Theory of non-leptonic B decays

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HQL 2006, München, October 19, 2006

- Theory of non-leptonic decays
  - Summary and comparison of different frameworks
  - New higher-order calculations
  - Power corrections
- Confronting data
  - Global comparison
  - Time-dependent CP asymmetries S_f

Hadronic B decays – $B \to M_1 M_2$ (2-body, mesons P or V, charmless)

- large variety of transitions :
  flavour
  Dirac structure ($V\mp A, \ldots$)

  ↙ Determine CKM parameters

  ↘ New Physics could be flavour-specific

- SM :

$$\mathcal{L}_{\rm eff} = -\frac{G_F}{\sqrt{2}} \sum_i \mathrm{CKM} \cdot \mathcal{O}_i$$

weak interactions for $\mu \ll M_W$
+ QCD + QED

Challenge of strong interaction

$$\langle M_1 M_2 | \mathcal{O}_i | \bar{B} \rangle$$

↙ SU(3) [isospin]
$\hat{=}$ graphical amplitudes T, C, P, ...

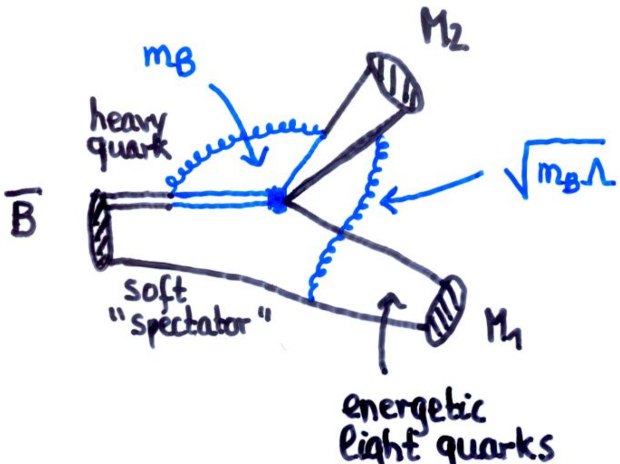
[Zeppenfeld (1981),
....
Chau (1991),
Gronau et al. (1994)
⋮]

↘ Factorization ($1/m_b$ exp.)

[Bauer, Stech, Wirbel (1985),
.....
from QCD: MB, Buchalla, Neubert, Sachrajda (1999)
⋮]

Theory of hadronic decays (factorization)

Scales and factorization



Scales (M_W integrated out)

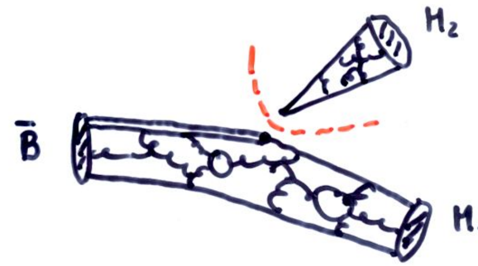
m_b	hard	} for large m_b α_s is small at these scales $\longrightarrow$ perturbation theory applies!
$\sqrt{m_b \Lambda}$	hard-collinear	
Λ	soft or collinear	

↑

long-distance

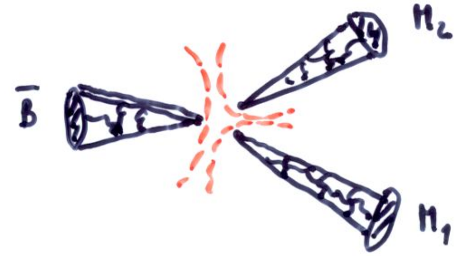
Factorization utilizes the heavy quark and collinear expansion (Λ/m_b, Λ/E)

Want to show – at leading order in $1/m_b$ – that the long-distance contributions look like



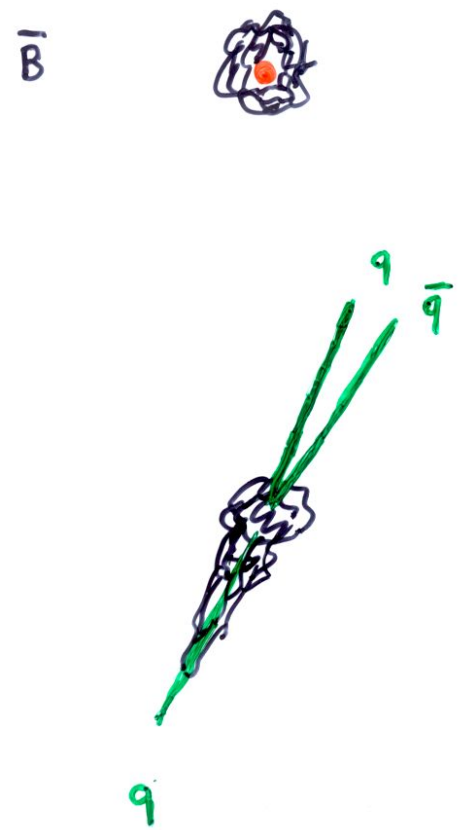
$\langle M_2 | .. | 0 \rangle$
$\langle M_1 | .. | \bar{B} \rangle$

or even



$\langle M_1 | .. | 0 \rangle$
$\langle M_2 | .. | 0 \rangle$
$\langle 0 | .. | \bar{B} \rangle$

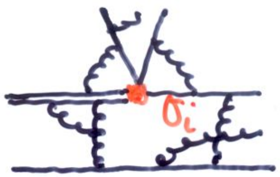
Factorization works (at leading power in $1/m_b$), because .....



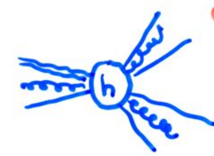
... energetic, low-invariant mass colour-singlet ($\to$ compact) escapes $\bar{B}$ remnant and hadronizes far away independent from the hadronization of q + remnant

Integrating out the scale m_b [QCD → SCET$_{\mathrm{I}}$]

(BBNS ; Chay, Kim ; MB, Feldmann, Bauer et al.)



Which hard subgraphs



are leading ?

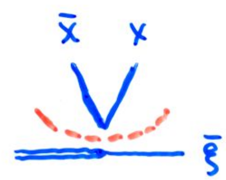
Result:

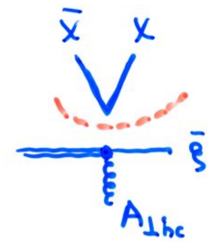
Correction to naive fact.

