Theory and Phenomenology of Neutrino Mixing

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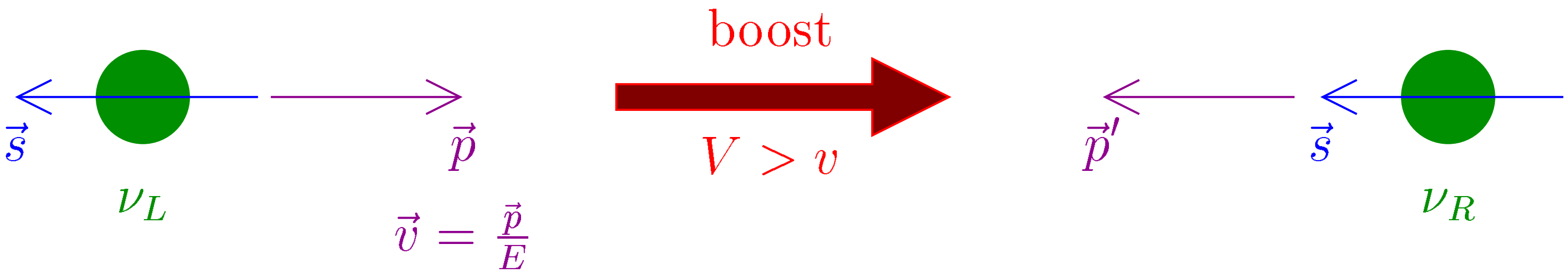
Introduction to Neutrino Masses, Mixing and Oscillations

Solar $\nu_e \to \nu_\mu, \nu_\tau$ + Atmospheric $\nu_\mu \to \nu_\tau \Longrightarrow$ 3-ν Mixing

Absolute Scale of Neutrino Masses

Neutrino Mass



Standard Model: $\nu_L, \nu_R^c \Longrightarrow$ no Dirac mass term $\mathcal{L}^{\mathrm{D}} \sim m^{\mathrm{D}} \left(\overline{\nu_L} \nu_R + \overline{\nu_R} \nu_L \right)$

(no ν_R, ν_L^c)

Majorana Neutrino: $\nu \equiv \nu^c$

$$\nu_R^c \equiv \nu_R \Longrightarrow \text{Majorana mass term } \mathcal{L}^{\mathrm{M}} \sim m^{\mathrm{M}} \left(\overline{\nu_L} \nu_R^c + \overline{\nu_R^c} \nu_L \right)$$

Standard Model: Majorana mass term not allowed by $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$

(no Higgs triplet)

Standard Model can be extended with ν_R $(e_L, e_R;\ u_L, u_R;\ d_L, d_R;\ \ldots)$

$\nu_L + \nu_R \Rightarrow$ Dirac mass term $\mathcal{L}^{\mathrm{D}} \sim m^{\mathrm{D}} \left(\overline{\nu_L} \nu_R + \overline{\nu_R} \nu_L \right) \Rightarrow m^{\mathrm{D}} \lesssim 10^2\,\mathrm{GeV}$

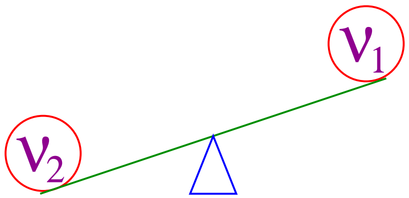
surprise: Majorana mass for ν_R is allowed! $\quad \mathcal{L}_R^{\mathrm{M}} \sim m_R^{\mathrm{M}} \left(\overline{\nu_L^c} \nu_R + \overline{\nu_R} \nu_L^c \right)$

total neutrino mass term $\quad \mathcal{L}^{\mathrm{D+M}} \sim \begin{pmatrix} \overline{\nu_L} & \overline{\nu_L^c} \end{pmatrix} \begin{pmatrix} 0 & m^{\mathrm{D}} \\ m^{\mathrm{D}} & m_R^{\mathrm{M}} \end{pmatrix} \begin{pmatrix} \nu_R^c \\ \nu_R \end{pmatrix}$

m_R^{M} can be arbitrarily large (not protected by SM symmetries)

$m_R^{\mathrm{M}} \sim$ scale of new physics beyond Standard Model $\Rightarrow m_R^{\mathrm{M}} \gg m^{\mathrm{D}}$

diagonalization of $\begin{pmatrix} 0 & m^{\mathrm{D}} \\ m^{\mathrm{D}} & m_R^{\mathrm{M}} \end{pmatrix} \quad \Rightarrow \quad m_1 \simeq \dfrac{(m^{\mathrm{D}})^2}{m_R^{\mathrm{M}}}, \qquad m_2 \simeq m_R^{\mathrm{M}}$



see-saw mechanism

natural explanation of
smallness of neutrino masses

massive neutrinos are Majorana!

[Minkowski, PLB 67 (1977) 42; Yanagida (1979); Gell-Mann, Ramond, Slansky (1979); Mohapatra, Senjanovic, PRL 44 (1980) 912]

Standard Model: Lepton numbers are conserved

	L_e	L_μ	L_τ
$(\nu_e\,,\,e^-)$	$+1$	0	0
$(\nu_\mu\,,\,\mu^-)$	0	$+1$	0
$(\nu_\tau\,,\,\tau^-)$	0	0	$+1$

	L_e	L_μ	L_τ
$(\nu_e^c\,,\,e^+)$	-1	0	0
$(\nu_\mu^c\,,\,\mu^+)$	0	-1	0
$(\nu_\tau^c\,,\,\tau^+)$	0	0	-1

$$\boxed{L = L_e + L_\mu + L_\tau}$$

Dirac mass term $m^{\mathrm{D}}\overline{\nu_L}\nu_R \Rightarrow \begin{pmatrix}\overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}}\end{pmatrix} \begin{pmatrix} m^{\mathrm{D}}_{ee} & m^{\mathrm{D}}_{e\mu} & m^{\mathrm{D}}_{e\tau} \\ m^{\mathrm{D}}_{\mu e} & m^{\mathrm{D}}_{\mu\mu} & m^{\mathrm{D}}_{\mu\tau} \\ m^{\mathrm{D}}_{\tau e} & m^{\mathrm{D}}_{\tau\mu} & m^{\mathrm{D}}_{\tau\tau} \end{pmatrix} \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix}$

L_e, L_μ, L_τ are not conserved, but L is conserved $L(\nu_{\alpha R}) = L(\nu_{\beta L}) \Rightarrow |\Delta L| = 0$

Majorana mass term $m^{\mathrm{M}}\overline{\nu_L}\nu_R^c \Rightarrow \begin{pmatrix}\overline{\nu_{eL}} & \overline{\nu_{\mu L}} & \overline{\nu_{\tau L}}\end{pmatrix} \begin{pmatrix} m^{\mathrm{M}}_{ee} & m^{\mathrm{M}}_{e\mu} & m^{\mathrm{M}}_{e\tau} \\ m^{\mathrm{M}}_{\mu e} & m^{\mathrm{M}}_{\mu\mu} & m^{\mathrm{M}}_{\mu\tau} \\ m^{\mathrm{M}}_{\tau e} & m^{\mathrm{M}}_{\tau\mu} & m^{\mathrm{M}}_{\tau\tau} \end{pmatrix} \begin{pmatrix} \nu^c_{eR} \\ \nu^c_{\mu R} \\ \nu^c_{\tau R} \end{pmatrix}$

L, L_e, L_μ, L_τ are not conserved $L(\nu^c_{\alpha R}) = -L(\nu_{\beta L}) \Rightarrow |\Delta L| = 2$

Majorana Neutrino Mass?

New Physics Beyond the SM $\Longrightarrow$ New High Energy Scale Λ

Effective Low-Energy Lagrangian: $\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{g_5}{\Lambda}\,\mathcal{O}_5 + \frac{g_6}{\Lambda^2}\,\mathcal{O}_6 + \ldots$

[S. Weinberg, Phys. Rev. Lett. 43 (1979) 1566]

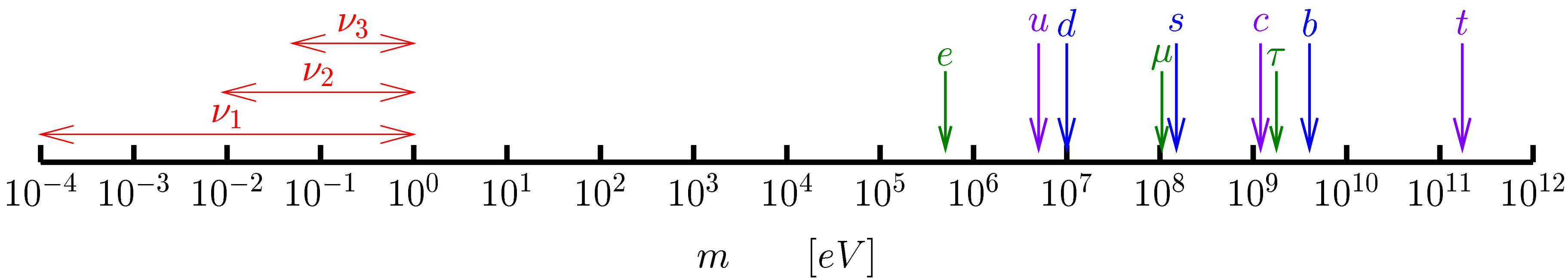
only one 5-D operator: $\mathcal{O}_5 \sim (HL)\,(HL) \to \Lambda_{\text{EW}}^2\,\nu\,\nu$ $\quad |\Delta L| = 2$

Majorana neutrino masses provide the most accessible window on New Physics Beyond the Standard Model

see-saw type generation of light Majorana neutrino masses

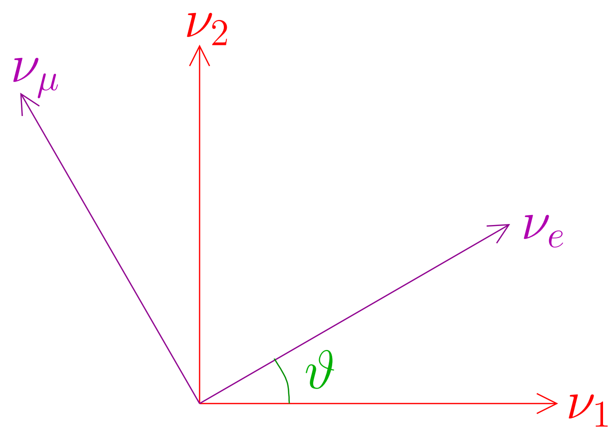
$$m_\nu \propto \frac{\Lambda_{\text{EW}}^2}{\Lambda} = \frac{\Lambda_{\text{EW}}}{\Lambda}\,\Lambda_{\text{EW}} \quad \ll \quad m_{\text{charged fermion}} \propto \Lambda_{\text{EW}} \sim 10^2\ \text{GeV}$$

natural explanation of smallness of ν masses



Two-Neutrino Mixing and Oscillations

$$|\nu_\alpha\rangle = \sum_{k=1}^{2} U_{\alpha k}\,|\nu_k\rangle \qquad (\alpha = e, \mu)$$



$$U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

$$\boxed{\begin{aligned} |\nu_e\rangle &= \cos\vartheta\,|\nu_1\rangle + \sin\vartheta\,|\nu_2\rangle \\ |\nu_\mu\rangle &= -\sin\vartheta\,|\nu_1\rangle + \cos\vartheta\,|\nu_2\rangle \end{aligned}}$$

$$\Delta m^2 \equiv \Delta m^2_{21} \equiv m_2^2 - m_1^2$$

Transition Probability: $$P_{\nu_e\to\nu_\mu} = P_{\nu_\mu\to\nu_e} = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

Survival Probabilities: $$P_{\nu_e\to\nu_e} = P_{\nu_\mu\to\nu_\mu} = 1 - P_{\nu_e\to\nu_\mu}$$

Experimental Evidences of Neutrino Oscillations

Solar

$\nu_e \to \nu_\mu, \nu_\tau$

$\begin{pmatrix} \text{Homestake} \\ \text{Kamiokande} \\ \text{GALLEX/GNO} \\ \text{SAGE} \\ \text{Super-Kamiokande} \end{pmatrix}$

Reactor

$\bar{\nu}_e$ disappearance

(KamLAND)

$\xrightarrow{2\sigma}$

$$\Delta m^2_{\text{SOL}} = 7.92\,(1 \pm 0.09) \times 10^{-5}\,\text{eV}^2$$

$$\sin^2\vartheta_{\text{SOL}} = 0.314\,\left(1^{+0.18}_{-0.15}\right)$$

[Fogli et al, PPNP 57 (2006) 742, hep-ph/0506083]

Atmospheric

$\nu_\mu \to \nu_\tau$

$\begin{pmatrix} \text{Kamiokande} \\ \text{IMB} \\ \text{Super-Kamiokande} \\ \text{MACRO} \\ \text{Soudan-2} \end{pmatrix}$

Accelerator

ν_μ disappearance

(K2K & MINOS)

$\xrightarrow{2\sigma}$

$$\Delta m^2_{\text{ATM}} = 2.6\,\left(1^{+0.14}_{-0.15}\right) \times 10^{-3}\,\text{eV}^2$$

$$\sin^2\vartheta_{\text{ATM}} = 0.45\,\left(1^{+0.35}_{-0.20}\right)$$

[Fogli et al, hep-ph/0608060]

Two scales of Δm^2: $\Delta m^2_{\text{ATM}} \simeq 30\,\Delta m^2_{\text{SOL}}$

Large mixings: $\vartheta_{\text{ATM}} \simeq 45^\circ\,, \quad \vartheta_{\text{SOL}} \simeq 34^\circ$

Three-Neutrino Mixing

$$\nu_{\alpha L} = \sum_{k=1}^{3} U_{\alpha k}\, \nu_{kL} \qquad (\alpha = e, \mu, \tau)$$

