Theory of Semi-Leptonic *B* Decays: Exclusive and Inclusive

Thomas Mannel

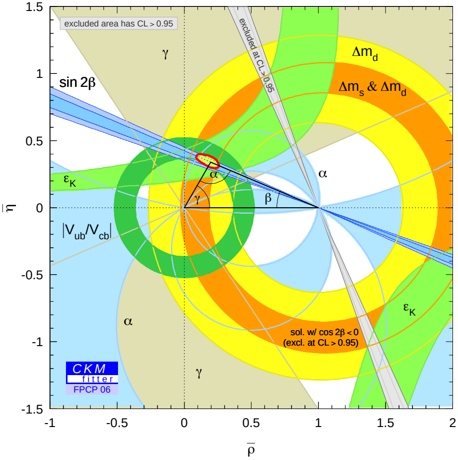
Theoretische Physik I
Universiät Siegen

Heavy Quarks and Leptons, Munich, October 2006

Contents

Why Semi-Leptonic *B* Decays?

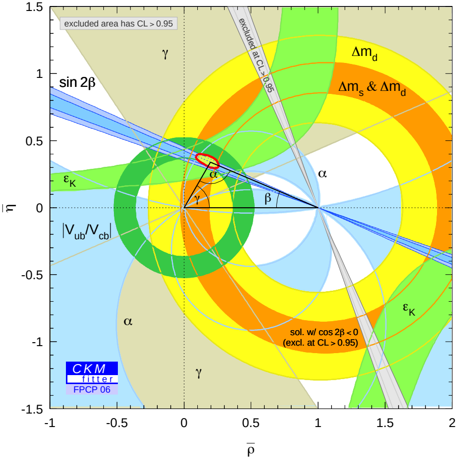
- Important Ingredient for the Unitarity Traingle



- “Unitarity Clock”: $|V_{ub}/V_{cb}|$ Buras
- Relation between Kaon CP violation and the Unitarity Trangle

Why Semi-Leptonic *B* Decays?

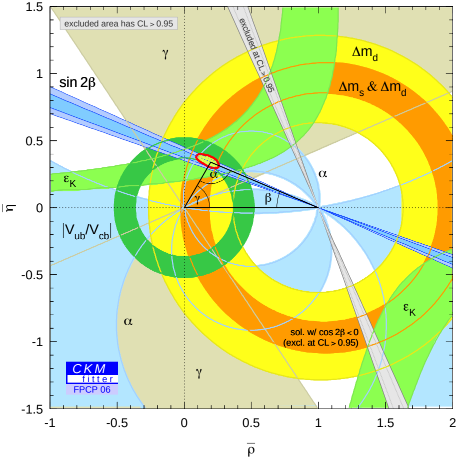
- Important Ingredient for the Unitarity Traingle



- "Unitarity Clock": $|V_{ub}/V_{cb}|$ Buras
- Relation between Kaon CP violation and the Unitarity Trangle

Why Semi-Leptonic B Decays?

- Important Ingredient for the Unitarity Traingle



- "Unitarity Clock": $|V_{ub}/V_{cb}|$ Buras
- Relation between Kaon CP violation and the Unitarity Trangle

Generalities of the Theoretical Methods

- Problem: Hadronic Matrix Elements of Quark Operators
- QCD Based theory: $1/m_b$ Expansion

  Isgur, Wise, Shifman, Voloshin, Vainshtain, Bigi, Uraltsev, Georgi, Grinstein, Falk, Luke, Neubert, M., ...
- Effective Theory / Operator Product Expansion
- Heavy Quark Effective Theory: HQET and Heavy Quark Expansion HQE

$$R = R_0 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right) R_1 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^2 R_2 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 R_3 + \cdots$$

Generalities of the Theoretical Methods

- Problem: Hadronic Matrix Elements of Quark Operators
- QCD Based theory: $1/m_b$ Expansion

Isgur, Wise, Shifman, Voloshin, Vainshtain, Bigi, Uraltsev, Georgi, Grinstein, Falk, Luke, Neubert, M., ...

- Effective Theory / Operator Product Expansion
- Heavy Quark Effective Theory: HQET and Heavy Quark Expansion HQE

$$R = R_0 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right) R_1 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^2 R_2 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 R_3 + \cdots$$

Generalities of the Theoretical Methods

- Problem: Hadronic Matrix Elements of Quark Operators
- QCD Based theory: $1/m_b$ Expansion

  Isgur, Wise, Shifman, Voloshin, Vainshtain, Bigi, Uraltsev, Georgi, Grinstein, Falk, Luke, Neubert, M., ...
- Effective Theory / Operator Product Expansion
- Heavy Quark Effective Theory: HQET and Heavy Quark Expansion HQE

$$R = R_0 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right) R_1 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^2 R_2 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 R_3 + \cdots$$

Generalities of the Theoretical Methods

- Problem: Hadronic Matrix Elements of Quark Operators
- QCD Based theory: $1/m_b$ Expansion

  Isgur, Wise, Shifman, Voloshin, Vainshtain, Bigi, Uraltsev, Georgi, Grinstein, Falk, Luke, Neubert, M., ...
- Effective Theory / Operator Product Expansion
- Heavy Quark Effective Theory: HQET and Heavy Quark Expansion HQE

$$R = R_0 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right) R_1 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^2 R_2 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 R_3 + \cdots$$

Generalities of the Theoretical Methods

- Problem: Hadronic Matrix Elements of Quark Operators
- QCD Based theory: $1/m_b$ Expansion

  Isgur, Wise, Shifman, Voloshin, Vainshtain, Bigi, Uraltsev, Georgi, Grinstein, Falk, Luke, Neubert, M., ...
- Effective Theory / Operator Product Expansion
- Heavy Quark Effective Theory: HQET and Heavy Quark Expansion HQE

$$R = R_0 + \left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right) R_1 + \left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right)^2 R_2 + \left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right)^3 R_3 + \cdots$$

Exclusive Semi-leptonic Decays

- Hadronic Matrix Elements = Form Factors: $0^- \to 0^-$

$$\langle M(p')|\bar{q}\gamma_\mu(1-\gamma_5)b|B(p)\rangle = f_+(q^2)(p+p')_\mu + f_-(q^2)(p-p')_\mu$$

- $q = c$: Heavy to Heavy transition $B \to D\ell\bar{\nu}_\ell$ and $B \to D^*\ell\bar{\nu}_\ell$
- $q = u$: Heavy to Light transition $B \to \pi\ell\bar{\nu}_\ell$ (and $B \to \rho\ell\bar{\nu}_\ell$
- Symmetries of HQET reduce the number of independent form factors in the infinite mass limit.
- Heavy Quark Symmetries are more efficient in the heavy to heavy case.

Exclusive Semi-leptonic Decays

- Hadronic Matrix Elements = Form Factors: $0^- \to 0^-$

$$\langle M(p')|\bar{q}\gamma_\mu(1-\gamma_5)b|B(p)\rangle = f_+(q^2)(p+p')_\mu + f_-(q^2)(p-p')_\mu$$

- $q = c$: Heavy to Heavy transition $B \to D\ell\bar{\nu}_\ell$ and $B \to D^*\ell\bar{\nu}_\ell$
- $q = u$: Heavy to Light transition $B \to \pi\ell\bar{\nu}_\ell$ (and $B \to \rho\ell\bar{\nu}_\ell$
- Symmetries of HQET reduce the number of independent form factors in the infinite mass limit.
- Heavy Quark Symmetries are more efficient in the heavy to heavy case.

