Lifetimes and oscillations of heavy mesons

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Outline

Theoretical framework

- **Heavy Quark Expansion**
- **State of the art**
- **Non-perturbative parameters**

Lifetimes

- τ_{B_+}/τ_{B_d}
- τ_{B_s}/τ_{B_d}
- τ_{B_c}

$B_q - \bar{B}_q$-Oscillations

- ΔM
- $\Delta\Gamma$ **and** $\Delta\Gamma/\Delta M$
- a_{fs}
- **New physics in mixing**

Outlook

Heavy Quark Expansion

Systematic expansion of the decay rate in powers of m_b^{-1} yields

$$\Gamma = \Gamma_0 + \frac{\Lambda^2}{m_b^2}\Gamma_2 + \frac{\Lambda^3}{m_b^3}\Gamma_3 + \ldots$$

Voloshin, Uraltsev, Kohze, Shifman

Γ_0: Decay of a free quark $\Rightarrow$ all b-hadrons have the same lifetime

Γ_2: First corrections due to kinetic and chromomagnetic operator

Γ_3: Weak annihilation and Pauli interference
Distinguish between different spectators $\Rightarrow$ Lifetime differences
numerically enhanced by phase space factor $16\pi^2$

State of the art

Meson vs Meson

$$\frac{\tau_1}{\tau_2} = 1 + \quad \frac{\Lambda^3}{m_b^3}\left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{(1)} + \ldots\right) + \quad \frac{\Lambda^4}{m_b^4}\left(\Gamma_4^{(0)} + \ldots\right) + \ldots$$

Baryon vs Meson

$$\frac{\tau_1}{\tau_2} = 1 + \quad \frac{\Lambda^2}{m_b^2}\left(\Gamma_2^{(0)} + \ldots\right) + \quad \frac{\Lambda^3}{m_b^3}\left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{(1)} + \ldots\right) + \quad \frac{\Lambda^4}{m_b^4}\left(\Gamma_4^{(0)} + \ldots\right) + \ldots$$

Neutral Mesons

$$\frac{\Delta\Gamma}{\Gamma} = \quad \frac{\Lambda^3}{m_b^3}\left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{(1)} + \ldots\right) + \quad \frac{\Lambda^4}{m_b^4}\left(\Gamma_4^{(0)} + \ldots\right) + \ldots$$

$$\Gamma_i^{(j)} = C_i^{(j)} \cdot \langle Q_i^{(j)} \rangle \propto f^2 \cdot B_i^{(j)} \cdot C_i^{(j)}$$

Perturbative corrections

- $C_3^{(0)}$: '79...'92
- $C_4^{(0)}$: '96...'03
- $C_3^{(1)}$: '98...'03; incomplete for Λ_b
- $C_5^{(0)}$: '03...'06

non-perturbative corrections

$\langle Q_3 \rangle$: prel. $n_f = 2+1$ for B-mixing
only one determination for τ_{B+}/τ_{B_d}
only prel. studies for Λ_b

$\langle Q_4 \rangle$: mostly VIA

$\langle Q_5 \rangle$: only naive estimates

Non-perturbative Parameters I: f_{B_s} - the easiest

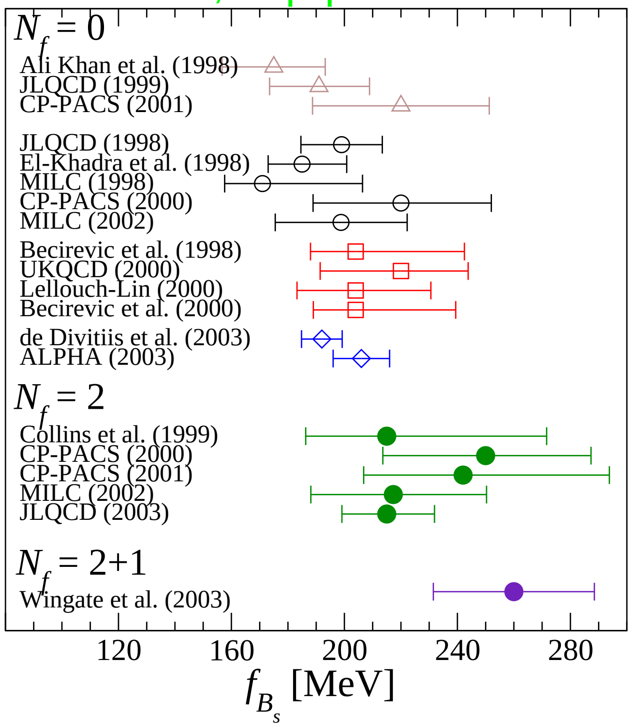
Sum rules

- ...
- 2002: Jamin, Lange, 244 ± 21 MeV

Lattice quenched

- ...
- 2000: Becirevic et al, 204^{+17}_{-15} MeV
- 2003: ALPHA, 205 ± 12 MeV
- 2006: Sommer et al, 191 ± 6 MeV