New effect: spectator scattering

$$(\bar u b)(\bar d u) \longrightarrow [\overset{\bar d}{\bar{\chi}^{(0)}_{(t n_-)}}\overset{u}{\chi^{(0)}}] * \Big(C^{\mathrm{I}} * [\overset{\bar u}{\bar{\xi}}(s n_+)\overset{b}{h_v}] + C^{\mathrm{II}} * [\bar{\xi}(s_1 n_+) A_{\perp}(s_2 n_+) h_v] \Big)$$

ϕ_{M_2} $\qquad$ $\xi(0)$ $[F^{BM_1}_{(0)}]$ $\qquad$ $\Xi(\tau; 0)$





- M_2 factorizes at scales $\mu < m_b$
- Strong phases in perturbative coefficient functions $C^{\mathrm{I},\mathrm{II}}$ ONLY
- Leaves out $1/m_b$ corrections (see below)

Comparison of different approaches / implementations

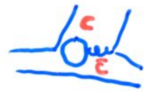
$$\langle M_1 M_2 | O_i | \bar{B} \rangle = \Big(\underset{①}{T^{I}} \cdot \underset{②}{F^{BM_1}_{(0)}} + \underset{③}{H^{II}} * \underset{④}{\Xi^{BM_1}_{(\tau,0)}} \Big) * \phi_{M_2}$$

	①	②	③	④
PQCD (Keum, Li, Sanda, 2000)	$\alpha_s^0 + \boxed{\alpha_s^1}$ '00 '05 New - see below	calculated ($k_\perp$ - fact.)	α_s^0 '00	calculated (α_s^1) '00
QCDF / SCET				
BBNS (1999)	$\alpha_s^0 + \alpha_s^1$ naive fact. '99	input	$\alpha_s^0 + \boxed{\alpha_s^1}$ '99 '05, '06 New - see below	calculated ($\alpha_s^1 + \alpha_s^2$) '99 '04
BPRS (Bauer et al., 2004)	α_s^0 (naive fact)	fit to data	α_s^0 (from BBNS)	fit to data

(BPRS approach cannot be improved beyond leading order in α_s, because in general $\Xi(\tau)$ is an unknown function; does not require perturbation theory at $\sqrt{m_b \Lambda}$)

The charming penguin saga

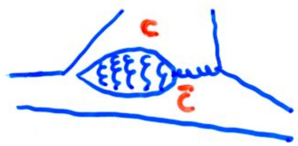
Ciuchini et al. (2001)

Large $1/m_b$ corrections from  or  (annihilation) to P^c

No theoretical arguments given.

↪ must be investigated by comparing calculations at leading power with data

BPRS



non-relativistic $c\bar{c}$ enhancement violates factorization <u>at leading power</u>

I believe this is wrong : $c\bar{c}$ threshold not relevant when integrated over smoothly; in any case there is always a Λ/m_b factor for the long-distance contribution

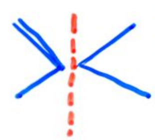
$\Rightarrow$ BPRS treat penguin amplitude P^c as complex fit parameter – one for every SU(2) multiplet.

$\rightarrow$ No CP asymmetry can be predicted from theory alone

Since also C, T are fitted (ϱ, Ξ) and $\arg(C/T)$ is set to zero, this approach has more in common with amplitude fits (like SU(3)) than with QCD / SCET calculations

Integrating out the scale $\sqrt{m_b \Lambda}$ ($SCET_I \to SCET_{II}$ matching)

$$\Xi \sim \langle M_1 | \bar\xi \not{A}_\perp h_v | \bar B \rangle = J * \phi_{B(\omega)} * \phi_{M_1}$$



J contains hard-collinear spectator interactions

$\Rightarrow$

$$\langle M_1 M_2 | O_i | \bar B \rangle = F^{BM_1}_{(0)} \cdot T_i^{I} * \phi_{M_2} + \phi_B * \left[H_i^{II} * J \right] * \phi_{M_1} * \phi_{M_2}$$

$+$ power corrections

(BBNS, 99)

Remarks

- establishes factorization, but often input parameters are not known with satisfactory accuracy, in particular

  $$F^{BM_1}_{(0)}, \quad m_s, \quad \lambda_B^{-1} \equiv \int \frac{d\omega}{\omega} \phi_{B(\omega)}$$

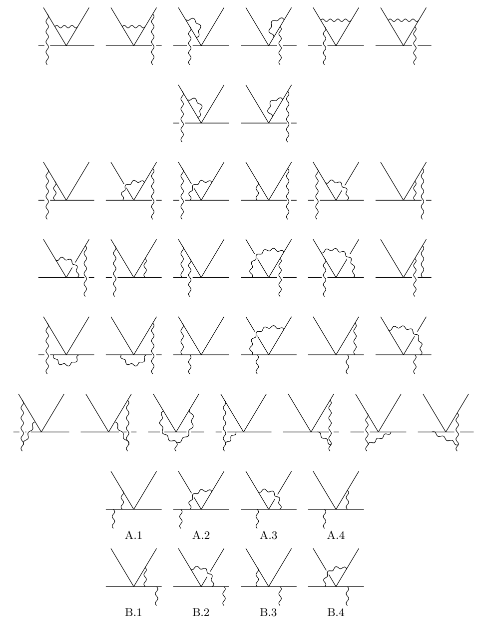
  so far philosophy was <u>not</u> to fit these to data

- there are important $1/m_b$ effects (see below)

New higher-order calculations

QCDF: NLO (α_s^2) spectator scattering

$$T_i^{\mathrm{II}} = H_i^{\mathrm{II}} \star J$$



- 1-loop J
  (Becher, Hill, Lee, Neubert 2004; MB, Yang 2005; Kirillin 2005)
  1-loop H^{II} tree amplitudes
  (MB, Jäger 2005; Kivel 2006 [error?])
  1-loop H^{II} penguin amplitudes
  (MB, Jäger 2006) [QCD penguin also: Li, Yang, 2005, but errors]

- Main results:
  - Perturbation theory well-behaved
  - Sizeable enhancement of the colour-suppressed tree amplitude (good!)
  - Negligible correction to QCD penguin amplitude (disappointing!)

Tree amplitudes and $\pi\pi$ branching fractions with NLO spectator scattering

- Requires smallish

$$|V_{ub}|\, f_+^{B\pi}(0) = 8.1 \cdot 10^{-4}$$

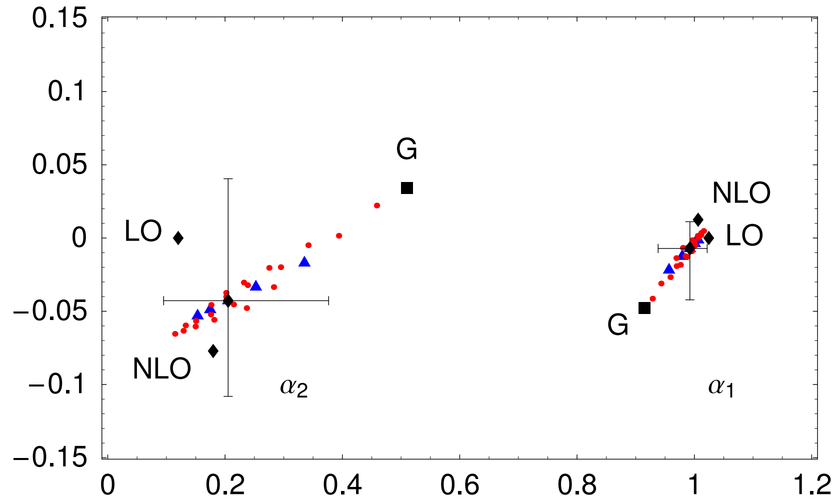
  and larger $f_B/(f_+^{B\pi}(0)\lambda_B)$ than expected. λ_B small?

