

Lattice QCD, Flavor Physics and the Unitarity Triangle Analysis



Vittorio Lubicz

OUTLINE

1. Impact of lattice calculations in the UT Fits
2. Review of lattice results
3. Determination of lattice hadronic parameters from UT Fits
4. V_{us} and K_{l3} decays from LQCD

Heavy Quarks and Leptons
Munich, 16-20 October, 2006

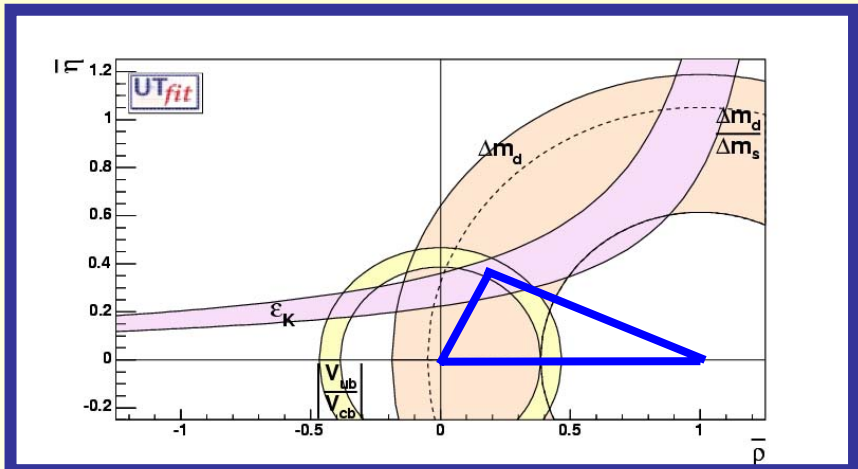


THE "UT-LATTICE" ANALYSIS:

UTA IN THE PRE-B
 FACTORIES ERA:
 CP-conserving sizes + ε_K

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

Hadronic matrix
 elements from
LATTICE QCD



$(b \rightarrow u)/(b \rightarrow c)$	$\bar{\rho}^2 + \bar{\eta}^2$	$f_+, F(1), \dots$
Δm_d	$(1 - \bar{\rho})^2 + \bar{\eta}^2$	$f_{B_d}^2 B_{B_d}$
$\Delta m_d / \Delta m_s$	$(1 - \bar{\rho})^2 + \bar{\eta}^2$	ξ
ε_K	$\bar{\eta} [(1 - \bar{\rho}) + P]$	B_K

4 CONSTRAINTS
2 PARAMETERS

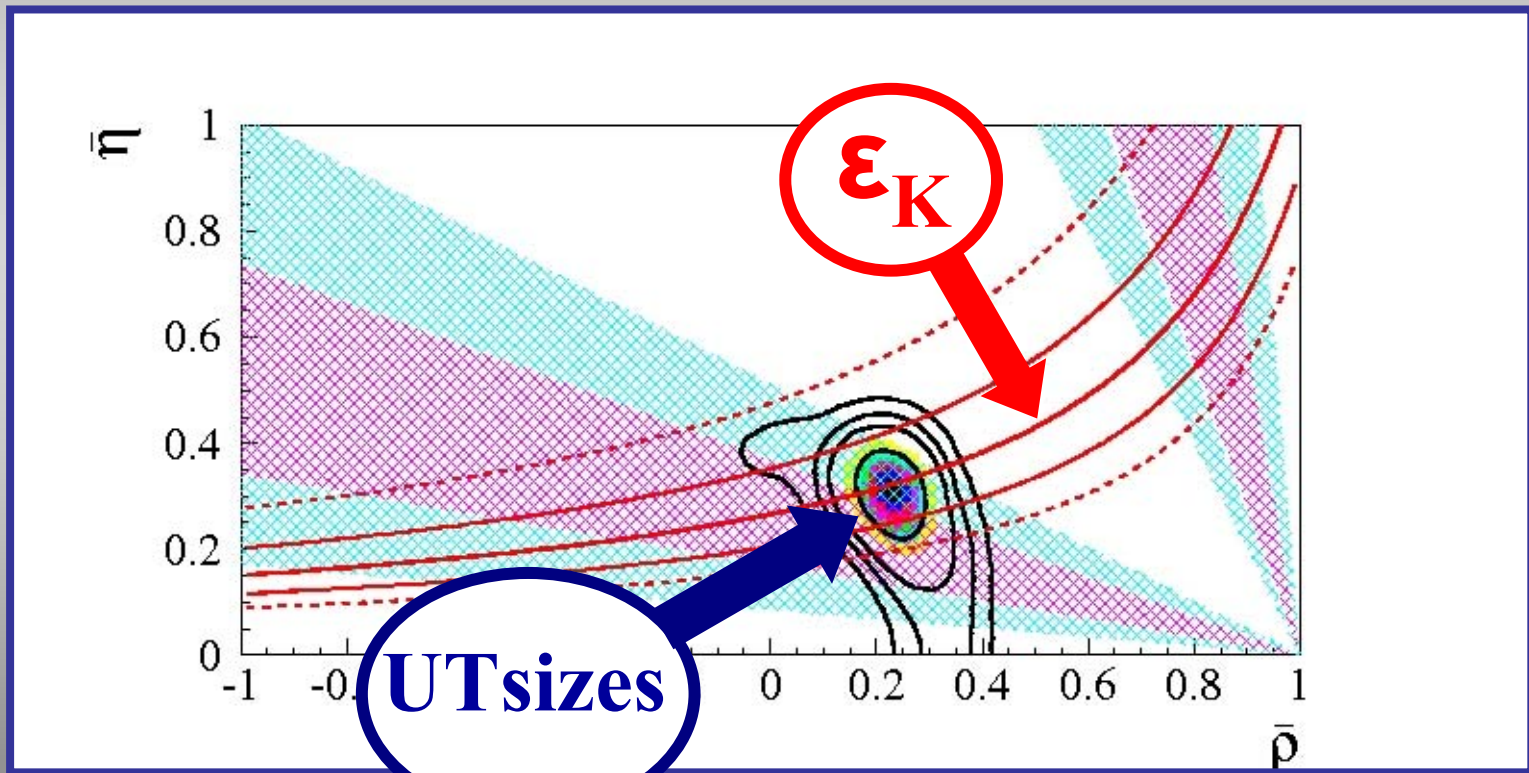
Already before the starting of the B factories
**3 IMPORTANT RESULTS FOR
FLAVOUR PHYSICS**

- 1) Confirmation of the **CKM** origin of ~~\mathcal{CP}~~ in **K- \bar{K}** mixing
- 2) Prediction of **$\sin 2\beta$**
- 3) Prediction of **Δm_s**

A great success of (quenched)
Lattice QCD calculations !!