Exclusive Semi-leptonic Decays

- Hadronic Matrix Elements = Form Factors: $0^- \to 0^-$

$$\langle M(p')|\bar{q}\gamma_\mu(1-\gamma_5)b|B(p)\rangle = f_+(q^2)(p+p')_\mu + f_-(q^2)(p-p')_\mu$$

- $q = c$: Heavy to Heavy transition $B \to D\ell\bar{\nu}_\ell$ and $B \to D^*\ell\bar{\nu}_\ell$
- $q = u$: Heavy to Light transition $B \to \pi\ell\bar{\nu}_\ell$ (and $B \to \rho\ell\bar{\nu}_\ell$
- Symmetries of HQET reduce the number of independent form factors in the infinite mass limit.
- Heavy Quark Symmetries are more efficient in the heavy to heavy case.

Exclusive Semi-leptonic Decays

- Hadronic Matrix Elements = Form Factors: $0^- \to 0^-$

$$\langle M(p')|\bar{q}\gamma_\mu(1-\gamma_5)b|B(p)\rangle = f_+(q^2)(p+p')_\mu + f_-(q^2)(p-p')_\mu$$

- $q = c$: Heavy to Heavy transition $B \to D\ell\bar{\nu}_\ell$ and $B \to D^*\ell\bar{\nu}_\ell$
- $q = u$: Heavy to Light transition $B \to \pi\ell\bar{\nu}_\ell$ (and $B \to \rho\ell\bar{\nu}_\ell$
- Symmetries of HQET reduce the number of independent form factors in the infinite mass limit.
- Heavy Quark Symmetries are more efficient in the heavy to heavy case.

Exclusive Semi-leptonic Decays

- Hadronic Matrix Elements = Form Factors: $0^- \to 0^-$

$$\langle M(p')|\bar{q}\gamma_\mu(1-\gamma_5)b|B(p)\rangle = f_+(q^2)(p+p')_\mu + f_-(q^2)(p-p')_\mu$$

- $q = c$: Heavy to Heavy transition $B \to D\ell\bar{\nu}_\ell$ and $B \to D^*\ell\bar{\nu}_\ell$
- $q = u$: Heavy to Light transition $B \to \pi\ell\bar{\nu}_\ell$ (and $B \to \rho\ell\bar{\nu}_\ell$
- Symmetries of HQET reduce the number of independent form factors in the infinite mass limit.
- Heavy Quark Symmetries are more efficient in the heavy to heavy case.

Exclusive Semi-leptonic Decays

- Hadronic Matrix Elements = Form Factors: $0^- \to 0^-$

$$\langle M(p')|\bar{q}\gamma_\mu(1-\gamma_5)b|B(p)\rangle = f_+(q^2)(p+p')_\mu + f_-(q^2)(p-p')_\mu$$

- $q = c$: Heavy to Heavy transition $B \to D\ell\bar{\nu}_\ell$ and $B \to D^*\ell\bar{\nu}_\ell$
- $q = u$: Heavy to Light transition $B \to \pi\ell\bar{\nu}_\ell$ (and $B \to \rho\ell\bar{\nu}_\ell$
- Symmetries of HQET reduce the number of independent form factors in the infinite mass limit.
- Heavy Quark Symmetries are more efficient in the heavy to heavy case.

Heavy to Heavy: $B \to D\ell\bar{\nu}_\ell$ and $B \to D^*\ell\bar{\nu}_\ell$

- Kinematic variable for a heavy quark: Four Velovity v
- Differential Rates

$$\frac{d\Gamma}{d\omega}(B \to D^*\ell\bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3}|V_{cb}|^2 m_{D^*}^3 (\omega^2 - 1)^{1/2} P(\omega) (\mathcal{F}(\omega))^2$$

$$\frac{d\Gamma}{d\omega}(B \to D\ell\bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3}|V_{cb}|^2 (m_B + m_D)^2 m_D^3 (\omega^2 - 1)^{3/2} (\mathcal{G}(\omega))^2$$

- with $\omega = vv'$ and
- $P(\omega)$: Calculable Phase space factor
- $\mathcal{F}$ and $\mathcal{G}$: Form Factors

Heavy to Heavy: $B \to D\ell\bar{\nu}_\ell$ and $B \to D^*\ell\bar{\nu}_\ell$

- Kinematic variable for a heavy quark: Four Velovity v
- Differential Rates

$$\frac{d\Gamma}{d\omega}(B \to D^*\ell\bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3}|V_{cb}|^2 m_{D^*}^3(\omega^2 - 1)^{1/2} P(\omega)(\mathcal{F}(\omega))^2$$

$$\frac{d\Gamma}{d\omega}(B \to D\ell\bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3}|V_{cb}|^2 (m_B + m_D)^2 m_D^3(\omega^2 - 1)^{3/2}(\mathcal{G}(\omega))^2$$

- with $\omega = vv'$ and
- $P(\omega)$: Calculable Phase space factor
- $\mathcal{F}$ and $\mathcal{G}$: Form Factors

Heavy to Heavy: $B \to D\ell\bar{\nu}_\ell$ and $B \to D^*\ell\bar{\nu}_\ell$

- Kinematic variable for a heavy quark: Four Velovity v
- Differential Rates

$$\frac{d\Gamma}{d\omega}(B \to D^*\ell\bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3}|V_{cb}|^2 m_{D^*}^3(\omega^2 - 1)^{1/2}P(\omega)(\mathcal{F}(\omega))^2$$

$$\frac{d\Gamma}{d\omega}(B \to D\ell\bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3}|V_{cb}|^2(m_B + m_D)^2 m_D^3(\omega^2 - 1)^{3/2}(\mathcal{G}(\omega))^2$$

- with $\omega = vv'$ and
- $P(\omega)$: Calculable Phase space factor
- $\mathcal{F}$ and $\mathcal{G}$: Form Factors

Heavy Quark Symmetries

- Normalization of the Form Factors is known at $vv' = 1$: (both initial and final meson at rest)
- Corrections can be calculated / estimated

$$\mathcal{F}(\omega) = \eta_{\rm QED}\eta_A \left[1 + \delta_{1/\mu^2} + \cdots\right](\omega - 1)\rho^2 + \mathcal{O}((\omega - 1)^2)$$

$$\mathcal{G}(1) = \eta_{\rm QED}\eta_V \left[1 + \mathcal{O}\left(\frac{m_B - m_D}{m_B + m_D}\right)\right]$$

- Parameter of HQS breaking: $\frac{1}{\mu} = \frac{1}{m_c} - \frac{1}{m_b}$
- $\eta_A = 0.960 \pm 0.007$, $\eta_V = 1.022 \pm 0.004$, $\delta_{1/\mu^2} = -(8 \pm 4)\%$, $\eta_{\rm QED} = 1.007$

Heavy Quark Symmetries

- Normalization of the Form Factors is known at $vv' = 1$: (both initial and final meson at rest)
- Corrections can be calculated / estimated

$$\mathcal{F}(\omega) = \eta_{\text{QED}}\eta_A \left[1 + \delta_{1/\mu^2} + \cdots\right] (\omega - 1)\rho^2 + \mathcal{O}((\omega - 1)^2)$$

$$\mathcal{G}(1) = \eta_{\text{QED}}\eta_V \left[1 + \mathcal{O}\left(\frac{m_B - m_D}{m_B + m_D}\right)\right]$$

- Parameter of HQS breaking: $\frac{1}{\mu} = \frac{1}{m_c} - \frac{1}{m_b}$
- $\eta_A = 0.960 \pm 0.007$, $\eta_V = 1.022 \pm 0.004$, $\delta_{1/\mu^2} = -(8 \pm 4)\%$, $\eta_{\text{QED}} = 1.007$

