated at loop level, ie., at $O\left(\alpha_{s}^{2} \Lambda / m_{B}\right)$. However, we notice that the residual
momentum carried by the $b$ quark in a nonfactorizable annihilation amplitude could result in a strong phase of $O\left(\alpha_{s} m_{0} \Lambda / m_{B}^{2}\right)$ [20], a new mechanism not included in [19].

The scalar penguin annihilation amplitude is also factorizable in $k_{T}$ factorization with the absence of the end-point singularity. Furthermore, it was almost imaginary in PQCD [6], whose corresponding mechanism is similar to the Bander-Silverman-Soni one [21]: when the $u$ or $c$ quark in a loop goes on mass shell, a strong phase is produced. In the case of the annihilation topology for heavy-to-light decays, the loop is formed by the virtual particles in the LO PQCD hard kernel and the infinitely many Sudakov gluons exchanged between two partons in a light meson. The virtual particle acquires the transverse momentum $k_{T}$ through the Sudakov gluon exchange. A sizable strong phase is then given, in terms of the principle-value prescription for the virtual particle propagator, by

$$
\begin{equation*}
\frac{1}{x m_{B}^{2}-k_{T}^{2}+i \epsilon}=\frac{P}{x m_{B}^{2}-k_{T}^{2}}-i \pi \delta\left(x m_{B}^{2}-k_{T}^{2}\right) \cdot(1 \tag{14}
\end{equation*}
$$

Therefore, the treatment and the effect of the scalar penguin annihilation amplitude are very different in QCDF, $\mathrm{SCET}_{\varphi}$, and PQCD.

Though the scalar penguin annihilation amplitude is factorizable in both PQCD and $\mathrm{SCET}_{\phi}$, it is almost imaginary in the former, but real in the latter. We argue that the above different opinions can be discriminated by comparing the direct CP asymmetries in the charged $B$ meson decays $B^{ \pm} \rightarrow K^{ \pm} \pi^{0}$ and $B^{ \pm} \rightarrow K^{ \pm} \rho^{0}$. The $B^{ \pm} \rightarrow K^{ \pm} \pi^{0}$ decays involve a $B$ meson transition to a pseudoscalar meson, so the penguin emission amplitude is proportional to the constructive combination of the Wilson coefficients $a_{4}+2\left(m_{0 K} / m_{B}\right) a_{6}$, where $m_{0 K}$ is the chiral enhancement scale associated with the kaon. The $B^{ \pm} \rightarrow K^{ \pm} \rho^{0}$ decays involve a $B$ meson transition to a vector meson, so the penguin emission amplitude is proportional to the destructive combination $a_{4}-2\left(m_{0 K} / m_{B}\right) a_{6}$. The annihilation effect is then less influential in the former than in the latter. If the scalar penguin annihilation is real, both decays will exhibit small direct CP asymmetries, ie., $A_{C P}\left(B^{ \pm} \rightarrow K^{ \pm} \pi^{0}\right) \approx A_{C P}\left(B^{ \pm} \rightarrow K^{ \pm} \rho^{0}\right)$. If the scalar penguin annihilation is imaginary, it will cause a larger direct CP asymmetry in $B^{ \pm} \rightarrow K^{ \pm} \rho^{0}$, ie., $A_{C P}\left(B^{ \pm} \rightarrow K^{ \pm} \pi^{0}\right) \ll A_{C P}\left(B^{ \pm} \rightarrow K^{ \pm} \rho^{0}\right)$. The current data $A_{C P}\left(B^{ \pm} \rightarrow K^{ \pm} \pi^{0}\right)=0.047 \pm 0.026$ and $A_{C P}\left(B^{ \pm} \rightarrow K^{ \pm} \rho^{0}\right)=0.31_{-0.10}^{+0.11}$ [22] seem to prefer an imaginary scalar penguin annihilation.