1) INDIRECT EVIDENCE OF \cancel{CP}

Ciuchini et al. ("pre-UTFit" paper), 2000



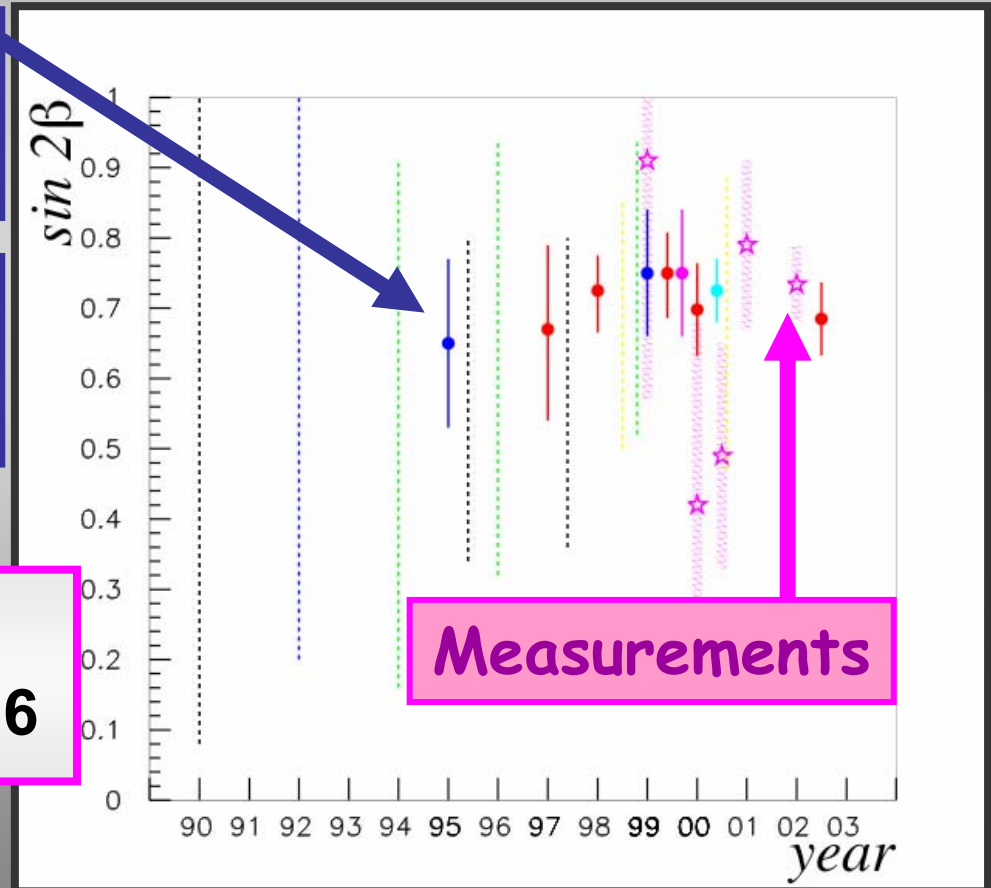
2) PREDICTION OF $\text{Sin}2\beta$

Predictions exist since 1995

Ciuchini et al., 1995:
 $\text{Sin}2\beta_{\text{UTA}} = 0.65 \pm 0.12$

Ciuchini et al., 2000:
 $\text{Sin}2\beta_{\text{UTA}} = 0.698 \pm 0.066$

ICHEP 2006:
 $\text{Sin}2\beta_{J/\psi K_0} = 0.675 \pm 0.026$



3) PREDICTION OF Δm_s

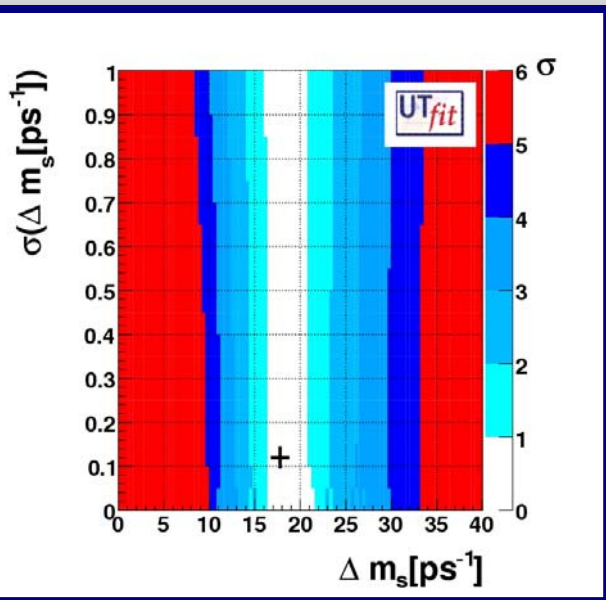
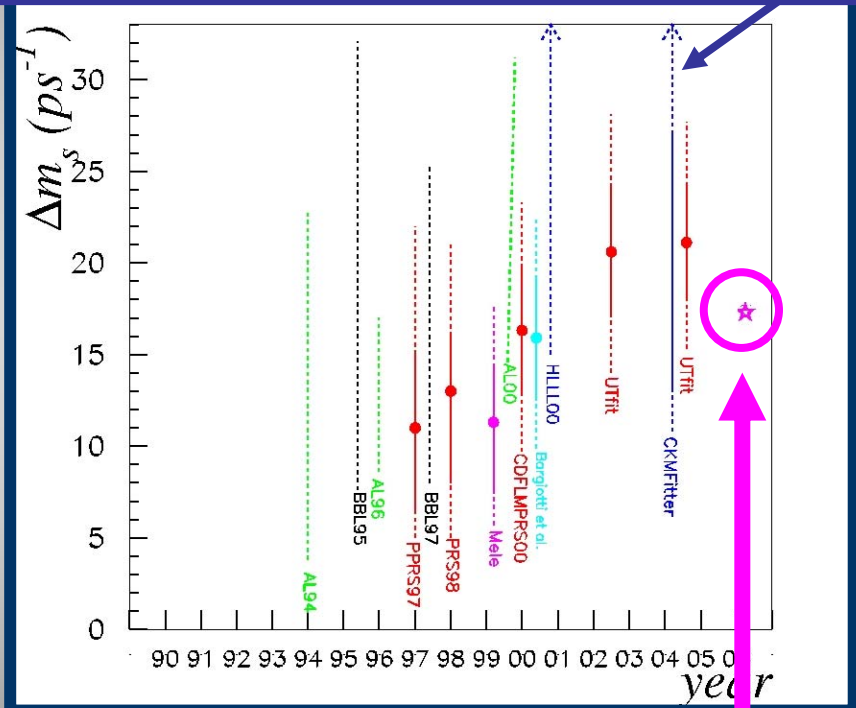
Ciuchini et al., 2000:

$$\Delta m_s = (16.3 \pm 3.4) \text{ ps}^{-1}$$

UTFit Coll., 2006:

$$\Delta m_s = (18.4 \pm 2.4) \text{ ps}^{-1}$$

The predicted range was very large in the frequentistic CKMFitter approach

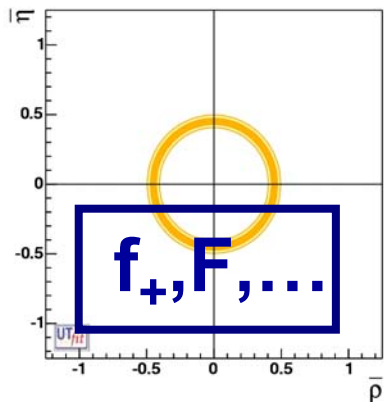


CDF, 2006:

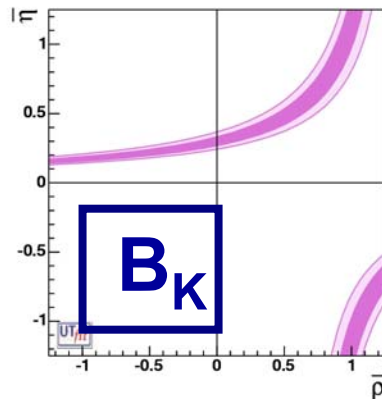
$$\Delta m_s = (17.77 \pm 0.10 \pm 0.07) \text{ ps}^{-1}$$

THE LATTICE INPUT PARAMETERS

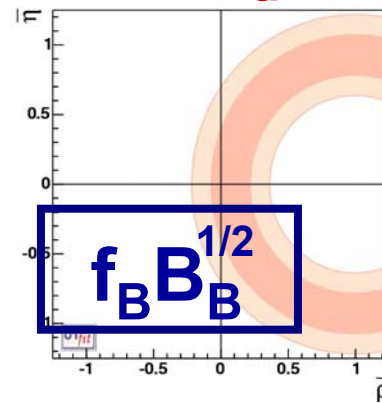
$$|V_{ub}/V_{cb}|$$



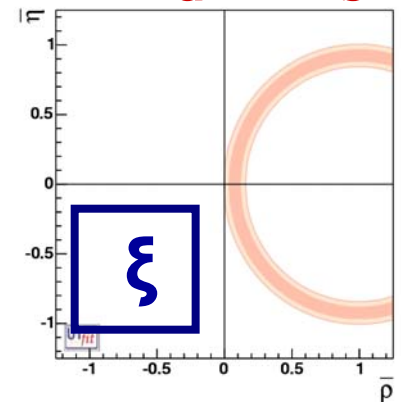
$$\epsilon_K$$



$$\Delta m_d$$



$$\Delta m_d / \Delta m_s$$



QUENCHING vs. UNQUENCHING

- Nf=0** most of the calculations
- Nf=2** few calculations
- Nf=2+1** one/two calculations

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Quenched calculations: unsatisfactory because $N_f=0$. But:

- other systematic uncertainties well under control (**non-perturbative renormalization**, continuum extrapolation,...)
- the **results checked by many groups**, using various gauge and quark lattice actions

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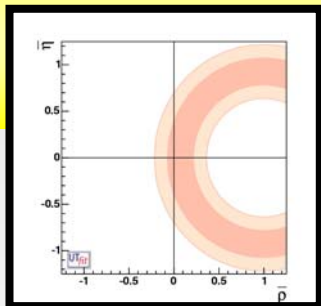
- other systematic uncertainties well under control (**non-perturbative renormalization**, continuum extrapolation,...)
- the **results checked by many groups**, using various gauge and quark lattice actions

Unquenched calculations: sound for being unquenched. But:

- dynamical quarks much heavier than the physical up and down quarks
- the consequences of the **fourth-root trick** (non-locality,...) are not clear
- **non-perturbative renormalization** is not carried out
- the **results have not been checked** by different groups.

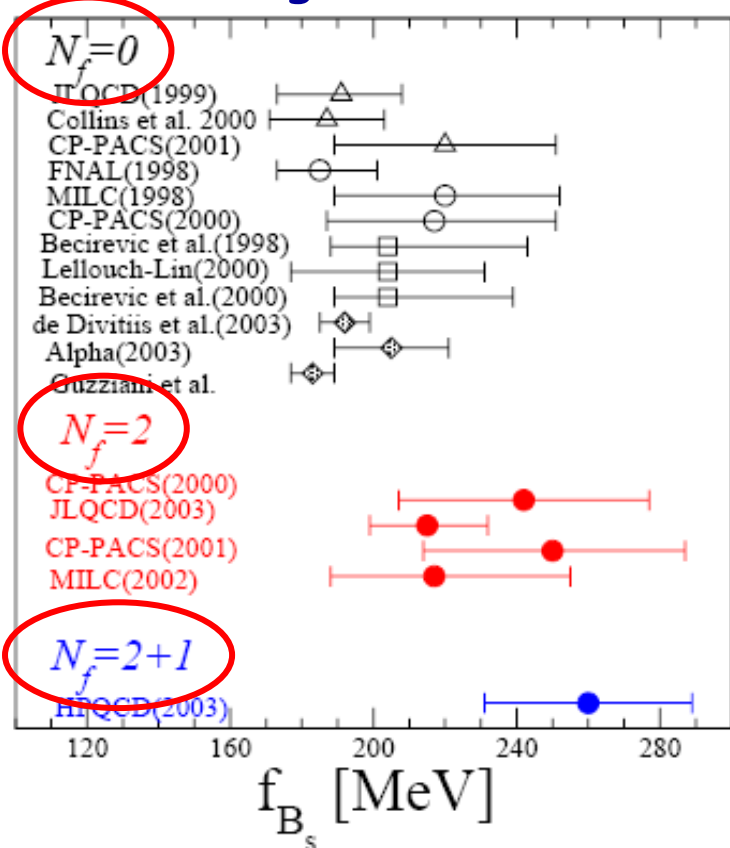
Unquenching is a work in progress

$B_{d,s}$ mixing: 1) f_{B_s}



The SM prediction for Δm_s relies on the lattice determination of f_{B_s} .

