Heavy Quark Expansion (HQE) Fits to Inclusive $B \to X_c l\nu$ and $B \to X_s\gamma$ Moment Measurements

Oliver Buchmüller CERN

- ***„R_b and sin2β tension“***
- ***Extraction of HQE parameters (m_b, m_c, μ_π^2, ...)***
- ***$|V_{cb}|$ at ~2% ; $|V_{ub}|$ at ~5% (inclusive)***
- ***A new world average for BR($b\to s\gamma$)***

The sides of the triangle

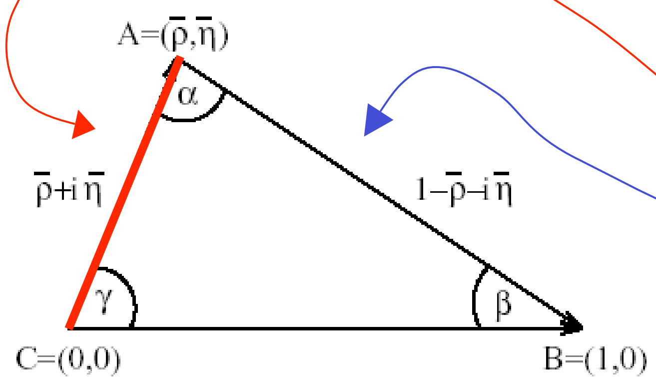
A measurement of a length of a triangle is as good as a measurement of an angle!

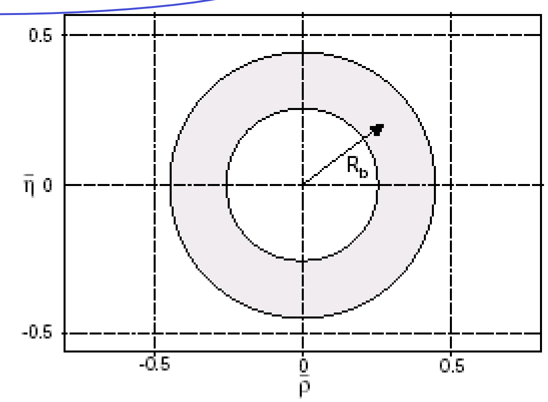
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$V_{ij}V^*_{jk} = \delta_{ik}$$

$$V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0$$

$$1 + \frac{V_{ud}V^*_{ub}}{V_{cd}V^*_{cb}} + \frac{V_{td}V^*_{tb}}{V_{cd}V^*_{cb}} = 0$$





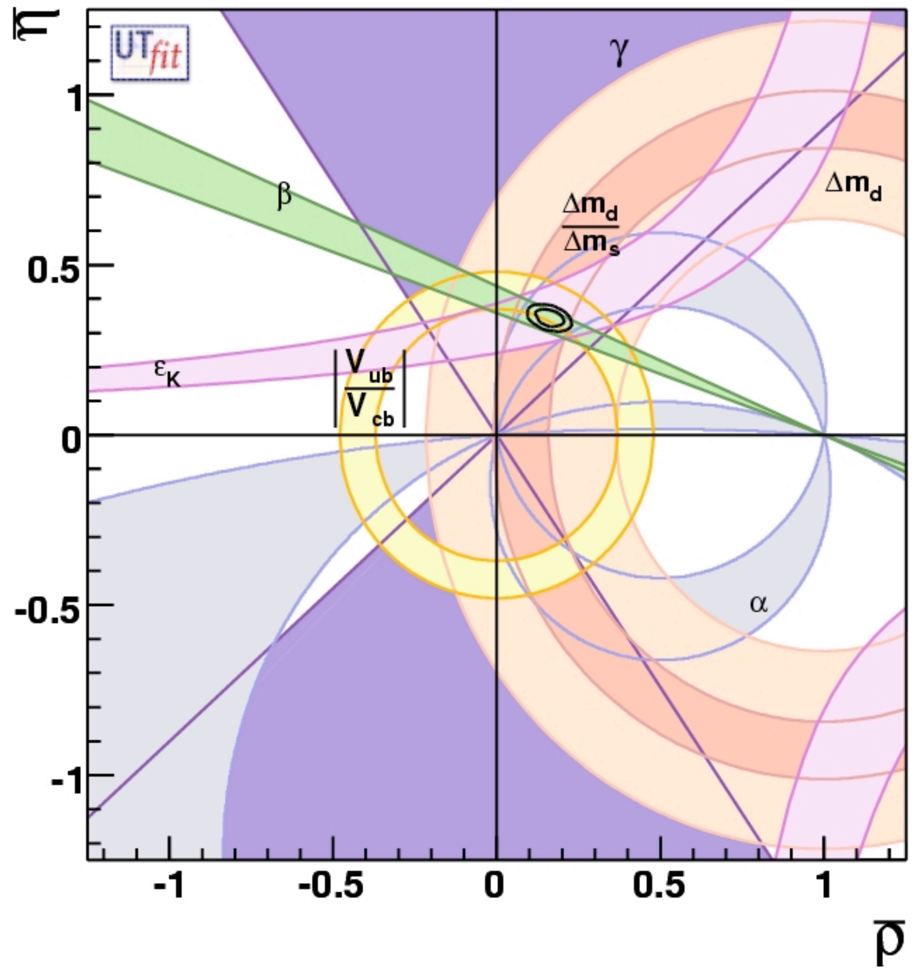
$R_b \sim |V_{ub}/V_{cb}|$ describes a circle in the $(\bar\rho,\bar\eta)$ plane

“R_b - sin2□ tension”

□ =0.163±0.028
□ =0.344± 0.016

Potential disagreement between R_b and sin2□.

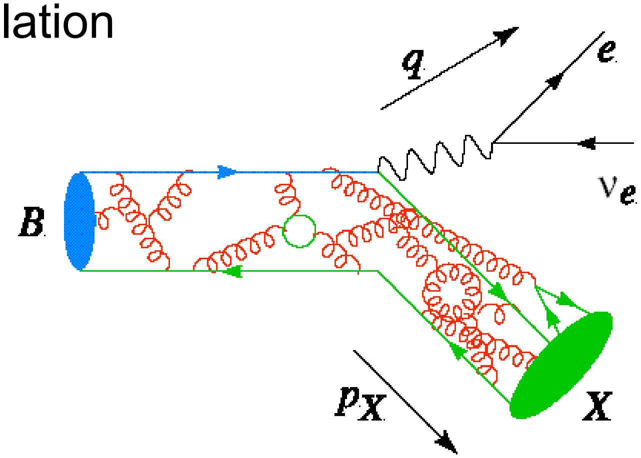
Yet, not significant but its interesting to see that the R_b constraint is comparabl to the one from sin2□



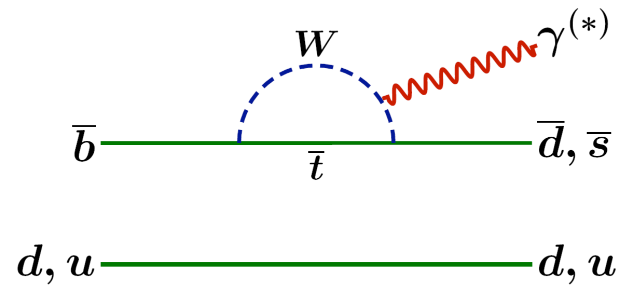
Semileptonic and Radiative B Decays

Why semileptonic decays?

$|V_{ub}|$ and $|V_{cb}|$ are crucial in testing CKM unitarity and SM mechanism for CP violation



Hadronic and leptonic currents factorise, theoretical uncertainties are under control giving access to $|V_{ub}|$ and $|V_{cb}|$



`radiative penguin'

- b → s,d transition is a Flavour Changing Neutral Current
  - forbidden in the standard model at tree-level
  - exists only at loop level
- heavy particles dominate in the loop
  - in SM: sensitive to 'top' CKM parameters: $V_{tb}V^*_{tq}$

Both decays can be treated in the framework of Heavy Quark Effective Theory, relating parton level decay rate to meson decay rate with the help of Operator Product Expansions

The Heavy Quark Expansion (HQE)

·Short-distance physics encoded in coefficients of operator products (to some order in □$_s$). Calculable!

·Long-distance physics encoded in exp. values of products of quark operators (to some order in $1/m_b$). NOT calculable! Must be determined empirically.

$$\Gamma_{c\ell\nu} = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2(1+A_{ew})A_{pert}(r,\mu)\times$$

$$\left[z_0(r)\left(1-\frac{\mu_\pi^2-\mu_G^2+\frac{\rho_D^3+\rho_{LS}^3}{m_b}}{2m_b^2}\right)\right.$$

$$\left.-2(1-r)^4\frac{\mu_G^2+\frac{\rho_D^3+\rho_{LS}^3}{m_b}}{m_b^2}+d(r)\frac{\rho_D^3}{m_b^3}+\mathcal{O}(1/m_b^4)\right].$$

$$r = m_c^2/m_b^2$$

□ Need to get access to the not predictable HQE parameters!

The Heavy Quark Expansion (HQE)

· Short-distance physics encoded in coefficients of operator products (to some order in α_s). Calculable!

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$$\Gamma_{c\ell\nu} = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2 (\ldots A_{pert}(r,\mu)) \times$$

$$\left[z_0(r) \ldots \frac{\ldots \mu_G^2 + \frac{\rho_D^3 + \rho_{LS}^3}{m_b}}{2m_b^2} \right)$$

$$-2(1-r)^4 \frac{\mu_G^2 + \ldots}{\ldots} + d(r)\frac{\rho_D^3}{m_b^3} + \mathcal{O}(1/m_b^4) \Big] .$$

$$r = m_c^2/m_b^2$$

Much more on the theoretical framework in the following talk from T. Mannel

⇒ Need to get access to the not predictable HQE parameters!

HQE Fit

$$\Gamma_{c\ell\nu} = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2(1+A_{ew})A_{pert}(r,\mu) \times \left[z_0(r)\left(1 - \frac{\mu_\pi^2 - \mu_G^2 + \frac{\rho_D^3+\rho_{LS}^3}{m_b}}{2m_b^2}\right) - 2(1-r)^4\frac{\mu_G^2 + \frac{\rho_D^3+\rho_{LS}^3}{m_b}}{m_b^2} + d(r)\frac{\rho_D^3}{m_b^3} + \mathcal{O}(1/m_b^4)\right]$$

$$r = m_c^2/m_b^2$$

Use dedicated HQE for every measured observable:

- ***$\Gamma_{cl\nu}$ (BR, lifetime)***
- ***Hadron Mass Moments $\langle M_x^n\rangle(E^{cut})$***
- ***Lepton Energy Moments $\langle E_l^n\rangle(E^{cut})$***

(Fit parameters are in red)

- $|V_{cb}|$ „master“ formula : $|V_{cb}|^2 = Br(B \to X_c e\nu)/\tau_B \; f_\ell^l(E_0, m_b, m_c, \mu_G^2, \mu_\pi^2, \rho_{LS}^3, \rho_D^3)$
- $Br(B \to X_c e\nu, E_l > E_0)$: $M_0^l(E_0)/Br(B \to X_c e\nu) = f_0^l(E_0, m_b, m_c, \mu_G^2, \mu_\pi^2, \rho_{LS}^3, \rho_D^3)$
- i-th central E_l moment for $E_l > E_0$: $M_i^l(E_0) = f_i^l(E_0, m_b, m_c, \mu_G^2, \mu_\pi^2, \rho_{LS}^3, \rho_D^3)\ (i = 1..3)$
- i-th M_x moment and $E_l > E_0$: $M_i^X(E_0) = f_i^X(E_0, m_b, m_c, \mu_G^2, \mu_\pi^2, \rho_{LS}^3, \rho_D^3)\ (i = 1..4)$
- i-th E_γ moment and $E_\gamma > E_0$: $f_0^l(E_0, m_b, m_c, \mu_G^2, \mu_\pi^2, \rho_{LS}^3, \rho_D^3)$ From $B \to X_s\gamma$

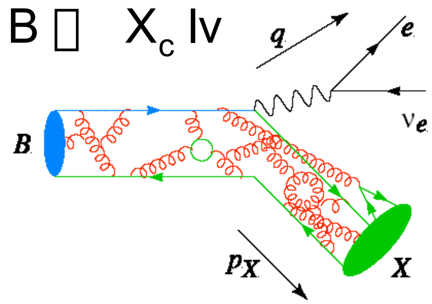
→ Extract the HQE parameter $m_b, m_c, \mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3$ and $|V_{cb}|$ as well as BR_{clv} from a simultaneous fit to all moment measurements (N+1*).

Experimental and theoretical errors and their correlations are all accounted for in the fit.