Hashimoto, hep-ph/0411126



Lattice unquenched, $n_f = 2$

- 1999: Collins et al, 212^{+64}_{-29} MeV
- 2000: CP-PACS, 250^{+18}_{-16} MeV
- 2001: CP-PACS, 242^{+52}_{-35} MeV
- 2002: MILC, 220^{+34}_{-22} MeV
- 2003: JLQCD, 216^{+31}_{-28} MeV

Lattice unquenched, $n_f = 2 + 1$

- 2003: Wingate et al, 260 ± 29 MeV
- 2005: HP QCD, 259 ± 32 MeV

Conservative estimate:

$$f_{B_s} = 240 \pm 40 \text{ MeV}$$

Non perturbative Parameters II

$$\langle \bar{B}_s | (\bar{s}b)_{V-A} (\bar{s}b)_{V-A} | B_s \rangle = \frac{8}{3} M_{B_s} f_{B_s}^2 B \, , \, ...$$

- B, B_S with $n_f = 2$ from JLQCD,01 & 03
- only 1 published determination of $\tilde{B}_S$: Becirevic et al. 01

New: Preliminary results with $n_f = 2 + 1$ (Shigemitsu, HPQCD, Lattice 06):
Combined determination of $f_{B_s} \sqrt{B_x} \Rightarrow$ smaller error

f_{B_d} - extrapolation to small quark masses: e.g. HPQCD ($n_f = 2 + 1$) vs. Belle

$$f_{B_d} = (216 \pm 22) \text{ MeV vs. } \left(229^{+47}_{-46}\right) \text{ MeV}$$

Clean ratio? $\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}} = 1.210^{+0.047}_{-0.035}$ (Okamoto, Lattice 05)

see talk by Vittorio Lubicz

τ_{B^+}/τ_{B_d} in NLO-QCD I

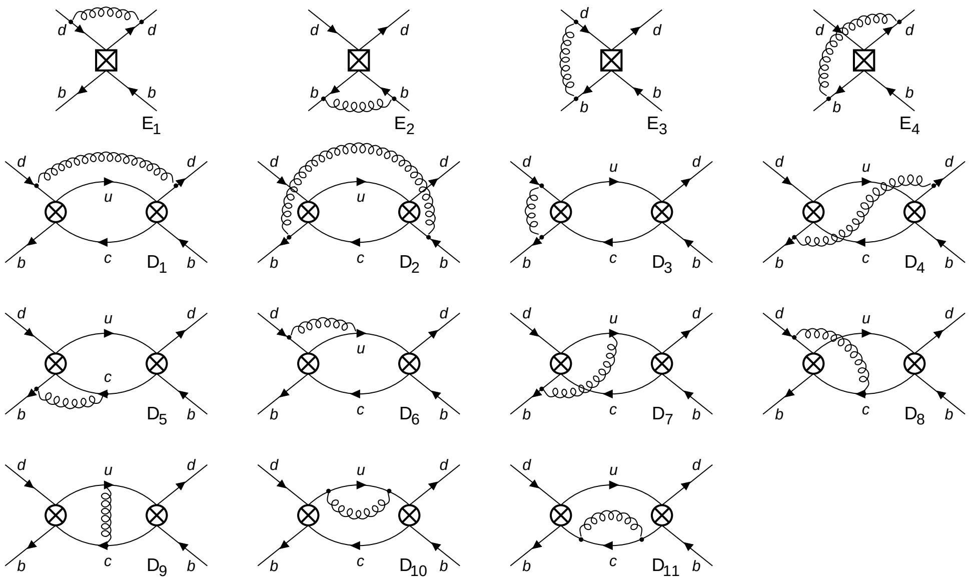
$$\frac{\tau_1}{\tau_2} = 1 + \left(\frac{\Lambda}{m_b}\right)^3 \left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{(1)} + ...\right) + \left(\frac{\Lambda}{m_b}\right)^4 \left(\Gamma_4^{(0)} + ...\right) + ...$$

$\Gamma_3^{(0)}$: Shifman, Voloshin; Uraltsev; Bigi, Vainshtein; Neubert, Sachrajda

$\Gamma_4^{(0)}$: Gabbiani, Onishchenko, Petrov; Greub, A.L., Nierste (unpublished)