Onogi@Latt'06



- The slight difference between $N_f=2$ and $N_f=0$ results depends on the quantity used to set the lattice scale

- There is some discrepancy ($\sim 1\sigma$) between the $N_f=2+1$ result of HPQCD and the $N_f=2$ JLQCD result

- Is this difference the effect of the strange sea quark or of an (underestimated) systematic error?

$B_{d,s}$ mixing: 1) f_{B_s}

Recent lattice "averages" are:

$$f_{B_s} = 230 \pm 30 \text{ MeV}$$

Hashimoto@ICHEP'04

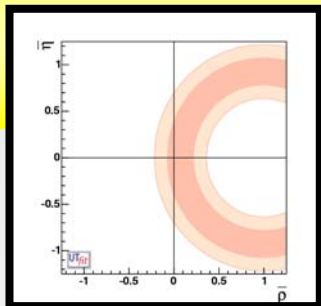
$$f_{B_s} = 260 \pm 29 \text{ MeV}$$

Okamoto@LATT'05
Onogi@LATT'06

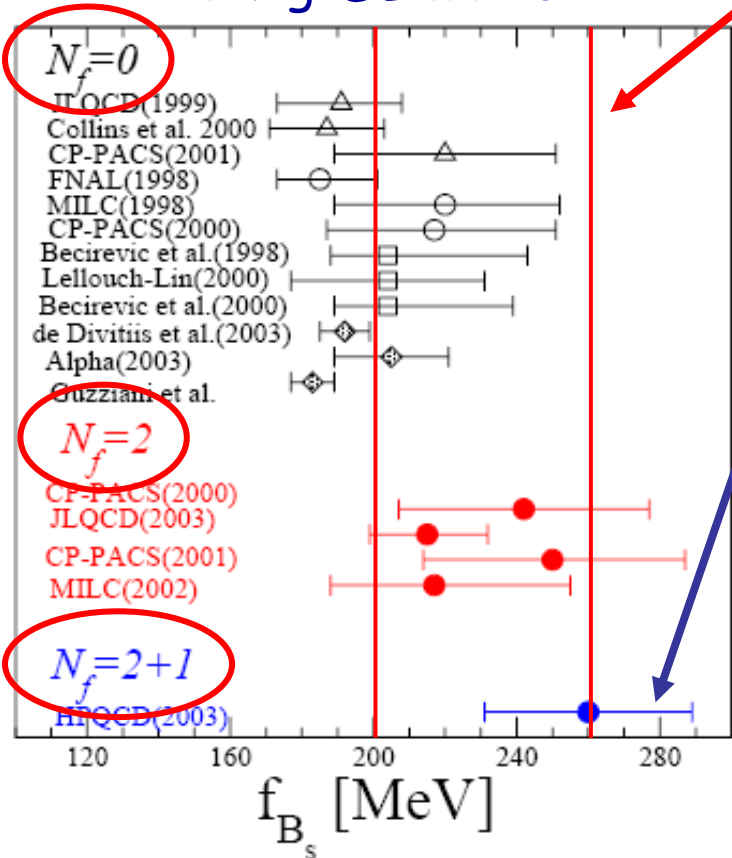
HPQCD

If the difference is due to the strange sea quark effect then we should only consider the $N_f=2+1$ result. But:

- i) $N_f=0 \ll N_f=2 < N_f=2+1$
- ii) $m_s \gg m_{u,d}$

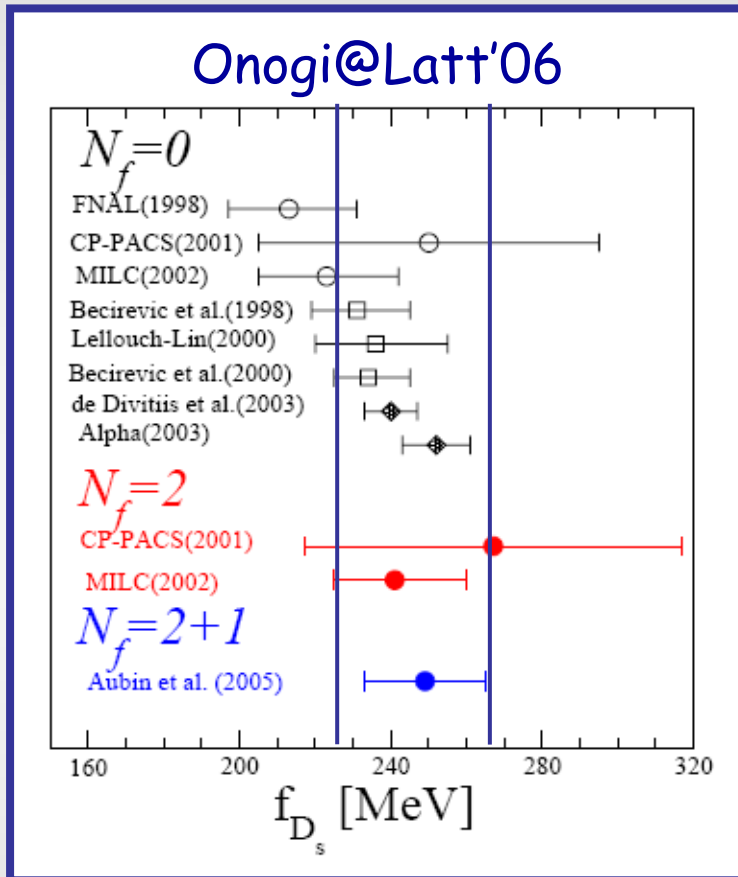


Onogi@Latt'06



D-mesons decay constants: f_D, f_{D_s}

Important test for Lattice QCD



Results from ICHEP06:

$$f_D = (222 \pm 17 \pm 3) \text{ MeV}$$

$$f_{D_s} = (280 \pm 12 \pm 6) \text{ MeV}$$

$$f_{D_s}/f_D = 1.26 \pm 0.11 \pm 0.03$$

Lattice QCD averages

$$f_{D_s} = (245 \pm 20) \text{ MeV} \quad , \quad f_{D_s}/f_D = 1.23 \pm 0.06$$

$B_{d,s}$ mixing: 2) f_{B_s}/f_B and ξ

f_B and therefore the ratio f_{B_s}/f_B are affected by the “potentially large” effect of chiral logarithms:

Recent history of ξ

“Old” estimates (<2002):

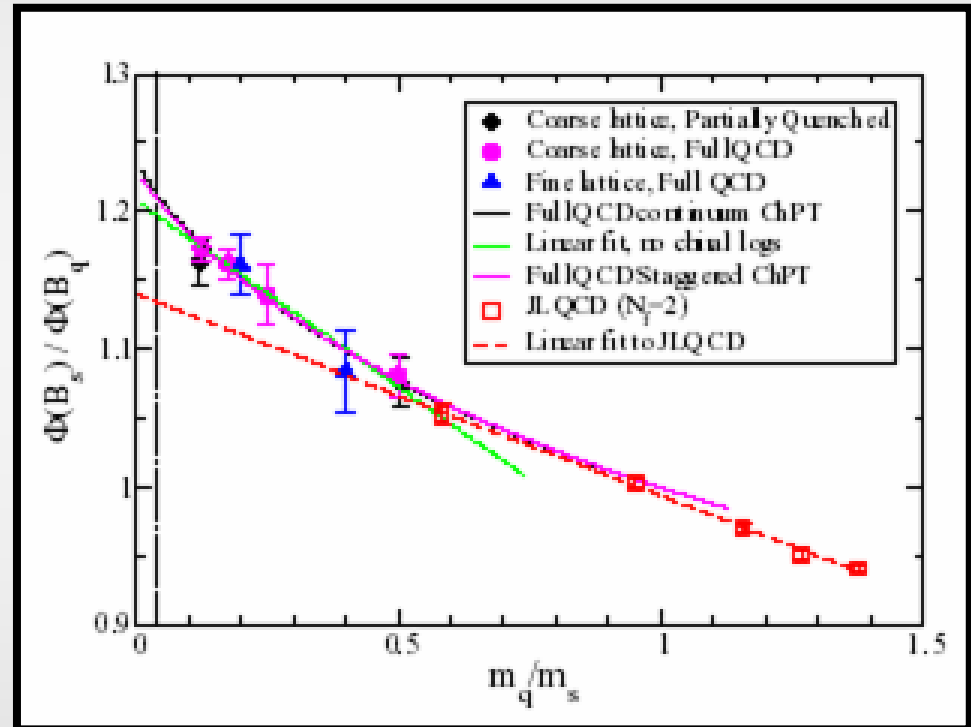
$$f_{B_s}/f_B \approx 1.14 - 1.18$$

Kronfeld and Ryan (2002)

$$f_{B_s}/f_B = 1.32 \pm 0.10$$

HPQCD (2005)