Heavy Quark Symmetries

- Normalization of the Form Factors is known at $vv' = 1$: (both initial and final meson at rest)
- Corrections can be calculated / estimated

$$\mathcal{F}(\omega) = \eta_{\rm QED}\eta_A \left[1 + \delta_{1/\mu^2} + \cdots\right] (\omega - 1)\rho^2 + \mathcal{O}((\omega-1)^2)$$

$$\mathcal{G}(1) = \eta_{\rm QED}\eta_V \left[1 + \mathcal{O}\left(\frac{m_B - m_D}{m_B + m_D}\right)\right]$$

- Parameter of HQS breaking: $\frac{1}{\mu} = \frac{1}{m_c} - \frac{1}{m_b}$
- $\eta_A = 0.960 \pm 0.007$, $\eta_V = 1.022 \pm 0.004$, $\delta_{1/\mu^2} = -(8 \pm 4)\%$, $\eta_{\rm QED} = 1.007$

Heavy Quark Symmetries

- Normalization of the Form Factors is known at $vv' = 1$: (both initial and final meson at rest)
- Corrections can be calculated / estimated

$$\mathcal{F}(\omega) = \eta_{\mathrm{QED}}\eta_A \left[1 + \delta_{1/\mu^2} + \cdots\right] (\omega - 1)\rho^2 + \mathcal{O}((\omega - 1)^2)$$

$$\mathcal{G}(1) = \eta_{\mathrm{QED}}\eta_V \left[1 + \mathcal{O}\left(\frac{m_B - m_D}{m_B + m_D}\right)\right]$$

- Parameter of HQS breaking: $\frac{1}{\mu} = \frac{1}{m_c} - \frac{1}{m_b}$
- $\eta_A = 0.960 \pm 0.007$, $\eta_V = 1.022 \pm 0.004$, $\delta_{1/\mu^2} = -(8 \pm 4)\%$, $\eta_{\mathrm{QED}} = 1.007$

Form Factors from the Lattice

- Unquenched Calculations become available!
- Heavy Mass Limit is not used
- Lattice Calculations of the deviation from unity

$$\boxed{\mathcal{F}(1) = 0.91^{+0.03}_{-0.04}}$$

$$\boxed{\mathcal{G}(1) = 1.074 \pm 0.018 \pm 0.016}$$

A. Kronfeld et al.

Form Factors from the Lattice

- Unquenched Calculations become available!
- Heavy Mass Limit is not used
- Lattice Calculations of the deviation from unity

$$\boxed{\mathcal{F}(1) = 0.91^{+0.03}_{-0.04}}$$

$$\boxed{\mathcal{G}(1) = 1.074 \pm 0.018 \pm 0.016}$$

A. Kronfeld et al.

Form Factors from the Lattice

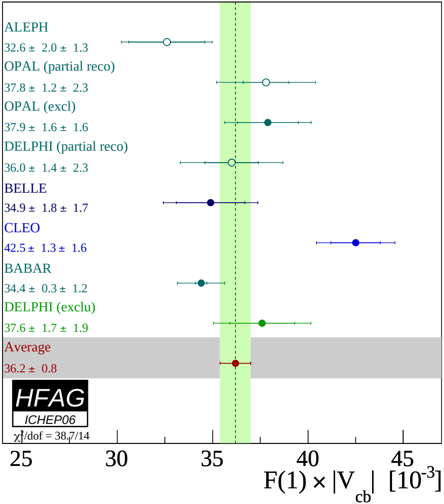
- Unquenched Calculations become available!
- Heavy Mass Limit is not used
- Lattice Calculations of the deviation from unity

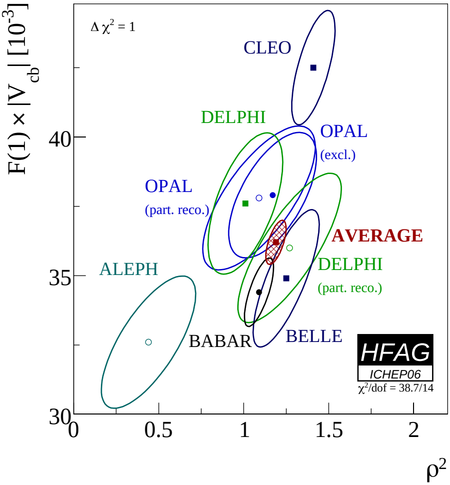
$$\mathcal{F}(1) = 0.91^{+0.03}_{-0.04}$$

$$\mathcal{G}(1) = 1.074 \pm 0.018 \pm 0.016$$

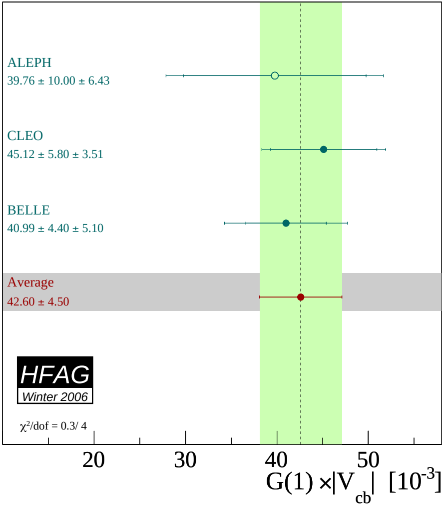
A. Kronfeld et al.

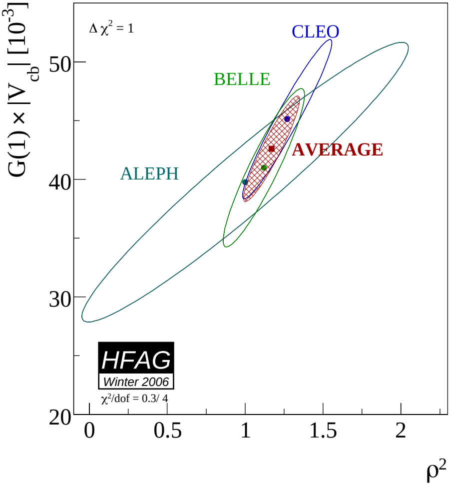
$B \to D^* \ell \bar{\nu}_\ell$





$B \to D\ell\bar{\nu}_\ell$





$$V_{cb,excl} = (39.4 \pm 0.87^{+1.56}_{-1.24}) \times 10^{-3}$$

Bob Kowalewski @ ICHEP06

- Central Value went slightly down due to the new BaBar Measurement

$$\boxed{V_{cb,excl} = (39.4 \pm 0.87^{+1.56}_{-1.24}) \times 10^{-3}}$$

Bob Kowalewski @ ICHEP06

- Central Value went slightly down due to the new BaBar Measurement

Heavy to Light: $B \longrightarrow \pi \ell \bar{\nu}_\ell$

- Main ingredient: Form Factors
- Heavy Quark Symmetries "less effiecient":
  No absolute Normalization
- Only relative Normalization between
  $B \rightarrow \pi$ and $D \rightarrow \pi$ form factors
- Rate (for vanishing lepton mass)

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 V_{ub}}{24\pi^3} |\vec{p}_\pi|^3 |f_+(q^2)|^2$$

Heavy to Light: $B \to \pi \ell \bar{\nu}_\ell$

- Main ingredient: Form Factors
- Heavy Quark Symmetries “less effiecient”:
  No absolute Normalization
- Only relative Normalization between
  $B \to \pi$ and $D \to \pi$ form factors
- Rate (for vanishing lepton mass)