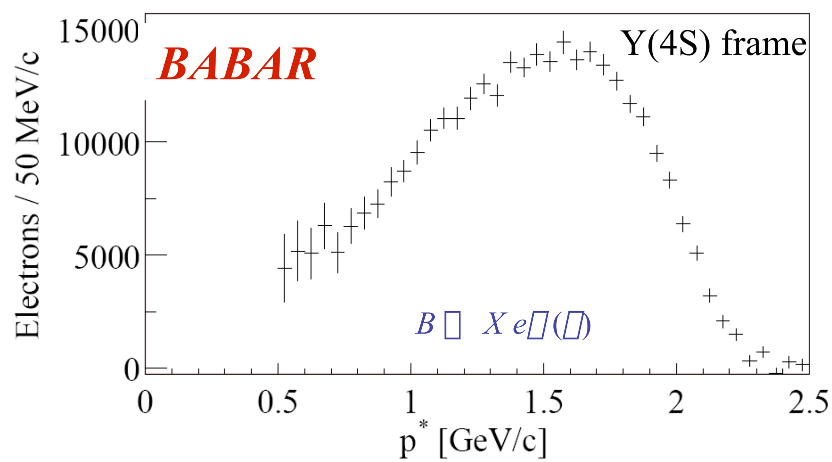
*** Only external input to the fit is:**
B lifetime τ_{B+B0} =1.585±0007 ps

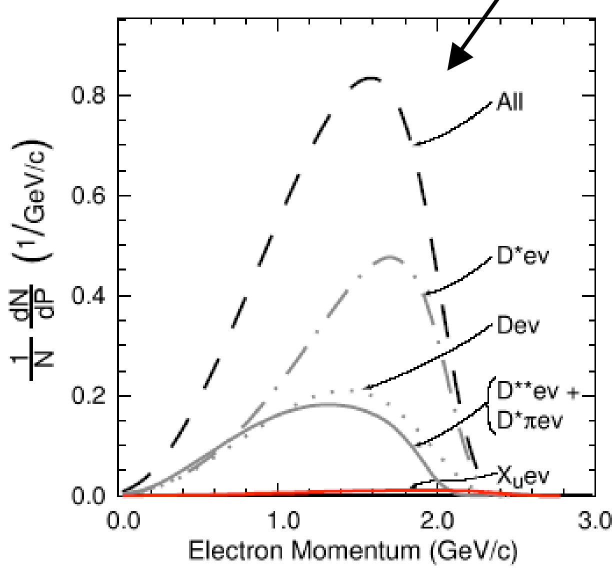
Lepton Energy Moments

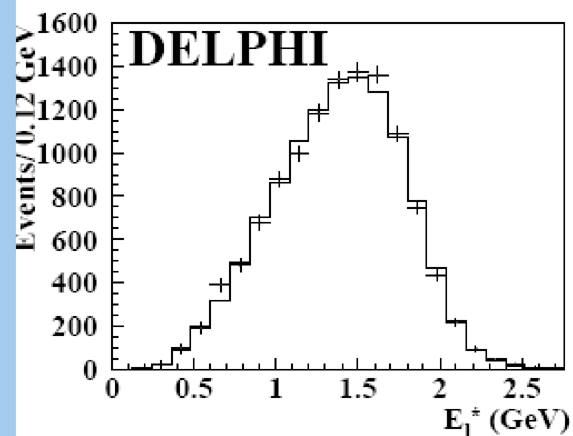
B → X_c lv

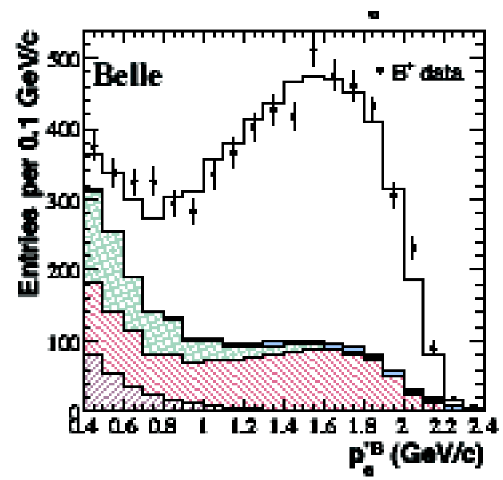


Observable





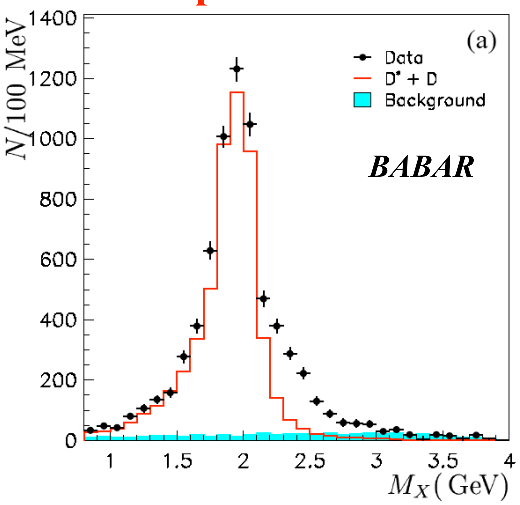




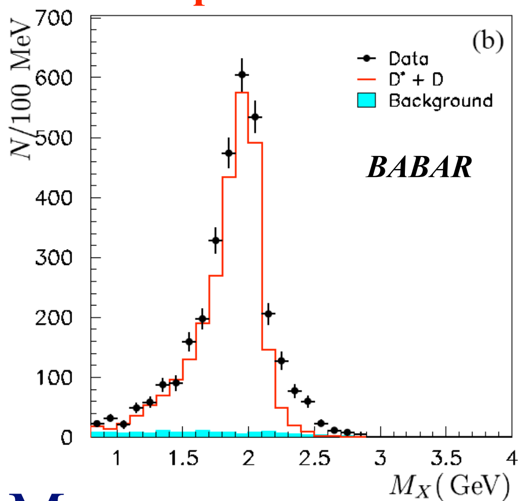
Hadronic Mass Moments

BaBar, Belle and CLEO measure full spectrum

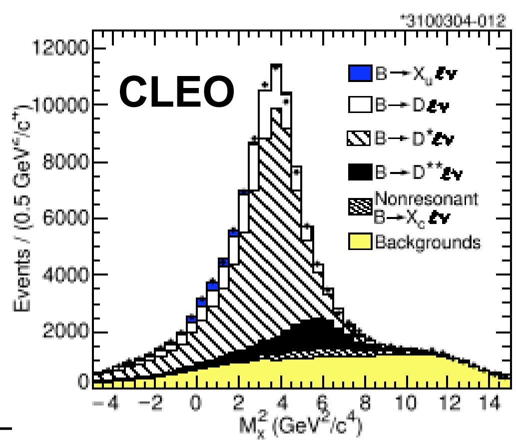
p* > 0.9 GeV

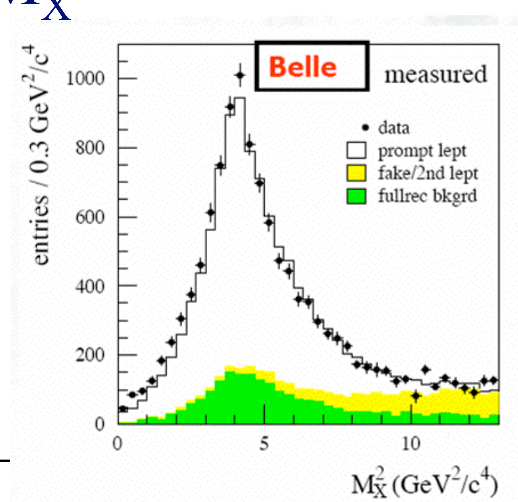


p* > 1.6 GeV

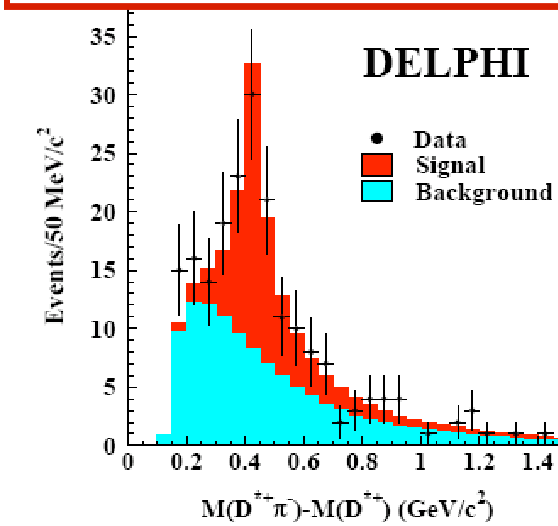


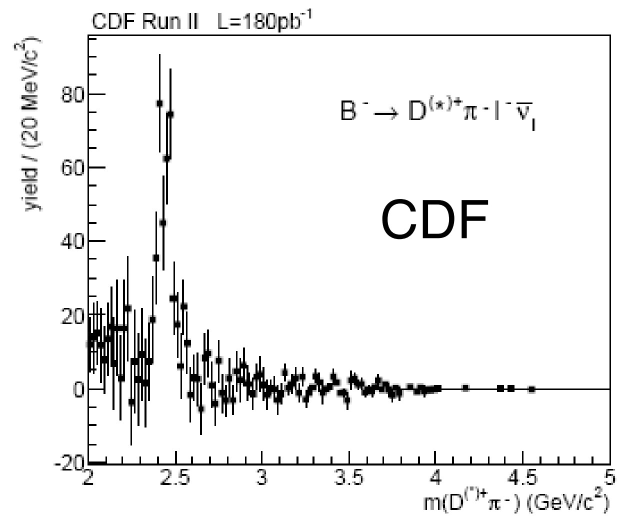
M_X





Delphi and CDF only measure higher resonances



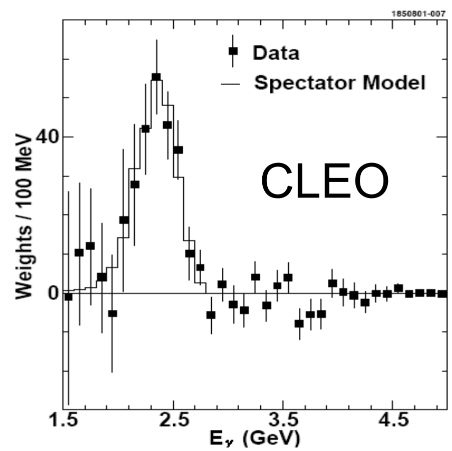


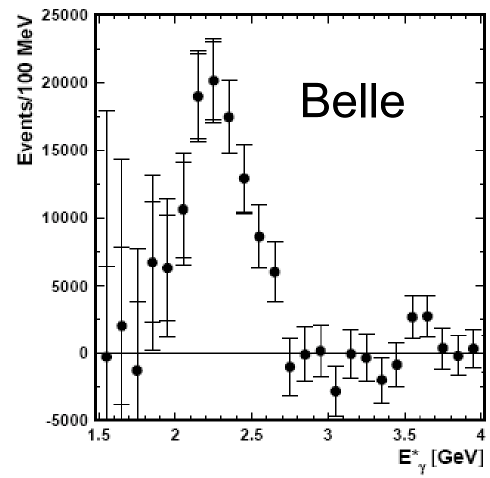
B→ sγ Spectra and Moments

Measure photon spectrum in b→sγ decays:

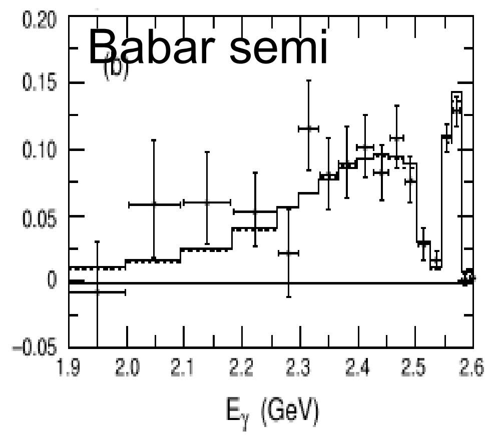
<u>Two main approaches:</u>

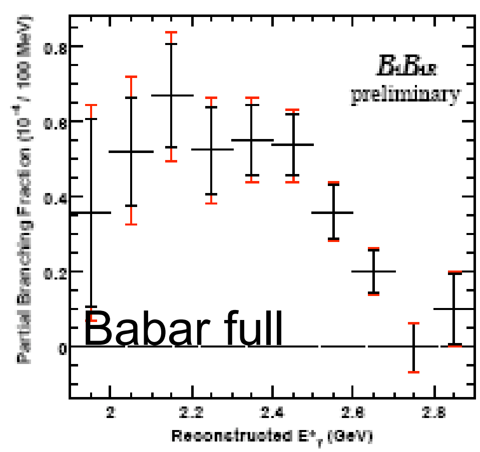
- Inclusive:
  - identify photon
- Semi-Inclusive:
  - reconstruct many exclusive final states (up to 38!)