## 4. Recent Results

### 4.1. The $B \rightarrow \pi \pi$ Puzzle

According to a naive estimate of the colorsuppressed tree amplitude, the hierarchy of the branching ratios $B\left(B^{0} \rightarrow \pi^{0} \pi^{0}\right) \sim O\left(\lambda^{2}\right) B\left(B^{0} \rightarrow\right.$ $\left.\pi^{\mp} \pi^{ \pm}\right)$is expected. However, the data [22]

$$
\begin{align*}
B\left(B^{0} \rightarrow \pi^{\mp} \pi^{ \pm}\right) & =(5.2 \pm 0.2) \times 10^{-6} \\
B\left(B^{0} \rightarrow \pi^{0} \pi^{0}\right) & =(1.31 \pm 0.21) \times 10^{-6} \tag{15}
\end{align*}
$$

show $B\left(B^{0} \rightarrow \pi^{0} \pi^{0}\right) \sim O(\lambda) B\left(B^{0} \rightarrow \pi^{\mp} \pi^{ \pm}\right)$, giving rise to the $B \rightarrow \pi \pi$ puzzle. As observed in [23], the NLO corrections, despite of increasing the color-suppressed tree amplitude significantly, are not enough to enhance the $B^{0} \rightarrow \pi^{0} \pi^{0}$ branching ratio to the measured value. A much larger color-suppressed tree amplitude, about the same order as the colorallowed tree amplitude, must be obtained in order to resolve the puzzle [24, 25]. To make sure that the above NLO effects are reasonable, the PQCD formalism has been applied to the $B \rightarrow \rho \rho$ decays [23], which also receive the color-suppressed tree contribution. It was found that the NLO PQCD predictions are in agreement with the data $B\left(B^{0} \rightarrow \rho^{0} \rho^{0}\right)=$ $(1.16 \pm 0.46) \times 10^{-6}$ [22]. We conclude that it is unlikely to accommodate the measured $B^{0} \rightarrow \pi^{0} \pi^{0}$ and $\rho^{0} \rho^{0}$ branching ratios simultaneously in PQCD, and that the $B \rightarrow \pi \pi$ puzzle remains.

It has been claimed that the $B \rightarrow \pi \pi$ puzzle is resolved in the QCDF approach [8] with an input from SCET [26, 27, 28]: the inclusion of the NLO jet function, the hard coefficient of $\mathrm{SCET}_{\mathrm{II}}$, into the QCDF formula for the color-suppressed tree amplitude gives sufficient enhancement of the $B^{0} \rightarrow \pi^{0} \pi^{0}$ branching ratio, if adopting the parameter scenario "S4" 29]. It is necessary to investigate whether the proposed new mechanism deteriorates the consistency of theoretical results with other data. The formalism in [26] has been extended to the $B \rightarrow \rho \rho$ decays as a check [23]. It was found that the NLO jet function overshoots the observed $B^{0} \rightarrow \rho^{0} \rho^{0}$ branching ratio very much as adopting "S4". That is, it is also unlikely to accommodate the $B \rightarrow \pi \pi$ and $\rho \rho$ data simultaneously in QCDF.

### 4.2. The $B \rightarrow \phi K^{*}$ Puzzle

For penguin-dominated $B \rightarrow V V$ decays, such as those listed in Table प 22], the polarization fractions deviate from the naive counting rules based on kinematics [30]. This is the so-called the $B \rightarrow \phi K^{*}$ puzzle. Many attempts to resolve the $B \rightarrow \phi K^{*}$ polarizations have been made 31], which include new physics 32, 33, 34, 35, 36], the annihilation contribution 37, 38] in the QCDF approach, the charming penguin in SCET [14], the rescattering effect [39, 40, 41], and

Table I Polarization fractions in the penguin-dominated $B \rightarrow V V$ decays.

| Mode | Belle | BaBar |
| :---: | :---: | :---: |
| $\phi K^{*+}$ | $0.52 \pm 0.08 \pm 0.03$ | $0.46 \pm 0.12 \pm 0.03$ |
| $\phi K^{* 0}$ | $0.45 \pm 0.05 \pm 0.02$ | $0.52 \pm 0.05 \pm 0.02$ |
| $K^{*+} \rho^{0}$ |  | $0.9 \pm 0.2[44]$ |
| $K^{* 0} \rho^{+}$ | $0.43 \pm 0.11_{-0.02}^{+0.05}$ | $0.52 \pm 0.10 \pm 0.04[44]$ |

the $b \rightarrow s g$ (the magnetic penguin) 42] and $b \rightarrow s \gamma$ 43] transitions. The annihilation contribution from the scalar penguin operators improves the consistency with the data, because it is of the same order for all the three final helicity states, and could enhance the transverse polarization fractions [30]. However, the PQCD analysis of the scalar penguin annihilation amplitudes indicates that the $B \rightarrow \phi K^{*}$ puzzle can not be resolved completely [31]. A reduction of the $B \rightarrow K^{*}$ form factor $A_{0}$, which is associated with the longitudinal polarization, further helps accommodating the data [45]. Note that there has not yet been any measurement, which constrains $A_{0}$. Hence, the value of $A_{0}$ should have a large uncertainty.