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 V_{ub}}{24\pi^3} |\vec{p}_\pi|^3 |f_+(q^2)|^2$$

Heavy to Light: $B \longrightarrow \pi \ell \bar{\nu}_\ell$

- Main ingredient: Form Factors
- Heavy Quark Symmetries “less effiecient”:
  No absolute Normalization
- Only relative Normalization between
  $B \to \pi$ and $D \to \pi$ form factors
- Rate (for vanishing lepton mass)

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 V_{ub}}{24\pi^3} |\vec{p}_\pi|^3 |f_+(q^2)|^2$$

Heavy to Light: $B \longrightarrow \pi \ell \bar{\nu}_\ell$

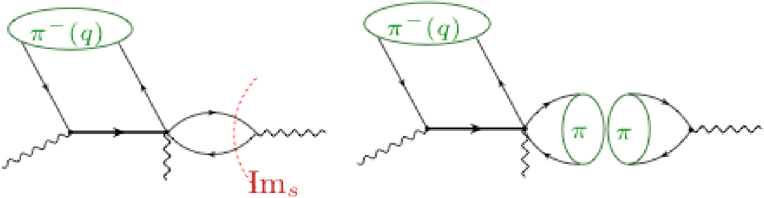
- Main ingredient: Form Factors
- Heavy Quark Symmetries “less effiecient”: No absolute Normalization
- Only relative Normalization between $B \to \pi$ and $D \to \pi$ form factors
- Rate (for vanishing lepton mass)

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 V_{ub}}{24\pi^3} |\vec{p}_\pi|^3 |f_+(q^2)|^2$$

$f_+(q^2)$ from QCD Sum Rules (Ball, Zwicky, Khodjamirian, ...)

- Dispersion Relation and Light Cone Expansion
- Study a Correlation Function

$$F_\lambda(p,q) = i \int d^4x \; e^{ipx} \langle \pi^+(q) | T\{\bar{u}\gamma_\lambda b(x) \; m_b \bar{b} i\gamma_5 d(0)\} | 0 \rangle$$

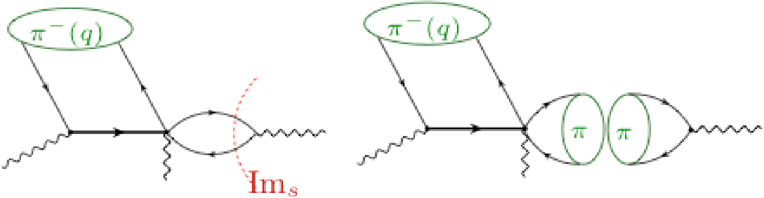


- Yields an estimate for $f_B f_+(q^2)$
- Limited to small q^2

$f_+(q^2)$ from QCD Sum Rules (Ball, Zwicky, Khodjamirian, ...)

- Dispersion Relation and Light Cone Expansion
- Study a Correlation Function

$$F_\lambda(p,q) = i \int d^4x \; e^{ipx} \langle \pi^+(q) | T\{\bar{u}\gamma_\lambda b(x) \; m_b \bar{b} i\gamma_5 d(0)\} | 0 \rangle$$

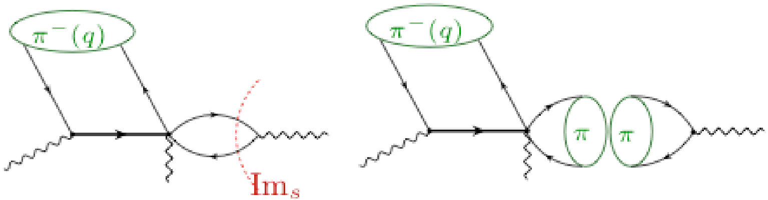


- Yields an estimate for $f_B f_+(q^2)$
- Limited to small q^2

$f_+(q^2)$ from QCD Sum Rules (Ball, Zwicky, Khodjamirian, ...)

- Dispersion Relation and Light Cone Expansion
- Study a Correlation Function

$$F_\lambda(p,q) = i \int d^4x \; e^{ipx} \langle \pi^+(q) | T\{\bar{u}\gamma_\lambda b(x)\; m_b \bar{b} i \gamma_5 d(0)\} | 0 \rangle$$



- Yields an estimate for $f_B f_+(q^2)$
- Limited to small q^2

Results from LCSR

- Uncertainties from
  - Higher Twists (≥ 4)
  - b quark mass and renormalization scale
  - Values of the condensates
  - Threshold and Borel parameters
  - Pion Distribution amplitude

$$f_+(0) = 0.27 \times \left[1 \pm (5\%)_{tw>4} \pm (3\%)_{m_b,\mu} \pm (3\%)_{\langle \bar{q}q \rangle} \pm (3\%)_{s_0^B,M} \pm (8\%)_{a^\pi_{2,4}}\right]$$

- Extrapolation to $q^2 \neq 0$ by a pole model

Results from LCSR

- Uncertainties from
  - Higher Twists (≥ 4)
  - b quark mass and renormalization scale
  - Values of the condensates
  - Threshold and Borel parameters
  - Pion Distribution amplitude

$$f_+(0) = 0.27 \times$$
$$\left[1 \pm (5\%)_{tw>4} \pm (3\%)_{m_b,\mu} \pm (3\%)_{\langle \bar{q}q \rangle} \pm (3\%)_{s_0^B,M} \pm (8\%)_{a^\pi_{2,4}}\right]$$

- Extrapolation to $q^2 \neq 0$ by a pole model

Results from LCSR

- Uncertainties from
  - Higher Twists (≥ 4)
  - b quark mass and renormalization scale
  - Values of the condensates
  - Threshold and Borel parameters
  - Pion Distribution amplitude

$$f_+(0) = 0.27 \times$$
$$\left[1 \pm (5\%)_{tw>4} \pm (3\%)_{m_b,\mu} \pm (3\%)_{\langle \bar{q}q \rangle} \pm (3\%)_{s_0^B,M} \pm (8\%)_{a^\pi_{2,4}}\right]$$

- Extrapolation to $q^2 \neq 0$ by a pole model

Results from LCSR

- Uncertainties from
  - Higher Twists (≥ 4)
  - b quark mass and renormalization scale
  - Values of the condensates
  - Threshold and Borel parameters
  - Pion Distribution amplitude

$$f_+(0) = 0.27 \times \left[1 \pm (5\%)_{tw>4} \pm (3\%)_{m_b,\mu} \pm (3\%)_{\langle\bar{q}q\rangle} \pm (3\%)_{s_0^B,M} \pm (8\%)_{a^\pi_{2,4}}\right]$$

- Extrapolation to $q^2 \neq 0$ by a pole model

Results from LCSR

- Uncertainties from
  - Higher Twists (≥ 4)
  - b quark mass and renormalization scale
  - Values of the condensates
  - Threshold and Borel parameters
  - Pion Distribution amplitude

$$f_+(0) = 0.27 \times \left[1 \pm (5\%)_{tw>4} \pm (3\%)_{m_b,\mu} \pm (3\%)_{\langle\bar{q}q\rangle} \pm (3\%)_{s_0^B,M} \pm (8\%)_{a^\pi_{2,4}}\right]$$

- Extrapolation to $q^2 \neq 0$ by a pole model

Results from LCSR

- Uncertainties from
  - Higher Twists (≥ 4)
  - b quark mass and renormalization scale
  - Values of the condensates
  - Threshold and Borel parameters
  - Pion Distribution amplitude

$$f_+(0) = 0.27 \times \left[1 \pm (5\%)_{tw>4} \pm (3\%)_{m_b,\mu} \pm (3\%)_{\langle\bar{q}q\rangle} \pm (3\%)_{s_0^B,M} \pm (8\%)_{a^\pi_{2,4}}\right]$$