$\Gamma_3^{(1)}$: Beneke, Buchalla, Greub, A.L., Nierste; Ciuchini,Franco, Lubicz, Mescia, Tarantino

lattice : Di Pierro, Sachrajda, Michael; Becirevic



τ_{B^+}/τ_{B_d} in NLO-QCD II

$$\frac{\tau(B^+)}{\tau(B_d^0)} - 1 = \tau(B^+)\left[\Gamma(B_d^0) - \Gamma(B^+)\right]$$
$$= 0.0325\,\frac{\tau(B^+)}{1.653\,\text{ps}}\left(\frac{|V_{cb}|}{0.04}\right)^2\left(\frac{m_b}{4.8\text{GeV}}\right)^2\left(\frac{f_B}{200\text{MeV}}\right)^2$$
$$\left[(1.0\pm 0.2)\,B_1 + (0.1\pm 0.1)\,B_2 - (18.4\pm 0.9)\,\epsilon_1 + (4.0\pm 0.2)\,\epsilon_2\right] + \delta_{1/m}$$

$(B_1, B_2, \epsilon_1, \epsilon_2) = (1.10\pm 0.20,\ 0.99\pm 0.10,\ -0.02\pm 0.02,\ 0.03\pm 0.01)$ '01: Becirevic

$$\left[\frac{\tau(B^+)}{\tau(B_d^0)}\right]_{\text{LO}} = 1.047\pm 0.049 \qquad \left[\frac{\tau(B^+)}{\tau(B_d^0)}\right]_{\text{NLO}} = 1.063\pm 0.027$$

NLO-QCD: '02: Beneke, Buchalla, A.L, Greub, Nierste; Ciuchini,Franco, Lubicz, Mescia, Tarantino

$1/m_b$: '03: Gabbiani, Onishchenko, Petrov; Greub, A.L, Nierste (unpublished): tiny ≤ 0.005

PDG 2006: $\left[\frac{\tau(B^+)}{\tau(B_d^0)}\right] = 1.071\pm 0.009$

The lifetime ratio τ_{B_s}/τ_{B_d}

$$\frac{\tau(B_s)}{\tau(B_d)} = 1.00 \pm 0.01$$

Neubert, Sachrajda; Beneke, Buchalla, Dunietz; Bigi, Blok, Shifman, Uraltsev, Vainshtein; U. Nierste, Y.-Y. Keum; M. Ciuchini, E. Franco, V. Lubicz, F. Mescia

Weak annihilation contributions for B_d and B_s have almost the same size.

Lifetime differences only due to small difference in phase space and by $SU(3)_F$ violations of the hadronic parameters.

NLO penguin contributions to τ_{B_s}/τ_{B_d} give a comparable effect $->$ search for new physics

HFAG 2006: $\left[\frac{\tau(B_s^0)}{\tau(B_d^0)}\right] = 0.957 \pm 0.027$

Lifetime of the double-heavy meson $\tau_{B_c^+}$

LO analysis gives

$$\tau(B_c) = 0.4 \ldots 0.7 \text{ ps}$$

Beneke, Buchalla;

Colangelo et al.; Anisimov et al.; Lusignoli, Masetti; Quigg; Kiselev et al; Chang et al.

Data: CDF 2006, 360 fb^{-1}, HFAG 2006

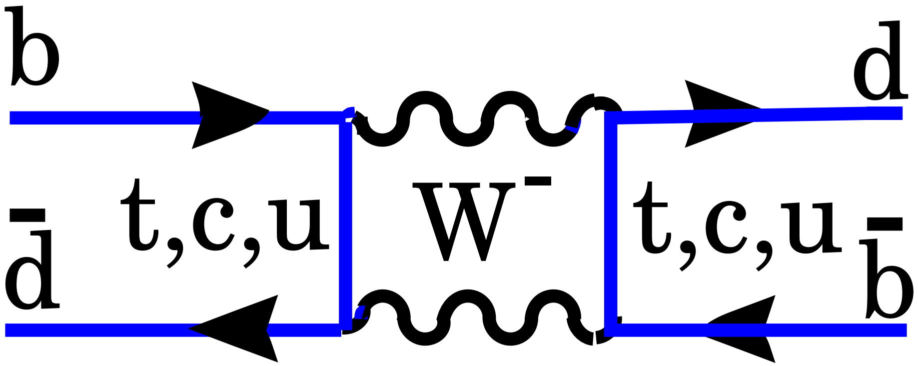
$$\tau(B_c) = 0.469 \pm 0.065 \text{ ps}$$