Difficult measurement:
Overwhelming background
from π^0s for $E_\gamma < 1.8$ GeV





Measurement of photon spectrum and its moments gives information about inner structure of B meson:

- **b quark mass(m_b)**
- **Fermi momentum (μ_π^2)**

Available moment measurements

Legend:

n = order of (central) moment of observable M_X, E_l and E_γ

l = min. lepton momentum

g = min. photon energy

☑ published with covariance matrix and used in fit

☒ not used in fit as covariance matrices not available

	Hadron Moments		Lepton Moments		Photon Moments	
BaBar	n=1,2,3,4 l=0.9-1.6	☑	n=0,1,2,3 l=0.6-1.5	☑	n=1,2,3 g=1.9-2.3	☑
Belle	n= 1,2 l=0.9-1.6	☒	n=1,2 l=0.6-1.5	☒	n=1,2 g=1.8	☑
CLEO	n=2,4 l=1.0-1.5	☑	n=1,2 l=0.6-1.5	☒	n=1,2 g=2.0	☑
Delphi	n=2,4,6 l=0.0	☑	n= 1,2,3 l=0.0	☑		
CDF	n=2,4 l=0.7	☑				

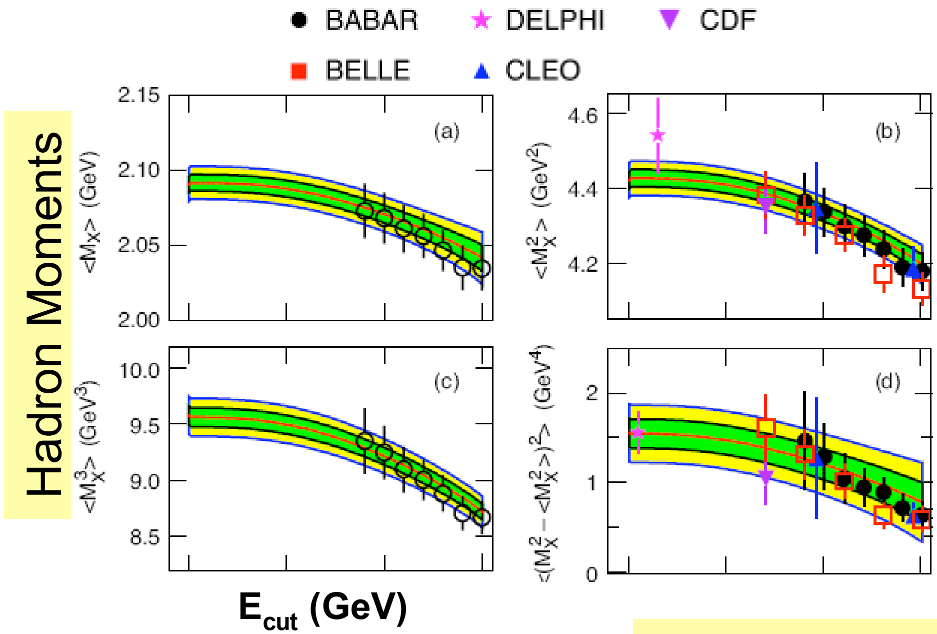
Total of 51 measurements!

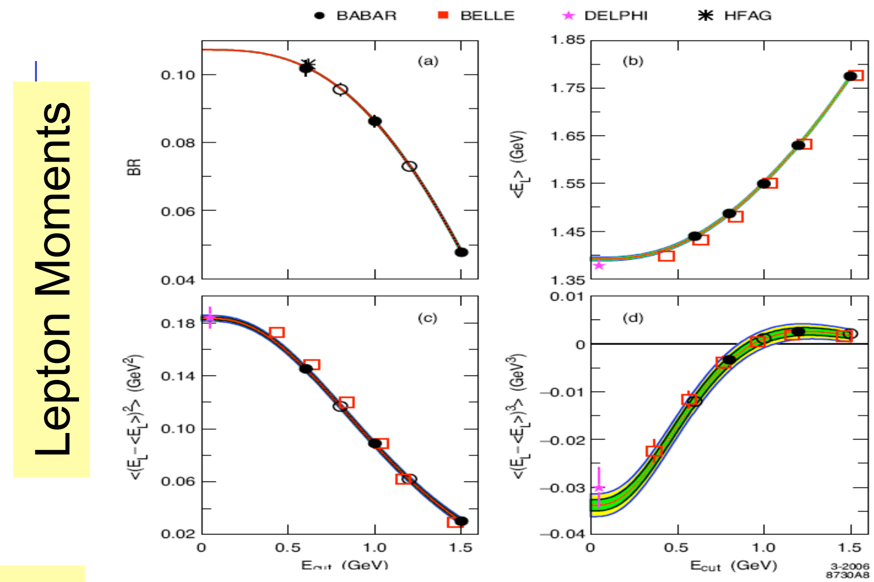
Important to take correlations between moments with different min. lepton/photon energies into account

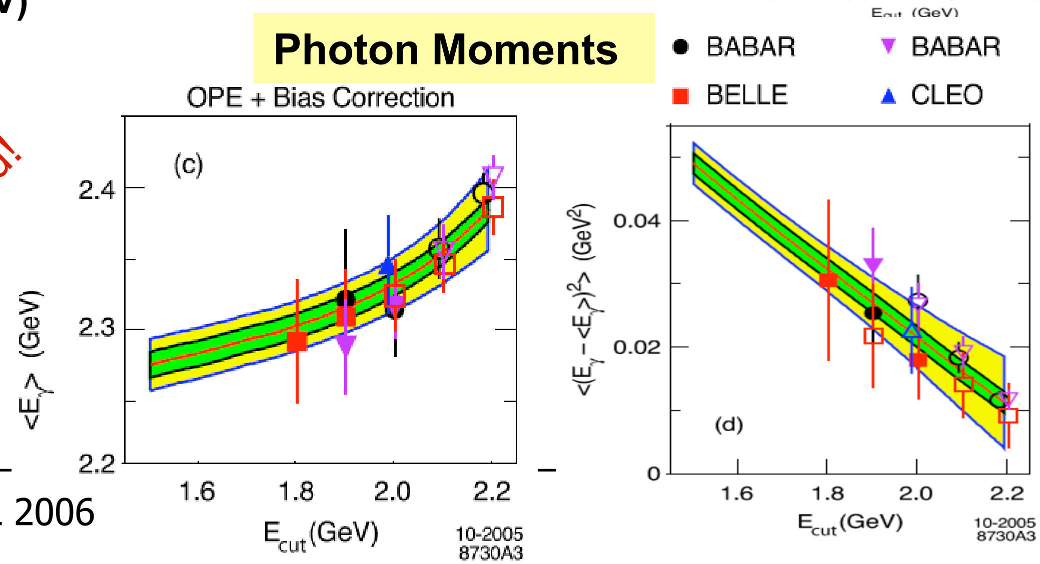
Inclusive |V_{cb}| - Fit to Moments

Based on calculations in kinetic scheme:

Benson, Bigi, Mannel & Uraltsev, hep-ph/0410080
Gambino & Uraltsev, hep-ph/0401063
Benson, Bigi & Uraltsev, hep-ph/0410080







Measurements highly correlated!

O.B.,H.Flacher.
PRD73:073008,2006

Inclusive $|V_{cb}|$

Result of fit to all moment measurements:

		exp	HQE	Γ_{SL}
$\|V_{cb}\|$ =	(41.96	± 0.23	± 0.35	± 0.59) 10^{-3}
m_b =	4.590	± 0.025	± 0.030	GeV
m_c =	1.142	± 0.037	± 0.045	GeV
μ_π^2 =	0.401	± 0.019	± 0.035	GeV^2
μ_G^2 =	0.297	± 0.024	± 0.046	GeV^2
ρ_D^3 =	0.174	± 0.009	± 0.022	GeV^3
ρ_{LS}^3 =	-0.183	± 0.054	± 0.071	GeV^3
BR_{clv} =	10.71	± 0.10	± 0.08	%

PRD73:073008,2006

$|V_{cb}|$ @ 2%
m_b < 1%
m_c @ 5%

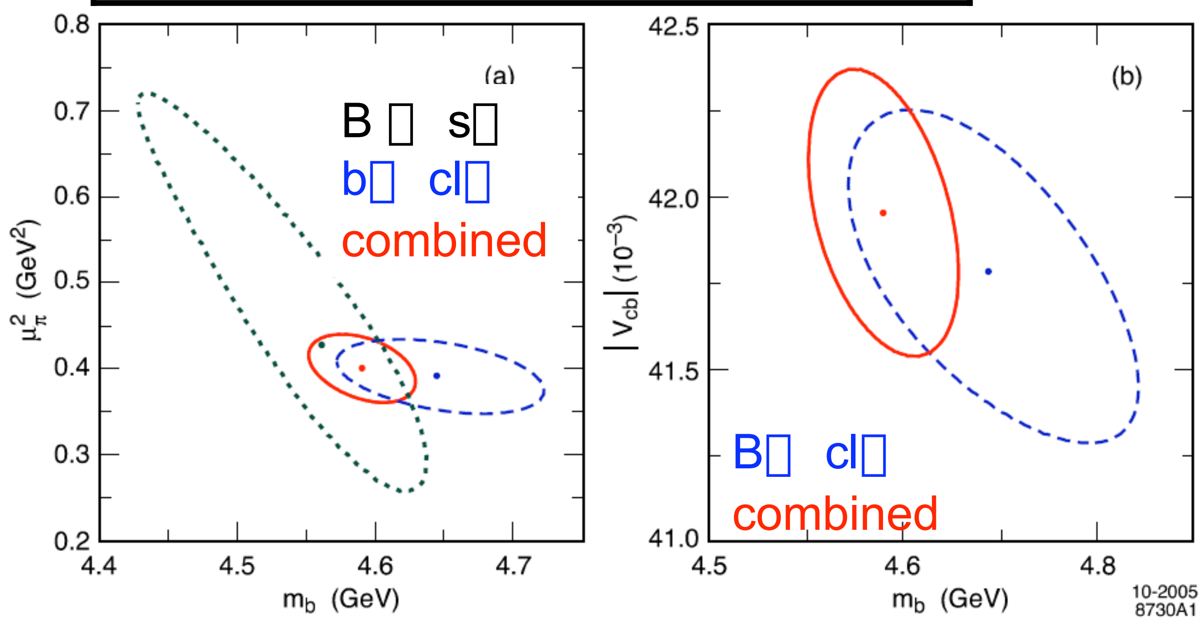
In $\overline{MS}$ scheme:

$\overline{m}_b(\overline{m}_b) = 4.20 \pm 0.04$ GeV

$\overline{m}_c(\overline{m}_c) = 1.24 \pm 0.07$ GeV

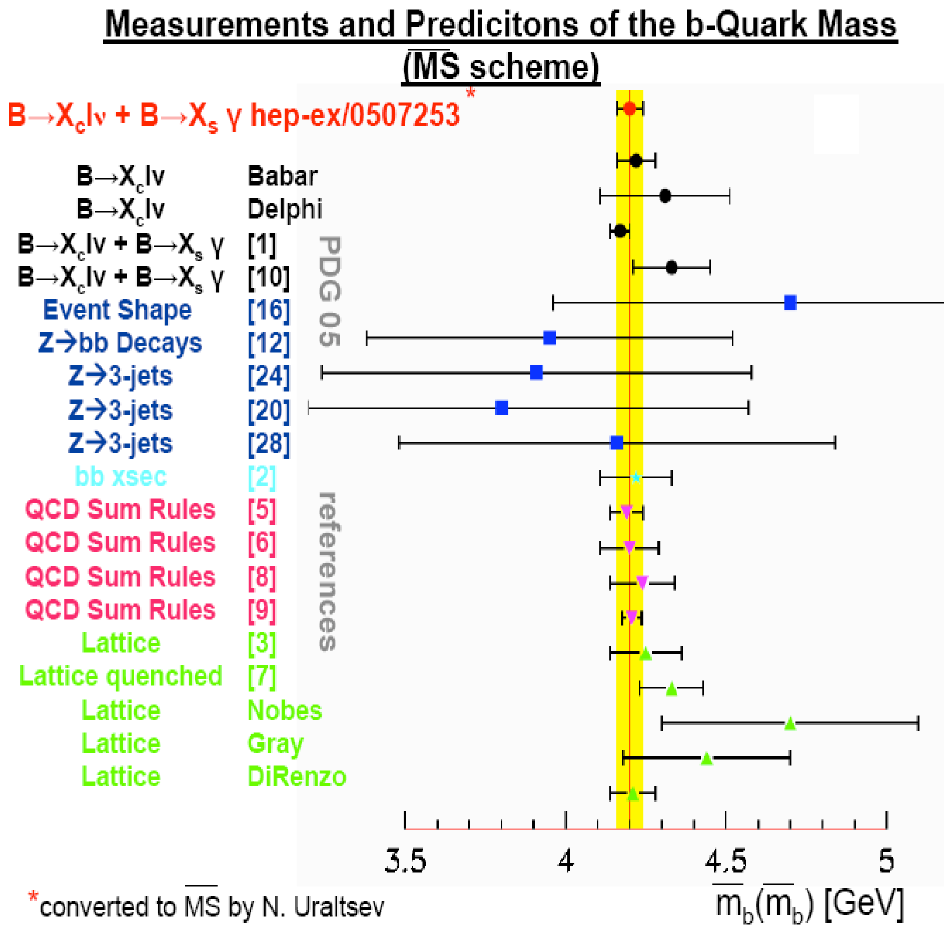
$\overline{m}_c(\mu)/\overline{m}_b(\mu) = 0.235 \pm 0.012$