- Extrapolation to $q^2 \neq 0$ by a pole model

Results from LCSR

- Uncertainties from
  - Higher Twists (≥ 4)
  - b quark mass and renormalization scale
  - Values of the condensates
  - Threshold and Borel parameters
  - Pion Distribution amplitude

$$f_+(0) = 0.27 \times$$
$$\left[1 \pm (5\%)_{tw>4} \pm (3\%)_{m_b,\mu} \pm (3\%)_{\langle\bar{q}q\rangle} \pm (3\%)_{s_0^B,M} \pm (8\%)_{a^\pi_{2,4}}\right]$$

- Extrapolation to $q^2 \neq 0$ by a pole model

Results from LCSR

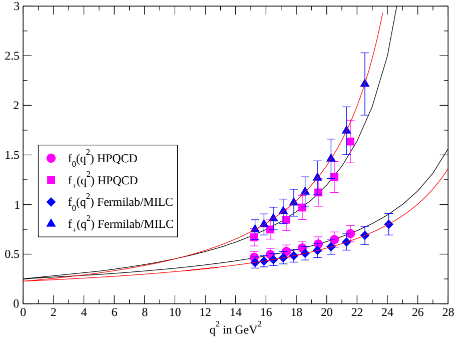
- Uncertainties from
  - Higher Twists (≥ 4)
  - b quark mass and renormalization scale
  - Values of the condensates
  - Threshold and Borel parameters
  - Pion Distribution amplitude

$$f_+(0) = 0.27 \times$$
$$\left[1 \pm (5\%)_{tw>4} \pm (3\%)_{m_b,\mu} \pm (3\%)_{\langle \bar{q}q \rangle} \pm (3\%)_{s_0^B,M} \pm (8\%)_{a^\pi_{2,4}}\right]$$

- Extrapolation to $q^2 \neq 0$ by a pole model

Lattice QCD for Heavy to Light Form Factors

- Results reliable for large q^2
- Unquenched results are available
- Extrapolation to small q^2 by a pole model Becirevic, Kaidalov



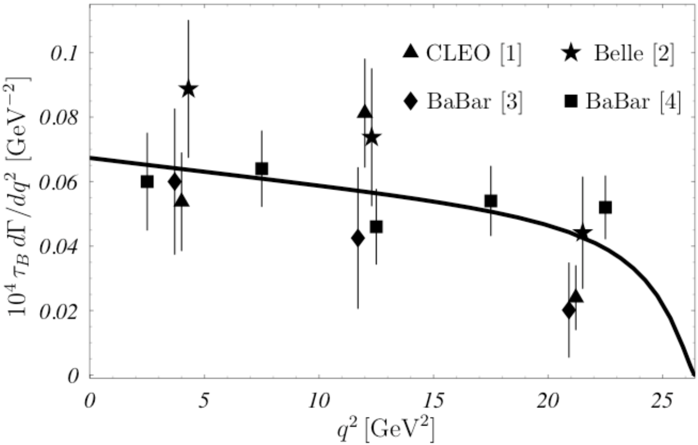
Rate for $q^2 \geq 16\,\mathrm{GeV}^2$

$$|V_{ub}|^2 \times (1.31 \pm 0.33)\,\mathrm{ps}^{-1}$$
$$|V_{ub}|^2 \times (1.80 \pm 0.48)\,\mathrm{ps}^{-1}$$

(HPQCD / Fermilab MILC)

Form Factors from Analyticity (Becher, Lange)

- Input of a single data point and input of analyticity and unitarity fixes the form factor almost completely



Inclusive Semi-leptonic Decays

Operator Product Expansion = Heavy Quark Expansion

(Chay, Georgi, Bigi, Shifman, Uraltsev, Vainstain, Manohar. Wise, Neubert, M,...)

$$\Gamma \propto \sum_X (2\pi)^4 \delta^4(P_B - P_X) |\langle X|\mathcal{H}_{eff}|B(v)\rangle|^2$$

$$= \int d^4x \, \langle B(v)|\mathcal{H}_{eff}(x)\mathcal{H}^\dagger_{eff}(0)|B(v)\rangle$$

$$= 2 \, \mathrm{Im} \int d^4x \, \langle B(v)|T\{\mathcal{H}_{eff}(x)\mathcal{H}^\dagger_{eff}(0)\}|B(v)\rangle$$

$$= 2 \, \mathrm{Im} \int d^4x \, e^{-im_b v\cdot x} \langle B(v)|T\{\widetilde{\mathcal{H}}_{eff}(x)\widetilde{\mathcal{H}}^\dagger_{eff}(0)\}|B(v)\rangle$$

- Last step: $p_b = m_b v + k$,
  Expansion in the residual momentum k

- Perform an OPE: m_b is much larger than any scale appearing in the matrix element

$$\int d^4x e^{im_b vx} T\{\widetilde{\mathcal{H}}_{eff}(x)\widetilde{\mathcal{H}}^\dagger_{eff}(0)\} = \sum_{n=0}^{\infty} \left(\frac{1}{2m_Q}\right)^n C_{n+3}(\mu)\mathcal{O}_{n+3}$$

$\longrightarrow$ The rate for $B \to X_c \ell \bar{\nu}_\ell$ can be written as

$$\Gamma = \Gamma_0 + \frac{1}{m_Q}\Gamma_1 + \frac{1}{m_Q^2}\Gamma_2 + \frac{1}{m_Q^3}\Gamma_3 + \cdots$$

- The Γ_i are power series in $\alpha_s(m_Q)$:
  $\longrightarrow$ Perturbaton theory!

- Γ_0 is the decay of a free quark (“Parton Model”)
- Γ_1 vanishing due to Luke’s theorem
- Γ_2 is expressed in terms of two parameters

$$\begin{aligned} 2M_H\mu_\pi^2 &= -\langle H(v)|\bar{Q}_v(iD)^2Q_v|H(v)\rangle \\ 2M_H\mu_G^2 &= \langle H(v)|\bar{Q}_v\sigma_{\mu\nu}(iD^\mu)(iD^\nu)Q_v|H(v)\rangle \end{aligned}$$

μ_π: Kinetic energy and μ_G: Chromomagnetic moment

- Γ_3 two more parameters

$$\begin{aligned} 2M_H\rho_D^3 &= -\langle H(v)|\bar{Q}_v(iD_\mu)(ivD)(iD^\mu)Q_v|H(v)\rangle \\ 2M_H\rho_{LS}^3 &= \langle H(v)|\bar{Q}_v\sigma_{\mu\nu}(iD^\mu)(ivD)(iD^\nu)Q_v|H(v)\rangle \end{aligned}$$

ρ_D: Darwin Term and ρ_{LS}: Chromomagnetic moment

New: $1/m_b^4$ Contribution Γ_4 (Dassinger, Turczyk, M.)