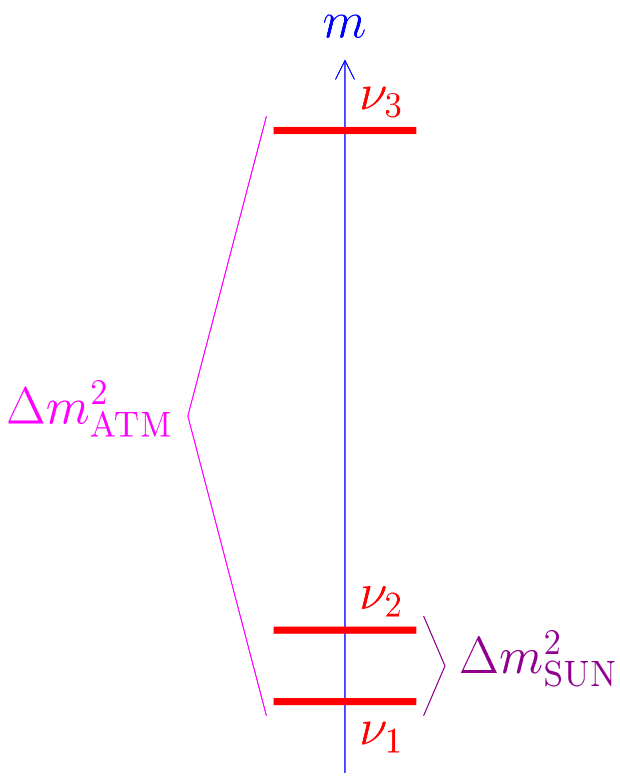
three flavor fields: ν_e, ν_μ, ν_τ

three massive fields: ν_1, ν_2, ν_3

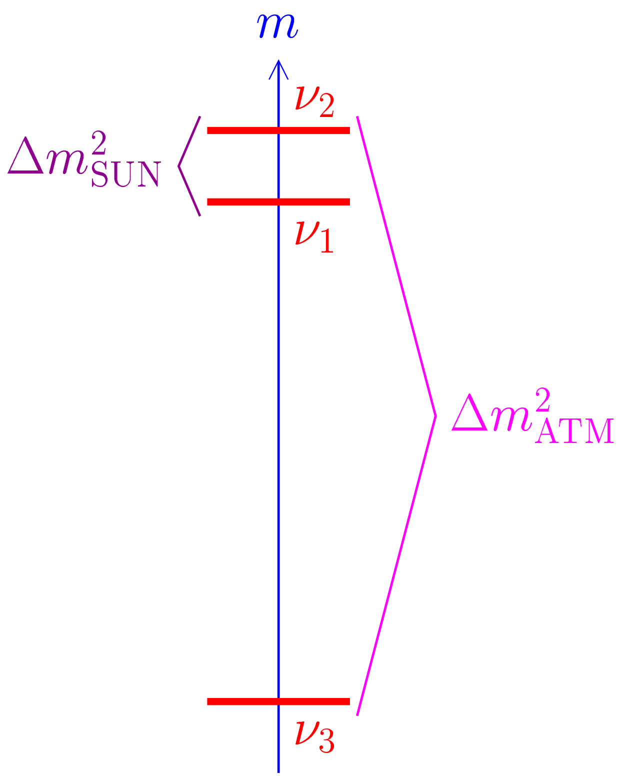
$$\Delta m^2_{\mathrm{SOL}} = \Delta m^2_{21} \simeq 8.0 \times 10^{-5}\,\mathrm{eV}^2$$

$$\Delta m^2_{\mathrm{ATM}} \simeq |\Delta m^2_{31}| \simeq |\Delta m^2_{32}| \simeq 2.5 \times 10^{-3}\,\mathrm{eV}^2$$

Allowed Three-Neutrino Schemes



"normal"



"inverted"

different signs of $\Delta m^2_{31} \simeq \Delta m^2_{32}$

absolute scale is not determined by neutrino oscillation data

Mixing Matrix

$$\boxed{\Delta m^2_{21} \ll |\Delta m^2_{31}|}$$

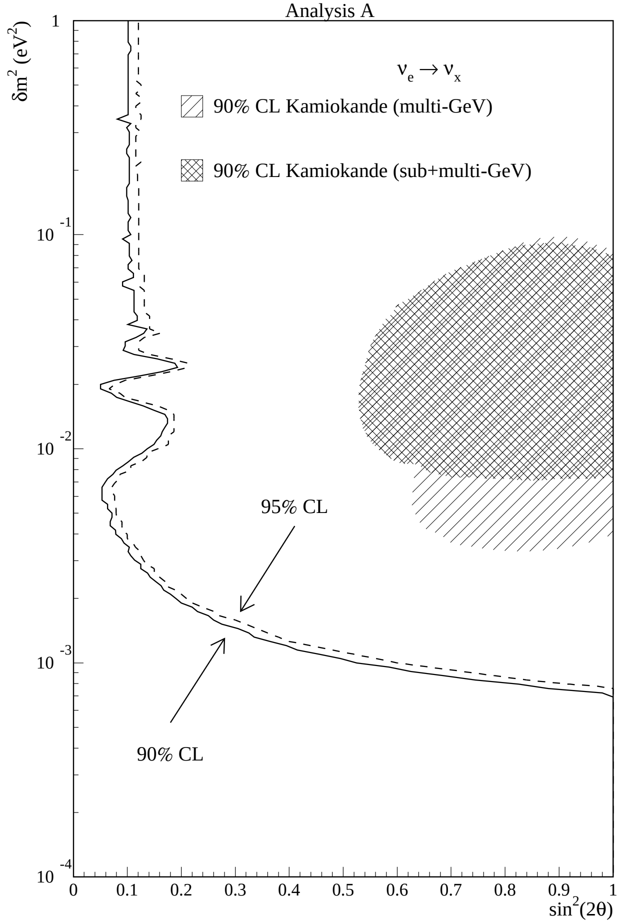
$$\begin{array}{rc} \text{SOL} \rightarrow & \\ & U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \\ & \uparrow \\ & \text{ATM} \end{array}$$

CHOOZ: $\left\{ \begin{array}{l} \Delta m^2_{\text{CHOOZ}} = \Delta m^2_{31} = \Delta m^2_{\text{ATM}} \\ \sin^2 2\vartheta_{\text{CHOOZ}} = 4|U_{e3}|^2(1 - |U_{e3}|^2) \end{array} \right.$

$$\Downarrow$$

$$\boxed{|U_{e3}|^2 < 5 \times 10^{-2} \ (99.73\% \text{ C.L.})}$$

[Fogli et al., PRD 66 (2002) 093008]



[CHOOZ, PLB 466 (1999) 415]

see also [Palo Verde, PRD 64 (2001) 112001]

SOLAR AND ATMOSPHERIC ν OSCILLATIONS ARE PRACTICALLY DECOUPLED!

TWO-NEUTRINO SOLAR and ATMOSPHERIC ν OSCILLATIONS ARE OK!

$$\sin^2 \vartheta_{\text{SOL}} = \frac{|U_{e2}|^2}{1 - |U_{e3}|^2} \simeq |U_{e2}|^2 \qquad \sin^2 \vartheta_{\text{ATM}} = |U_{\mu 3}|^2$$

[Bilenky, C.G, PLB 444 (1998) 379]

[Guo, Xing, PRD 67 (2003) 053002]

Standard Parameterization of Mixing Matrix

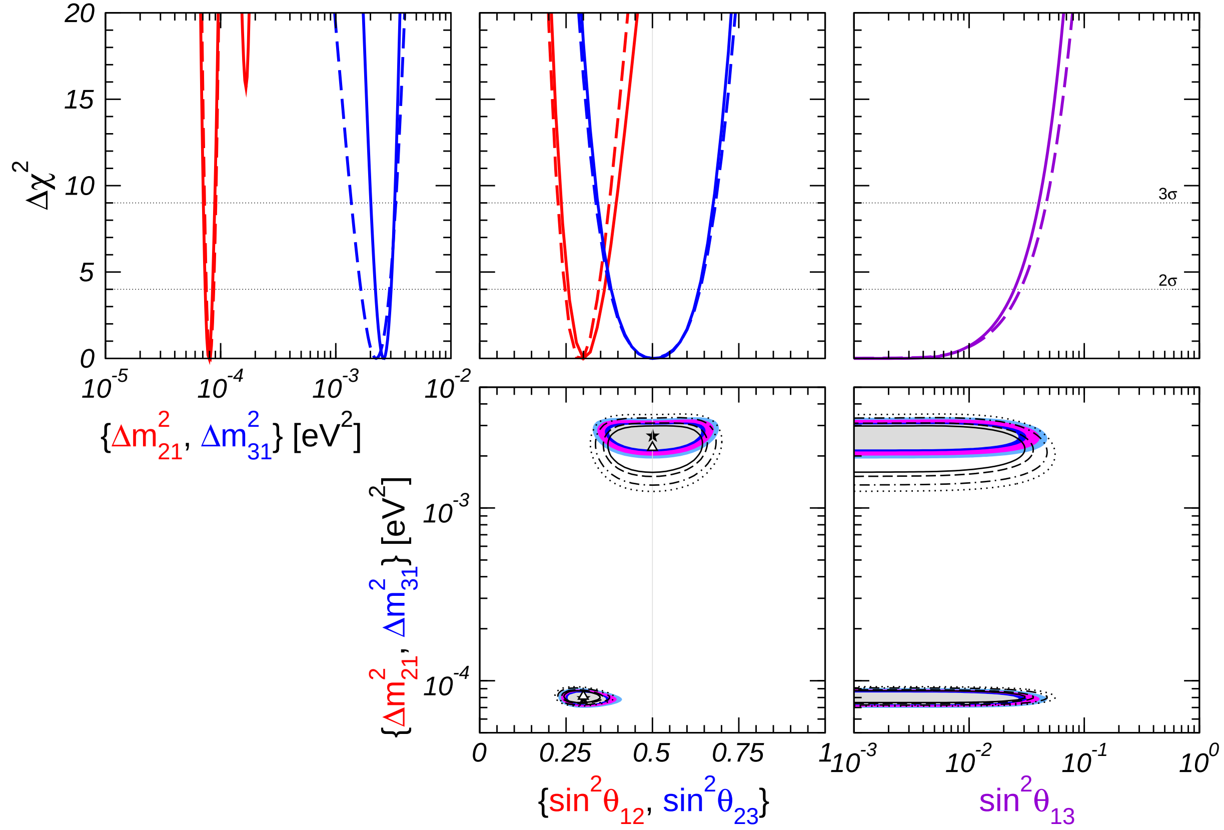
$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\vartheta_{23} \simeq \vartheta_{\text{ATM}}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix}}_{\vartheta_{13} \simeq \vartheta_{\text{CHOOZ}}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\vartheta_{12} \simeq \vartheta_{\text{SOL}}} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}}_{\beta\beta_{0\nu}}$$

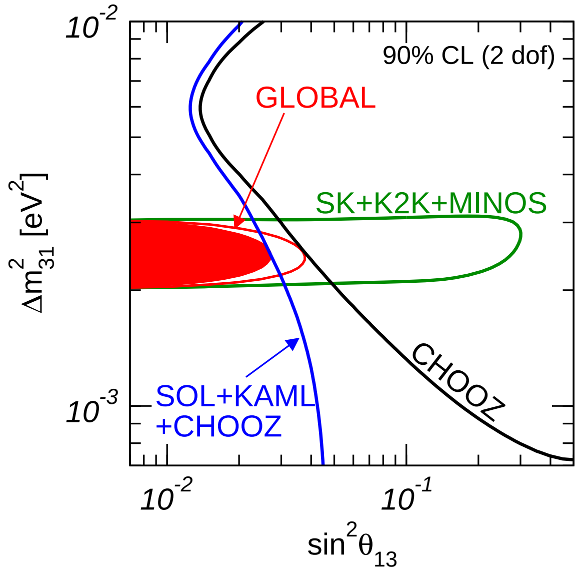
$$= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

$$\text{CHOOZ} + \text{SK} + \text{MINOS} \Longrightarrow \sin^2 \vartheta_{\text{CHOOZ}} = 0.008^{+0.023}_{-0.008} \ @ \ 2\sigma$$

[Fogli et al, hep-ph/0608060]



[Maltoni, Schwetz, Tortola, Valle, hep-ph/0405172 v5]



$$P_{ee}^{\text{SUN}} = c_{13}^4 P_{ee}^{(2)}(\Delta m^2_{21}, \vartheta_{12}, c_{13}^2 V) + s_{13}^4$$

$$\sin^2\vartheta_{13} < 0.020\,(90\%\ \text{CL})\,,\ 0.041\,(3\sigma)$$

[Schwetz, hep-ph/0606060]

Bilarge Mixing

$$\boxed{|U_{e3}|^2 \ll 1}$$

$$U \simeq \begin{pmatrix} c_{\vartheta_S} & s_{\vartheta_S} & 0 \\ -s_{\vartheta_S} c_{\vartheta_A} & c_{\vartheta_S} c_{\vartheta_A} & s_{\vartheta_A} \\ s_{\vartheta_S} s_{\vartheta_A} & -c_{\vartheta_S} s_{\vartheta_A} & c_{\vartheta_A} \end{pmatrix} \Longrightarrow \begin{cases} \nu_e = c_{\vartheta_S} \nu_1 + s_{\vartheta_S} \nu_2 \\ \nu_a^{(S)} = -s_{\vartheta_S} \nu_1 + c_{\vartheta_S} \nu_2 \\ \quad = c_{\vartheta_A} \nu_\mu - s_{\vartheta_A} \nu_\tau \end{cases}$$

$$\sin^2 2\vartheta_A \simeq 1 \Longrightarrow \vartheta_A \simeq \frac{\pi}{4} \Longrightarrow U \simeq \begin{pmatrix} c_{\vartheta_S} & s_{\vartheta_S} & 0 \\ -s_{\vartheta_S}/\sqrt{2} & c_{\vartheta_S}/\sqrt{2} & 1/\sqrt{2} \\ s_{\vartheta_S}/\sqrt{2} & -c_{\vartheta_S}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

Solar $\nu_e \to \nu_a^{(S)} \simeq \frac{1}{\sqrt{2}} (\nu_\mu - \nu_\tau)$

$$\frac{\Phi^{\text{SNO}}_{\text{CC}}}{\Phi^{\text{SSM}}_{\nu_e}} \simeq \frac{1}{3} \Longrightarrow \Phi_{\nu_e} \simeq \Phi_{\nu_\mu} \simeq \Phi_{\nu_\tau} \text{ for } E \gtrsim 6\,\text{MeV}$$

$$\sin^2 \vartheta_S \simeq \frac{1}{3} \Longrightarrow U \simeq \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Tri-Bimaximal Mixing