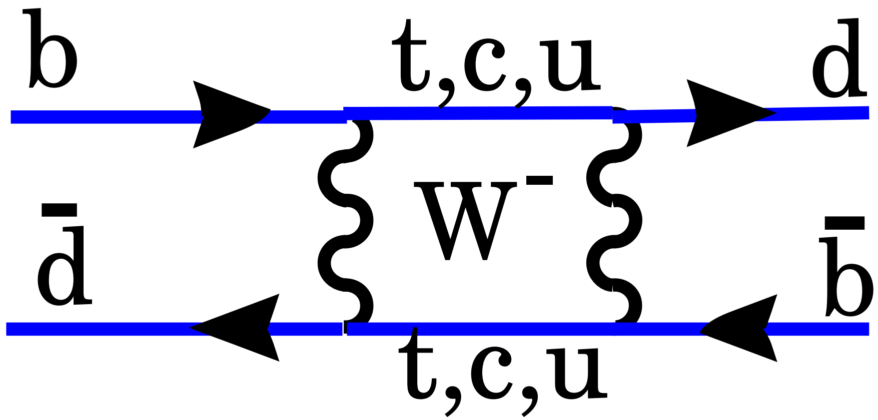
B-mixing I

Time evolution of a decaying particle: $B(t) = \exp\left[-i m_B t - \Gamma_B/2 t\right]$
can be written as

$$i \frac{d}{dt} \begin{pmatrix} |B(t)\rangle \\ |\bar{B}(t)\rangle \end{pmatrix} = \left(\hat{M} - \frac{i}{2}\hat{\Gamma}\right) \begin{pmatrix} |B(t)\rangle \\ |\bar{B}(t)\rangle \end{pmatrix}$$

BUT: In the neutral B-system transitions like $B_{d,s} \to \bar{B}_{d,s}$ are possible due to weak interaction: **Boxdiagrams**





$\Rightarrow$ **off-diagonal elements** in $\hat{M}$, $\hat{\Gamma}$: M_{12}, Γ_{12} (complex)

Diagonalization of $\hat{M}$, $\hat{\Gamma}$ gives the physical eigenstates B_H and B_L with the masses M_H, M_L and the deacy rates Γ_H, Γ_L

CP-odd: $B_H := p\, B + q\, \bar{B}$, CP-even: $B_L := p\, B - q\, \bar{B}$ with $|p|^2 + |q|^2 = 1$

B-mixing II

$|M_{12}|$, $|\Gamma_{12}|$ and $\Phi = \text{arg}(-M_{12}/\Gamma_{12})$ can be related to three observables:

- **Mass difference:** $\Delta M := M_H - M_L = 2|M_{12}| \left(1 + \frac{1}{8}\frac{|\Gamma_{12}|^2}{|M_{12}|^2}\sin^2\Phi + ...\right)$
  $|M_{12}|$: heavy internal particles: t, SUSY, ...

- **Decay rate difference:** $\Delta\Gamma := \Gamma_L - \Gamma_H = 2|\Gamma_{12}|\cos\Phi \left(1 - \frac{1}{8}\frac{|\Gamma_{12}|^2}{|M_{12}|^2}\sin^2\Phi + ...\right)$
  $|\Gamma_{12}|$: light internal particles: u, c, ... (almost) no NP!!!

- **Flavor specific/semileptonic CP asymmetries:**
  $\bar{B}_q \to f$ and $B_q \to \bar{f}$ forbidden
  No direct CP violation: $|\langle f|B_q\rangle| = |\langle \bar{f}|\bar{B}_q\rangle|$
  e.g. $B_s \to D_s^-\pi^+$ or $B_q \to Xl\nu$ (*semileptonic*)

$$a_{sl} \equiv a_{fs} = \frac{\Gamma(\overline{B}_q(t) \to f) - \Gamma(B_q(t) \to \overline{f})}{\Gamma(\overline{B}_q(t) \to f) + \Gamma(B_q(t) \to \overline{f})} = -2\left(\left|\frac{q}{p}\right| - 1\right) = \text{Im}\frac{\Gamma_{12}}{M_{12}} = \frac{\Delta\Gamma}{\Delta M}\tan\Phi$$

The mass difference Δ_M

Calculating the Boxdiagram with an internal top-quark yields

$$M_{12,q} = \frac{G_F^2}{12\pi^2}(V_{tq}^* V_{tb})^2 M_W^2 S_o(x_t) B_{B_q} f_{B_q}^2 M_{B_q} \hat{\eta}_B$$

(Inami, Lim ’81)

- Hadronic matrix element: $\frac{8}{3} B_{B_q} f_{B_q}^2 M_{B_q} = \langle \bar{B}_q | (\bar{b}q)_{V-A} (\bar{b}q)_{V-A} | B_q \rangle$
- Perturbative QCD corrections $\hat{\eta}_B$ (Buras, Jamin, Weisz, ’90)

$\Delta M_s = 19.3 \pm 6.7\,\text{ps}^{-1}$ better: $\frac{\Delta M_s}{\Delta M_d} = \frac{M_{B_s}}{M_{B_d}} \left| \frac{V_{ts}^2}{V_{td}^2} \right| \frac{f_{B_s}^2 B_{B_s}}{f_{B_d}^2 B_{B_d}}$