- Five new parameters:

$\langle \vec{E}^2 \rangle$: Chromoelectric Field squared

$\langle \vec{B}^2 \rangle$: Chromomagnetic Field squared

$\langle (\vec{p}^2)^2 \rangle$: Fourth power of the residual *b* quark momentum

$\langle (\vec{p}^2)(\vec{\sigma} \cdot \vec{B}) \rangle$: Mixed Chromomag. Mom. and res. Momentum

$\langle (\vec{p} \cdot \vec{B})(\vec{\sigma} \cdot \vec{p}) \rangle$: Mixed Chromomag. field and res. helicity

- Some of these can be estimated in naive factorization

- Complete triple differential rate has been calculated
- Contribution to the total rate:

$$\Gamma^{(4)} = \Gamma_0 \left[13.2 \frac{g_s^2 \langle \vec{E}^2 \rangle}{m_b^4} - 2.08 \frac{g_s^2 \langle \vec{B}^2 \rangle}{m_b^4} \right.$$
$$\left. -4.53 \frac{\langle (\vec{p}^2)^2 \rangle}{m_b^4} + 4.53 \frac{\langle g_s (\vec{\sigma} \cdot \vec{B})(\vec{p}^2) \rangle}{m_b^4} \right]$$

with $\Gamma_0 = \frac{G_F^2 m_b^5 |V_{cb}|^2}{192\pi^3}$

Heavy to Heavy: $B \to X_c \ell \bar{\nu}_\ell$

$$\Gamma = |V_{cb}|^2 \hat{\Gamma}_0 m_b^5(\mu)(1 + A_{ew}) A^{\rm pert}(r, \mu) \left[z_0(r) + z_2(r) \left(\frac{\mu_\pi^2}{m_b^2}, \frac{\mu_G^2}{m_b^2} \right) + z_3(r) \left(\frac{\rho_D^3}{m_b^2}, \frac{\rho_{\rm LS}^3}{m_b^2} \right) + \dots \right]$$

- State of the art:
  - $1/m_b$ Expansion at tree level up to $1/m_b^3$ (New $1/m_b^4$ still "too new")
  - Complete $\mathcal{O}(\alpha_s)$ corrections for the partonic rate ($1/m_b^0$)
  - and partial $\mathcal{O}(\alpha_s^2)$
  - $\mathcal{O}(\alpha_s)$ for $1/m_b^2$ terms under consideration
- Radiative Corrections: Scheme Dependence

Scheme Dependence

- Pole mass introduces large radiative corrections
- $\rightarrow$ use a suitably defined short distance mass
- Two Schemes are commonly used:
  - Kinetic Scheme — Bigi, Uraltsev, Shifman ...
    $m_{\text{kin}}(\mu)$ defined from a sum rule
    for the kinetic energy of the heavy quark
  - $1S$ Scheme: — Manohar, Hoang, Bauer, Ligeti ...
    m_{1S} defiend from a (perturbative) calculation
    of the $\Upsilon(1S)$ mass
- Both Schemes yield comparable
  and small uncertainties.

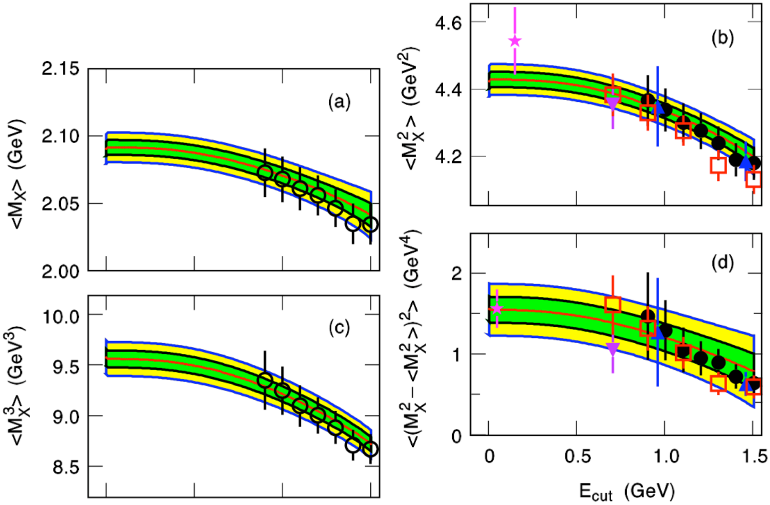
Heavy to Heavy: $B \to X_c \ell \bar{\nu}_\ell$

- Determine the HQE parameters from
  - Charged lepton energy spectrum
  - Hadronic invariant mass spectrum
- From the theoretical side:
  Calculation of moments of the spectra

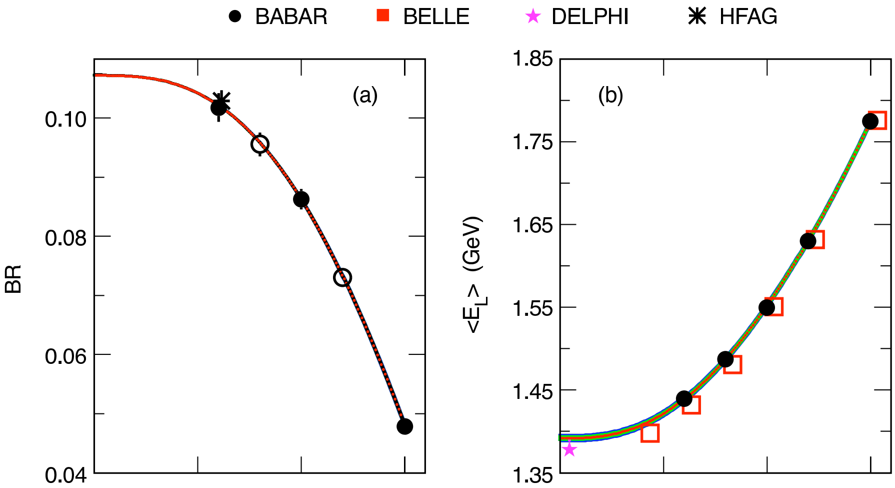
$$\langle M_X^n \rangle = \frac{1}{\Gamma} \int dM_X \, M_X^n \int_{E_{\rm cut}} dE_\ell \, \frac{d^2\Gamma}{dM_x \, dE_\ell}$$

$$\langle E_\ell^n \rangle = \frac{1}{\Gamma} \int dM_X \int_{E_{\rm cut}} dE_\ell \, E_\ell^n \frac{d^2\Gamma}{dM_x \, dE_\ell}$$

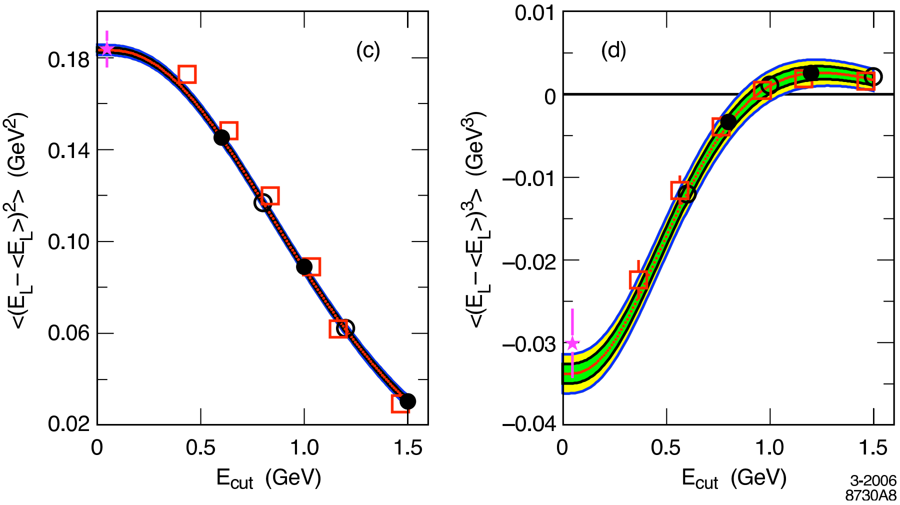
Hadronic Invariant Mass Moments (Buchmüller, Flächer)



Lepton Energy Moments I (Buchmüller, Flächer)



Lepton Energy Moments II (Buchmüller, Flächer)



Perspectives

- Currently: $\frac{\delta V_{cb}}{V_{cb}} \sim (2\%)_{\rm theo}$
- Main sources of uncertainties:
  - Mass of the b quark: $\delta m_b \sim 50$ MeV
  - Higher order QED and QCD radiative corrections
  - Higher Order of the $1/m_b$ expansion
  - Mass ratio $r = m_c^2/m_b^2$
  - Extraction of the HQE Parameters
  - Parton Hadron Duality (?)