-

$$C/T = \alpha_2/\alpha_1 = 0.55 + 0.07i$$

- $\pi^-\pi^0$, $\pi^+\pi^-$ are ok, $\pi^0\pi^0$ still somewhat low

- $A_{\rm CP}(\pi^+\pi^-) = 0.39 \pm 0.19$ remains a problem (see below)



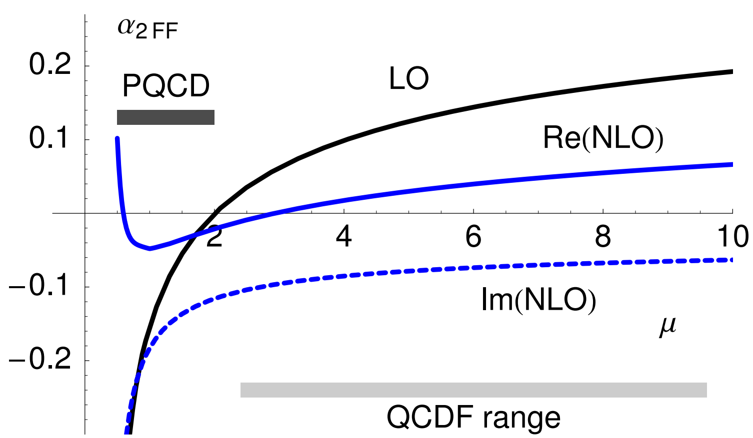
(MB, Jäger, 2005)

10^6 Br$_{\rm Av}$	Theory (NLO$_{\rm sp}$)	Exp.
$\pi^-\pi^0$	$5.5^{+0.3}_{-0.3}$(CKM)$^{+0.5}_{-0.4}$(hadr.)$^{+0.9}_{-0.8}$(pow.)	5.7 ± 0.5
$\pi^+\pi^-$	$5.0^{+0.8}_{-0.9}$(CKM)$^{+0.3}_{-0.5}$(hadr.)$^{+1.0}_{-0.5}$(pow.)	5.2 ± 0.2
$\pi^0\pi^0$	$0.73^{+0.27}_{-0.24}$(CKM)$^{+0.52}_{-0.21}$(hadr.)$^{+0.35}_{-0.25}$(pow.)	1.31 ± 0.21

PQCD: (partial) NLO (Li, Mishima, Sanda, 2005)

- First NLO corrections (partially) included using vertex and penguin kernels from BBNS (1999).
- Same diagrams, but very different numbers.
  Consider colour-suppressed tree amplitude: large negative correction in BBNS, but huge enhancement in LMS: $C_{\pi\pi} = 0.8e^{2.6i} \rightarrow 4.3e^{-1.1i}$
- What is going on?

$$a_{2\mathrm{FF}}(\mu) = C_2 + \frac{C_1}{N_c} + \underbrace{\frac{\alpha_s C_F}{4\pi}\frac{C_1}{N_c}\left[12\ln\frac{m_b}{\mu} - \frac{37}{2} - 3i\pi\right]}_{\text{NLO correction (asymptotic LCDA)}}$$



- Wilson coefficients are evaluated at scales down to 500 MeV. This is conceptually incorrect. Running stops around m_b.
- Correction is evaluated at scales, where perturbation theory breaks down. Numerics is unstable against including higher-order corrections. No scale variations are included in theoretical errors.
- I believe this is a general problem of the PQCD approach and therefore – despite its phenomenological successes – do not consider it as a theoretical framework on the same footing as QCDF/SCET.

$1/m_b$-suppressed effects

Most important $1/m_b$ effect : scalar QCD penguins

$$P^c_{\pi\pi} \sim \underbrace{\overset{\vdots}{\underset{\text{V-A}}{a_4}} + \overset{\vdots}{\underset{\text{operators that Fierz to S-P}}{r_\chi a_6}} + \overset{\cdot\cdot}{\underset{\text{penguin annihilation}}{\beta_3}}}_{1/m_b\text{- suppressed}} \approx \underbrace{\overset{a_4}{[-0.03-0.01i]} + \overset{r_\chi a_6}{[-0.07-0.01\,i]}}_{\text{calculable}} + \underbrace{\overset{\beta_3}{[-0.01-0.02\,e^{i\cdot\text{any}}]}}_{\text{model parameter } (\varrho_A)}$$

- Strict $1/m_b$ expansion is a phenomenological disaster!
- a_6 is fortunately calculable.
  [confirmed - roughly - by data : PP $a_4 + r_\chi a_6$, PV $\approx a_4$, VP $a_4 - r_\chi a_6$]
- Still need to understand whether a_6 factorizes to all orders.
  Probably not, but the effect may be small (MB, Neubert)

Weak annihilation

Several annihilation amplitudes : $\underbrace{\beta_1, \beta_2}_{\text{tree}}$, $\underbrace{\beta_3, \beta_4}_{\text{QCD penguin}}$, ...

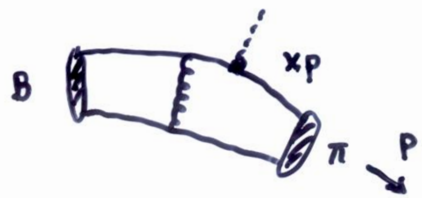
So far no unambiguous <u>empirical</u> evidence that any β_i is relevant at all for charmless decays :

- Some extra contribution to P^c required in QCDF and PQCD, but cannot separate a_6 from β_3 in data
- Constraints from $B \to KK$ only imply that β_i cannot be very large

Theory frameworks	PQCD	BBNS	BPRS	QCD sum rules
	calculable; large	modelled; not very large	neglected	calculable; not very large
	but effect is artificially enhanced by evaluating Wilson coefficients at low scale	makes A_{CP} somewhat model-dependent	except for β_3 fitted with P^c	(Khodjamirian et al., 2005)

Why does factorization usually not work for power corrections?

Actually for $B \to \pi$ form factor does not even work at leading power (Brodsky et al.)



$$\supset \int_0^1 dx \, \frac{\phi_\pi(x)}{(1-x)^2} \sim \int_0^1 \frac{dx}{1-x} = \infty$$

Factorization was derived under the assumption that quarks in π are energetic, but the result shows that the dominant contribution arises when the $\bar{q}$ is soft $(x \to 1)$

- $k_\perp$ - factorisation in PQCD makes integrals convergent, but does not imply that soft contribution can be neglected

- Up to now no regularisation and factorization scheme for endpoint singularities is known.

Recently proposal to implement this in SCET (Manohar, Stewart, 2006)
Illustrate for weak annihilation (Arnesen et al., 2006)

$$\supset \alpha_s \int_0^1 dx \, \frac{\phi_{M_2}(x)}{(1-x)^2} \xrightarrow[\text{expression with } x=1 \text{ region subtracted}]{\text{replaced by}} \alpha_s \int_0^1 dx \, \frac{\phi_{M_2}(x)}{[1-x]^2_{\emptyset}} \equiv \alpha_s \int_0^1 dx \, \frac{\phi_{M_2}(x) + \bar{x}\,\phi'_{M_2}(1)}{\bar{x}^2} - \phi'_{M_2}(1) \cdot \alpha_s \ln \frac{m_B}{\mu_-}$$

convergent

some kind of cut-off "zero-bin"

new cut-off! scale-dependence

compare BBNS

$$\alpha_s \int_0^1 dx \, \frac{\phi_{M_2}(x) + \bar{x}\,\phi'_{M_2}(1)}{\bar{x}^2} + 6\,\underbrace{(1+\varrho_A e^{i\phi_A})}_{\text{phenomenological model}} \cdot \alpha_s \ln \frac{m_B}{\Lambda_{QCD}}$$

Arnesen et al. set $\mu_- = m_b$, but $\alpha_s \ln \frac{m_B}{\mu_-}$ should be interpreted as $\mathcal{O}(1)$.
μ_- dependence is not consistently cancelled in this framework.
$\phi'(1)\,\alpha_s \cdot \log$ should be identified with a yet undefined non-perturbative object.