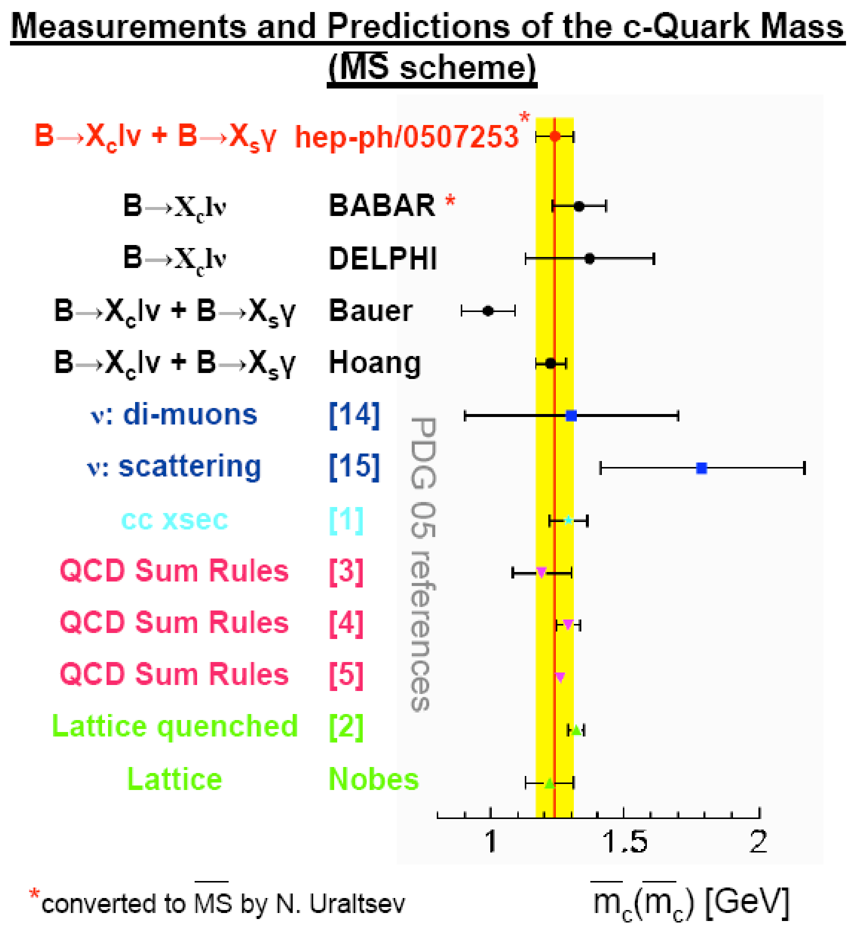
courtesy of N.Uraltsev



Good agreement with other similar analyses:
Bauer et al. hep-ph/0408002
DELPHI hep-ex/0510024

m_b and m_c





$\overline{m}_b(\overline{m}_b) = 4.20 \pm 0.04$ GeV

$\overline{m}_c(\overline{m}_c) = 1.24 \pm 0.07$ GeV

$\overline{m}_c(\overline{m}_c) = 1.22 \pm 0.02 \pm 0.04$ GeV (from 1S Fit
Hoang & Manohar Phys.Lett.B633:526-532,2006)

Inclusive V_{cb}: Summary

>100 moment measurements and many HQE fit results …

Fits in kinetic scheme

Based on hep-ph/0401063

BABAR
Phys.Rev.Lett.93:011803,2004
Vcb = 41.7 $\pm$ 0.4$_{exp}$ $\pm$ 0.4$_{HQE}$ $\pm$ 0.6$_{\Gamma SL}$ *

DELPHI
Eur.Phys.J.C45:35-59,2006
Vcb = 41.9 $\pm$ 0.6$_{exp}$ $\pm$ 0.6$_{FIT}$ $\pm$ 0.6$_{\Gamma SL}$ *

BELLE
ICHEP06 (preliminary)
Vcb = 41.9 $\pm$ 0.7$_{fit}$ $\pm$ 0.5$_{\alpha s}$ $\pm$ 0.6$_{\Gamma SL}$

OB & HF
Phys.Rev.D73:073008,2006
Vcb = 42.0 $\pm$ 0.2$_{exp}$ $\pm$ 0.4$_{HQE}$ $\pm$ 0.6$_{\Gamma SL}$

Used by HFAG

Fits in 1S scheme

Based on hep-ph/0408002

Bauer et al.
Phys.Rev.D70:094017,2004
Vcb = 41.7 $\pm$ 0.6$_{fit}$ $\pm$ 0.1$_{\square}$ *

BELLE
ICHEP06 (preliminary)
Vcb = 41.5 $\pm$ 0.5$_{fit}$ $\pm$ 0.2$_{\square}$

* Scaled to the same lifetime
τ_B=1.585 $\pm$ 0.007 ps
All Vcb numbers x10^{-3}

Very good consistency
V_{cb} @ <2%
established

Inclusive V_{cb}: Summary

>100 moment measurements and many HQE fit results …

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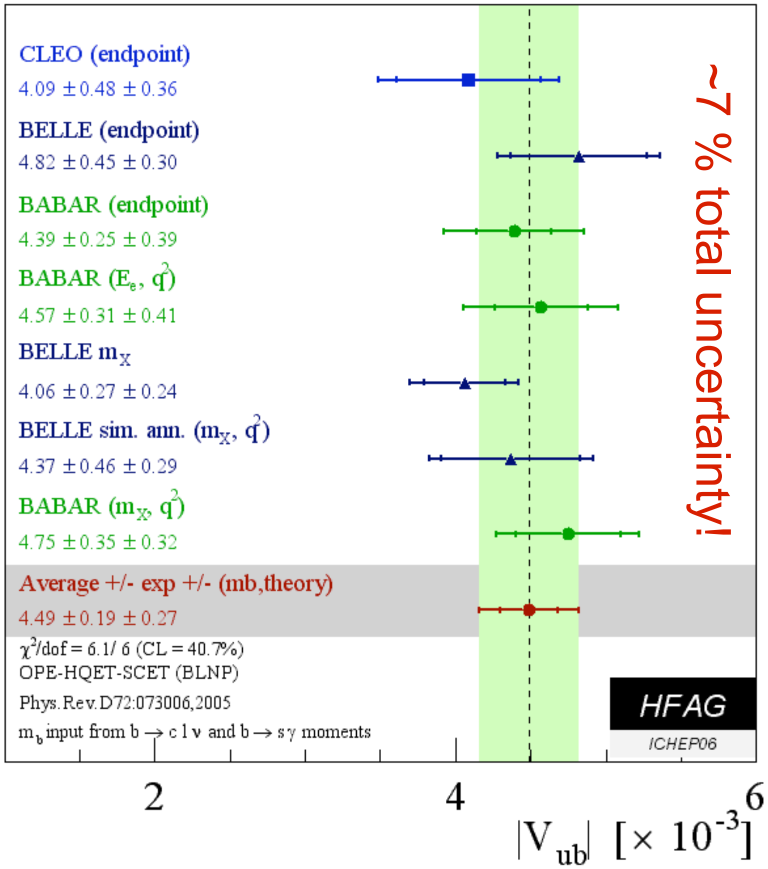
All Vcb numbers x10^{-3}

"Pre-HQE Fit Era" (~2000):
V_{cb} ~4-6%

$|V_{ub}|$ Summary and UT Constraints from Sides and Tree Processes

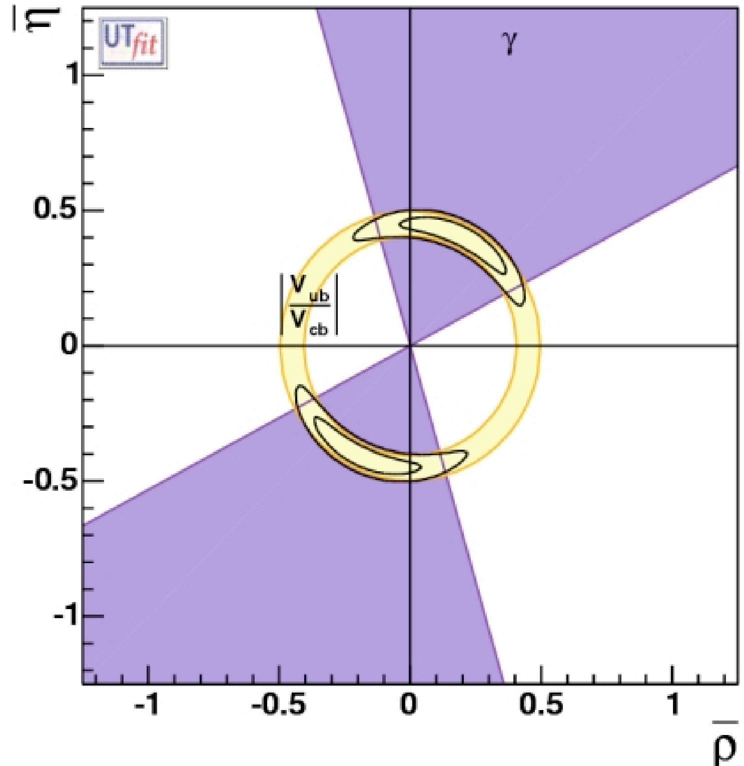
SF from PRD73:073008,2006

Inclusive $|V_{ub}|$:



$|V_{ub}| = (4.49 \pm 0.19 \pm 0.27) \times 10^{-3}$

Main improvement due to better knowledge of "shape function" parameters (including m_b)



□= ± 0.18±0.12
□ = ± 0.39± 0.06

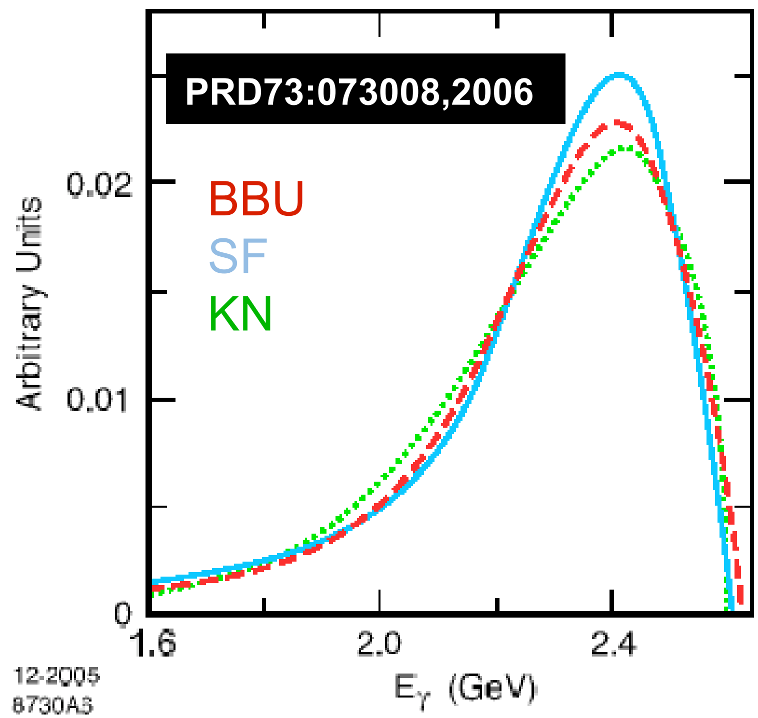
sin2□=0.783±0.072
(preferred solution)