Experimental status: Heavy Flavor Averaging Group, 2006

$$\Delta M_d = 0.507 \pm 0.004\ ps^{-1}$$

ALEPH, CDF, D0, DELPHI, L3, OPAL, BABAR, BELLE, ARGUS, CLEO

$$\Delta M_s = 17.77 \pm 0.10(stat) \pm 0.07(syst)\ ps^{-1}$$

CDF hep-ex/0609040, D0

Important bounds on the unitarity triangle and new physics

Γ_{12} in NLO-QCD

$$\Gamma_{12} = \left(\frac{\Lambda}{m_b}\right)^3 \left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{(1)} + ...\right) + \left(\frac{\Lambda}{m_b}\right)^4 \left(\Gamma_4^{(0)} + ...\right) + ...$$

$\Gamma_3^{(0)}$	:	Hagelin; Buras, Slominski, Steger; Datta, Paschos, Türke,Wu;
		Voloshin, Uraltsev, Khoze, Shifman; Chau; Franco,Lusignoli, Pugliese; (1981 . . .)
$\Gamma_3^{(1)}$	:	Beneke, Buchalla, Greub, A.L., Nierste (1998,2003)
		Ciuchini, Franco, Lubicz, Mescia, Tarantino (2003)
$\langle \| \| \rangle$	:	HPQCD, JLQCD, Becirevic et al.; Gimenez, Reyes; Jamin, Lange
		Huang, Zhang, Zhou; ... (1999 . . .)
$\Gamma_4^{(0)}$	:	Beneke, Buchalla, Dunietz (1996); Dighe, Hurth, Kim, Yoshikawa (2001)
$\Gamma_5^{(0)}$	:	A.L., Nierste preliminary: small Wilson coefficients
$\langle \| \| \rangle$	:	part of operators of dim 7 and 8 Becirevic et al (2001)

New determination of Γ_{12}: A.L., Nierste, 2006

- In the calculation of Γ_{12} 4 operators arise: $Q, Q_S, \tilde{Q}, \tilde{Q}_S$
- They are not independent: ($\alpha_i = 1 + \mathcal{O}(\alpha_s)$)

$$\tilde{Q} = Q \quad \text{and} \quad R_0 = Q_S + \alpha_1 \tilde{Q}_S + \frac{\alpha_2}{2} Q = \mathcal{O}\left(\frac{1}{m_b}\right)$$

- Old Basis: $\{Q, Q_S\} \Longleftrightarrow$ New Basis $\{Q, \tilde{Q}_S\}$

Problems in the old basis:

:-(Almost complete cancellation in coefficient of Q

:-(Huge $1/m_b$-corrections

:-(Large α_s-corrections

Γ_{12}^{new}:

+ New basis free of the above shortcomings
+ Use also $\overline{MS}$-scheme for m_b
+ Sum $z \ln z$ to all orders
+ Include subleading CKM structures

$\Delta\Gamma_s$ and $\Delta\Gamma_s/\Delta M_s$

Old basis and pole scheme of m_b

$$\Delta\Gamma_s = \left(\frac{f_{B_s}}{240\,\text{MeV}}\right)^2 \left[0.002 B + 0.094 B'_S - \left(0.033 B_{\tilde{R}_2} + 0.019 B_{R_0} + 0.005 B_R\right)\right]$$

$$\frac{\Delta\Gamma_s}{\Delta M_s} = 10^{-4} \cdot \left[0.9 + 40.9 \frac{B'_S}{B} - \left(14.4 \frac{B_{\tilde{R}_2}}{B} + 8.5 \frac{B_{R_0}}{B} + 2.1 \frac{B_R}{B}\right)\right]$$

New basis, sum up $z \ln z$, average of pole and $\overline{\text{MS}}$-scheme for m_b

$$\Delta\Gamma_s = \left(\frac{f_{B_s}}{240\,\text{MeV}}\right)^2 \left[0.105 B + 0.024 \tilde{B}'_S - \left(0.030 B_{\tilde{R}_2} - 0.006 B_{R_0} + 0.003 B_R\right)\right]$$

$$\frac{\Delta\Gamma_s}{\Delta M_s} = 10^{-4} \cdot \left[46.2 + 10.6 \frac{B'_S}{B} - \left(13.2 \frac{B_{\tilde{R}_2}}{B} - 2.5 \frac{B_{R_0}}{B} + 1.2 \frac{B_R}{B}\right)\right]$$

Now a precise determination of $\Delta\Gamma/\Delta M$ is possible!