[Harrison, Perkins, Scott, hep-ph/0202074]

Global Fit of Oscillation Data: Bilarge Mixing

$$\Delta m^2_{21} = 7.92\,(1 \pm 0.09) \times 10^{-5}\,\text{eV}^2 \qquad \sin^2\vartheta_{12} = 0.314\,(1^{+0.18}_{-0.15})$$

$$|\Delta m^2_{31}| = 2.6\,(1^{+0.14}_{-0.15}) \times 10^{-3}\,\text{eV}^2 \qquad \sin^2\vartheta_{23} = 0.45\,(1^{+0.35}_{-0.20})$$

$$\sin^2\vartheta_{13} = 0.008^{+0.023}_{-0.008}$$

[Fogli et al, hep-ph/0608060]

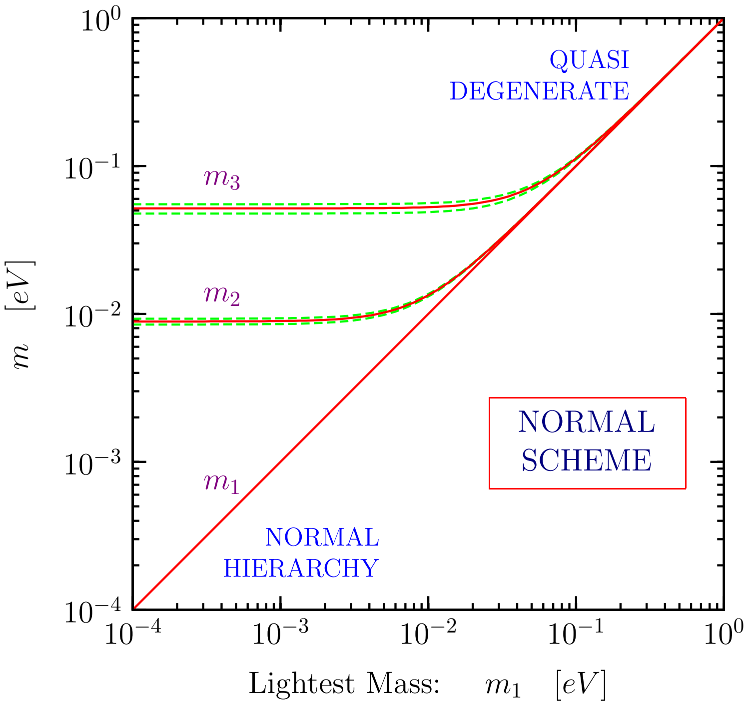
$$|U|_{\text{bf}} \simeq \begin{pmatrix} 0.82 & 0.56 & 0.09 \\ 0.37-0.47 & 0.58-0.65 & 0.67 \\ 0.32-0.43 & 0.52-0.59 & 0.74 \end{pmatrix}$$

$$|U|_{2\sigma} \simeq \begin{pmatrix} 0.78-0.86 & 0.51-0.61 & 0.00-0.18 \\ 0.21-0.57 & 0.41-0.74 & 0.59-0.78 \\ 0.19-0.56 & 0.39-0.72 & 0.62-0.80 \end{pmatrix}$$

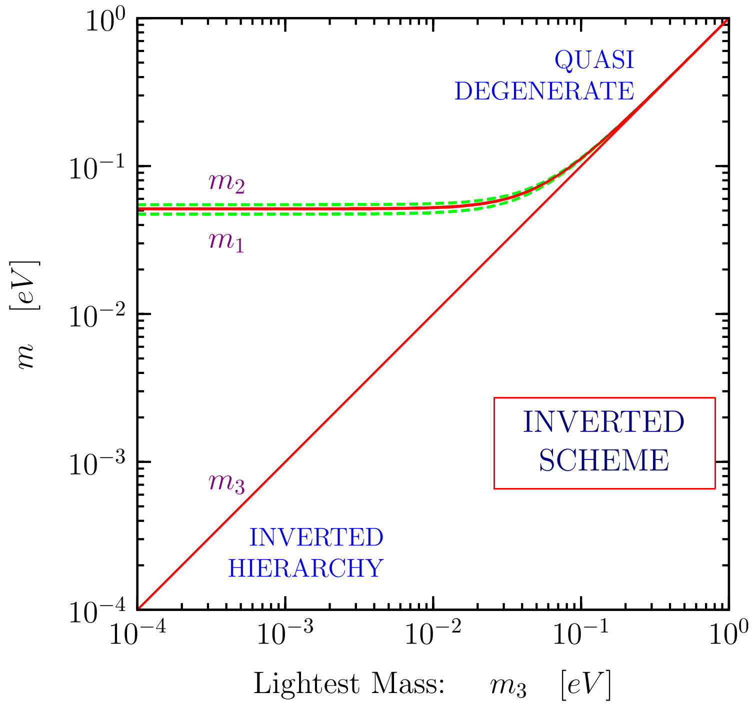
future: measure $\vartheta_{13} \neq 0 \Longrightarrow$ CP violation, matter effects, mass hierarchy

Absolute Scale of Neutrino Masses

normal scheme



inverted scheme



$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_{\text{SOL}}^2$$
$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_{\text{ATM}}^2$$

$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_{\text{ATM}}^2$$
$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_{\text{ATM}}^2$$

Quasi-Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gg \sqrt{\Delta m_{\text{ATM}}^2} \simeq 5 \times 10^{-2}\,\text{eV}$

Tritium Beta-Decay

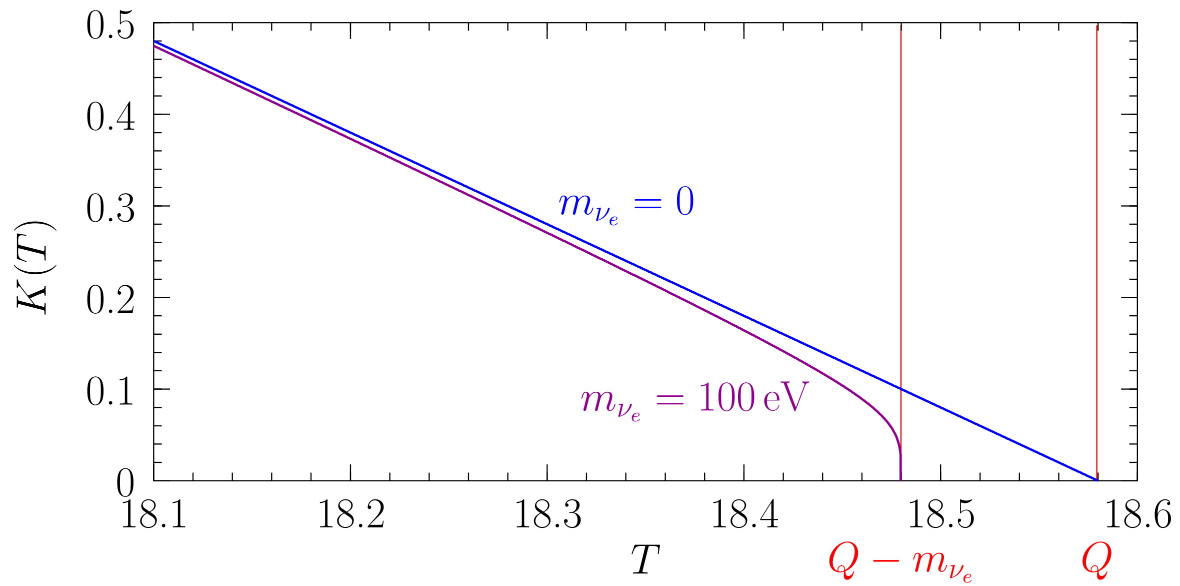
$^3\mathrm{H} \to {}^3\mathrm{He} + e^- + \bar{\nu}_e$

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}T} = \frac{(\cos\vartheta_C G_{\mathrm{F}})^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE\, (Q-T) \sqrt{(Q-T)^2 - m_{\nu_e}^2}$$

$$Q = M_{^3\mathrm{H}} - M_{^3\mathrm{He}} - m_e = 18.58\,\mathrm{keV}$$

Kurie plot

$$K(T) = \sqrt{\frac{\mathrm{d}\Gamma/\mathrm{d}T}{\frac{(\cos\vartheta_C G_{\mathrm{F}})^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE}} = \left[(Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2}\right]^{1/2}$$



$$\boxed{m_{\nu_e} < 2.2\,\mathrm{eV} \quad (95\%\ \mathrm{C.L.})}$$

Mainz & Troitsk

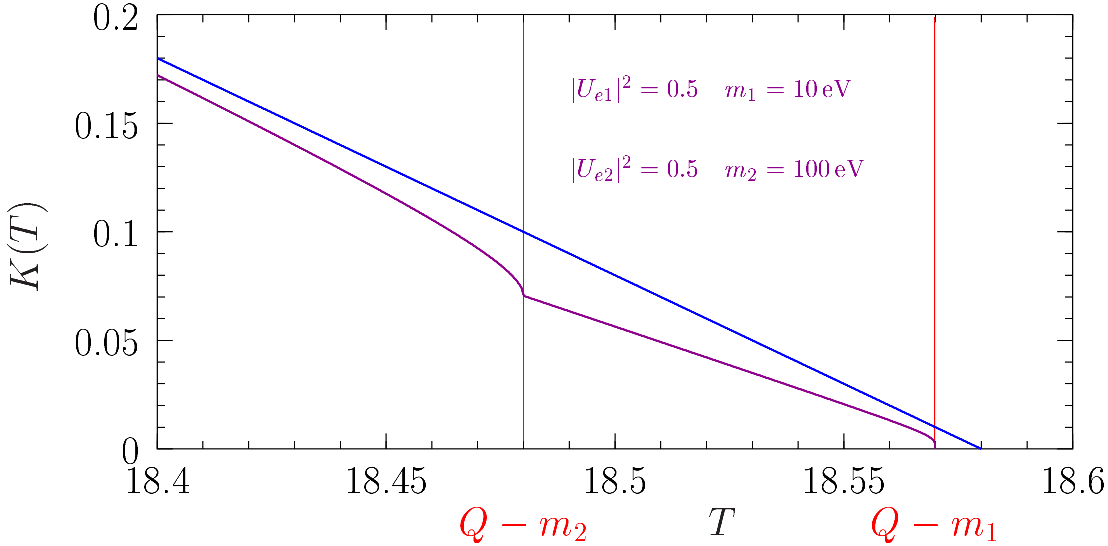
[Weinheimer, hep-ex/0210050]

future: KATRIN (start 2010)

[hep-ex/0109033] [hep-ex/0309007]

sensitivity: $m_{\nu_e} \simeq 0.2\,\mathrm{eV}$

Neutrino Mixing $\Longrightarrow K(T) = \left[(Q-T) \sum_k |U_{ek}|^2 \sqrt{(Q-T)^2 - m_k^2} \right]^{1/2}$



analysis of data is different from the no-mixing case: $2N-1$ parameters $\left(\sum_k |U_{ek}|^2 = 1 \right)$

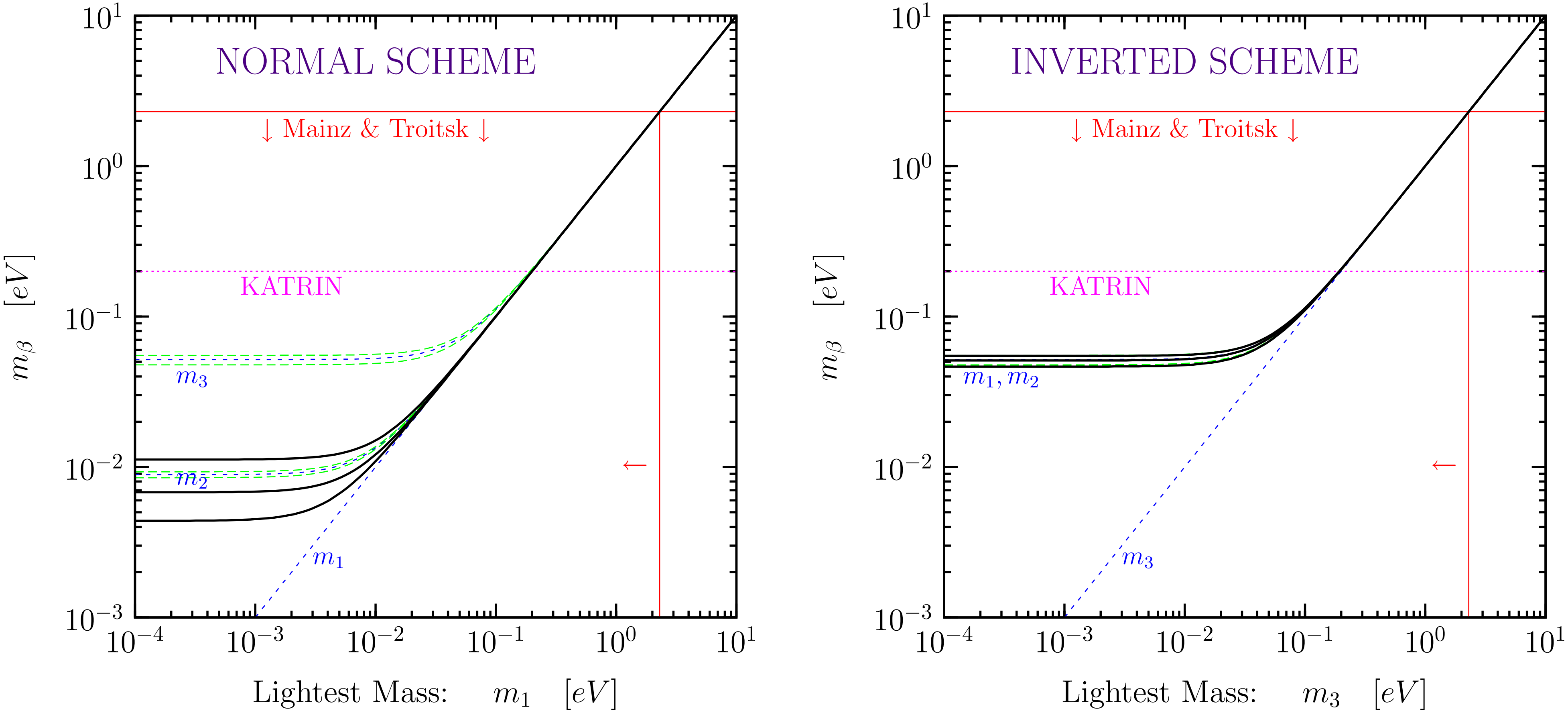
if experiment is not sensitive to masses $(m_k \ll Q - T)$

effective mass: $\boxed{m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2}$

$$K^2 = (Q-T)^2 \sum_k |U_{ek}|^2 \sqrt{1 - \frac{m_k^2}{(Q-T)^2}} \simeq (Q-T)^2 \sum_k |U_{ek}|^2 \left[1 - \frac{1}{2} \frac{m_k^2}{(Q-T)^2} \right]$$

$$= (Q-T)^2 \left[1 - \frac{1}{2} \frac{m_\beta^2}{(Q-T)^2} \right] \simeq (Q-T) \sqrt{(Q-T)^2 - m_\beta^2}$$