$b \to s\gamma$ Branching Fraction

- Partial branching fractions are measured above different photon energies
- Need to be extrapolated to $E_\gamma > 1.6$ GeV to compare with theory
- Extrapolation factors based on HQE fit to clv and bsg moments

Mode	Reported $\mathcal{B}$	E_{min}	$\mathcal{B}$ at E_{min}
CLEO Inc. [3]	$321 \pm 43 \pm 27^{+18}_{-10}$	2.0	$306 \pm 41 \pm 26$
Belle Semi.[4]	$336 \pm 53 \pm 42^{+50}_{-54}$	2.24	–
Belle Inc.[5]	$355 \pm 32^{+30+11}_{-31-7}$	1.8	$351 \pm 32 \pm 29$
BABAR Semi.[6]	$335 \pm 19^{+56+4}_{-41-9}$	1.9	$327 \pm 18^{+55+4}_{-43-9}$
BABAR Inc.[7]	–	1.9	$367 \pm 29 \pm 34 \pm 29$

Photon energy spectrum constraint with m_b and μ_π^2 from the global HQE fit



New World Average from HFAG:

$BR(B \to X_s\gamma) =$
$(3.55 \pm 0.24 \pm 0.10 \pm 0.03)\ 10^{-4}$

7% uncertainty

SM prediction:
3.57 ± 0.3 $\times 10^{-4}$ Buras et al. (hep-ph/0203135)
3.44 ± 0.4 $\times 10^{-4}$ Neubert (hep-ph/0408179)
3.61 ± 0.42 $\times 10^{-4}$ Hurth et al. (hep-ph/0312260)

b→ s γ Branching Fraction

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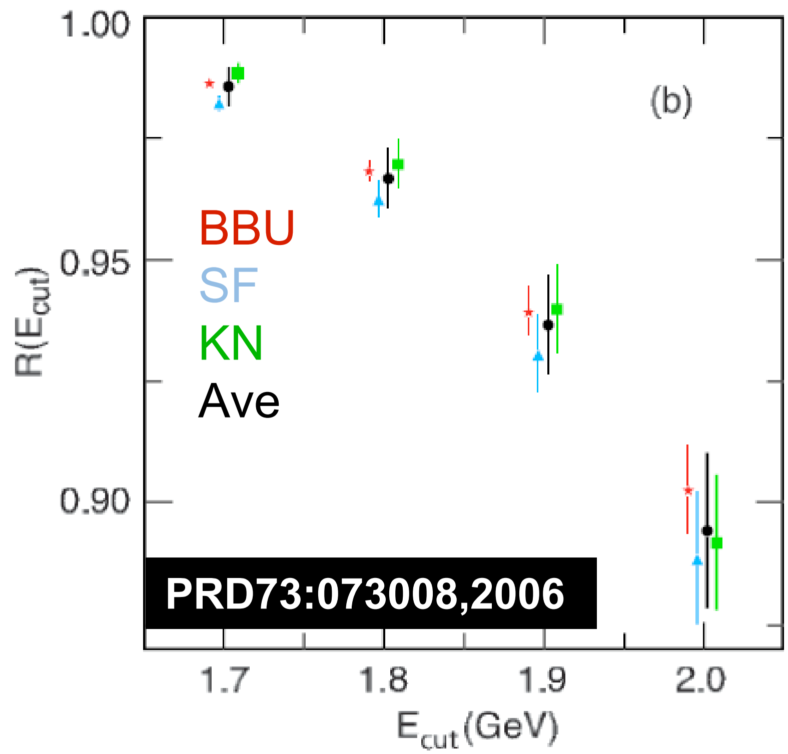
Extrapolation Factors for BF



New World Average from HFAG:

$BR(B \to X_s\gamma)$ =
$(3.55 \pm 0.24 \pm 0.10 \pm 0.03)\ 10^{-4}$

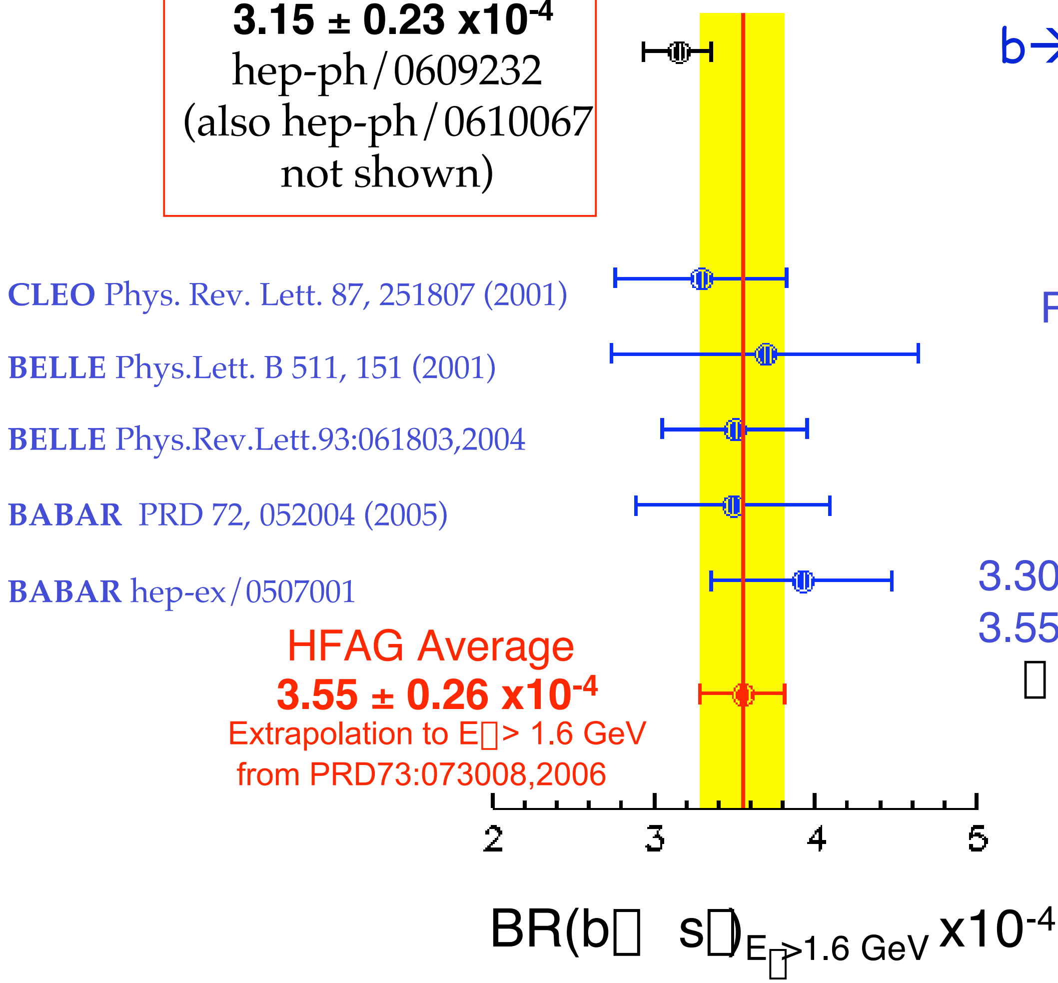
7% uncertainty

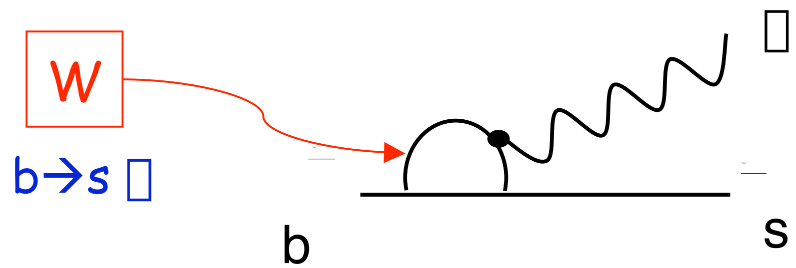
SM prediction:
3.57 ± 0.3 $\times 10^{-4}$ Buras et al. (hep-ph/0203135)
3.44 ± 0.4 $\times 10^{-4}$ Neubert (hep-ph/0408179)
3.61 ± 0.42 $\times 10^{-4}$ Hurth et al. (hep-ph/0312260)

BR($b \to s\gamma$) average

NEW!

NNLO SM Prediction
$3.15 \pm 0.23 \times 10^{-4}$
hep-ph/0609232
(also hep-ph/0610067 not shown)





For comparison:
PDG2004 Average: $3.30 \pm 0.40 \times 10^{-4}$

Improvement within 1 year!

Experiment	SM prediction
$3.30 \pm 0.40 \times 10^{-4}$ (2005) $3.55 \pm 0.26 \times 10^{-4}$ (2006) → 35% improved	NLO vs NNLO → ~40–50% improved

Expected to improve further

Conclusions

Extracting fundamental QCD parameter (m_b, m_c, μ_π^2, …) from HQE fits to semileptonic and radiative B Decays has become an established procedure providing important input for many measurements.

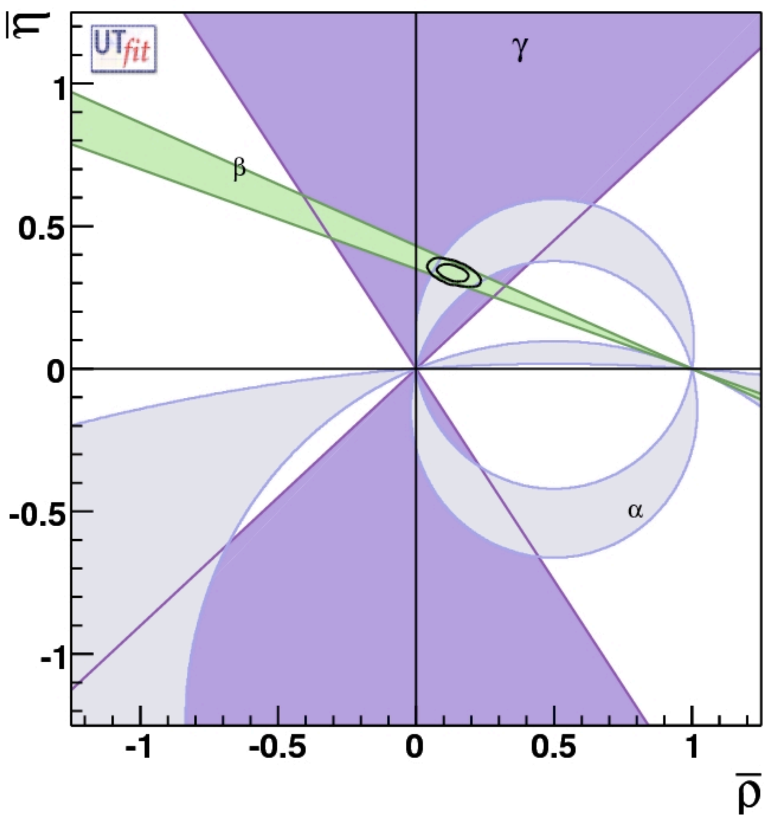
- Consistency between experimental results
- Good agreement of results from semileptonic and radiative B decays

- Precision determination of SM parameters:
  - $|V_{cb}|$ at <2% level established
  - $|V_{ub}|$ at ~7% probing consistency with $\sin(2\beta)$ and hence SM
  - m_b (<1%) and m_c (5%)
- Radiative B decays
  - $BR(B \rightarrow X_s \gamma)$ @ 7% - important constraint on many NP models