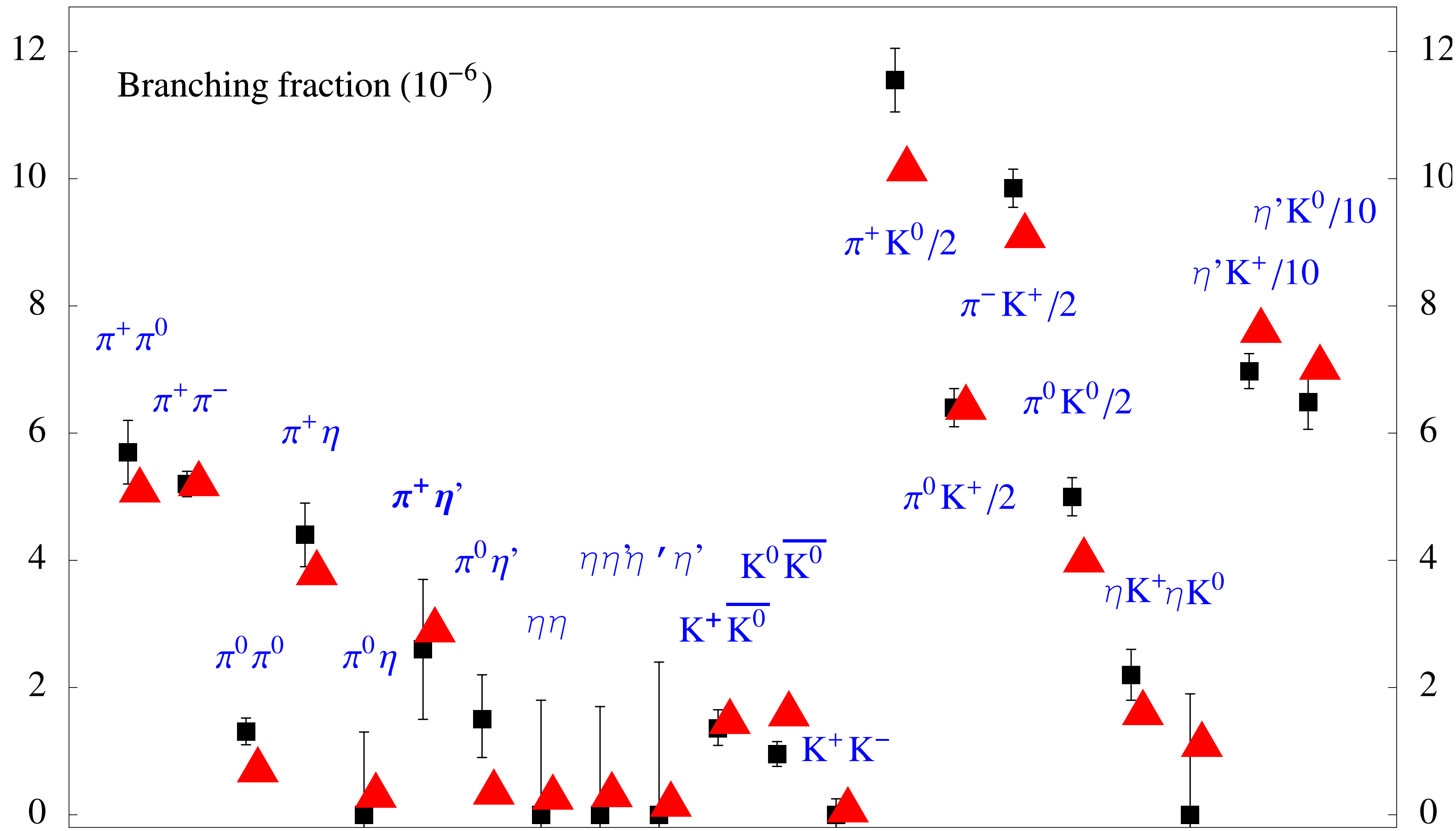
I do not see how "zero-bin" factorization could possibly be correct. More work needed on this very important (and general) problem of endpoint factorization. Prerequisite for further studies in soft-collinear effective theory. (Feldmann, Hurth, 2004)

Confronting data

Overall comparison

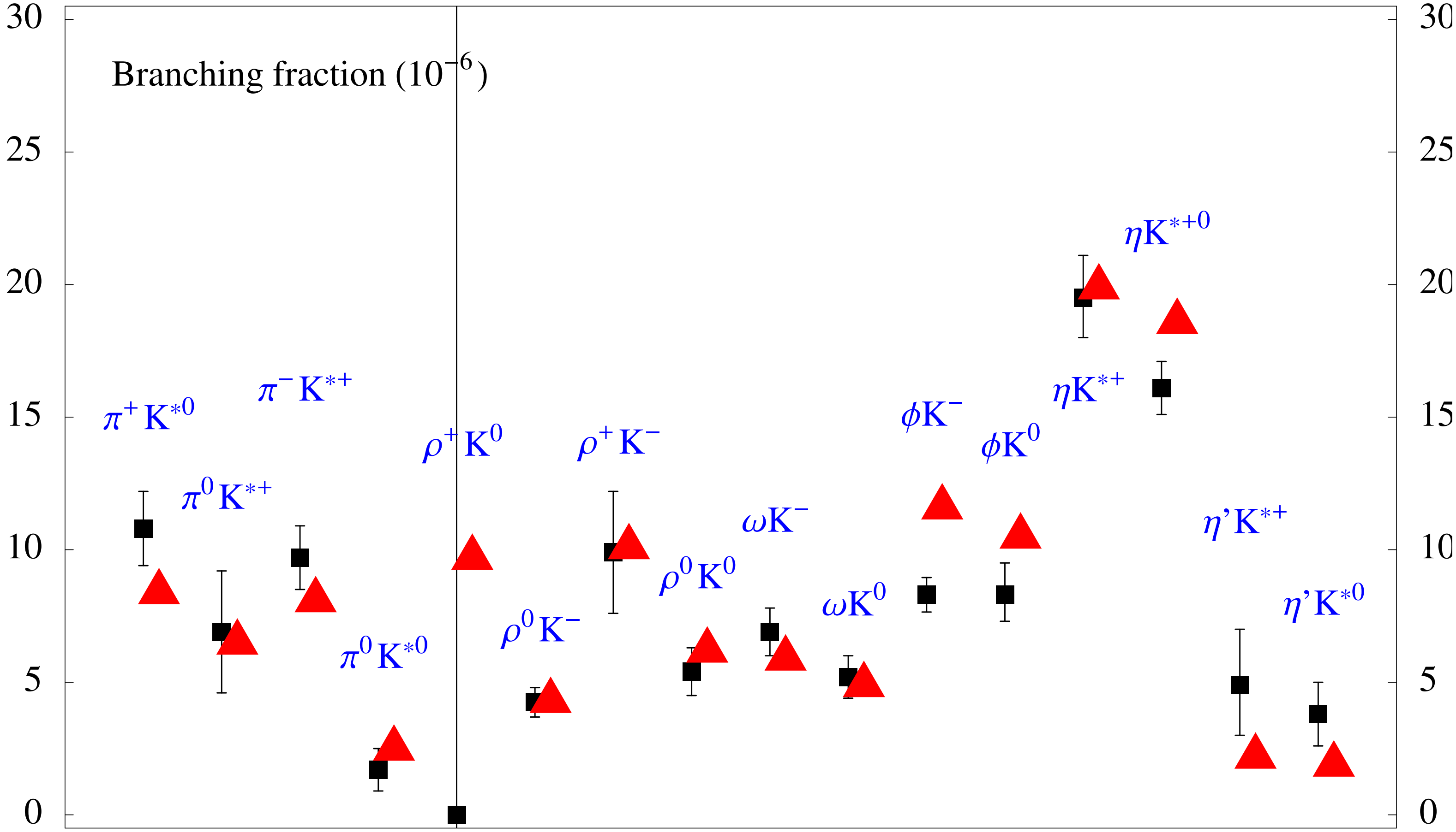
Results for many modes available from the BBNS (BBNS, 2001; Du, Yang, Zhu, 2002; MB, Neubert, 2003), PQCD (Li and collaborators, 2000ff) and the BPRS approach. (Bauer, Rothstein, Stewart, 2005; Williamson, Zupan, 2006)
Apologies for not collecting all (too difficult – scattered over many papers [PQCD] or output changes with new data [BPRS]).
Here show QCDF results and focus on global features which I believe are common to all.