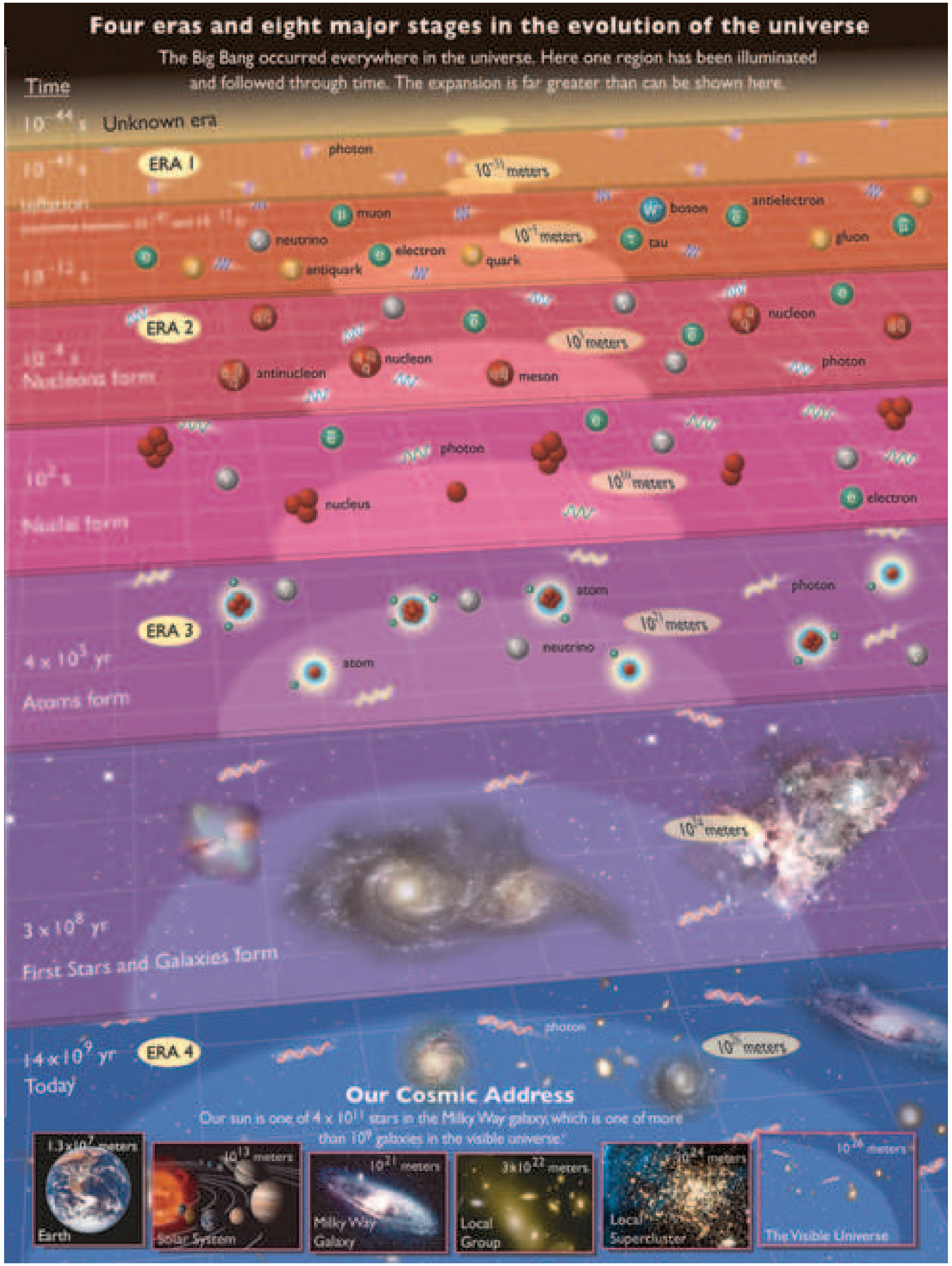
$$m_\beta^2 = |U_{e1}|^2 \, m_1^2 + |U_{e2}|^2 \, m_2^2 + |U_{e3}|^2 \, m_3^2$$



Quasi-Degenerate: $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \implies m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$

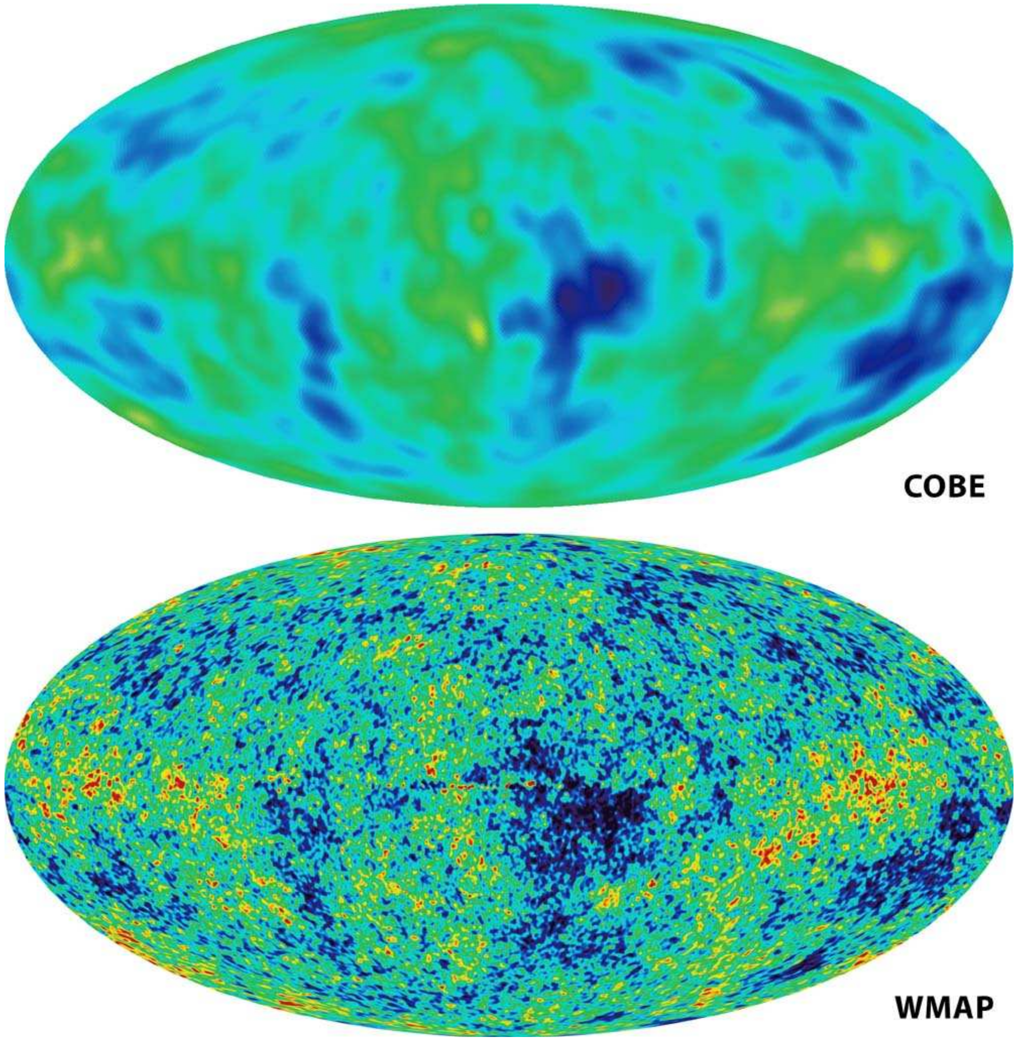
FUTURE: IF $m_\beta \lesssim 4 \times 10^{-2}\,\text{eV} \implies$ NORMAL HIERARCHY

Cosmological Bound on Neutrino Masses



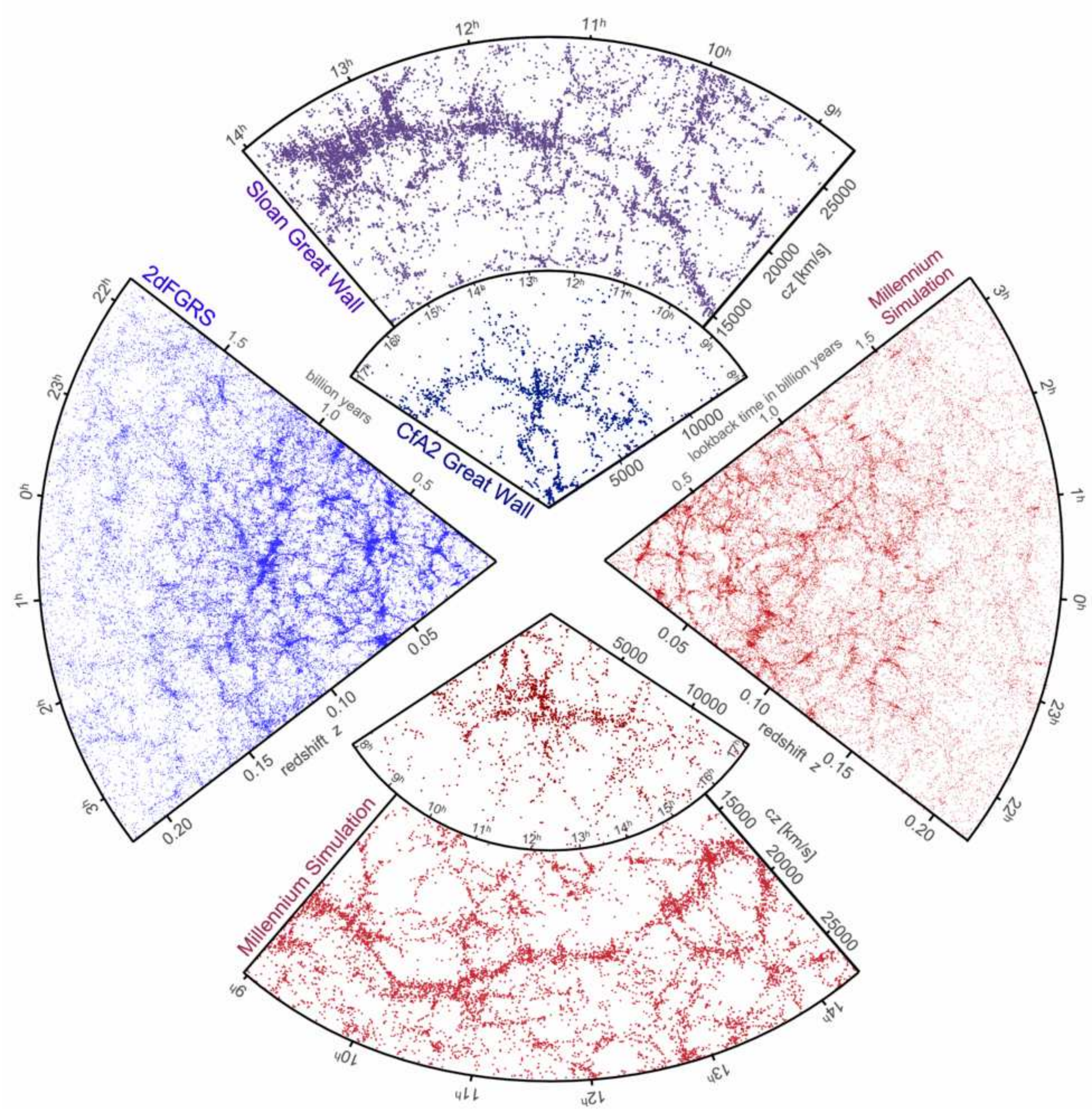
[CPEP, http://www.cpepweb.org/]

WMAP (Wilkinson Microwave Anisotropy Probe)