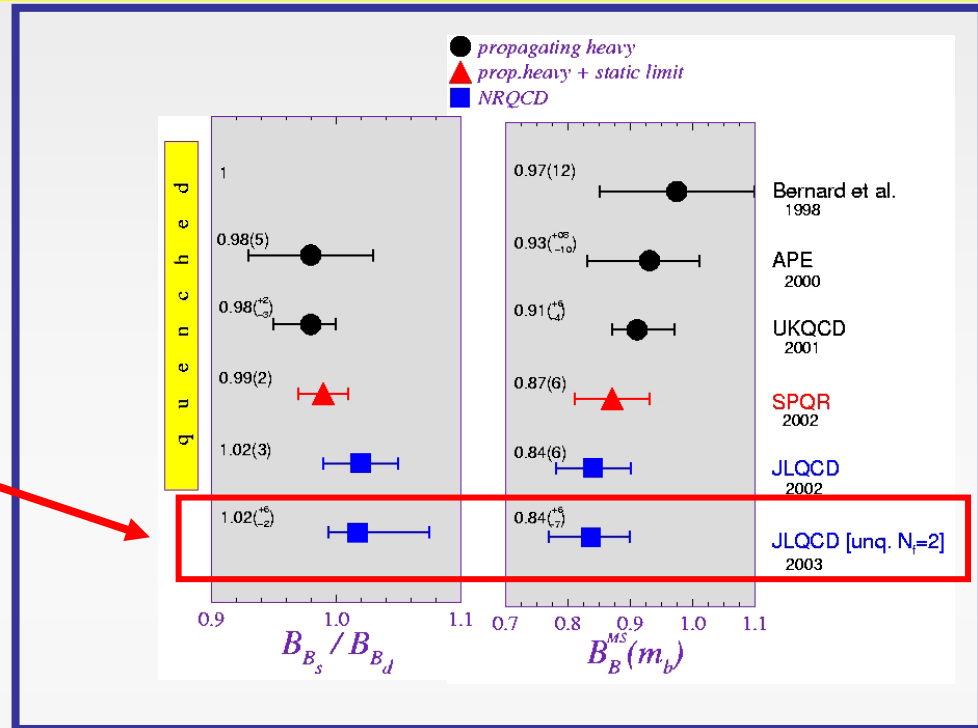
$$f_{B_s}/f_B = 1.20 \pm 0.03$$



But the present estimate still relies on a single calculation. Further determinations at low quark masses are required.

B_d and B_s mixing: 3) $B_{d,s}$

- No large chiral logs effects
- The $N_f=2$ result and a preliminary $N_f=2+1$ result consistent with quenched estimates
- Combining the $N_f=2$ result with the Hashimoto's averages of decay constants one gets:



$$f_{B_s} \sqrt{B_{B_s}} = 262 \pm 35 \text{ MeV}$$

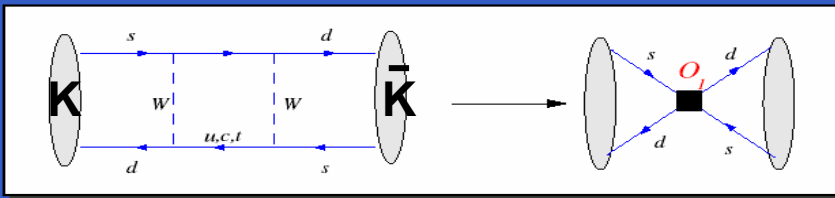
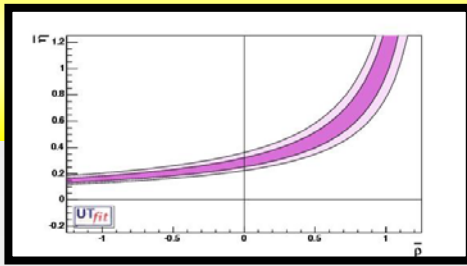
$$\xi = 1.23 \pm 0.06$$

Preliminary $N_f=2+1$
HPQCD@Lattice06:

$$f_{B_s} \sqrt{B_{B_s}} = 281 \pm 21 \text{ MeV}$$

$$\xi = 1.25 \pm 0.05$$

K-K̄ mixing: ϵ_K and B_K



$$\langle \bar{K}^0 | Q(\mu) | K^0 \rangle = \frac{8}{3} f_K^2 m_K^2 B_K(\mu)$$

QUENCHED ERROR

$$B_K = 0.58 \pm 0.03 \pm 0.06$$

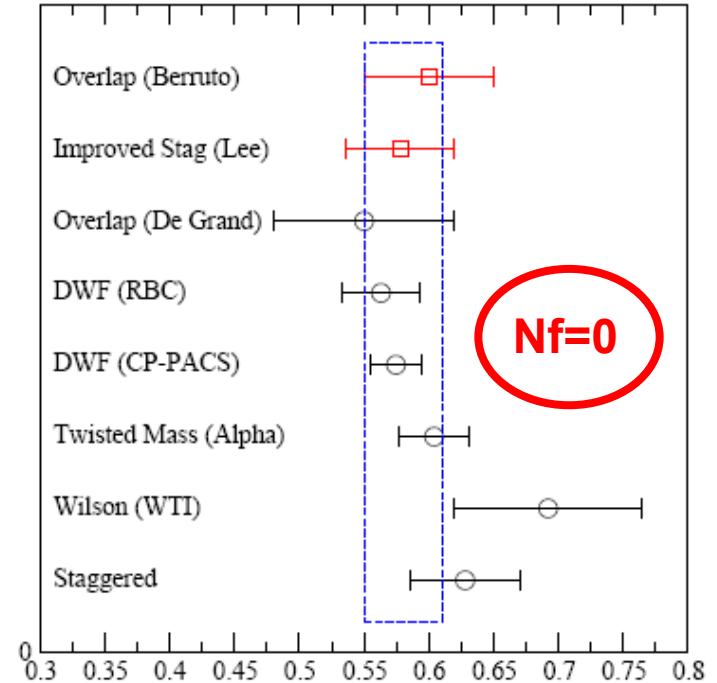
$$\hat{B}_K = 0.79 \pm 0.04 \pm 0.09$$

C.Dawson@Latt'05

$$\hat{B}_K = 0.90 \pm 0.20 \quad (\text{Gavela et al.})$$

1987

C.Dawson@Latt'05



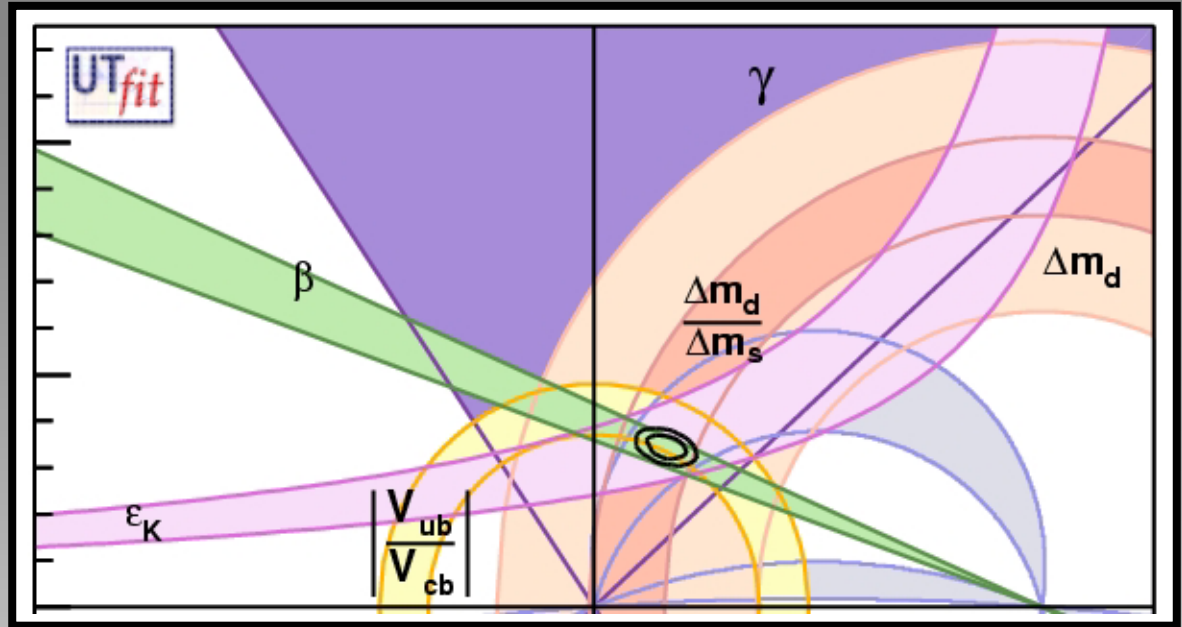
$$B_K = 0.50 \pm 0.02 \pm ?? \quad \text{Stat.} \quad \text{Nf=2, RBC}$$

$$B_K = 0.62 \pm 0.14 \quad \text{Nf=2+1}$$

HPQCD & UKQCD

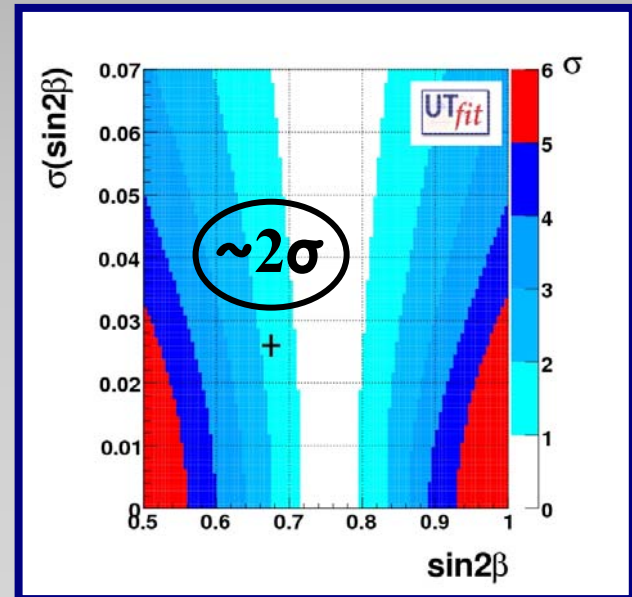
$b \rightarrow u$ decays and the V_{ub} puzzle