- Br, $A_{\rm CP}$ and some S-parameters calculated for all 96 $B_{u,d,s} \longrightarrow PP, PV$ decays at NLO. (MB, Neubert, 2003)
- Noted that smaller $B \longrightarrow \pi$ form factor, small λ_B and some penguin annihilation contribution to P^c provided a globally better description of the data $\longrightarrow$ defines 'scenario' S4.
- Not a fit.
  Still very successful. No update since 2003, but many new data points.



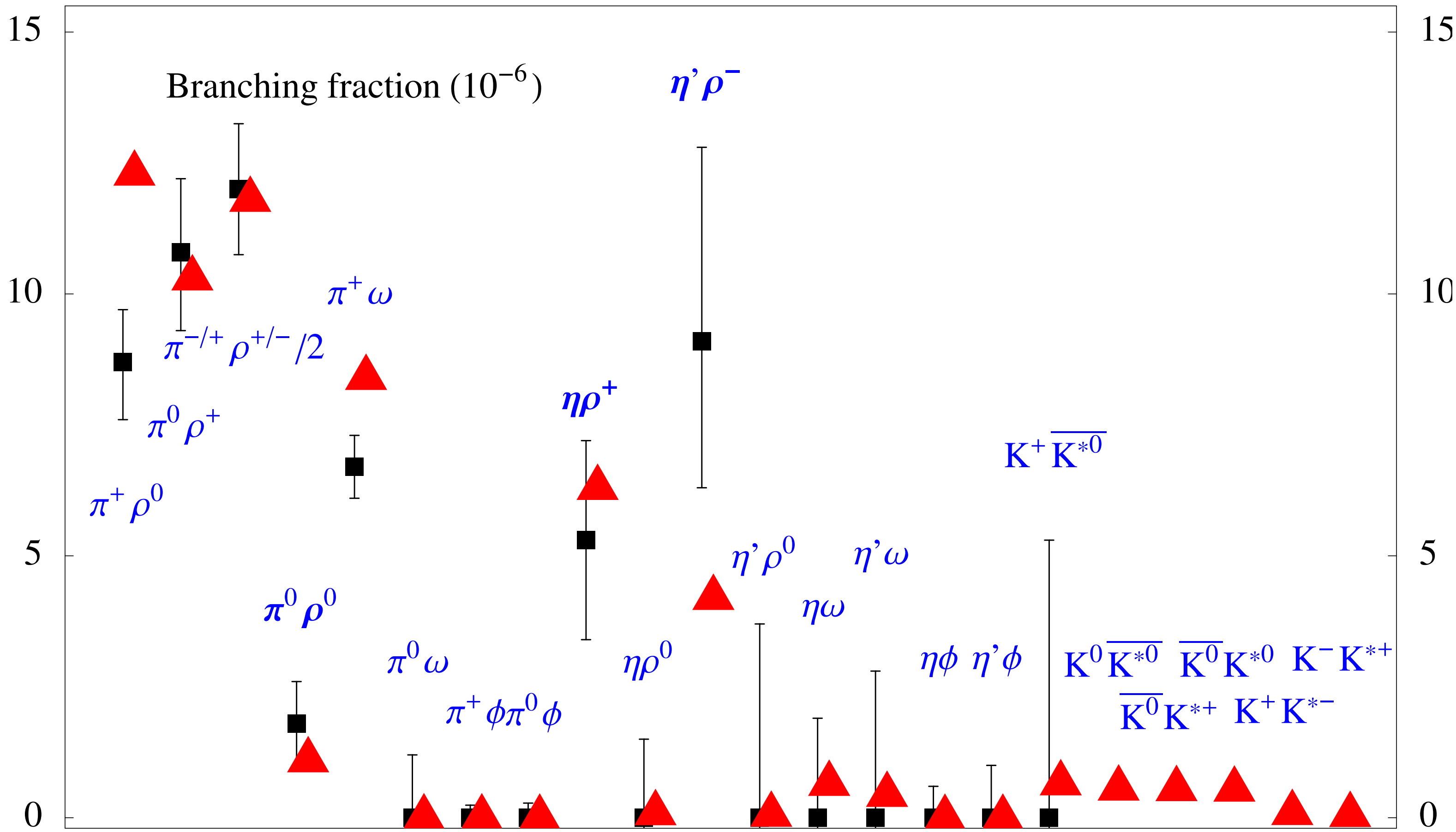
CP-averaged $B \to PP$ branching fractions.

Red triangles: Theory (S4) from MB, M. Neubert, Nucl. Phys. B675 (2003) 333.