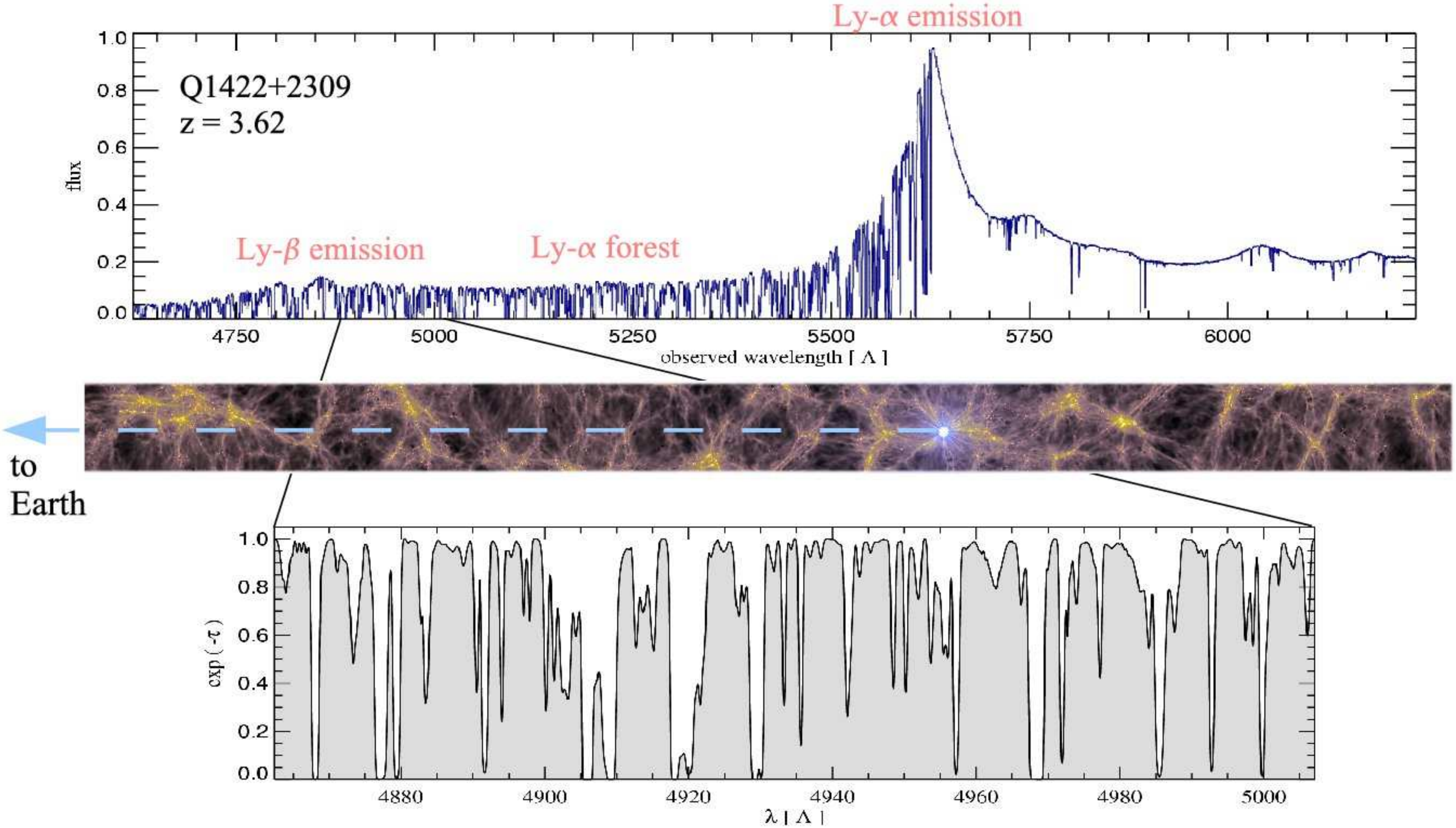
[WMAP, http://map.gsfc.nasa.gov]

Galaxy Redshift Surveys



[Springel, Frenk, White, astro-ph/0604561]

Lyman-alpha Forest



[Springel, Frenk, White, astro-ph/0604561]

Rest-frame Lyman α, β, γ wavelengths: $\lambda_\alpha^0 = 1215.67$ Å, $\lambda_\beta^0 = 1025.72$ Å, $\lambda_\gamma^0 = 972.54$ Å

Lyman-α forest: The region in which only Lyα photons can be absorbed: $[(1+z_q)\lambda_\beta^0, (1+z_q)\lambda_\alpha^0]$

Relic Neutrinos

neutrinos are in equilibrium in primeval plasma through weak interaction reactions

$$\nu\bar{\nu} \leftrightarrows e^+e^- \quad \overset{(-)}{\nu} e \leftrightarrows \overset{(-)}{\nu} e \quad \overset{(-)}{\nu} N \leftrightarrows \overset{(-)}{\nu} N \quad \nu_e n \leftrightarrows p e^- \quad \bar{\nu}_e p \leftrightarrows n e^+ \quad n \leftrightarrows p e^- \bar{\nu}_e$$

weak interactions freeze out

$$\Gamma_{\text{weak}} = N\sigma v \sim G_F^2 T^5 \sim T^2/M_P \sim \sqrt{G_N T^4} \sim \sqrt{G_N \rho} \sim H \Longrightarrow \underset{\text{neutrino decoupling}}{T_{\text{dec}} \sim 1\,\text{MeV}}$$

$$\text{Relic Neutrinos:} \quad T_\nu = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_\gamma \simeq 1.945\,\text{K} \Longrightarrow k\,T_\nu \underset{(T_\gamma = 2.725 \pm 0.001\,\text{K})}{\simeq} 1.676 \times 10^{-4}\,\text{eV}$$

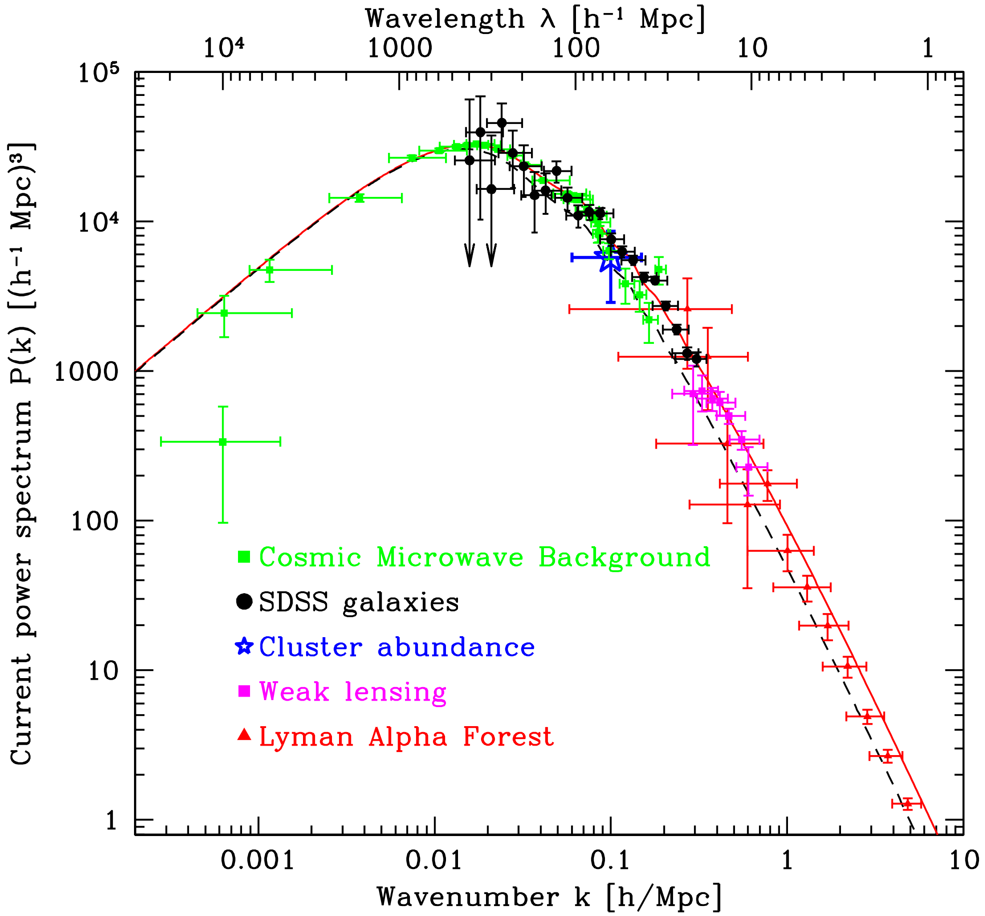
$$\text{number density:} \quad n_f = \frac{3}{4}\frac{\zeta(3)}{\pi^2} g_f T_f^3 \Longrightarrow n_{\nu_k,\bar{\nu}_k} \simeq 0.1827\,T_\nu^3 \simeq 112\,\text{cm}^{-3}$$

$$\underset{\left(\rho_c = \frac{3H^2}{8\pi G_N}\right)}{\text{density contribution:}} \quad \Omega_k = \frac{n_{\nu_k,\bar{\nu}_k}\,m_k}{\rho_c} \simeq \frac{1}{h^2}\frac{m_k}{94.14\,\text{eV}} \Longrightarrow \boxed{\Omega_\nu h^2 = \frac{\sum_k m_k}{94.14\,\text{eV}}}$$

[Gershtein, Zeldovich, JETP Lett. 4 (1966) 120] [Cowsik, McClelland, PRL 29 (1972) 669]

$$h \sim 0.7, \quad \Omega_\nu \lesssim 0.3 \qquad \Longrightarrow \qquad \sum_k m_k \lesssim 14\,\text{eV}$$

Power Spectrum of Density Fluctuations



[Tegmark, hep-ph/0503257]

Solid Curve: flat ΛCDM model

$(\Omega_{\rm M}^0 = 0.28\,,\ h = 0.72\,,\ \Omega_{\rm B}^0/\Omega_{\rm M}^0 = 0.16)$

Dashed Curve: $\displaystyle\sum_{k=1}^{3} m_k = 1\,\text{eV}$

hot dark matter
prevents early galaxy formation

$$\delta(\vec{x}) \equiv \frac{\rho(\vec{x}) - \overline{\rho}}{\overline{\rho}}$$

$$\langle \delta(\vec{x}_1)\delta(\vec{x}_2)\rangle = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{x}}\, P(\vec{k})$$

small scale suppression

$$\frac{\Delta P(k)}{P(k)} \approx -8\,\frac{\Omega_\nu}{\Omega_m} \approx -0.8 \left(\frac{\sum_k m_k}{1\,\text{eV}}\right)\left(\frac{0.1}{\Omega_m h^2}\right)$$

for

$$k \gtrsim k_{\rm nr} \approx 0.026 \sqrt{\frac{m_\nu}{1\,\text{eV}}}\sqrt{\Omega_m}\, h\,\text{Mpc}^{-1}$$

[Hu, Eisenstein, Tegmark, PRL 80 (1998) 5255]

WMAP, AJ SS 148 (2003) 175, astro-ph/0302209

CMB (WMAP, ...) + LSS (2dFGRS) + HST + SNIa $\Longrightarrow$ ΛCDM

$T_0 = 13.7 \pm 0.1\,\text{Gyr}$ $\quad h = 0.71^{+0.04}_{-0.03}$

$\Omega_0 = 1.02 \pm 0.02$ $\quad \Omega_B h^2 = 0.0224 \pm 0.0009$ $\quad \Omega_M h^2 = 0.135^{+0.008}_{-0.009}$

$$\Omega_\nu h^2 < 0.0076 \quad (95\%\ \text{conf.}) \quad \Longrightarrow \quad \boxed{\sum_{k=1}^{3} m_k < 0.71\,\text{eV}}$$

WMAP, astro-ph/0603449

Flat ΛCDM $\quad$ (WMAP+HST: $\Omega_0 = 1.010^{+0.016}_{-0.009}$, $\Omega_\Lambda = 0.72 \pm 0.04$)

$$\sum_{k=1}^{3} m_k < \begin{cases} 2.0\,\text{eV} & \text{WMAP} \\ 0.91\,\text{eV} & \text{WMAP+SDSS} \\ 0.87\,\text{eV} & \text{WMAP+2dFGRS} \\ 0.68\,\text{eV} & \text{CMB+LSS+SNIa} \end{cases} \quad (95\%\ \text{conf.})$$

Goobar, Hannestad, Mortsell, Tu, JCAP 0606 (2006) 019, astro-ph/0602155

Flat ΛCDM

$$\sum_{k=1}^{3} m_k < \begin{cases} 0.70\,\text{eV} & \text{CMB+LSS+SNIa} \\ 0.48\,\text{eV} & \text{CMB+LSS+SNIa+BAO} \\ 0.27\,\text{eV} & \text{CMB+LSS+SNIa+BAO+Ly}\alpha \end{cases} \qquad \text{(95\% conf.)}$$

Seljak, Slosar, McDonald, astro-ph/0604335

Flat ΛCDM CMB+LSS+SNIa+BAO+Lyα

$$\sum_{k=1}^{3} m_k < 0.17\,\text{eV} \qquad \text{(95\% conf.)}$$

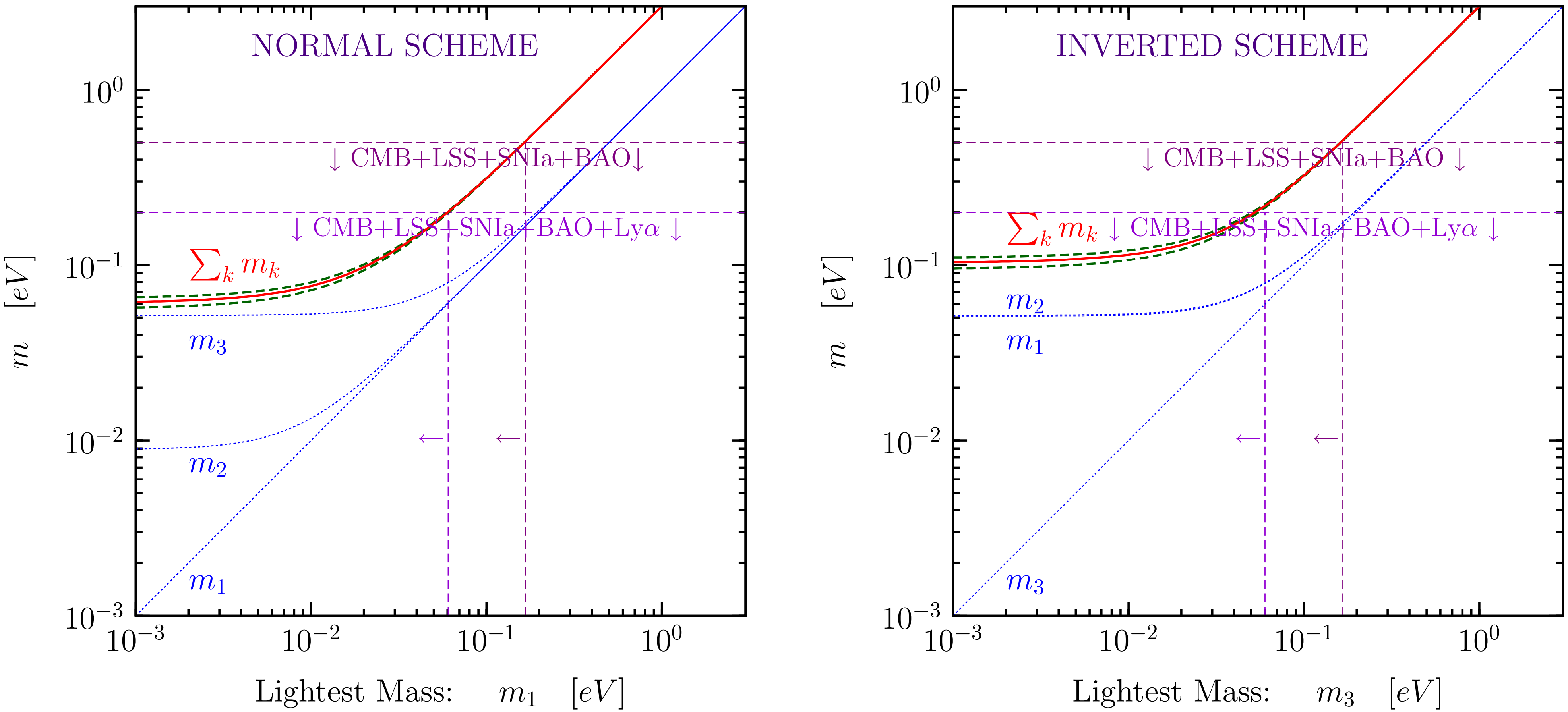
Fogli, Lisi, Marrone, Melchiorri, Palazzo, Serra, Silk, Slosar, hep-ph/0608060