There is some tension in the fit, particularly between $\sin 2\beta$ and V_{ub}

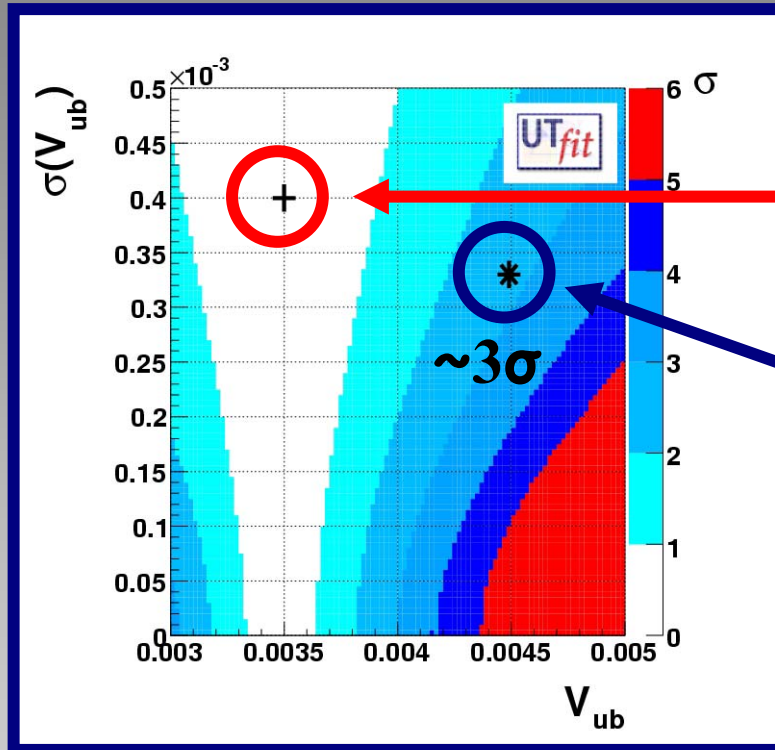


$\sin 2\beta = 0.675 \pm 0.026$
from the direct measurement

$\sin 2\beta = 0.755 \pm 0.039$
from the rest of the fit



The tension is between the **inclusive V_{ub}** and the rest of the fit



EXCLUSIVE

$$V_{ub}^{\text{excl.}} = (35.0 \pm 4.0) 10^{-4}$$

Form factors from LQCD and QCDSR

INCLUSIVE

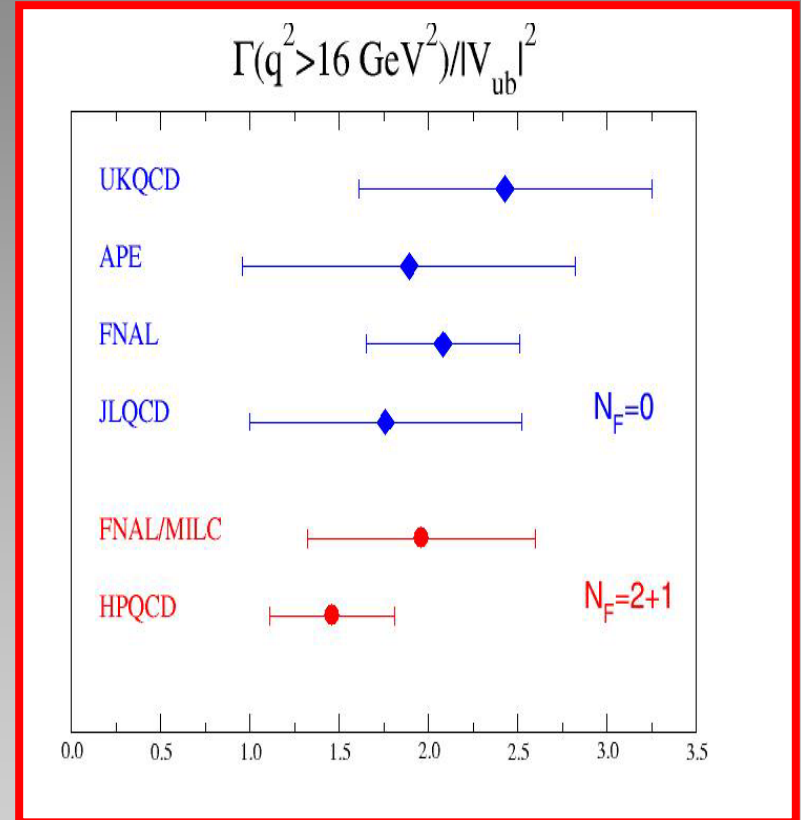
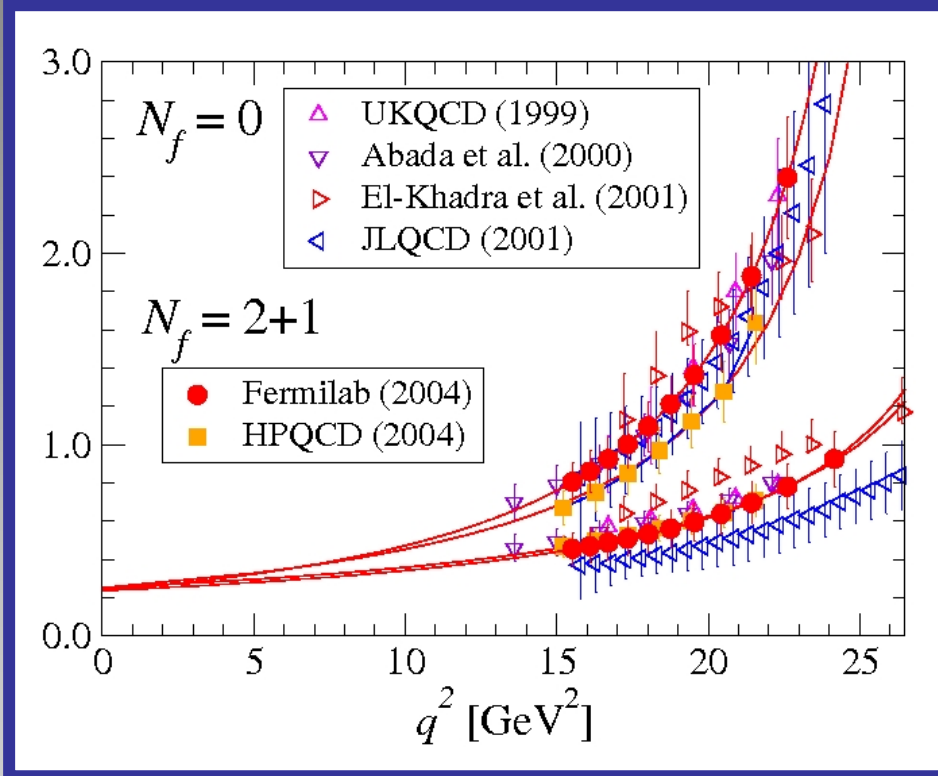
$$V_{ub}^{\text{incl.}} = (44.9 \pm 3.3) 10^{-4}$$

Model dependent (BLNP, DGE,..)
Non perturbative parameters most not from LQCD (fitted from experiments)

A **New Physics effect** is **unlikely** in this tree-level process

- ➔ i) Statistical fluctuation
- ii) Problem with the theoretical calculations and/or the estimate of the uncertainties