$\Delta\Gamma_s$ and $\Delta\Gamma_s/\Delta M_s$

Old basis and assume no new physics in B_s-mixing ($\equiv f_{B_s} = 230$ MeV)

$$\frac{\Delta\Gamma_s}{\Gamma_s} = \left(\frac{\Delta\Gamma_s}{\Delta M_s}\right)^{Theory} \cdot \Delta M_s^{Exp} \cdot \tau_B^{Exp} = 0.10 \pm 0.06$$

(= Bona et al, hep-ph/0605213;)

New basis:

$$\Delta\Gamma_s = (0.096 \pm 0.039)\ \text{ps}^{-1} \quad \Rightarrow \quad \frac{\Delta\Gamma_s}{\Gamma_s} = 0.147 \pm 0.060$$

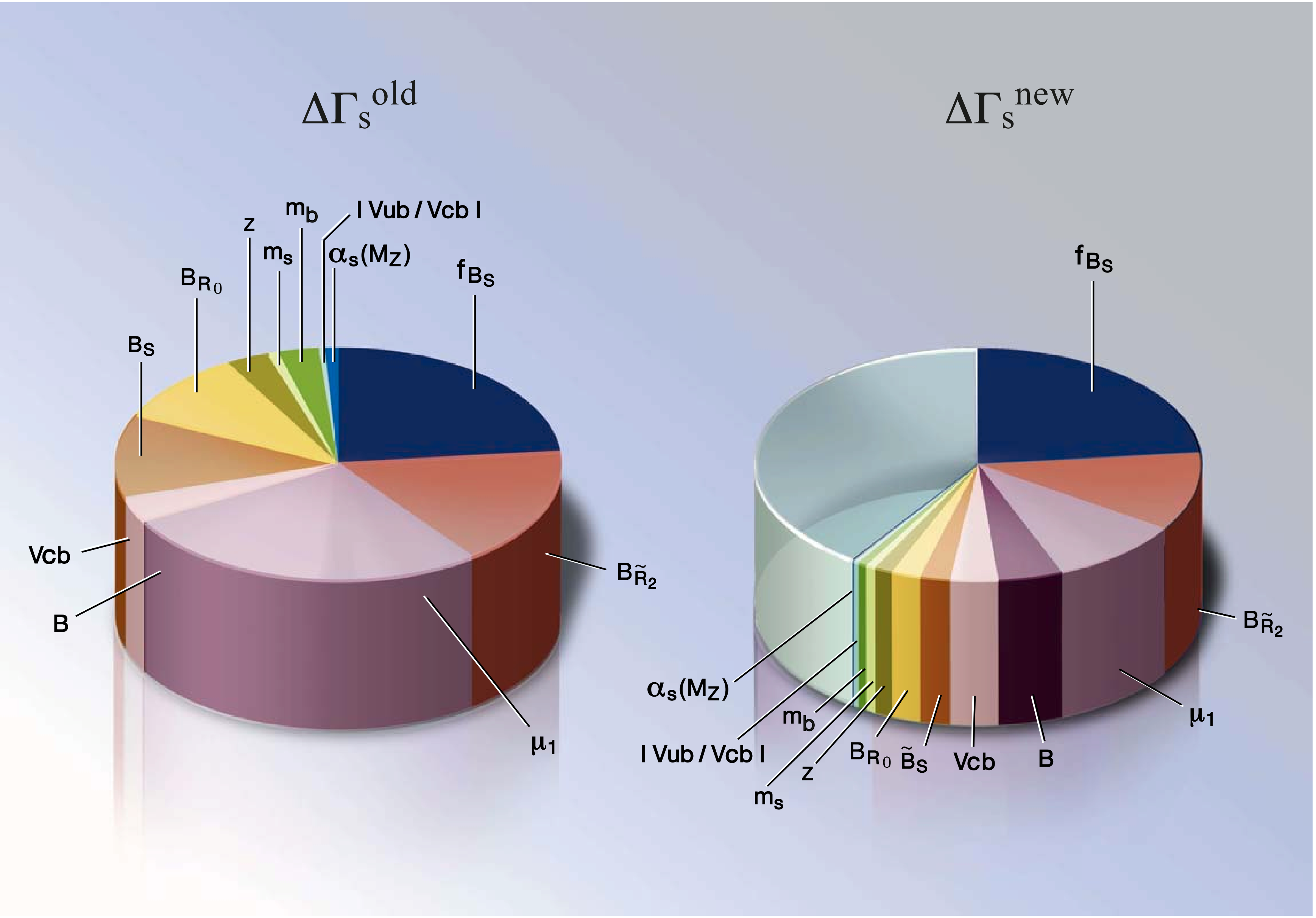
$$\frac{\Delta\Gamma_s}{\Delta M_s} = (49.7 \pm 9.4) \cdot 10^{-4} \quad \Rightarrow \quad \frac{\Delta\Gamma_s}{\Gamma_s} = 0.127 \pm 0.024$$

Unofficial world average: van Kooten, hep-ex/0606005 (without update from D0)