The penguin-dominated $B \rightarrow K^{*} \rho$ decays are expected to exhibit similar polarization fractions. This is the reason the longitudinal polarization fraction in the $B^{+} \rightarrow K^{* 0} \rho^{+}$decay, which contains only the penguin contribution, is close to $f_{L}\left(\phi K^{*}\right) \sim 0.5$ as listed in Table $\square$ Another mode $B^{+} \rightarrow K^{*+} \rho^{0}$, nevertheless, shows a large longitudinal polarization fraction almost unity. This mode involves tree amplitudes, which are subdominant, and should not cause a significant deviation from $f_{L} \sim 0.5$. Though the data of $f_{L}\left(K^{* 0} \rho^{+}\right)$ from BaBar still suffer a large error [44], the dramatically different longitudinal polarization fractions, $f_{L}\left(K^{*+} \rho^{0}\right) \neq f_{L}\left(K^{* 0} \rho^{+}\right)$, demand a deeper understanding. It is highly suggested to update or perform the measurement of $f_{L}\left(K^{*+} \rho^{0}\right)$. It is also worthwhile to investigate the $B \rightarrow K^{*} K^{*}$ decays [45, 46], whose measurement can help discriminating the various proposals for resolving the $B \rightarrow \phi K^{*}$ puzzle.

### 4.3. The $B \rightarrow K \pi$ Puzzle

The $B^{0} \rightarrow K^{ \pm} \pi^{\mp}$ decays depend on the tree amplitude $T^{\prime}$ and the QCD penguin amplitude $P^{\prime}$. The data of the direct CP asymmetry $A_{C P}\left(B^{0} \rightarrow K^{ \pm} \pi^{\mp}\right) \approx$ $-10 \%$ then imply a sizable relative strong phase between $T^{\prime}$ and $P^{\prime}$, which verifies the LO PQCD prediction made years ago [6]. The $B^{ \pm} \rightarrow K^{ \pm} \pi^{0}$ decays contain the additional color-suppressed tree amplitude $C^{\prime}$ and electroweak penguin amplitude $P_{e w}^{\prime}$. Since both $C^{\prime}$ and $P_{e w}^{\prime}$ are subdominant, the approximate equality for the direct CP asymmetries $A_{C P}\left(B^{ \pm} \rightarrow K^{ \pm} \pi^{0}\right) \approx A_{C P}\left(B^{0} \rightarrow K^{ \pm} \pi^{\mp}\right)$ is ex-
pected. However, this naive expectation is in conflict with the data [22],

$$
\begin{align*}
& A_{C P}\left(B^{0} \rightarrow K^{ \pm} \pi^{\mp}\right)=-0.093 \pm 0.015 \\
& A_{C P}\left(B^{ \pm} \rightarrow K^{ \pm} \pi^{0}\right)=0.047 \pm 0.026 \tag{16}
\end{align*}
$$

leading to one of the $B \rightarrow K \pi$ puzzles.
While LO PQCD gives a negligible $C^{\prime}$ [6, 7], it is possible that this supposedly tiny amplitude receives a significant subleading correction. Note that the small $C^{\prime}$ is attributed to the accidental cancellation between the Wilson coefficients $C_{1}$ and $C_{2} / N_{c}$ at the scale of the $b$ quark mass $m_{b}$. In [47] the important NLO contributions to the $B \rightarrow K \pi$ decays from the vertex corrections, the quark loops, and the magnetic penguins were calculated. It was observed that the vertex corrections increase $C^{\prime}$ by a factor of 3 , and induce a large phase about $-80^{\circ}$ relative to $T^{\prime}$. The large and imaginary $C^{\prime}$ then renders the total tree amplitude $T^{\prime}+C^{\prime}$ more or less parallel to the total penguin amplitude $P^{\prime}+P_{e w}^{\prime}$ in the $B^{ \pm} \rightarrow K^{ \pm} \pi^{0}$ decays, leading to nearly vanishing $A_{C P}\left(B^{ \pm} \rightarrow K^{ \pm} \pi^{0}\right)=\left(-1_{-6}^{+3}\right) \%$ at NLO (it is about $-8 \%$ at LO). We conclude that the $B \rightarrow K \pi$ puzzle has been alleviated, but not yet gone away completely. Whether new physics effects [48, 49] are called for will become clear when the data get precise. More detailed discussion on this subject can be found in [50].