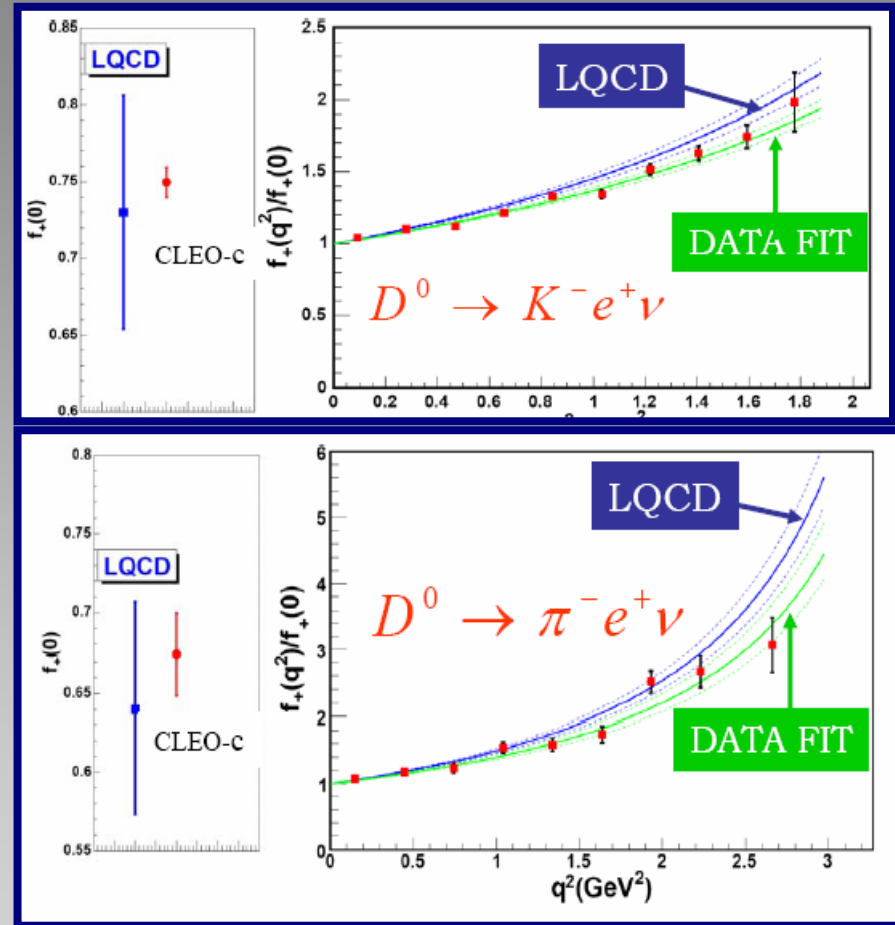
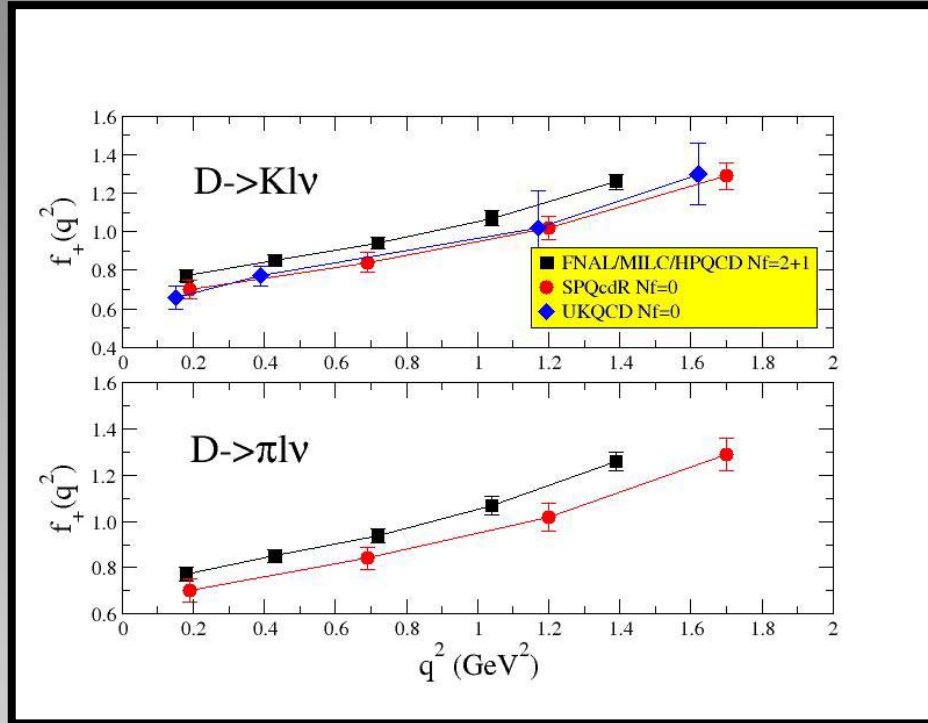
LATTICE QCD: improve V_{ub} exclusive to solve the tension



4 calculations with $N_f=0$
 2 calculations with $N_f=2+1$

Semileptonic $D \rightarrow K/\pi l \nu$ decays

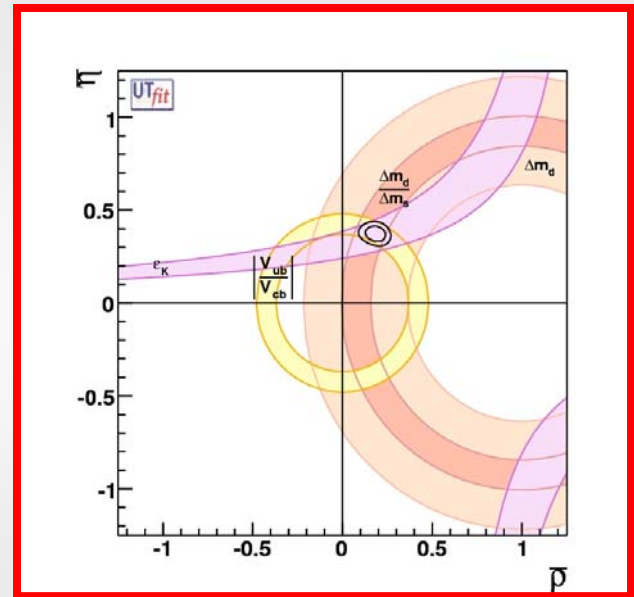
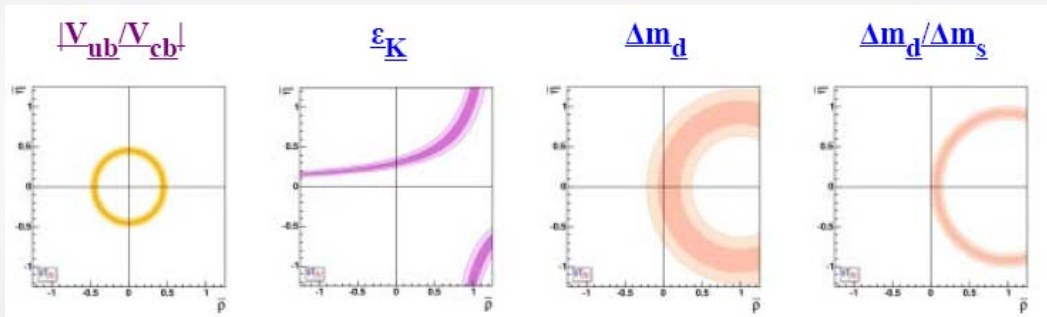
An important test for Lattice QCD



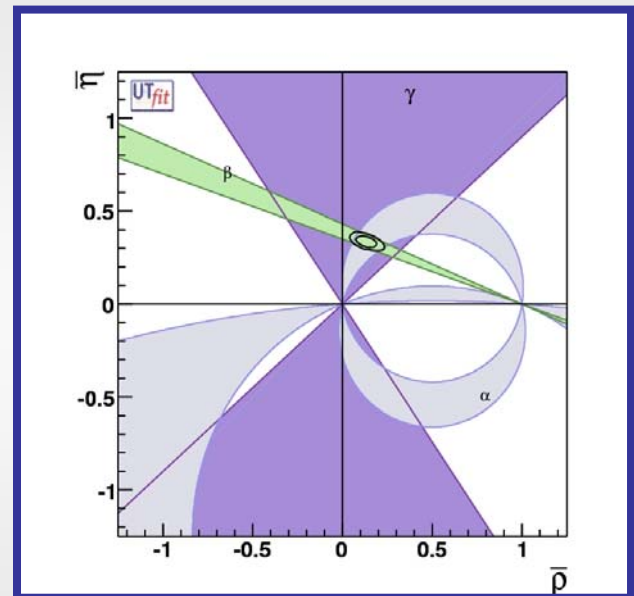
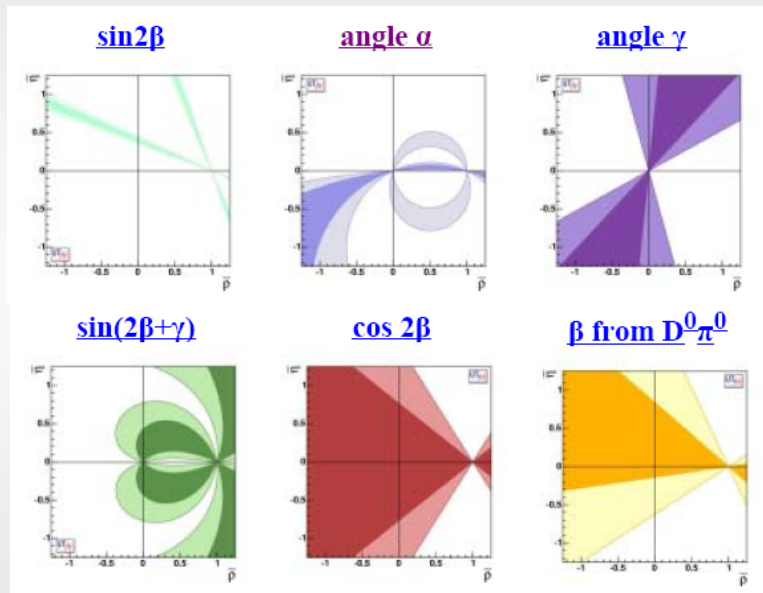
More (and possibly more accurate) LQCD calculations are needed, also for the D \rightarrow Vector channel