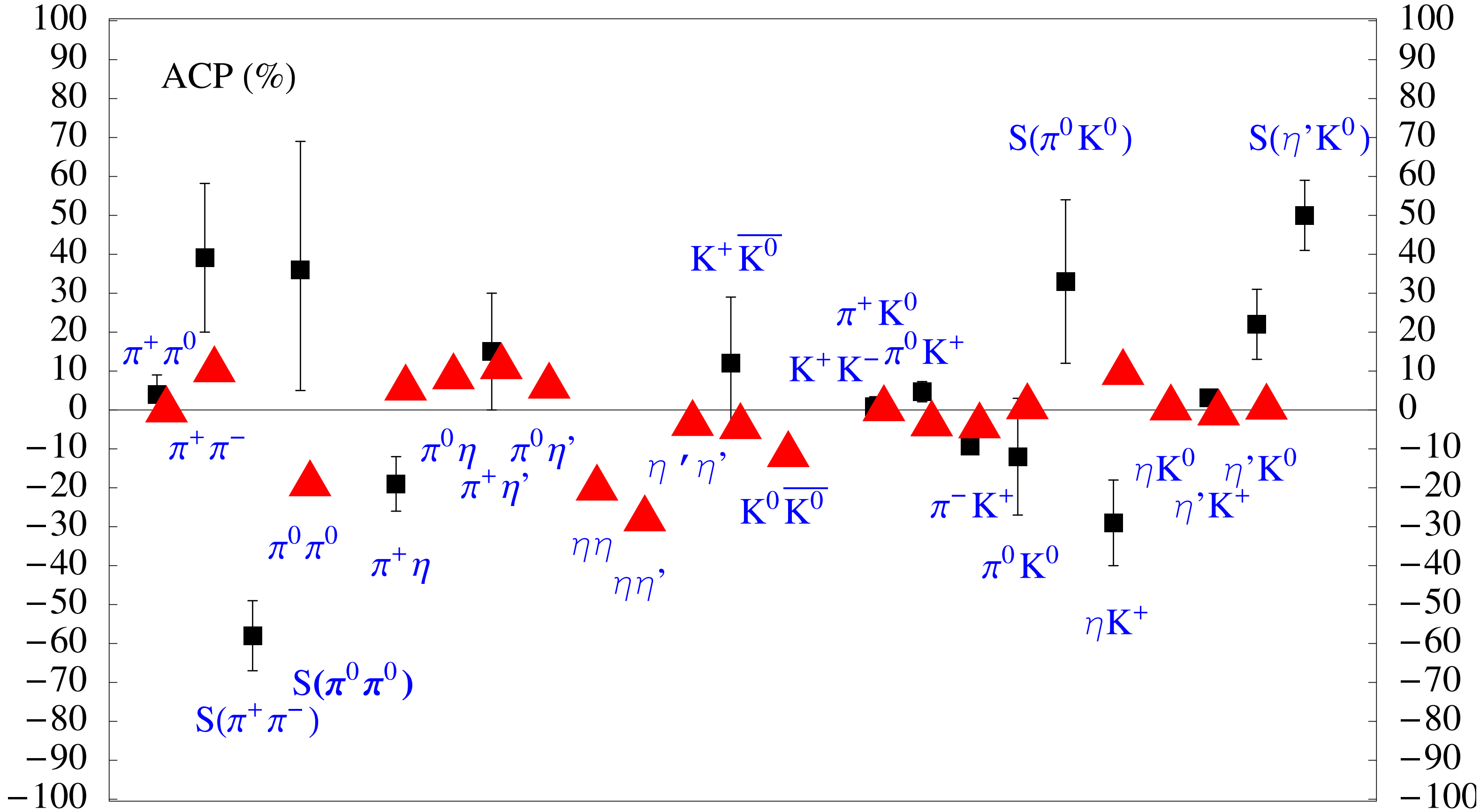
CP-averaged $\Delta S = 1$ $B \to PV$ branching fractions.

Red triangles: Theory (S4) from MB, M. Neubert, Nucl. Phys. B675 (2003) 333.



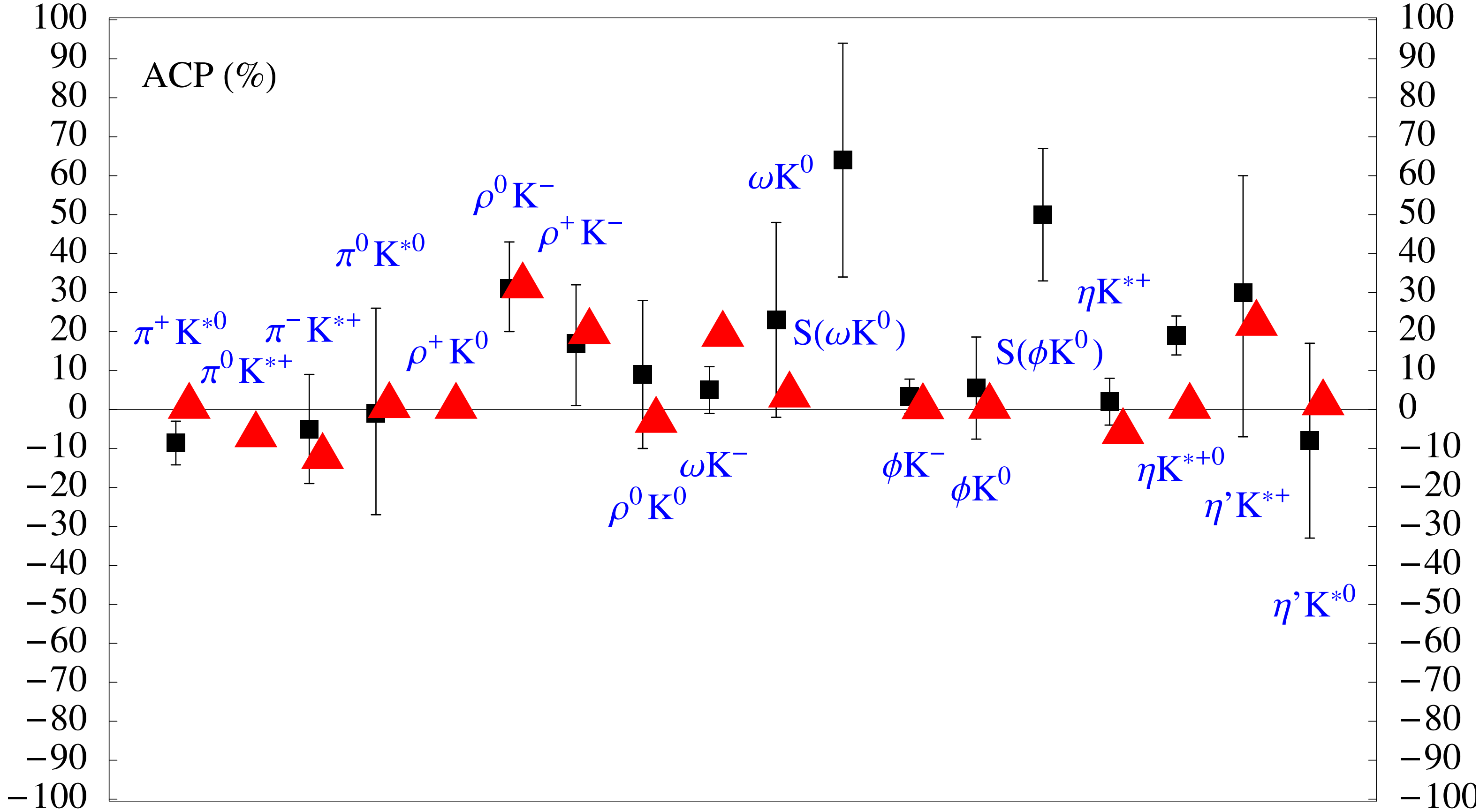
CP-averaged $\Delta S = 0$ $B \to PV$ branching fractions.