Backup Slides

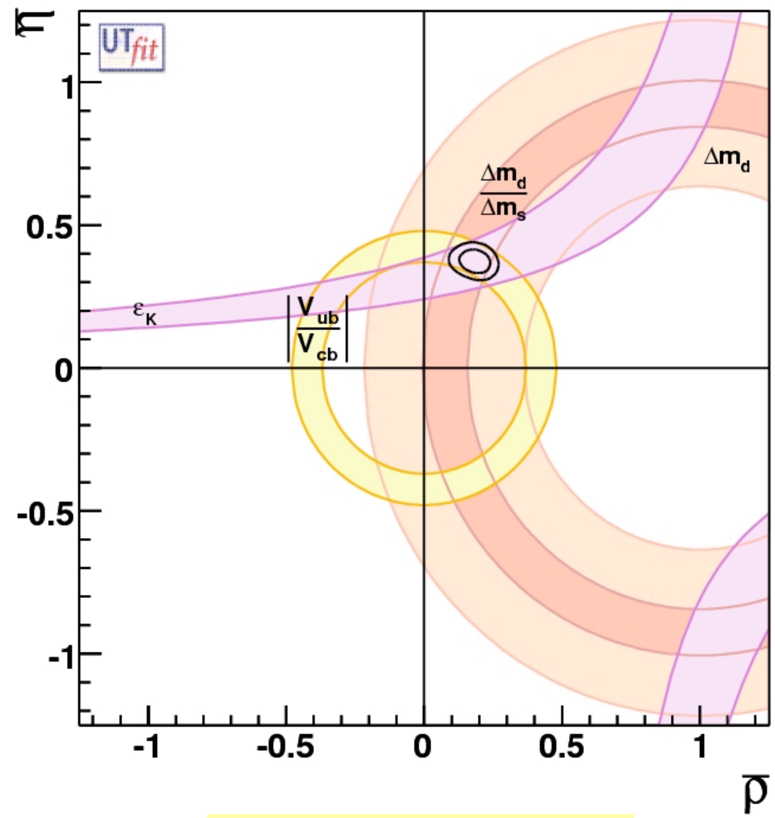
“R_b - sin2β tension”

Angles



ρ=0.134±0.039
η=0.335± 0.020

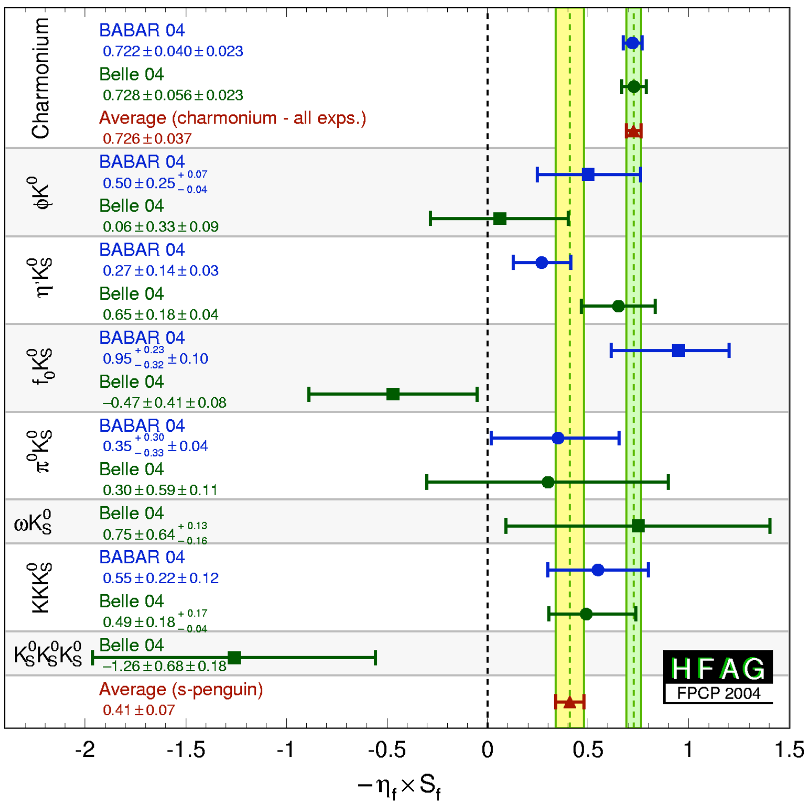
Sides



ρ=0.188±0.036
η=0.371± 0.027

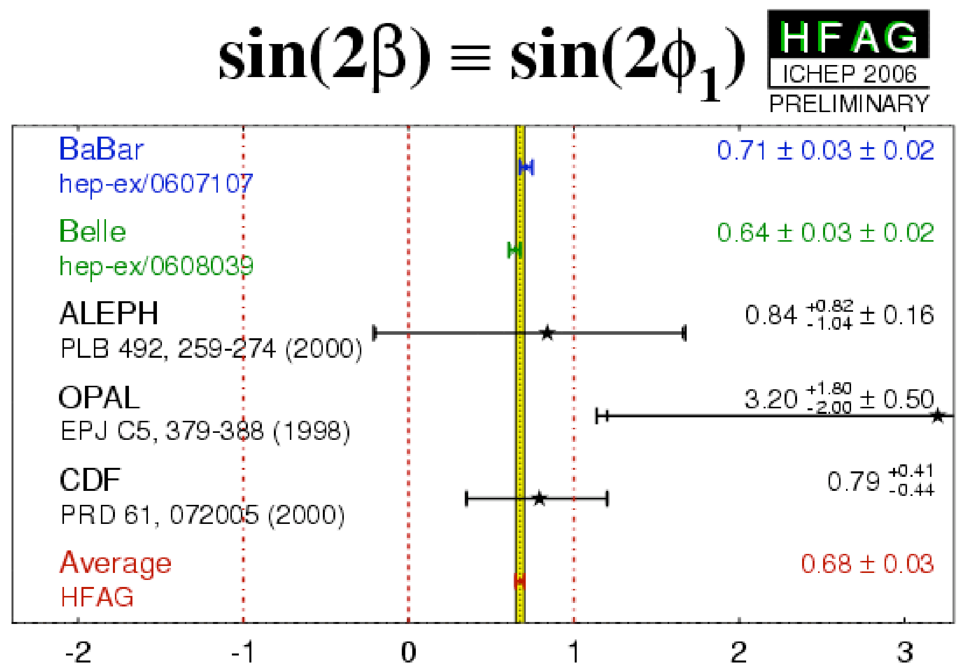
Sin2β: Recent improvements

2004



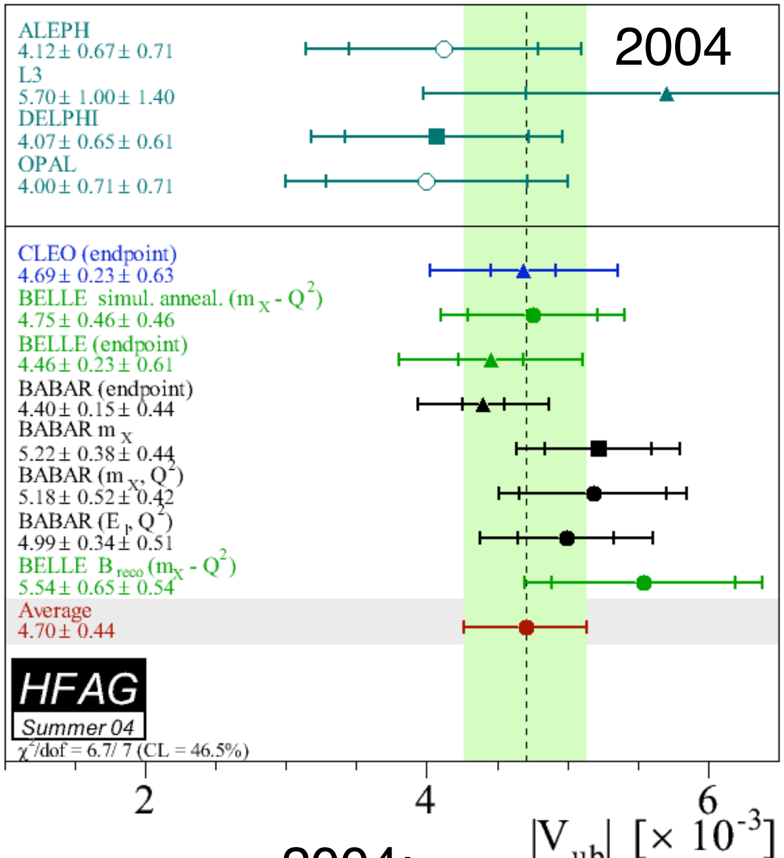
2004:
sin2β= 0.726± 0.037

2006



2006:
sin2β= 0.675± 0.026

V_{ub}^{inc}: Recent improvements

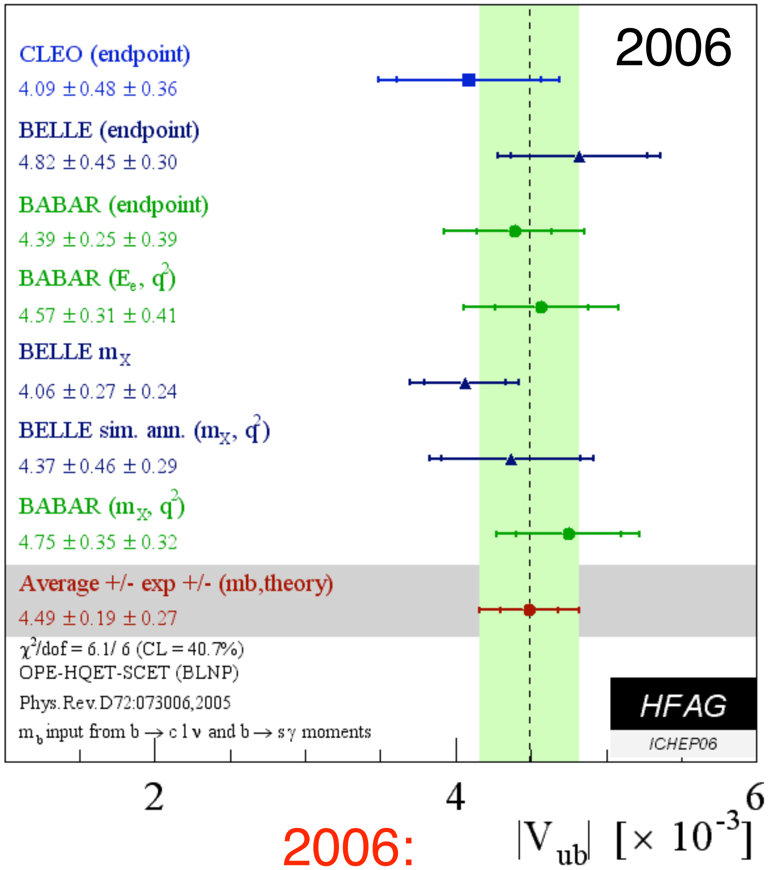


2004:

V_{ub}= 4.70± 0.21± 0.39

Dominate uncertainty:

m_b and shape function



2006:

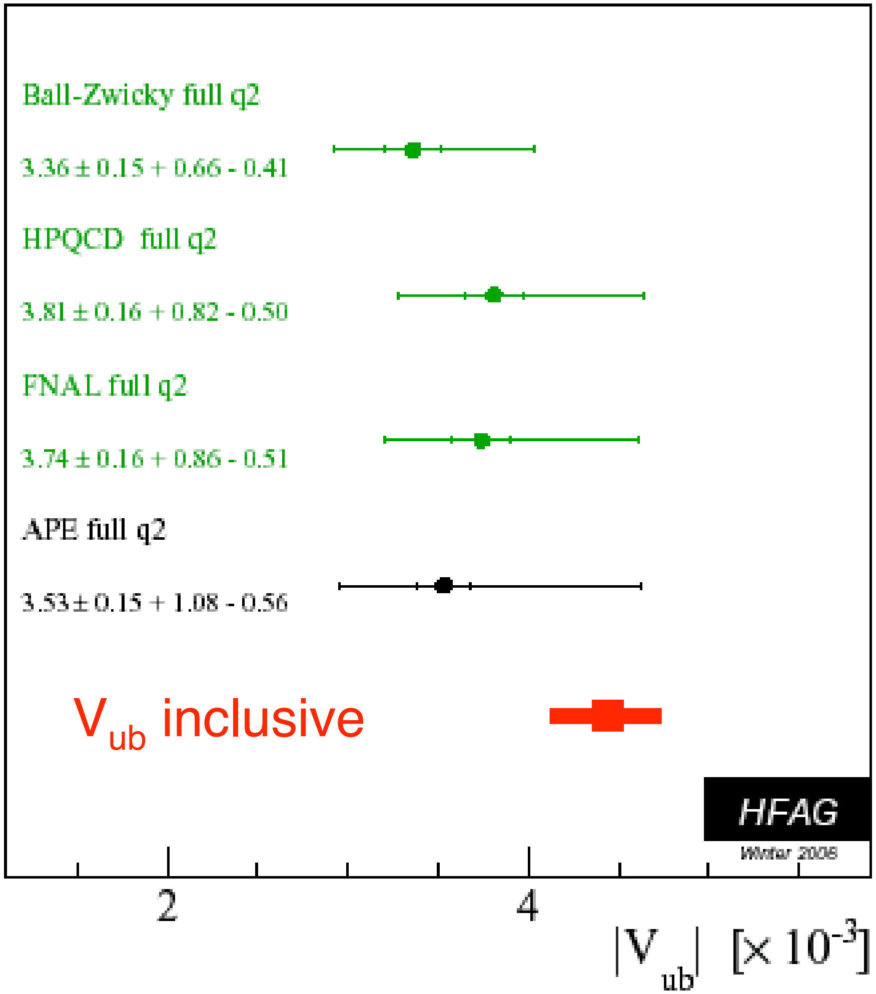
V_{ub}= 4.49± 0.19± 0.27

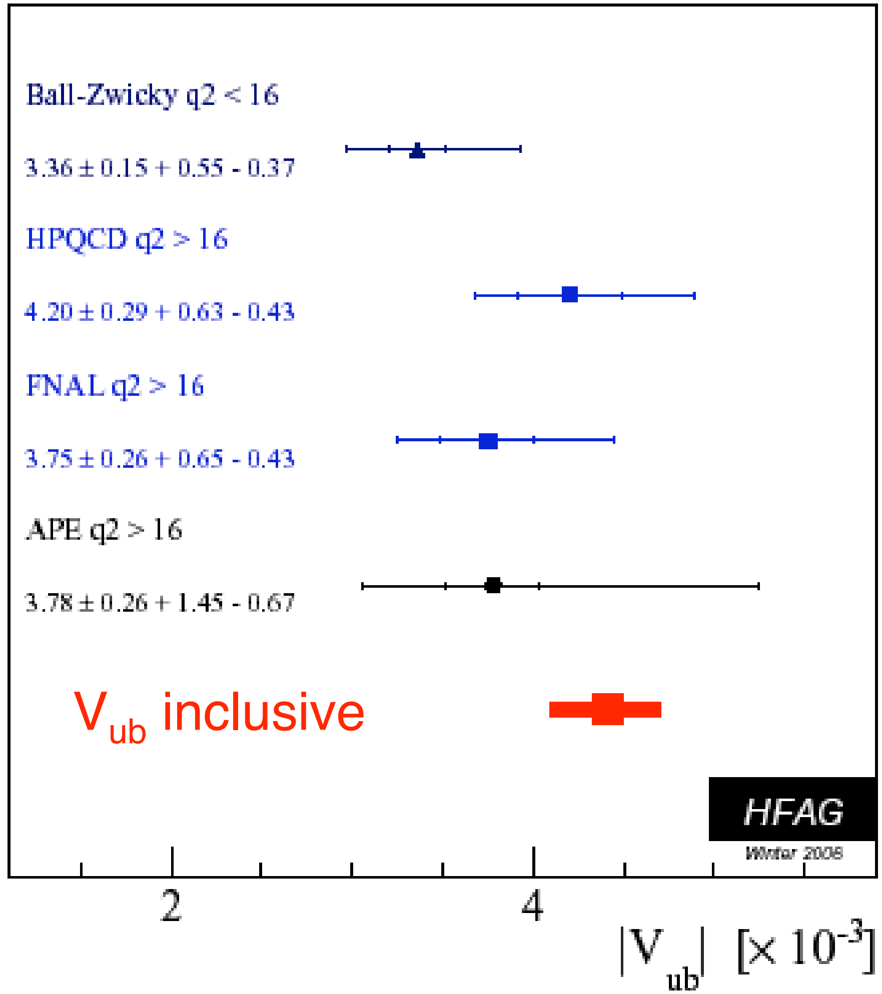
Significantly improved

m_b and shape function

uncertainties

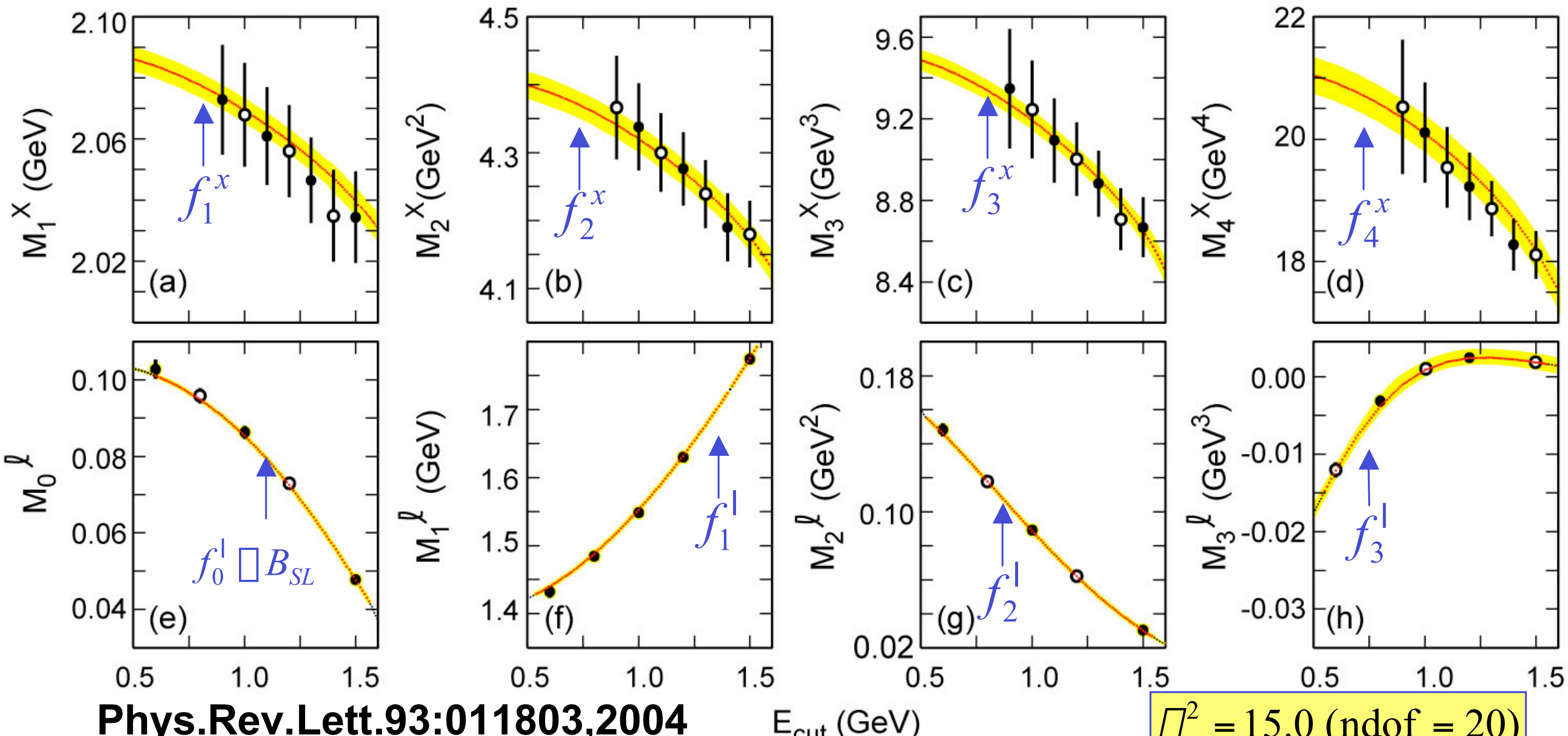
V_{ub}: exclusive vs. inclusive





HQE Fit from BABAR

- $|V_{cb}|$ „master“ formula : $|V_{cb}|^2 = Br(B \to X_c e\nu)/\Gamma_B \; f_{\Gamma}^{l}(E_0, m_b, m_c, \mu_G^2, \mu_\pi^2, \rho_{LS}^3, \rho_D^3)$
- $Br(B \to X_c e\nu, E_l > E_0)$: $M_0^{l}(E_0)/Br(B \to X_c e\nu) = f_0^{l}(E_0, m_b, m_c, \mu_G^2, \mu_\pi^2, \rho_{LS}^3, \rho_D^3)$
- i-th central E_l moment for $E_l > E_0$: $M_i^{l}(E_0) = f_i^{l}(E_0, m_b, m_c, \mu_G^2, \mu_\pi^2, \rho_{LS}^3, \rho_D^3)\;(i = 1..3)$
- i-th M_x moment and $E_l > E_0$: $M_i^{X}(E_0) = f_i^{X}(E_0, m_b, m_c, \mu_G^2, \mu_\pi^2, \rho_{LS}^3, \rho_D^3)\;(i = 1..4)$



Calculations taken from Gambino and Uraltsev, hep/ph 0401063

high correlation between measurements :
⇒ this fit uses solid points only

Phys.Rev.Lett.93:011803,2004

$\chi^2 = 15.0 \; (\text{ndof} = 20)$

BABAR Fit Results

$$|V_{cb}| = (41.4 \pm 0.4_{exp} \pm 0.4_{HQE} \pm 0.2_{\alpha_s} \pm 0.6_{\Gamma_{SL}}) \times 10^{-3}$$
$$Br(B \to X_c e\nu) = (10.61 \pm 0.16_{exp} \pm 0.06_{HQE})\%$$
$$m_b(1\ GeV) = (4.61 \pm 0.05_{exp} \pm 0.04_{HQE} \pm 0.02_{\alpha_s})\ GeV$$
$$m_c(1\ GeV) = (1.18 \pm 0.07_{exp} \pm 0.06_{HQE} \pm 0.02_{\alpha_s})\ GeV$$

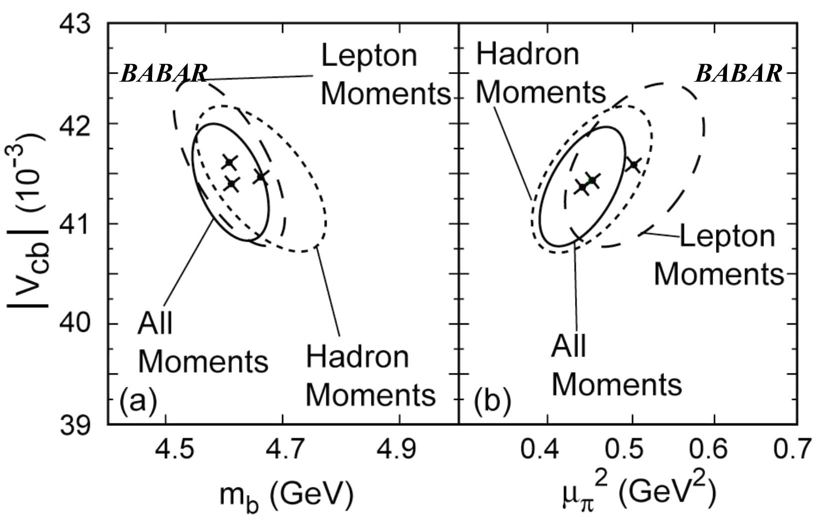
kinetic mass scheme

$$\mu_\pi^2 = (0.45 \pm 0.04_{exp} \pm 0.04_{HQE} \pm 0.01_{\alpha_s})\ GeV^2$$
$$\mu_G^2 = (0.27 \pm 0.06_{exp} \pm 0.03_{HQE} \pm 0.02_{\alpha_s})\ GeV^2$$
$$\rho_D^3 = (0.20 \pm 0.02_{exp} \pm 0.02_{HQE} \pm 0.00_{\alpha_s})\ GeV^3$$
$$\rho_{LS}^3 = (-0.09 \pm 0.04_{exp} \pm 0.07_{HQE} \pm 0.01_{\alpha_s})\ GeV^3$$

Phys. Rev. Lett. 93:011803,2004"

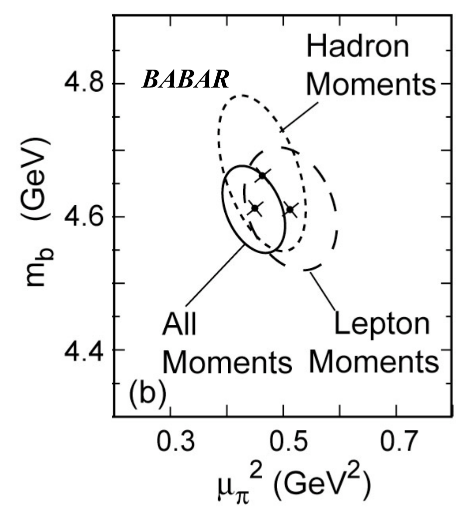
Strong correlation between m_b and m_c:

$m_b(1\ GeV) - m_c(1\ GeV) =$
$(3.44 \pm 0.03_{exp} \pm 0.02_{HQE} \pm 0.01_{\alpha_s})$ GeV



2D projections of the fit result:

$\Delta\chi^2=1$ ellipses



NEW Belle Result (ICHEP06)

Kinetic Scheme $|V_{cb}|$ and HQ parameters

Belle Conf 0669

Preliminary

$X_c \ell \nu + X_s g$ $\chi^2/\text{d.o.f.} = 17.8/24$

$|V_{cb}| = (41.93 \pm 0.65_{fit} \pm 0.48_{\alpha_s} \pm 0.63_{th}) \times 10^{-3}$