"EXPERIMENTAL"
DETERMINATION OF THE
LATTICE PARAMETERS

UT-LATTICE



UT-ANGLES



"EXPERIMENTAL" DETERMINATION OF LATTICE PARAMETERS

The **new measurements** of Δm_s and $BR(B \rightarrow \tau \nu_\tau)$ allows a simultaneous fit of the hadronic parameters:

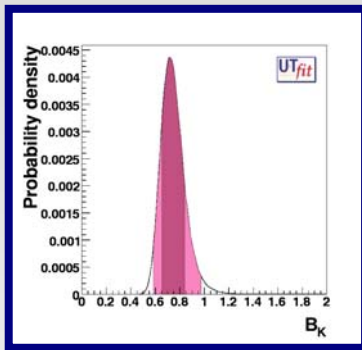
$$|\epsilon_K| = C_\epsilon A^2 \lambda^6 \bar{\eta} \left[-\eta_1 S(x_c) + \eta_2 S(x_t) \left(A^2 \lambda^4 (1 - \bar{\rho}) \right) + \eta_3 S(x_c, x_t) \right] \hat{B}_K$$

$$\Delta m_q = \frac{G_F^2}{6\pi^2} m_{B_q} M_W^2 \eta_B S_0(x_t) |V_{tq}|^2 \hat{B}_{B_q} f_{B_q}^2$$

$$BR(B^- \rightarrow \tau^- \bar{\nu}_\tau) = f_B^2 |V_{tb}|^2 \frac{G_F^2 m_B m_\tau^2}{8\pi} \left(1 - \frac{m_\tau^2}{m_B^2} \right)^2 \tau_B$$

Take the angles from experiments and extract

$f_{B_s} \sqrt{B_{B_s}}$, $f_B \sqrt{B_{B_d}}$ or ξ and f_B

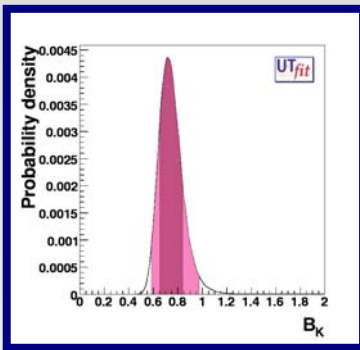


$$\hat{B}_K = 0.75 \pm 0.09$$

$$\hat{B}_K = 0.79 \pm 0.04 \pm 0.08$$

UTA

Lattice
[Dawson]

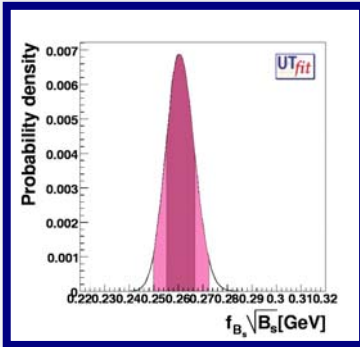


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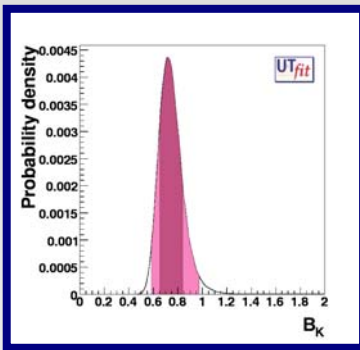
$$f_{B_s} \sqrt{B_{B_s}} = 261 \pm 6 \text{ MeV}$$

UTA

$$f_{B_s} \sqrt{B_{B_s}} = 262 \pm 35 \text{ MeV}$$

Lattice
[Hashimoto]

2%!

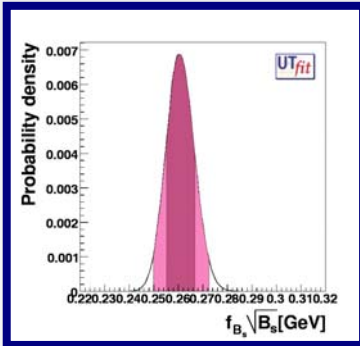


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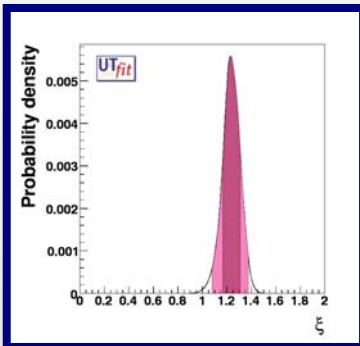
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Lattice
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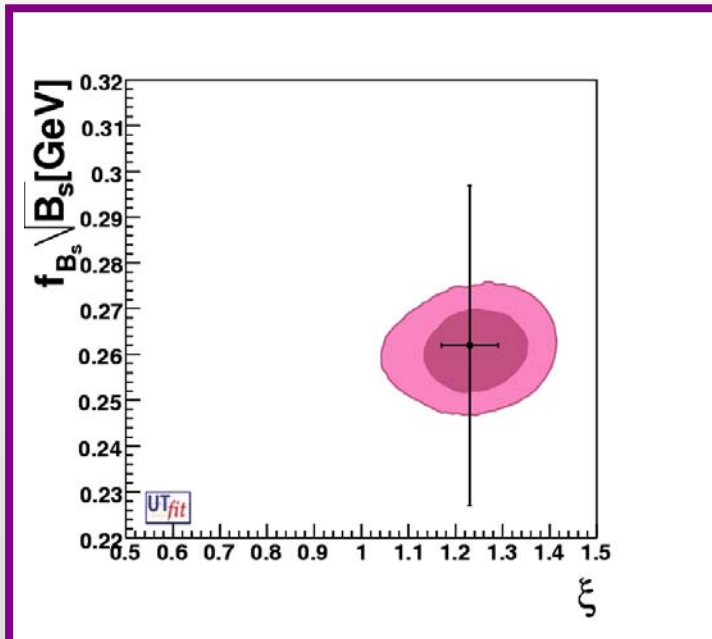
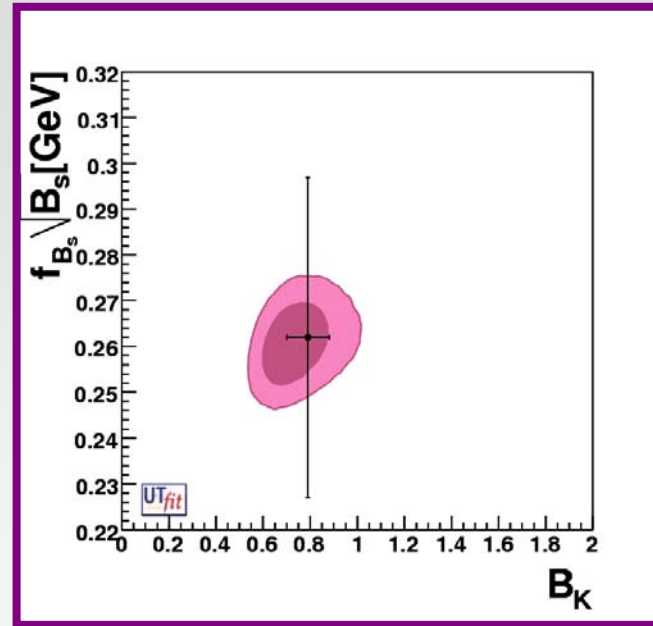
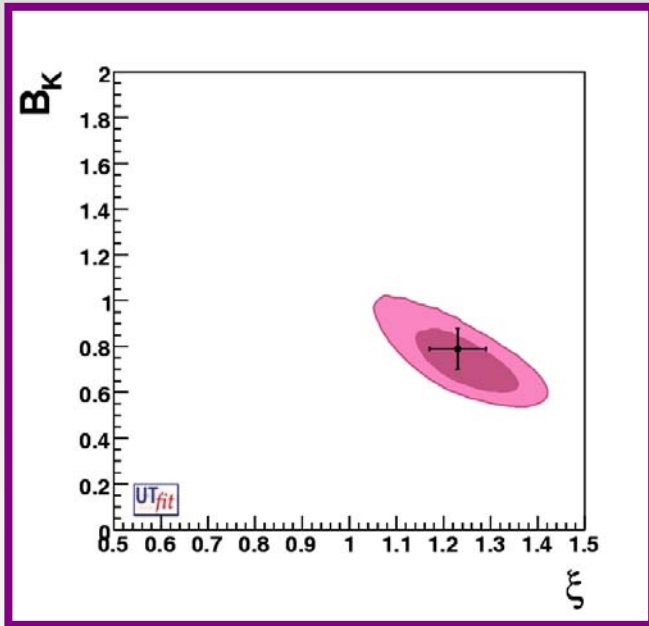
$$\xi = 1.24 \pm 0.08$$

UTA

$$\xi = 1.23 \pm 0.06$$

Lattice
[Hashimoto]

The agreement is spectacular!



Note: the scale is 6 times larger!!