Red triangles: Theory (S4) from MB, M. Neubert, Nucl. Phys. B675 (2003) 333.



$B \to PP$ CP asymmetries.

Red triangles: Theory (S4) from MB, M. Neubert, Nucl. Phys. B675 (2003) 333.



$\Delta S = 1$ $B \to PV$ CP asymmetries.

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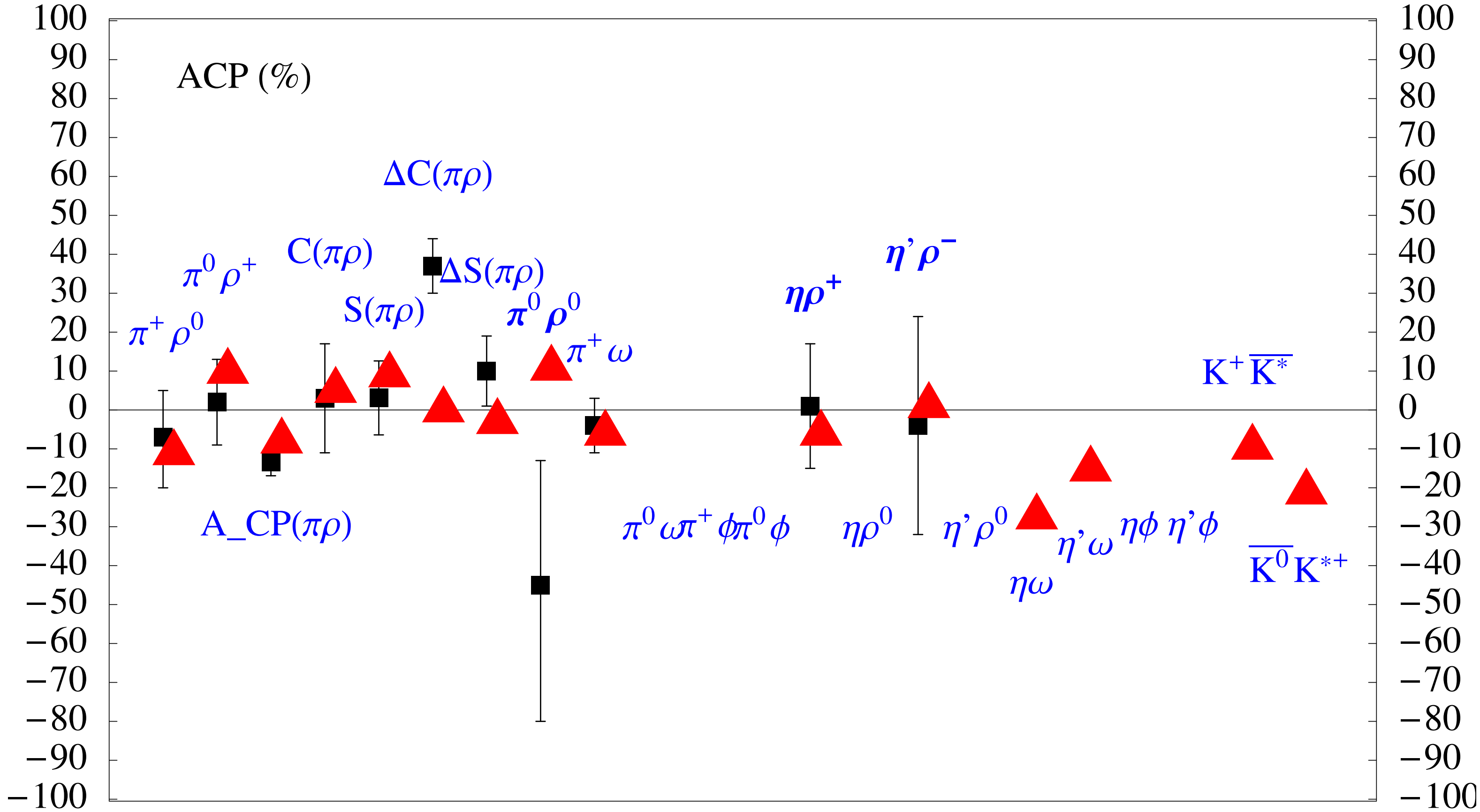


$\Delta S = 0$ $B \to PV$ CP asymmetries and $\Delta C_{\pi\rho}$, $\Delta S_{\pi\rho}$.

Red triangles: Theory (S4) from MB, M. Neubert, Nucl. Phys. B675 (2003) 333.

Summary of the global comparison

- Hierarchy of branching fractions ranging from $1 \cdot 10^{-6}$ to $70 \cdot 10^{-6}$ $(\eta' K)$ is well predicted/reproduced.

- Direct CP asymmetries are generally found small in agreement with expectations. Some predictions are quantitatively very good ($\pi\rho$, ρK, πK^* vs πK, $\eta' K^*$), but there are also serious discrepancies ($\pi^+\pi^-$, $\pi^+ K^-$, $\pi^+\eta$, ηK).
  Expect all kinds of corrections to be more important for direct CP asymmetries, because leading terms starts with α_s.

- Devil is in details difficult to see in the global comparison: the $\pi^0\pi^0$ rate, the $\pi^+ K^-$ and $\pi^+\pi^-$ CP asymmetry, ...

  Also required annihilation with strong phase to improve the comparison – some model-dependence!

$\gamma\,[\alpha]$ from time-dependent CP asymmetries S in $b \to d$ transitions

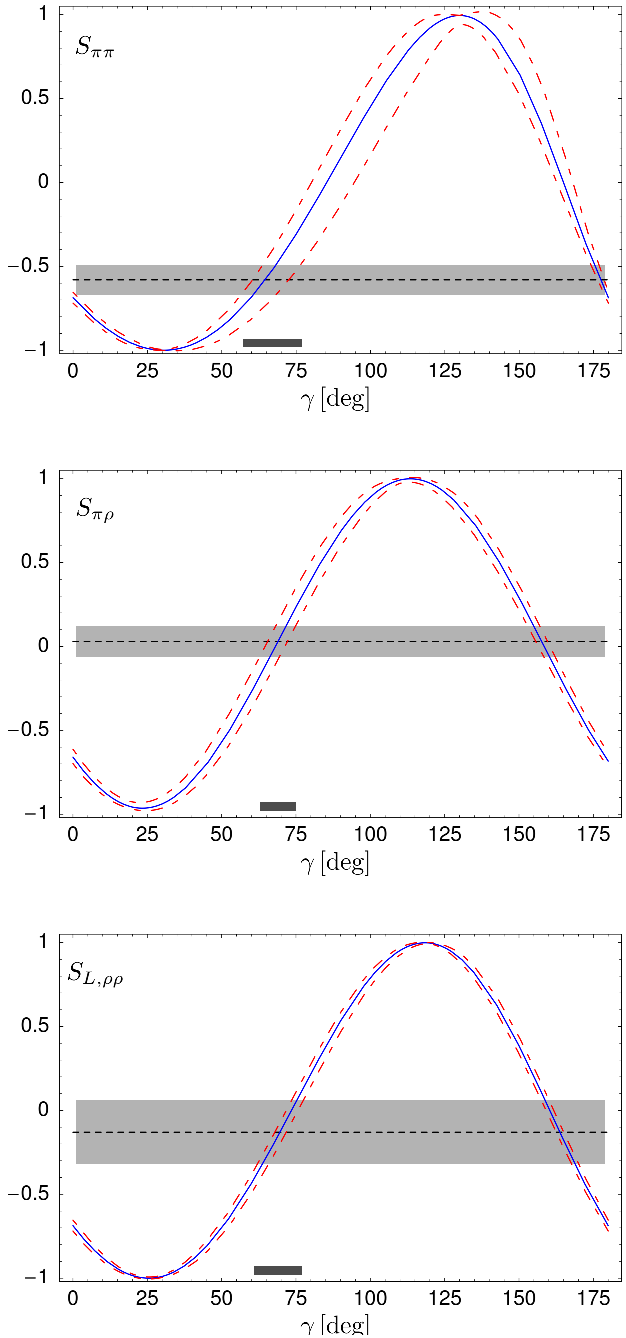
S_f = coefficient of $\sin(\Delta m_B t)$ in the time-dependent $B(t) \longrightarrow f$ decay rate