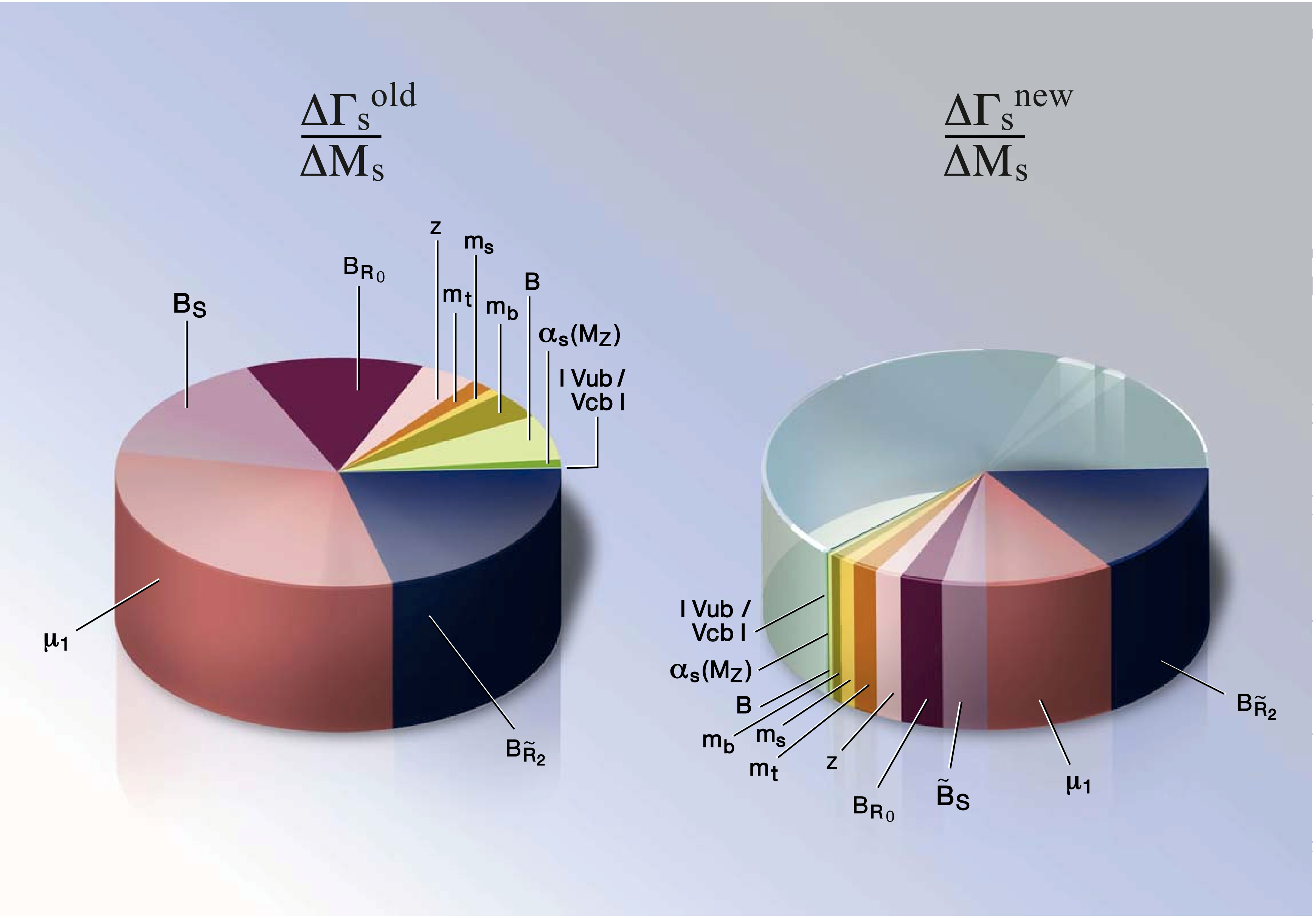
$$|\Delta\Gamma_s| = (0.097 \pm 0.042)\ \text{ps}^{-1}$$

$$\left|\frac{\Delta\Gamma_s}{\Delta M_s}\right| = (55 \pm 24) \cdot 10^{-4}$$

Error budget for $\Delta\Gamma_s$



Error budget for $\Delta\Gamma_s/\Delta M_s$



Semileptonic CP-asymmetries a_{fs} and $\Delta\Gamma_d$

SM expectations:

$$
\begin{aligned}
a_{fs}^s &= (2.1 \pm 0.5) \cdot 10^{-5} \\
\Phi_s &= 0.24^\circ \pm 0.04 \\
a_{fs}^d &= -(5.0 \pm 1.1) \cdot 10^{-4} \\
\frac{\Delta\Gamma_d}{\Gamma_d} &= (3.0 \pm 1.2) \cdot 10^{-3}
\end{aligned}
$$