Flat ΛCDM

$$\sum_{k=1}^{3} m_k < \begin{cases} 0.75\,\text{eV} & \text{CMB+LSS+SNIa} \\ 0.58\,\text{eV} & \text{CMB+LSS+SNIa+BAO} \\ 0.17\,\text{eV} & \text{CMB+LSS+SNIa+BAO+Ly}\alpha \end{cases} \qquad \text{(95\% conf.)}$$

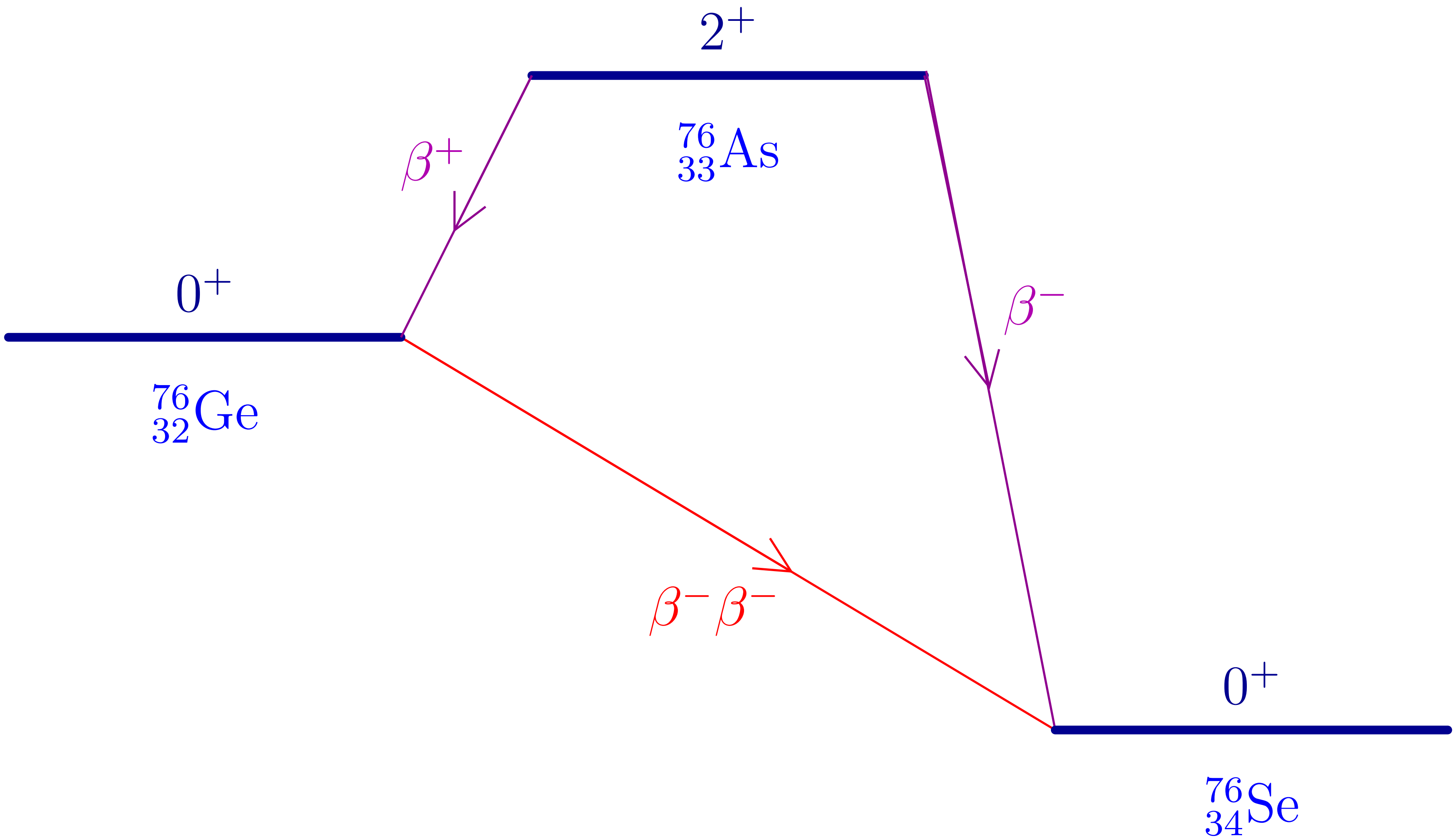
$$\sum_{k=1}^{3} m_k \lesssim 0.5\,\mathrm{eV} \qquad (\sim 2\sigma) \qquad \text{CMB+LSS+SNIa+BAO}$$

$$\sum_{k=1}^{3} m_k \lesssim 0.2\,\mathrm{eV} \qquad (\sim 2\sigma) \qquad \text{CMB+LSS+SNIa+BAO+Ly}\alpha$$



$$\text{FUTURE: IF} \quad \sum_{k=1}^{3} m_k \lesssim 9 \times 10^{-2}\,\mathrm{eV} \Longrightarrow \text{NORMAL HIERARCHY}$$

Neutrinoless Double-Beta Decay

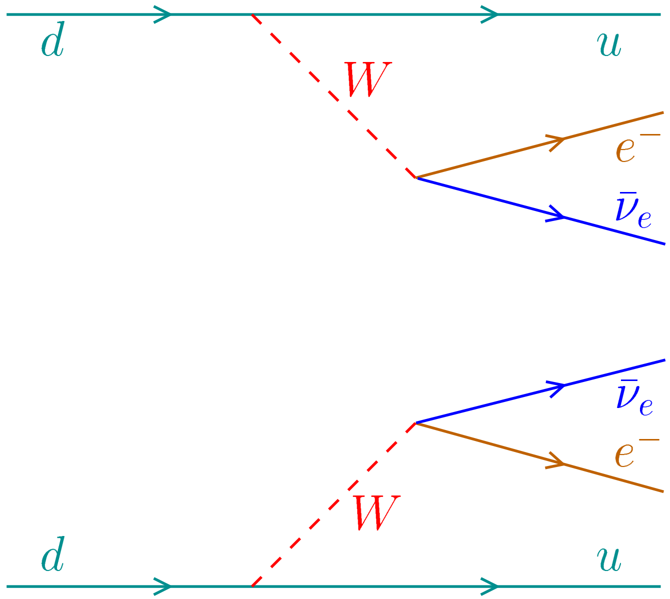


Two-Neutrino Double-β Decay: $\Delta L = 0$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

$$(T^{2\nu}_{1/2})^{-1} = G_{2\nu}\,|\mathcal{M}_{2\nu}|^2$$

second order weak interaction process
in the Standard Model



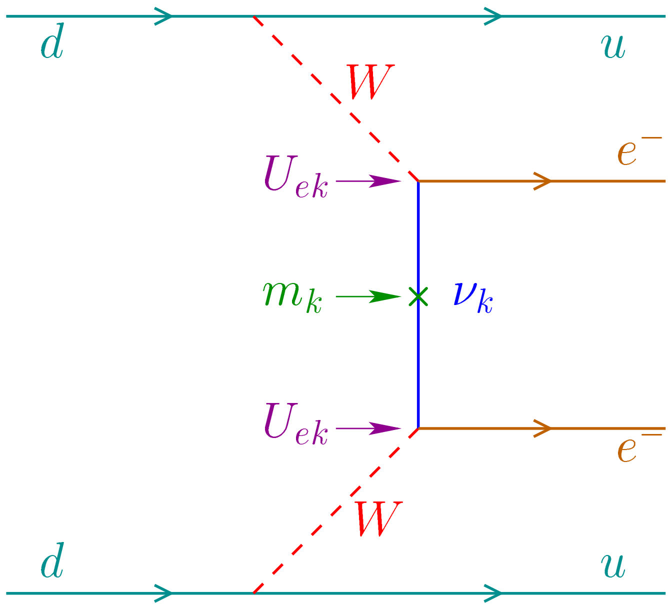
Neutrinoless Double-β Decay: $\Delta L = 2$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^-$$

$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu}\,|\mathcal{M}_{0\nu}|^2\,|m_{\beta\beta}|^2$$

effective
Majorana
mass

$$m_{\beta\beta} = \sum_k U_{ek}^2\, m_k$$

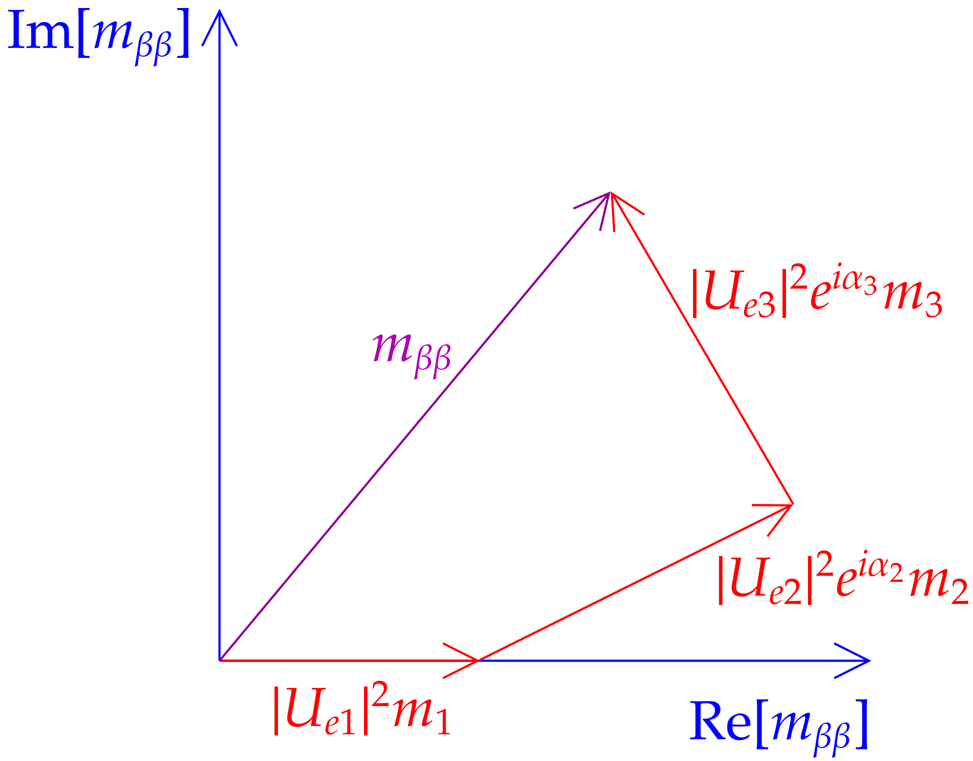


Effective Majorana Neutrino Mass

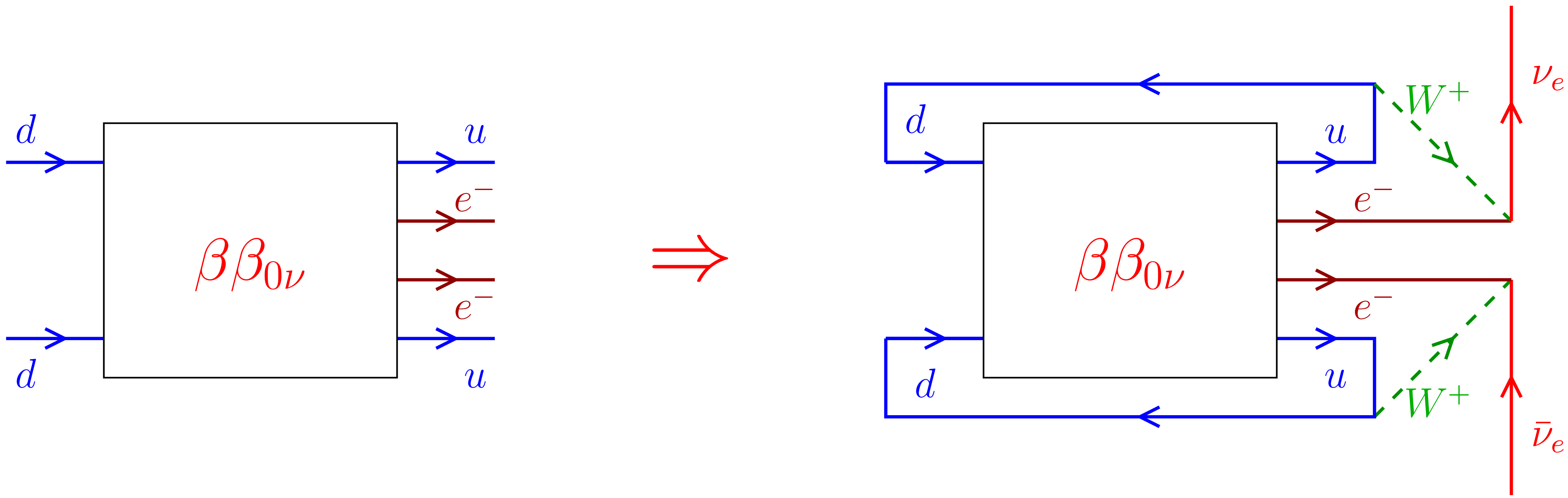
$$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k$$ complex U_{ek} $\Rightarrow$ possible cancellations

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$

$$\alpha_2 = 2\lambda_2 \qquad \alpha_3 = 2\,(\lambda_3 - \delta_{13})$$



Majorana Neutrino Mass $\Leftrightarrow$ $\beta\beta_{0\nu}$ Decay



[Schechter, Valle, PRD 25 (1982) 2951] [Takasugi, PLB 149 (1984) 372]