The extraction of the **decay constants** can be obtained by using the Lattice QCD determinations of the B parameters: $\hat{B}_{B_d} = \hat{B}_{B_s} = 1.28 \pm 0.05 \pm 0.09$

$$f_{B_s} = 229 \pm 9 \text{ MeV}$$

$$f_B = 190 \pm 14 \text{ MeV}$$

UTA

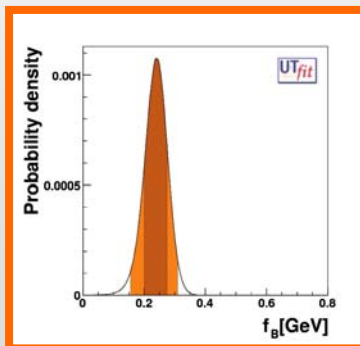
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$$f_B = 189 \pm 27 \text{ MeV}$$

Lattice

f_B can also be extracted using V_{ub} and the recent determination of the **leptonic branching fraction**:

$$\text{Br}(B \rightarrow \tau \nu_\tau) = (1.31 \pm 0.48) \times 10^{-4} \quad \text{Belle + BaBar average}$$



$$f_B = 237 \pm 37 \text{ MeV}$$

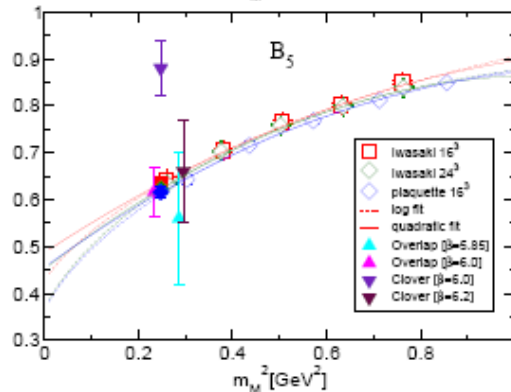
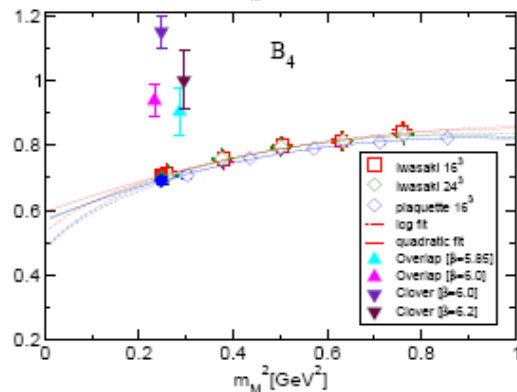
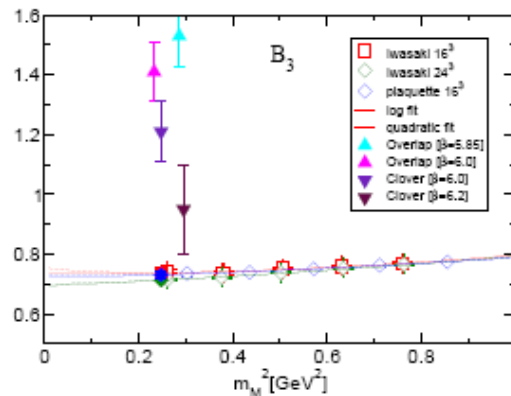
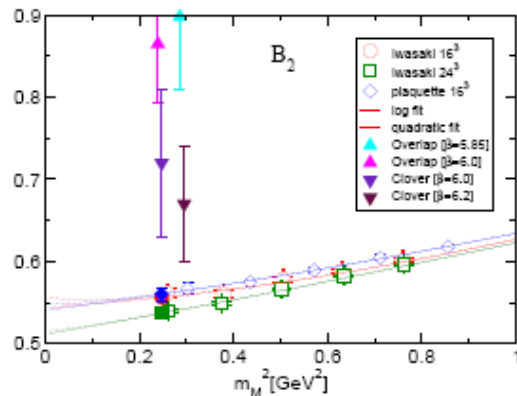
(V_{ub} from the UTA)

You may have got the impression that Lattice QCD calculations are becoming irrelevant for the UTA.
But this is not true...

They are crucial to perform the UTA in the context of **New Physics** scenarios.

K- \bar{K} MIXING IN NP MODELS

$$\begin{aligned} \mathcal{O}_1 &= \bar{s}^a \gamma_\mu (1 - \gamma_5) d^a \bar{s}^b \gamma_\mu (1 - \gamma_5) d^b, \\ \mathcal{O}_2 &= \bar{s}^a (1 - \gamma_5) d^a \bar{s}^b (1 - \gamma_5) d^b, \\ \mathcal{O}_3 &= \bar{s}^a (1 - \gamma_5) d^b \bar{s}^b (1 - \gamma_5) d^a, \\ \mathcal{O}_4 &= \bar{s}^a (1 - \gamma_5) d^a \bar{s}^b (1 + \gamma_5) d^b, \\ \mathcal{O}_5 &= \bar{s}^a (1 - \gamma_5) d^b \bar{s}^b (1 + \gamma_5) d^a \end{aligned}$$



▼ ▼ Clover
APE 1999

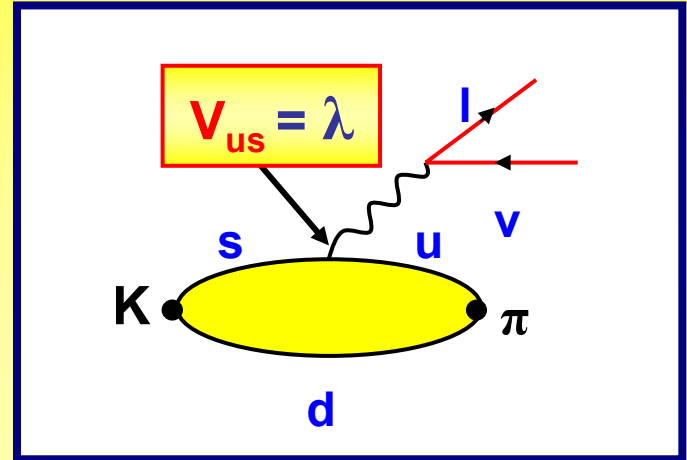
▲ ▲ Overlap
Babich et al. 2006

◆ ■ Domain wall
CP-PACS 2006

V_{us} AND K_{l3} DECAYS

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

The most stringent **unitarity test**



$$\Gamma(K \rightarrow \pi l \nu(\gamma)) = \frac{G_F^2 M_K^5}{192\pi^3} \cdot$$

$$C_K^2 |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 I_l S_{ew} (1 + \delta_l)^2$$

Ademollo-Gatto:

$$f_+(0) = 1 - O(m_s - m_u)^2$$

$$f_+(0) = 1 + f_2 + f_4 + O(p^8)$$

Vector Current Conservation

$f_2 = -0.023$
Independent of L_i
(Ademollo-Gatto)

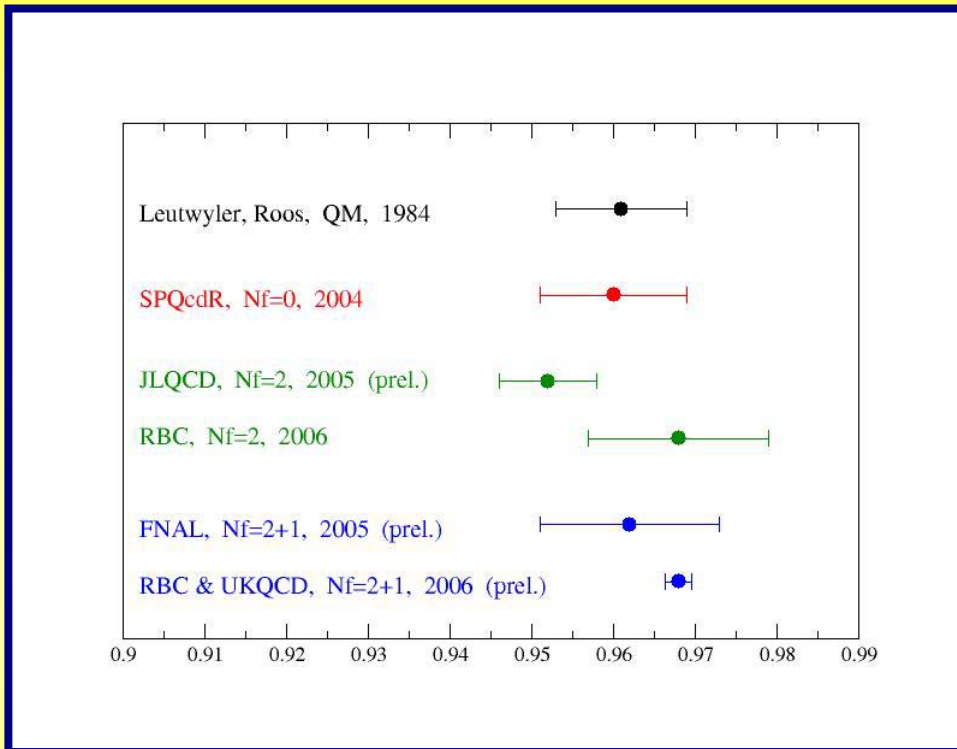
THE LARGEST UNCERTAINTY

“Standard” estimate:

Leutwyler, Roos (1984)
(QUARK MODEL)

$$f_4 = -0.016 \pm 0.008$$

Several lattice QCD calculations in the last 2 years:



The old quark model calculation by Leutwyler and Roos has been now confirmed in QCD.

But lattice QCD calculations have the capability to further improve the theoretical accuracy

$$f_+(0) = 0.960 \pm 0.009$$

CONCLUSIONS

- The accuracy of lattice QCD calculation is improving, particularly because the quenched approximation is being abandoned. But **unquenching** is still a work in progress...
- A special effort must be done for the **semileptonic form factors** necessary to the extraction of V_{ub}
- Extraordinary experimental progresses allow the **extraction of several hadronic quantities from the data**, assuming the SM. The results are in very good agreement with LQCD determinations
- LQCD calculations of **many** hadronic parameters are required to study **flavor physics** in the framework of **New Physics scenarios**.