Perspectives

- Currently: $\frac{\delta V_{cb}}{V_{cb}} \sim (2\%)_{\text{theo}}$
- Main sources of uncertainties:
  - Mass of the b quark: $\delta m_b \sim 50$ MeV
  - Higher order QED and QCD radiative corrections
  - Higher Order of the $1/m_b$ expansion
  - Mass ratio $r = m_c^2/m_b^2$
  - Extraction of the HQE Parameters
  - Parton Hadron Duality (?)

Perspectives

- Currently: $\frac{\delta V_{cb}}{V_{cb}} \sim (2\%)_{\text{theo}}$
- Main sources of uncertainties:
  - Mass of the b quark: $\delta m_b \sim 50$ MeV
  - Higher order QED and QCD radiative corrections
  - Higher Order of the $1/m_b$ expansion
  - Mass ratio $r = m_c^2/m_b^2$
  - Extraction of the HQE Parameters
  - Parton Hadron Duality (?)

Perspectives

- Currently: $\boxed{\frac{\delta V_{cb}}{V_{cb}} \sim (2\%)_{\rm theo}}$
- Main sources of uncertainties:
  - Mass of the b quark: $\delta m_b \sim 50$ MeV
  - Higher order QED and QCD radiative corrections
  - Higher Order of the $1/m_b$ expansion
  - Mass ratio $r = m_c^2/m_b^2$
  - Extraction of the HQE Parameters
  - Parton Hadron Duality (?)

Perspectives

- Currently: $\frac{\delta V_{cb}}{V_{cb}} \sim (2\%)_{\rm theo}$
- Main sources of uncertainties:
  - Mass of the b quark: $\delta m_b \sim 50$ MeV
  - Higher order QED and QCD radiative corrections
  - Higher Order of the $1/m_b$ expansion
  - Mass ratio $r = m_c^2/m_b^2$
  - Extraction of the HQE Parameters
  - Parton Hadron Duality (?)

Perspectives

- Currently: $\frac{\delta V_{cb}}{V_{cb}} \sim (2\%)_{\text{theo}}$
- Main sources of uncertainties:
  - Mass of the b quark: $\delta m_b \sim 50$ MeV
  - Higher order QED and QCD radiative corrections
  - Higher Order of the $1/m_b$ expansion
  - Mass ratio $r = m_c^2/m_b^2$
  - Extraction of the HQE Parameters
  - Parton Hadron Duality (?)

Perspectives

- Currently: $\frac{\delta V_{cb}}{V_{cb}} \sim (2\%)_{\rm theo}$
- Main sources of uncertainties:
  - Mass of the b quark: $\delta m_b \sim 50$ MeV
  - Higher order QED and QCD radiative corrections
  - Higher Order of the $1/m_b$ expansion
  - Mass ratio $r = m_c^2/m_b^2$
  - Extraction of the HQE Parameters
  - Parton Hadron Duality (?)

Heavy to Light

- In principle, the OPE is the same as Heavy to Heavy, but
- Suppression of the large $b \to c$ background eliminates a lot of phase space
- Theory Problem: OPE is not valid in some of the relevant corners of phase space (such as $y \to 1$)

$$\frac{d\Gamma}{dy} \stackrel{y\to 1}{=} \frac{G_F^2 |V_{ub}^2| m_b^5}{96\pi^3} \left[\Theta(1-y) + \frac{\mu_\pi^2 - \mu_G^2}{6m_b^2}\delta(1-y) + \frac{\mu_\pi^2}{6m_b^2}\delta'(1-y) + \cdots \right]$$

Shape- or Light-Cone Distribution Functions

- Resummation into a shape function or light cone distribution function (Bigi, Shifman, Uraltsev, Neubert, M., ...)

$$2M_B f(\omega) = \langle B(v)|\bar{b}_v \delta(\omega + i(n \cdot D))|B(v)\rangle$$

such that

$$\frac{d\Gamma}{dy} = \frac{G_F^2 |V_{ub}^2| m_b^5}{96\pi^3} \int d\omega\, \Theta(m_b(1-y) - \omega) f(\omega)$$

- Moment Expansion of f in terms of HQE parameters:

$$f(\omega) = \delta(\omega) + \frac{\mu_\pi^2}{6m_b^2}\delta''(\omega) - \frac{\rho_D^3}{18m_b^3}\delta'''(\omega) + \cdots$$

Radiative and $1/m_b$ Corrections: SCET

(Sterman, Korchemski, Bauer, Stewart, Pirjol, Beneke, Feldmann, ...)

- Inclusive Rates in the Endpoint become

$$d\Gamma = H * J * S$$

- H: Hard Coefficient Function, Scales $\mathcal{O}(m_b)$
- J: Jet Function, Scales $\mathcal{O}(\sqrt{m_b \Lambda_{\rm QCD}})$
- S: Shape function, Scales $\mathcal{O}(\Lambda_{\rm QCD})$
- State of the art:
  - $1/m_b$ terms have been investigated (Bauer, Luke, Beneke, Pecjak, Campanario, M.)
  - NNLO QCD Radiative corrections available (Becher, Lange, Neubert, Paz)
  - Models for the shape function: BLNP and DGE (Gadi)

Radiative and $1/m_b$ Corrections: SCET

(Sterman, Korchemski, Bauer, Stewart, Pirjol, Beneke, Feldmann, ...)

- Inclusive Rates in the Endpoint become

$$d\Gamma = H * J * S$$

- H: Hard Coefficient Function, Scales $\mathcal{O}(m_b)$
- J: Jet Function, Scales $\mathcal{O}(\sqrt{m_b \Lambda_{\rm QCD}})$
- S: Shape function, Scales $\mathcal{O}(\Lambda_{\rm QCD})$
- State of the art:
  - $1/m_b$ terms have been investigated (Bauer, Luke, Beneke, Pecjak, Campanario, M.)
  - NNLO QCD Radiative corrections available (Becher, Lange, Neubert, Paz)
  - Models for the shape function: BLNP and DGE (Gadi)

Radiative and $1/m_b$ Corrections: SCET

(Sterman, Korchemski, Bauer, Stewart, Pirjol, Beneke, Feldmann, ...)

- Inclusive Rates in the Endpoint become

$$d\Gamma = H * J * S$$

- H: Hard Coefficient Function, Scales $\mathcal{O}(m_b)$
- J: Jet Function, Scales $\mathcal{O}(\sqrt{m_b \Lambda_{\mathrm{QCD}}})$
- S: Shape function, Scales $\mathcal{O}(\Lambda_{\mathrm{QCD}})$
- State of the art:
  - $1/m_b$ terms have been investigated (Bauer, Luke, Beneke, Pecjak, Campanario, M.)
  - NNLO QCD Radiative corrections available (Becher, Lange, Neubert, Paz)
  - Models for the shape function: BLNP and DGE (Gadi)

Radiative and $1/m_b$ Corrections: SCET

(Sterman, Korchemski, Bauer, Stewart, Pirjol, Beneke, Feldmann, ...)