### 4.4. Nonleptonic $B_{s}$ Decays

Two-body nonleptonic $B_{s}$ meson decays are interesting, since their study can test $\mathrm{SU}(3)$ or U-spin symmetry. The framework for these decays is basically identical to that for $B_{u, d}$ meson decays. The results of two-body nonleptonic $B_{s}$ meson decays from different factorization approaches can be found in [29] for QCDF, in [51] for SCET, and in [52, 53] for PQCD. Roughly speaking, the branching ratios predicted by QCDF and by PQCD are similar, but the predicted direct CP asymmetries are usually opposite in sign.

## 5. Conclusion

The factorization approaches are systematic theoretical tools for exclusive $B$ meson decays, in which hadronic inputs are universal, and the hard kernels can be computed order by order. NLO corrections have been obtained for some $B$ meson decay modes, and the consistency between the theoretical predictions and the experimental data is improved in general. More need to be done in order to pin down QCD uncertainty especially for those quantities exhibiting puzzling behaviors. Higher-power corrections are another important source of QCD uncertainty, which deserves a careful investigation. The recent development
in $\mathrm{SCET}_{9}$ is encouraging, whose counting rules become consistent with those in PQCD. However, the arbitrary logarithms $\ln \mu_{ \pm}$resulting from the zero-bin subtraction needs to be handled (recall that the double logarithm $\ln ^{2}\left(m_{B} / k_{T}\right)$ from the smearing of the end-point singularity has been resummed in PQCD). We did not review the progress on the $\Delta S$ puzzle appearing in the extraction of the weak phase $\sin \left(2 \phi_{1}\right)$ from penguin-dominated $B$ meson decays. For the detail, refer to [54].