Majorana Mass Term

$$\boxed{\mathcal{L}_L^{\mathrm{M}} = -\frac{1}{2}\, m \left(\overline{\nu_L^c}\, \nu_L + \overline{\nu_L}\, \nu_L^c \right) = \frac{1}{2}\, m \left(\nu_L^T\, \mathcal{C}^\dagger\, \nu_L + \nu_L^\dagger\, \mathcal{C}\, \nu_L^* \right)}$$

two conditions: $\begin{cases} u,\ d,\ e \text{ are massive} \\ \text{standard left-handed weak interaction exists} \end{cases}$

cancellation with other diagrams is very unlikely

(no symmetry, unstable under perturbative expansion)

Experimental Bound

Heidelberg-Moscow (^{76}Ge) [EPJA 12 (2001) 147]

$$\boxed{T^{0\nu}_{1/2} > 1.9 \times 10^{25}\,\text{y} \quad (90\%\ \text{C.L.})} \implies \boxed{|m_{\beta\beta}| \lesssim 0.32 - 1.0\,\text{eV}}$$

IGEX (^{76}Ge) [PRD 65 (2002) 092007]

$$\boxed{T^{0\nu}_{1/2} > 1.57 \times 10^{25}\,\text{y} \quad (90\%\ \text{C.L.})} \implies \boxed{|m_{\beta\beta}| \lesssim 0.33 - 1.35\,\text{eV}}$$

CUORICINO (^{130}Te) [PRL 95 (2005) 142501]

$$\boxed{T^{0\nu}_{1/2} > 1.8 \times 10^{24}\,\text{y} \quad (90\%\ \text{C.L.})} \implies \boxed{|m_{\beta\beta}| \lesssim 0.2 - 1.1\,\text{eV}}$$

NEMO 3 (^{100}Mo) [PRL 95 (2005) 182302]

$$\boxed{T^{0\nu}_{1/2} > 4.6 \times 10^{23}\,\text{y} \quad (90\%\ \text{C.L.})} \implies \boxed{|m_{\beta\beta}| \lesssim 0.7 - 2.8\,\text{eV}}$$

FUTURE EXPERIMENTS

NEMO 3, CUORICINO, COBRA, XMASS, CAMEO, CANDLES

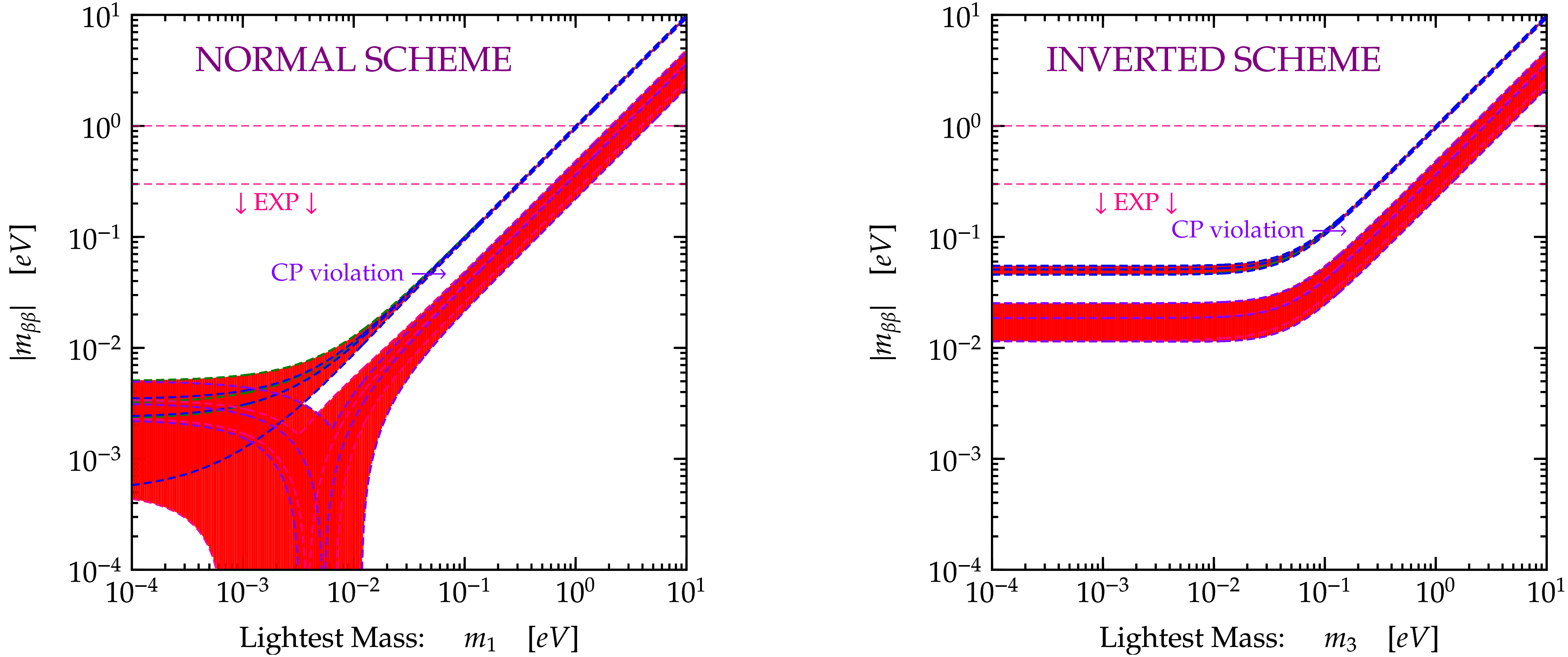
$$|m_{\beta\beta}| \sim \text{few}\ 10^{-1}\,\text{eV}$$

EXO, MOON, Super-NEMO, CUORE, Majorana, GEM, GERDA

$$|m_{\beta\beta}| \sim \text{few}\ 10^{-2}\,\text{eV}$$

Bounds from Neutrino Oscillations

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$

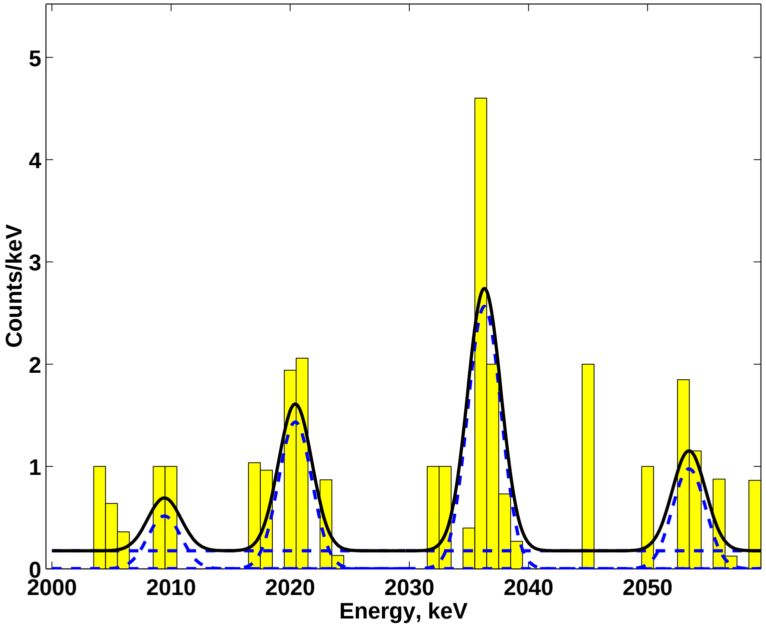


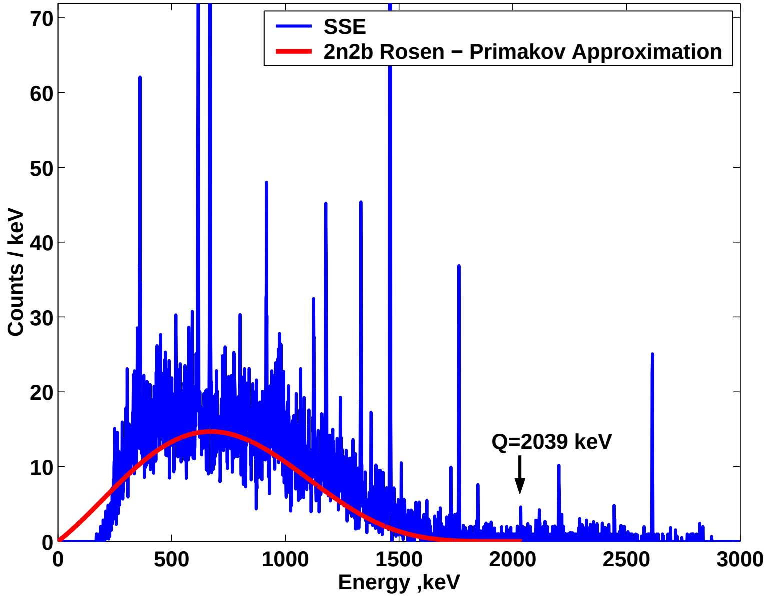
FUTURE: IF $|m_{\beta\beta}| \lesssim 10^{-2}$ eV $\Longrightarrow$ NORMAL HIERARCHY

Experimental Positive Indication

[Klapdor et al., MPLA 16 (2001) 2409; FP 32 (2002) 1181; NIMA 522 (2004) 371; PLB 586 (2004) 198]

$T_{1/2}^{0\nu\,\mathrm{bf}} = 1.19 \times 10^{25}\,\mathrm{y}$ $\quad T_{1/2}^{0\nu} = (0.69 - 4.18) \times 10^{25}\,\mathrm{y}\,(3\sigma)$ $\quad 4.2\sigma$ evidence





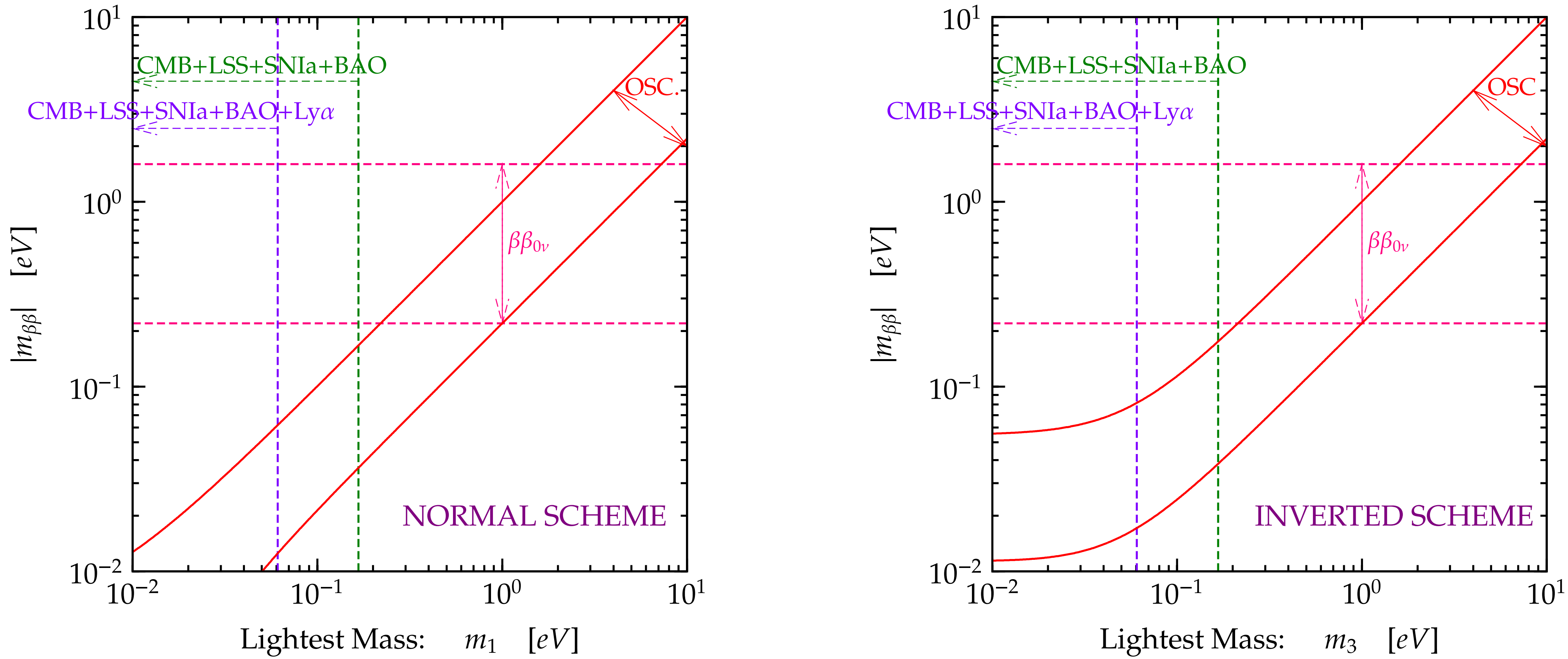
pulse-shape selected spectrum $\quad$ 3.8σ evidence $\quad$ [PLB 586 (2004) 198]

the indication must be checked by other experiments

$$1.35 \lesssim |\mathcal{M}_{0\nu}| \lesssim 4.12 \quad \Longrightarrow \quad 0.22\,\mathrm{eV} \lesssim |m_{\beta\beta}| \lesssim 1.6\,\mathrm{eV}$$