- Inclusive Rates in the Endpoint become

$$d\Gamma = H * J * S$$

- H: Hard Coefficient Function, Scales $\mathcal{O}(m_b)$
- J: Jet Function, Scales $\mathcal{O}(\sqrt{m_b \Lambda_{\rm QCD}})$
- S: Shape function, Scales $\mathcal{O}(\Lambda_{\rm QCD})$
- State of the art:
  - $1/m_b$ terms have been investigated (Bauer, Luke, Beneke, Pecjak, Campanario, M.)
  - NNLO QCD Radiative corrections available (Becher, Lange, Neubert, Paz)
  - Models for the shape function: BLNP and DGE (Gadi)

Radiative and $1/m_b$ Corrections: SCET

(Sterman, Korchemski, Bauer, Stewart, Pirjol, Beneke, Feldmann, ...)

- Inclusive Rates in the Endpoint become

$$d\Gamma = H * J * S$$

- H: Hard Coefficient Function, Scales $\mathcal{O}(m_b)$
- J: Jet Function, Scales $\mathcal{O}(\sqrt{m_b \Lambda_{\rm QCD}})$
- S: Shape function, Scales $\mathcal{O}(\Lambda_{\rm QCD})$
- State of the art:
  - $1/m_b$ terms have been investigated (Bauer, Luke, Beneke, Pecjak, Campanario, M.)
  - NNLO QCD Radiative corrections available (Becher, Lange, Neubert, Paz)
  - Models for the shape function: BLNP and DGE (Gadi)

Radiative and $1/m_b$ Corrections: SCET

(Sterman, Korchemski, Bauer, Stewart, Pirjol, Beneke, Feldmann, ...)

- Inclusive Rates in the Endpoint become

$$d\Gamma = H * J * S$$

- H: Hard Coefficient Function, Scales $\mathcal{O}(m_b)$
- J: Jet Function, Scales $\mathcal{O}(\sqrt{m_b \Lambda_{\mathrm{QCD}}})$
- S: Shape function, Scales $\mathcal{O}(\Lambda_{\mathrm{QCD}})$
- State of the art:
  - $1/m_b$ terms have been investigated (Bauer, Luke, Beneke, Pecjak, Campanario, M.)
  - NNLO QCD Radiative corrections available (Becher, Lange, Neubert, Paz)
  - Models for the shape function: BLNP and DGE (Gadi)

Radiative and $1/m_b$ Corrections: SCET

(Sterman, Korchemski, Bauer, Stewart, Pirjol, Beneke, Feldmann, ...)

- Inclusive Rates in the Endpoint become

$$d\Gamma = H * J * S$$

- H: Hard Coefficient Function, Scales $\mathcal{O}(m_b)$
- J: Jet Function, Scales $\mathcal{O}(\sqrt{m_b \Lambda_{\rm QCD}})$
- S: Shape function, Scales $\mathcal{O}(\Lambda_{\rm QCD})$
- State of the art:
  - $1/m_b$ terms have been investigated (Bauer, Luke, Beneke, Pecjak, Campanario, M.)
  - NNLO QCD Radiative corrections available (Becher, Lange, Neubert, Paz)
  - Models for the shape function: BLNP and DGE (Gadi)

Radiative and $1/m_b$ Corrections: SCET

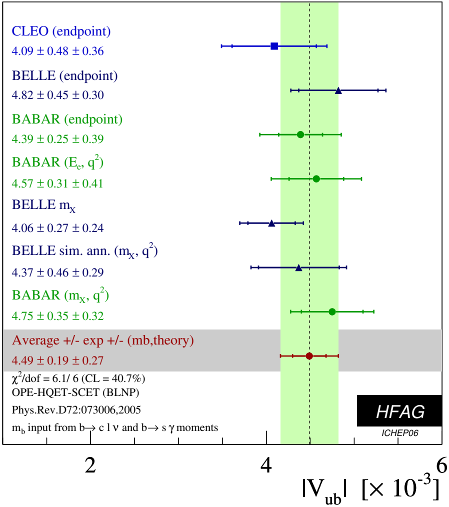
(Sterman, Korchemski, Bauer, Stewart, Pirjol, Beneke, Feldmann, ...)

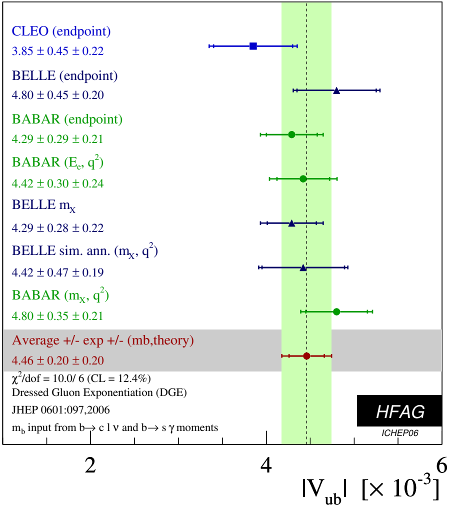
- Inclusive Rates in the Endpoint become

$$d\Gamma = H * J * S$$

- H: Hard Coefficient Function, Scales $\mathcal{O}(m_b)$
- J: Jet Function, Scales $\mathcal{O}(\sqrt{m_b \Lambda_{\rm QCD}})$
- S: Shape function, Scales $\mathcal{O}(\Lambda_{\rm QCD})$
- State of the art:
  - $1/m_b$ terms have been investigated (Bauer, Luke, Beneke, Pecjak, Campanario, M.)
  - NNLO QCD Radiative corrections available (Becher, Lange, Neubert, Paz)
  - Models for the shape function: BLNP and DGE (Gadi)

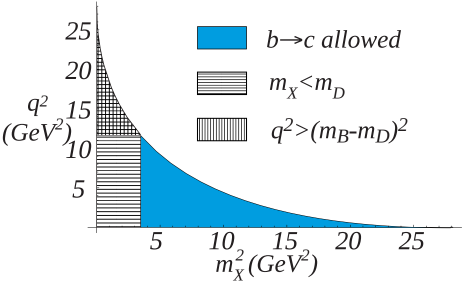
Results for V_{ub}

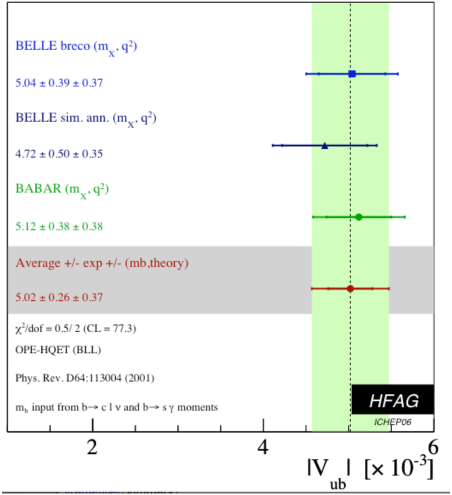




Shape Function insensitive methods (Bauer, Luke, Ligeti)

- Cut on the dilepton invariant mass and on hadronic invariant mass: This reduces the sensitivity to the shape function





Summary and Perspectives

- Theory of heavy semi-leptonics is in a mature state
- Determination of V_{cb}:
  - Currently relative theoretical uncertainties at the 2% level
  - Small improvements through α_s/m_b^2
  - and $1/m_b^4$
  - Exclusive decays with lattice form factors
- Determination of V_{ub}:
  - Currently relative theoretical uncertainties at the 8% level
  - Improvements through more data: Shape functions
  - Lattice data for exclusive decays
- Exclusive Decays are catching up ...