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## References

[1] M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C 29, 637 (1985); 34, 103 (1987).
[2] M. Nagashima and H-n. Li, Phys. Rev. D 67, 034001 (2003).
[3] H-n. Li, Phys. Rev. D 64, 014019 (2001); M. Nagashima and H-n. Li, Eur. Phys. J. C 40, 395 (2005).
[4] S. Nandi and H-n. Li, arXiv:0704.3790, to appear in Phys. Rev. D.
[5] H-n. Li and H.L. Yu, Phys. Rev. Lett. 74, 4388 (1995); Phys. Lett. B 353, 301 (1995); Phys. Rev. D 53, 2480 (1996).
[6] Y.Y. Keum, H-n. Li, and A.I. Sanda, Phys. Lett. B 504, 6 (2001); Phys. Rev. D 63, 054008 (2001); Y.Y. Keum and H-n. Li, Phys. Rev. D63, 074006 (2001).
[7] C. D. Lü, K. Ukai, and M. Z. Yang, Phys. Rev. D 63, 074009 (2001).
[8] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Phys. Rev. Lett. 83, 1914 (1999); Nucl. Phys. B591, 313 (2000).
[9] C.W. Bauer, S. Fleming, and M. Luke, Phys. Rev. D 63, 014006 (2001).
[10] C.W. Bauer, S. Fleming, D. Pirjol, and I.W. Stewart, Phys. Rev. D 63, 114020 (2001).
[11] A. Szczepaniak, E.M. Henley, and S. Brodsky, Phys. Lett. B 243, 287 (1990).
[12] M. Beneke and T. Feldmann, Nucl. Phys. B592, 3 (2000).
[13] C.W. Bauer, D. Pirjol, and I.W. Stewart, Phys. Rev. D 67, 071502 (2003); R.J. Hill, T. Becher, S.J. Lee, and M. Neubert, JHEP 0407, 081 (2004).
[14] C.W. Bauer, D. Pirjol, I.Z. Rothstein, and I.W. Stewart, Phys. Rev. D 70, 054015 (2004).
[15] T. Kurimoto, H-n. Li, and A.I. Sanda, Phys. Rev D 65, 014007 (2002).
[16] J. Botts and G. Sterman, Nucl. Phys. B325, 62 (1989).
[17] H-n. Li and G. Sterman, Nucl. Phys. B381, 129 (1992).
[18] A.V. Manohar and I.W. Stewart, hep-ph/0605001.
[19] C.M. Arnesen, Z. Ligeti, I.Z. Rothstein, I.W. Stewart, hep-ph/0607001.
[20] J. Chay, H-n. Li, and S. Mishima, in preparation.
[21] M Bander, D. Silverman and A. Soni, Phys. Rev. Lett. 43, 242 (1979).
[22] Heavy Flavor Averaging Group, http://www.slac.stanford.edu/xorg/hfag.
[23] H-n. Li and S. Mishima, Phys. Rev. D 73, 114014 (2006).
[24] Y.Y. Charng and H-n. Li, Phys. Rev. D 71, 014036 (2005).
[25] T.N. Pham, hep-ph/0610063
[26] M. Beneke and D. Yang, Nucl. Phys. B736, 34 (2006).
[27] M. Beneke and S. Jager, hep-ph/0512101.
[28] M. Beneke and S. Jager, Nucl. Phys. B751, 160 (2006).
[29] M. Beneke and M. Neubert, Nucl. Phys. B675, 333 (2003).
[30] C.H. Chen, Y.Y. Keum, and H-n. Li, Phys. Rev. D 66, 054013 (2002).
[31] H-n. Li and S. Mishima, Phys. Rev. D 71, 054025 (2005).
[32] Y. Grossman, Int. J. Mod. Phys. A 19, 907 (2004).
[33] Y.D. Yang, R.M. Wang, and G.R. Lu, Phys. Rev. D 72, 015009 (2005).
[34] P.K. Das and K.C. Yang, Phys. Rev. D 71, 094002 (2005).
[35] C.H. Chen and C.Q. Geng, Phys. Rev. D 71, 115004 (2005).
[36] C.S. Huang, P. Ko, X.H. Wu, and Y.D. Yang, Phys. Rev. D 73, 034026 (2006).
[37] A.L. Kagan, Phys. Lett. B 601, 151 (2004); hep-ph/0407076.
[38] M. Beneke, J. Rohrer, and D. Yang, hep-ph/0612290
[39] P. Colangelo, F. De Fazio, and T.N. Pham, Phys. Lett. B 597, 291 (2004).
[40] M. Ladisa, V. Laporta, G. Nardulli, and P. Santorelli, Phys. Rev. D 70, 114025 (2004).
[41] H.Y. Cheng, C.K. Chua, and A. Soni, Phys. Rev. D 71, 014030 (2005).
[42] W.S. Hou and M. Nagashima, hep-ph/0408007.
[43] M. Beneke, J. Rohrer, and D. Yang, Phys. Rev. Lett. 96, 141801 (2006).
[44] The BABAR Collaboration, B. Aubert, et. al, Phys. Rev. Lett. 97, 201801 (2006).
[45] H-n. Li, Phys. Lett. B 622 63, (2005).
[46] A. Datta et al., arXiv:0705.3915.
[47] H-n. Li, S. Mishima, and A.I. Sanda, Phys. Rev. D 72, 114005 (2005).
[48] A.J. Buras, R. Fleischer, S. Recksiegel, and F. Schwab, Eur. Phys. J. C 45, 701 (2006); R. Fleischer, S. Recksiegel, and F. Schwab, hep-ph/0702275
[49] S. Baek and D. London, arXiv:hep-ph/0701181.
[50] M. Gronau, arXiv:0706.2156, talk presented at Flavor Physics and CP Violation Conference, Bled, 2007.
[51] A.R. Williamson and J. Zupan, Phys. Rev. D 74, 014003 (2006).
[52] Z.J. Xiao, X.F. Chen, and D.Q. Guo, hep-ph/0608222 X.F. Chen, D.Q. Guo, and Z.J. Xiao, hep-ph/0701146.
[53] A. Ali et al., hep-ph/0703162.
[54] J. Zupan, arXiv:0707.1323, talk presented at Flavor Physics and CP Violation Conference, Bled, 2007.