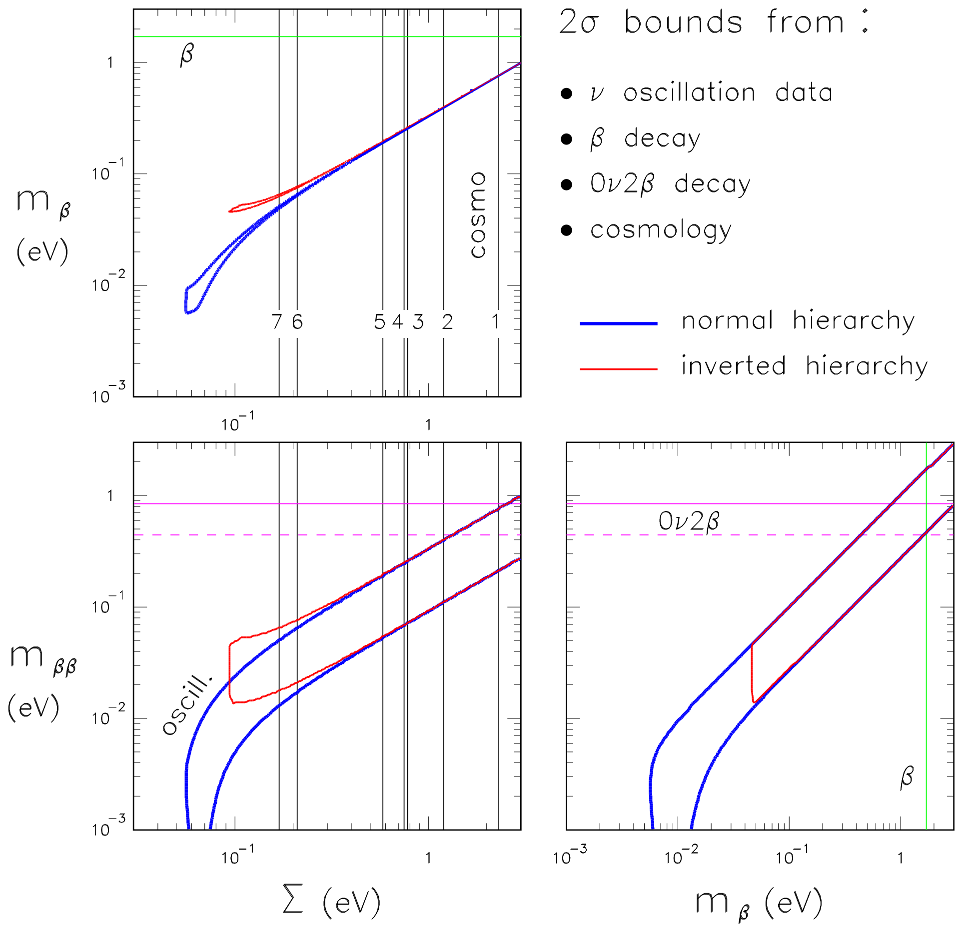
if confirmed, very exciting (Majorana ν and large mass scale)

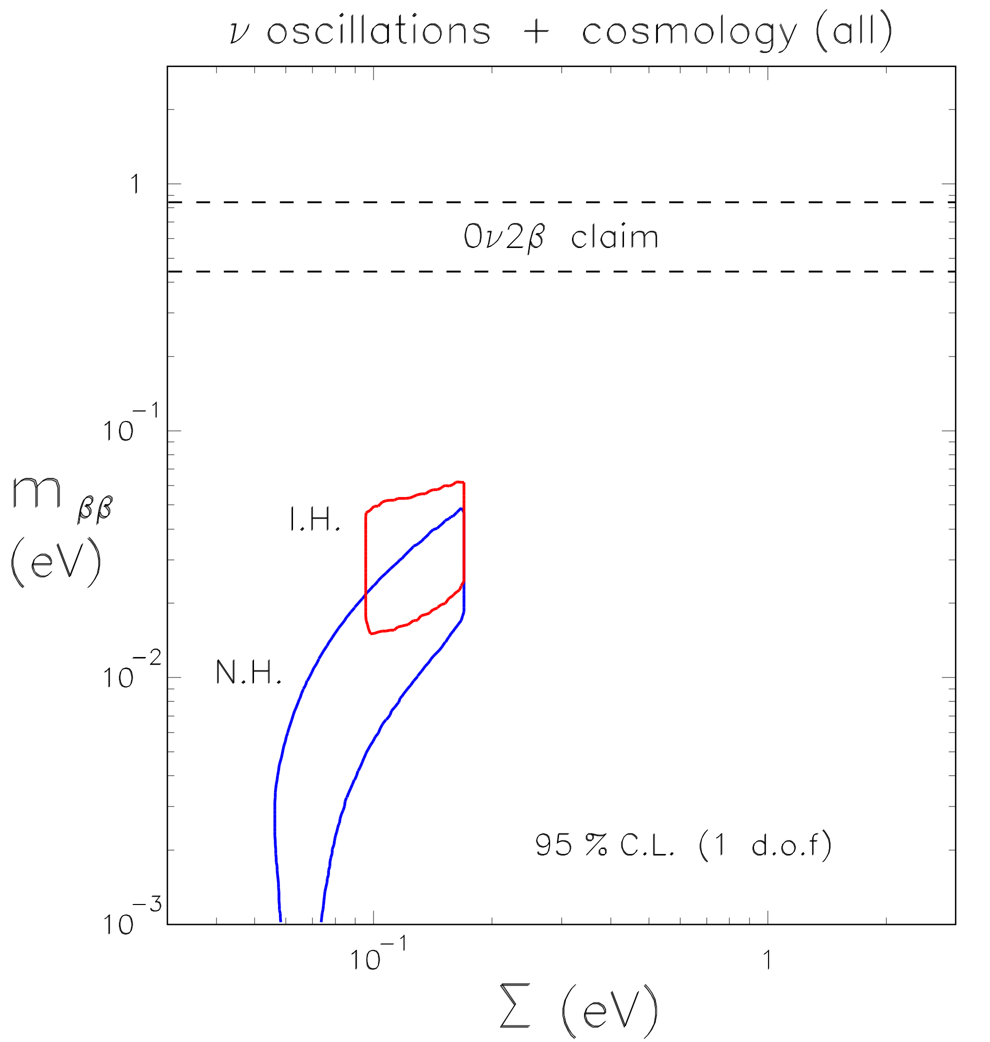
Indication of $\beta\beta_{0\nu}$ Decay: $0.22\,\text{eV} \lesssim |m_{\beta\beta}| \lesssim 1.6\,\text{eV}$ $(\sim 3\sigma$ range$)$



tension among oscillation data, CMB+LSS+BAO(+Lyα) and $\beta\beta_{0\nu}$ signal

Case	Cosmological data set	Σ bound (2σ)
1	WMAP	< 2.3 eV
2	WMAP + SDSS	< 1.2 eV
3	WMAP + SDSS + $\mathrm{SN}_{\mathrm{Riess}}$ + HST + BBN	< 0.78 eV
4	CMB + LSS + $\mathrm{SN}_{\mathrm{Astier}}$	< 0.75 eV
5	CMB + LSS + $\mathrm{SN}_{\mathrm{Astier}}$ + BAO	< 0.58 eV
6	CMB + LSS + $\mathrm{SN}_{\mathrm{Astier}}$ + Ly-α	< 0.21 eV
7	CMB + LSS + $\mathrm{SN}_{\mathrm{Astier}}$ + BAO + Ly-α	< 0.17 eV

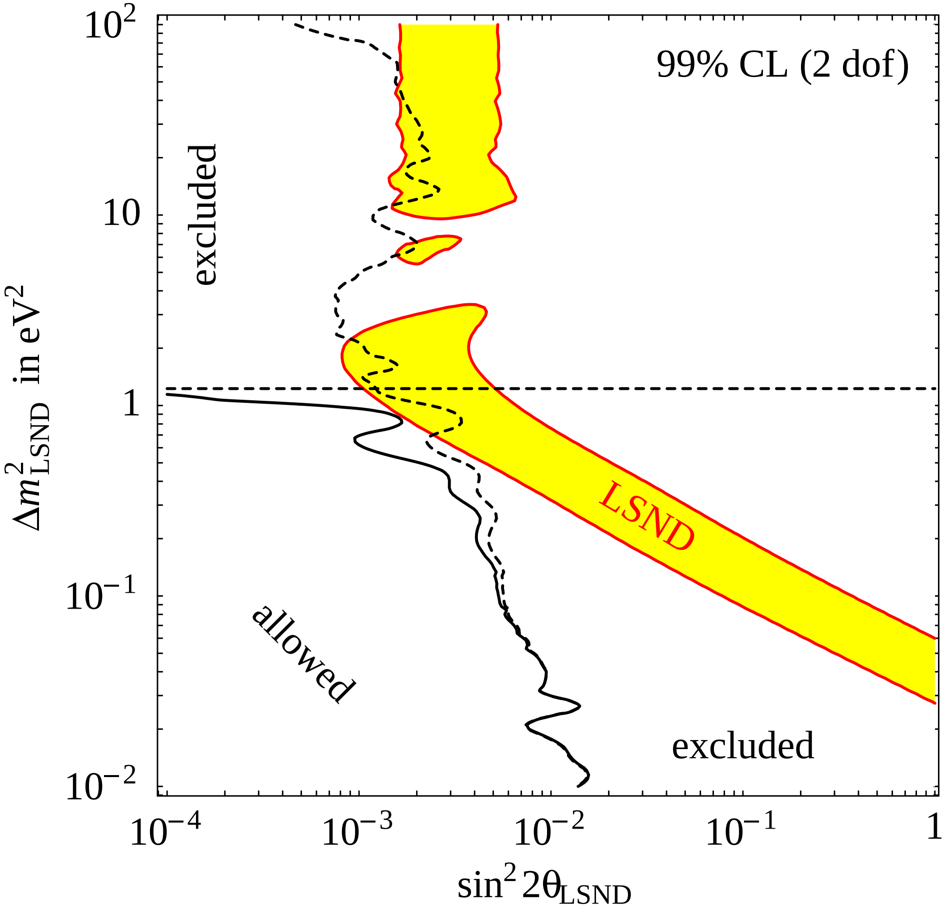




[Fogli, Lisi, Marrone, Melchiorri, Palazzo, Serra, Silk, Slosar, hep-ph/0608060]

LSND

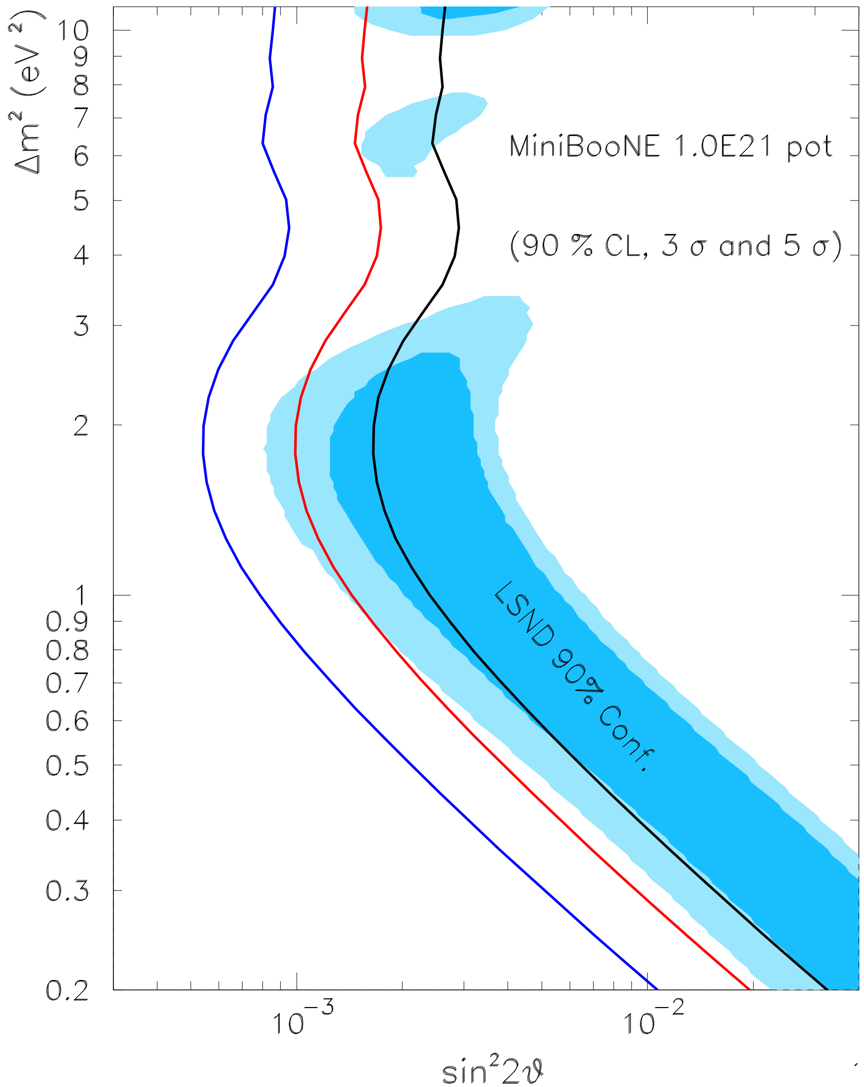
$\bar{\nu}_\mu \to \bar{\nu}_e$ $\Delta m^2_{\text{LSND}} \gtrsim 0.1\,\text{eV}^2$ $(\gg \Delta m^2_{\text{ATM}} \gg \Delta m^2_{\text{SOL}})$



[Strumia, Vissani, hep-ph/0606054]

3+1 scheme

bounds from ν experiments
and cosmology



MiniBooNE $\nu_\mu \to \nu_e$ [hep-ex/0406048]

Conclusions

$\nu_e \to \nu_\mu, \nu_\tau$ with $\Delta m^2_{\text{SOL}} \simeq 8.3 \times 10^{-5}\,\text{eV}^2$ (solar ν, KamLAND)

$\nu_\mu \to \nu_\tau$ with $\Delta m^2_{\text{ATM}} \simeq 2.4 \times 10^{-3}\,\text{eV}^2$ (atm. ν, K2K, MINOS)

$\Downarrow$

Bilarge 3ν-Mixing with $|U_{e3}|^2 \ll 1$ (CHOOZ)

β Decay, Cosmology, $\beta\beta_{0\nu}$ Decay $\Longrightarrow m_\nu \lesssim 1\,\text{eV}$

FUTURE

Theory: Why lepton mixing $\neq$ quark mixing?

(Due to Majorana nature of ν's?)

Why only $|U_{e3}|^2 \ll 1$?

Improve uncertainties in calculation of $\mathcal{M}_{0\nu}$!

Exp.: LSND?

Measure $|U_{e3}| > 0 \Rightarrow$ CP viol., matter effects, mass hierarchy

Check $\beta\beta_{0\nu}$ signal at Quasi-Degenerate mass scale

Improve β Decay, Cosmology, $\beta\beta_{0\nu}$ Decay measurements