$$S_{\pi\rho} = \frac{2R}{1+R^2}\sin 2\alpha - \frac{2R}{1+R^2}\left\{a\cos\delta_a\left(\frac{2\sin 2\alpha}{1+R^2}\cos\gamma + \sin(2\beta+\gamma)\right)\right.$$
$$\left. - \, b\cos\delta_b\left(\frac{2R^2\sin 2\alpha}{1+R^2}\cos\gamma + \sin(2\beta+\gamma)\right)\right\} + \ldots \qquad (\alpha \equiv \pi - \beta - \gamma)$$

$$A_{\rho\pi}T_{\rho\pi}/(A_{\pi\rho}T_{\pi\rho}) = R\,e^{i\delta_T} \quad R = 0.91\,^{+0.26}_{-0.21},\;\; \delta_T \approx 0$$
$$P_{\pi\rho}/T_{\pi\rho} = a\,e^{i\delta_a},\;\; P_{\rho\pi}/T_{\rho\pi} = -b\,e^{i\delta_b}, \quad a \approx b \approx 0.1,\;\; \cos\delta_{a,b} \approx 1$$

For $S_{\pi\pi}$ $[S_{\rho\rho}]$ put $R = 1$, $\delta_T = 0$, $a = -b \approx 0.3\,[0.1]$, $\delta_a = \delta_b$.

- S parameters have large sensitivity to γ if γ is near 70°.
- Theoretical uncertainties enter only in the sub-leading correction term, which is especially small for $\pi\rho$ and $\rho\rho$. Strong phases enter only as $\cos$.



$$S_{\pi\pi} = -0.58 \pm 0.09$$
$$\Rightarrow \quad \gamma = (65\,^{+12}_{-8})^\circ$$

$$S_{\pi\rho} = 0.03 \pm 0.09$$
$$\Rightarrow \quad \gamma = (69\,^{+6}_{-6})^\circ$$

$$S_{\rho\rho} = -0.13 \pm 0.19$$
$$\Rightarrow \quad \gamma = (69\,^{+8}_{-8})^\circ$$

Mutually consistent

$$\gamma = (68 \pm 4)^\circ,$$

consistent with the standard unitarity triangle fit (UTfit, 2006):

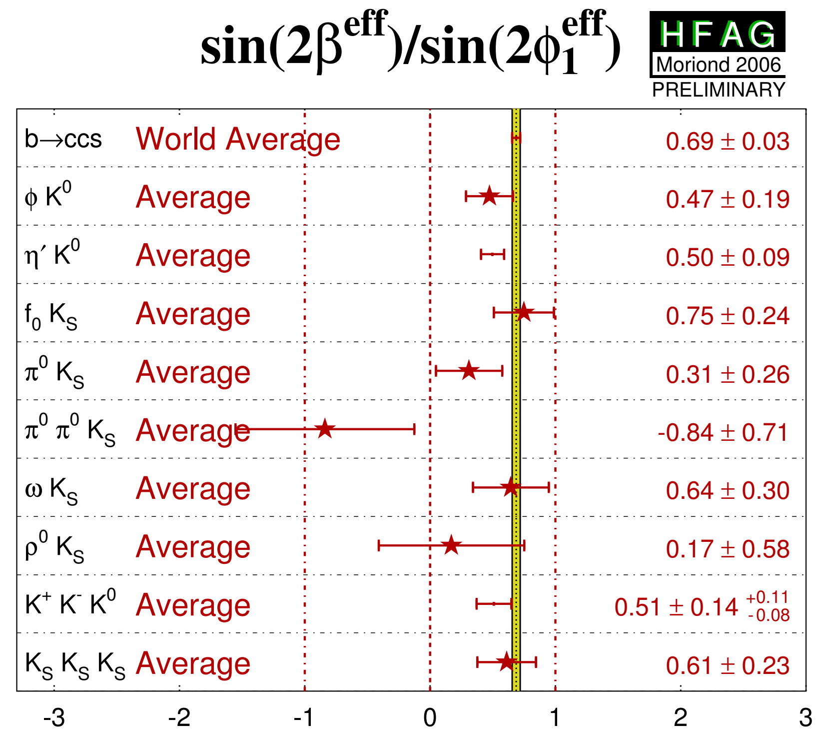
$$\gamma = (61 \pm 5)^\circ$$

$\sin(2\beta)$ from $b \to s$ transitions (QCDF: MB, Neubert 2003; MB 2005 ; Cheng, Chua, Soni 2005)

$$\Delta S_f \equiv S_{f(b\to s)} - \sin(2\beta)_{J/\psi K_S}$$

should be small, since $b \to c\bar{c}s$ and $b \to s\bar{s}s$ have (nearly) the same weak phase.

Mode	ΔS_f (Theory)	ΔS_f [Range*]
$\pi^0 K_S$	$0.07^{+0.05}_{-0.04}$	$[+0.02, 0.15]$
$\rho^0 K_S$	$-0.08^{+0.08}_{-0.12}$	$[-0.29, 0.02]$
$\eta' K_S$	$0.01^{+0.01}_{-0.01}$	$[+0.00, 0.03]$
ηK_S	$0.10^{+0.11}_{-0.07}$	$[-1.67, 0.27]$
ϕK_S	$0.02^{+0.01}_{-0.01}$	$[+0.01, 0.05]$
ωK_S	$0.13^{+0.08}_{-0.08}$	$[+0.01, 0.21]$



ΔS_f is positive except for ρK_S and ηK_S.
$\Rightarrow$ Many speculations on anomalous CP violation in $b \to s\bar{s}s$.

* from a random scan of $2 \cdot 10^5$ input parameter sets and requiring that experimental branching fractions are reproduced within $\pm 3\sigma$

Summary

1 We have learned a lot about hadronic dynamics

- ⋆ Now calculate observables that were once thought to be intractable with rigorous methods
- ⋆ The subject has been an extremely fertile ground for developing new theoretical concepts (such as SCET)
- ⋆ Recent issues: higher-order calculations (conceptually clear but hard work) and power corrections (still unclear)

2 We have learnt a lot about the CKM angle γ from charmless hadronic final states. There should be some way to include this information in the CKM fit beyond the few standard methods!