Experimental bounds

$$
\begin{aligned}
a_{fs}^s &= -(0.3 \pm 9.0) \cdot 10^{-3} && \text{(D0, e.g. Beauty 2006)} \\
\Phi_s &= -0.56 \pm 0.44 && \text{(D0, e.g. Beauty 2006)} \\
a_{fs}^d &= (1.1 \pm 5.5) \cdot 10^{-3} && \text{(Belle, BaBar, e.g. Beauty 2006)} \\
\frac{\Delta\Gamma_d}{\Gamma_d} &= (0.9 \pm 3.7) \cdot 10^{-3} && \text{(HFAG 2006)}
\end{aligned}
$$

typical enhancement due to NP: $a_{fs}^s \approx 5 \cdot 10^{-3}$ close to exp. error!

New physics in B-mixing I

There is still plenty of room for a "pinch " of new physics in B-mixing

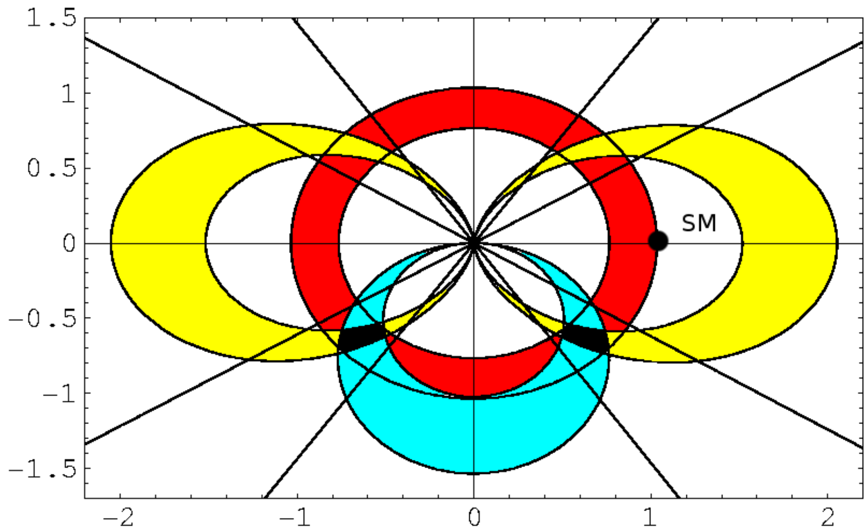
New physics in mixing II

$$\Gamma_{12,s} = \Gamma_{12,s}^{\rm SM}\,, \qquad M_{12,s} = M_{12,s}^{\rm SM}\cdot \Delta_s\,; \qquad \Delta_s = 1 + \frac{S_s^{\rm new}}{S_0(x_t)} =: |\Delta_s| e^{i\Phi_s^\Delta}$$

$$\Delta_s = r_s^2 e^{2i\theta_s} = C_{B_s} e^{2i\phi_{B_s}} = 1 + h_s e^{2i\sigma_s}$$

$$\begin{aligned}
\Delta M_s &= 2|M_{12,s}^{\rm SM}|\cdot|\Delta_s| \\
\Delta\Gamma_s &= 2|\Gamma_{12,s}|\cdot\cos\left(\Phi_s^{\rm SM}+\Phi_s^\Delta\right) \\
\frac{\Delta\Gamma_s}{\Delta M_s} &= \frac{|\Gamma_{12,s}|}{|M_{12,s}^{\rm SM}|}\cdot\frac{\cos\left(\Phi_s^{\rm SM}+\Phi_s^\Delta\right)}{|\Delta_s|} \\
a_{fs}^s &= \frac{|\Gamma_{12,s}|}{|M_{12,s}^{\rm SM}|}\cdot\frac{\sin\left(\Phi_s^{\rm SM}+\Phi_s^\Delta\right)}{|\Delta_s|}
\end{aligned}$$

For $|\Delta_s| = 0.9$ and $\Phi_s^\Delta = -\pi/4$ one gets the following bounds in the complex Δ-plane:



Outlook - Wishlist

Main message:

- HQE seems to work perfect
- Clean prediction of $\Delta\Gamma/\Delta M$ in new basis
- precise non-perturbative parameters needed!
- No signal of duality violation

Non-perturbative parameters

- f_{B_s} **for** B_s**-mixing and** τ_{B_s}
- f_{B_d} **for** τ_{B_d} **and** B_d**-mixing**
- $B_1, B_2, \epsilon_1, \epsilon_2$ **for** τ_{B+}/τ_{B_d}
- $1/m_b$ **operators for** B**-mixing**
- $B, \tilde{B}_S$ **for** B**-mixing**

Perturbative Calculations

- α_s^2**-corrections to** Γ_{12}
- α_s/m_b**-corrections to** Γ_{12}
- **...**