$m_b = 4.564 \pm 0.076$ GeV

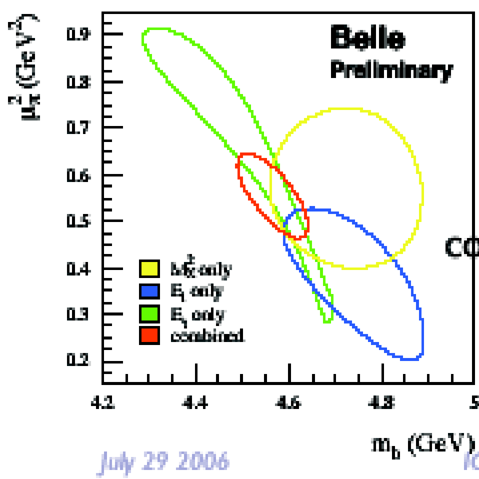
$m_c = 1.105 \pm 0.116$ GeV

$Br(B \to X_c \ell \nu) = 10.59 \pm 0.16$ %

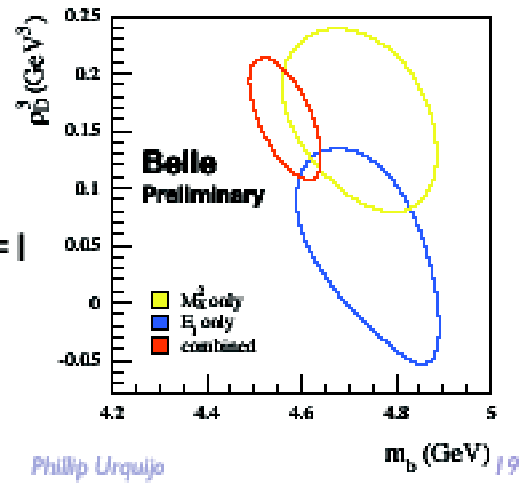
$\mu^2_\pi = 0.557 \pm 0.092$ GeV^2

$\tilde{\rho}^3_D = 0.162 \pm 0.054$ GeV^3

$\Delta|V_{cb}| < 2.5\%$, $\Delta m_b^{kin} < 2\%$, Δm_c @ 10%



contours $\Delta\chi^2 = 1$



July 29 2006 ICHEP Moscow 2006 Phillip Urquijo 19

http://belle.kek.jp/belle/talks/ICHEP2006/Urquijo.pdf

NEW Belle Result (ICHEP06)

Kinetic Scheme $|V_{cb}|$ and HQ parameters

Belle Conf 0669

Preliminary

$X_c \ell \nu + X_s \gamma$ χ^2/d.o.f. = 17.8/24

$|V_{cb}| = (41.93 \pm 0.65_{fit} \pm 0.48_{\alpha_s} \pm 0.63_{th}) \times 10^{-3}$

$m_b = 4.564 \pm 0.076$ GeV

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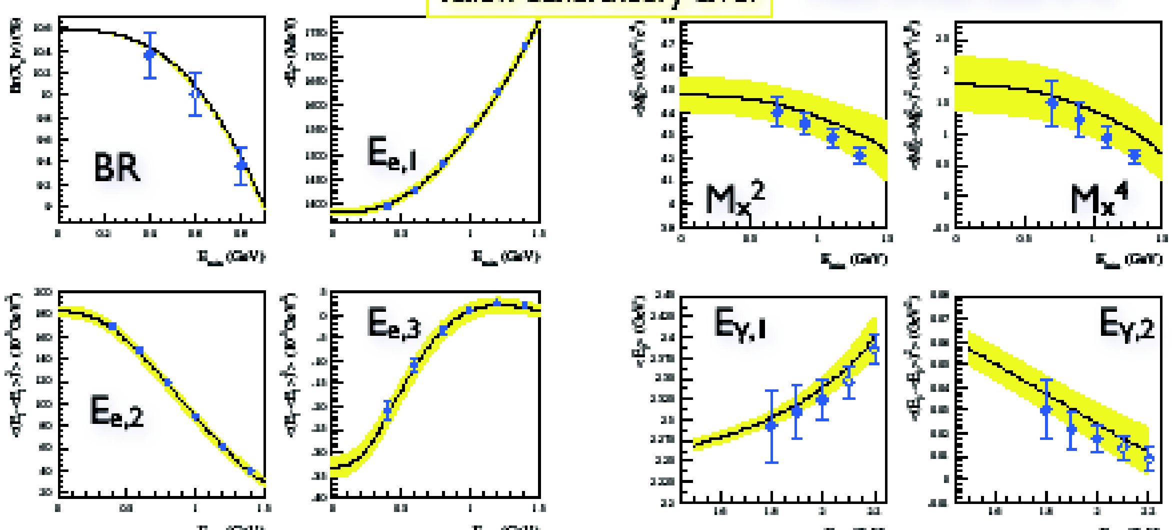
$\mu^2_\pi = 0.557 \pm 0.092$ GeV^2

$\tilde{\rho}^3_D = 0.162 \pm 0.054$ GeV^3

Yellow band: theory error

Filled circles: used in fit



Lepton moments in hep-ex/0610012 - hadron moments soon to come

NEW Belle Result (ICHEP06)

1S Scheme

$|V_{cb}|$ and HQ parameters

Belle Conf 0669

Preliminary

$|V_{cb}| = (41.5 \pm 0.5_{fit} \pm 0.2_{T}) \times 10^{-3}$ $\Delta|V_{cb}|<1.5\%$

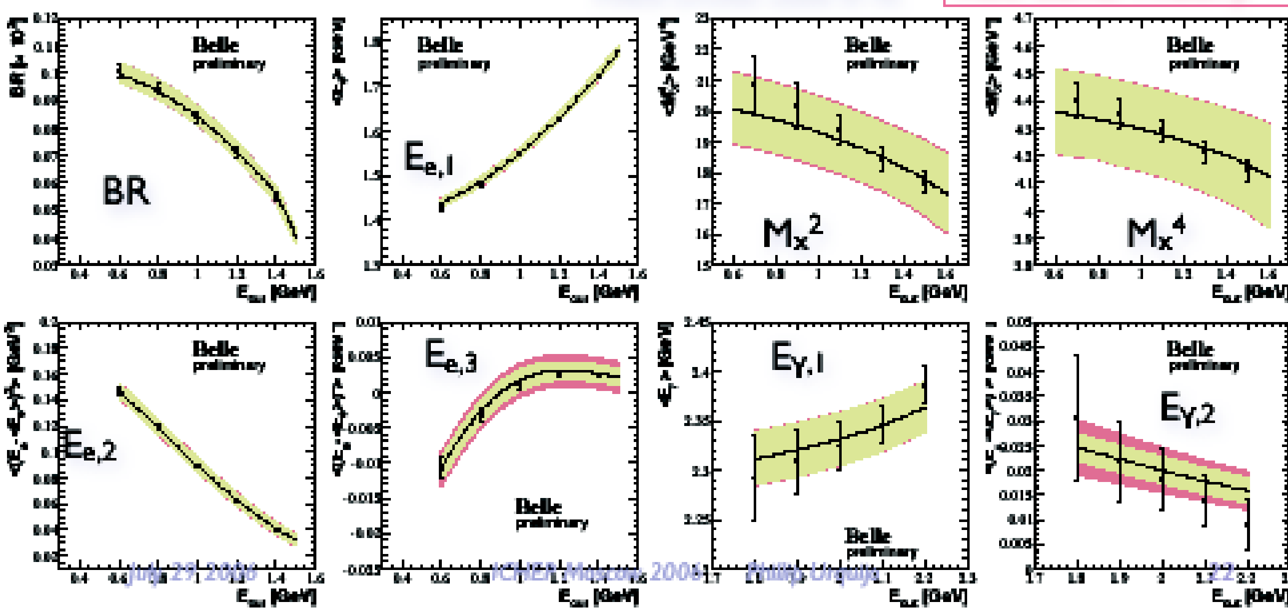
$m_b^{1s} = 4.73 \pm 0.05$ GeV $\Delta m_b^{1s}<1.5\%$

$\lambda_1 = -0.30 \pm 0.04$ GeV^2

Yellow band: fit error

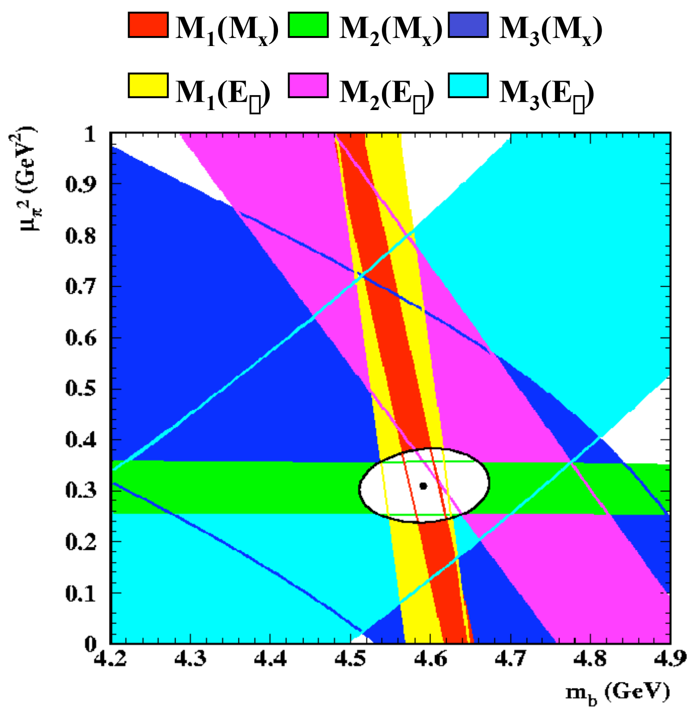
Red band: Theory + Fit

Filled circles: used in fit



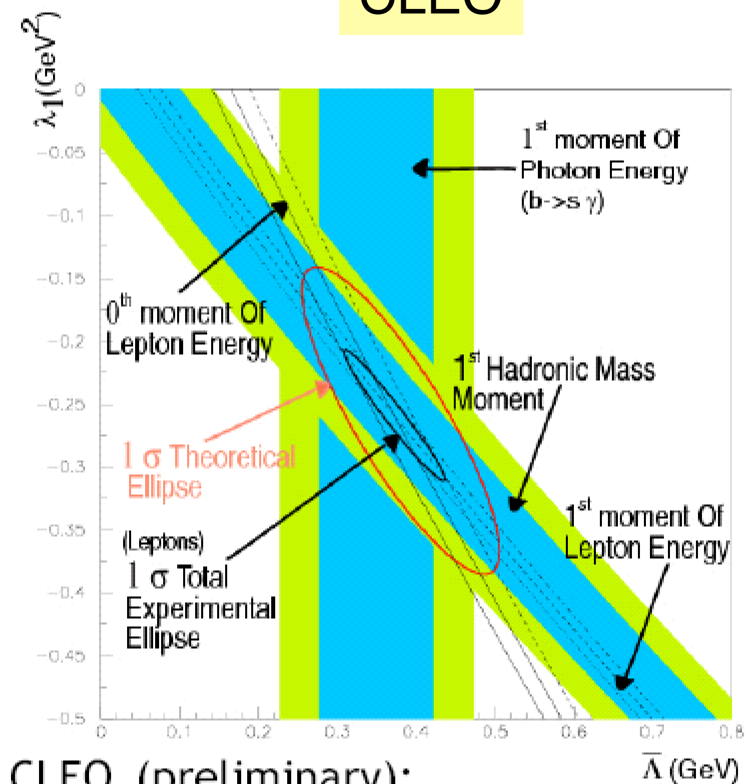
Pioneering Work

DELPHI



$m_{b,kin}$ (1GeV)= 4.58 $\pm 0.06_{fit} \pm 0.05_{sys}$ GeV
$m_{c,kin}$ (1GeV)= 1.15 $\pm 0.09_{fit} \pm 0.08_{sys}$ GeV
μ_π^2 (1GeV) = 0.41 $\pm 0.04_{fit} \pm 0.04_{sys}$ GeV^2
ρ_D^3 (1GeV) = 0.05 $\pm 0.02_{fit} \pm 0.01_{sys}$ GeV^3

CLEO



CLEO (preliminary):

$$\overline{\Lambda} = 0.39 \pm 0.03 \pm 0.06 \pm 0.12\,\mathrm{GeV}$$

$$\lambda_1 = -0.25 \pm 0.02 \pm 0.05 \pm 0.142\,\mathrm{GeV}^2$$

Multi parameter fits to moments but sensitivity was limited …