K_{e4} decays and Wigner cusp

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On behalf of the NA48/2 collaboration

Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Wien

Outline

1. Introduction
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   - The NA48/2 experiment: beamline and detector
2. $K^\pm \to \pi^+\pi^- e^\pm \nu_e$
   - Form factors and $\pi\pi$ scattering lengths
3. $K^\pm \to \pi^0\pi^0 e^\pm \nu_e$
   - Branching fraction and form factors
4. $K^\pm \to \pi^0\pi^0\pi^\pm$
   - The “cusp” effect
   - Slopes and $\pi\pi$ scattering lengths
5. Conclusions

$\pi\pi$ scattering lengths: why and how measure them?

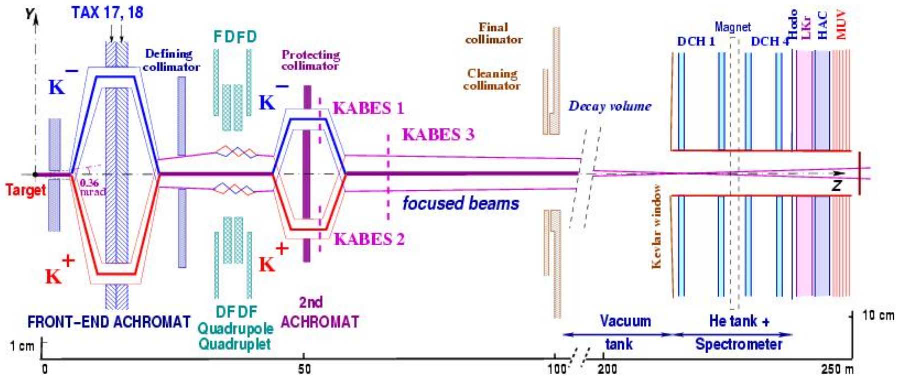
WHY?

- Very precise theoretical predictions from χPT (2%) and generalised χPT, depending only on one free parameter: **the quark condensate $\langle \bar{q}q \rangle_0$**
- $\langle \bar{q}q \rangle_0$ must be determined experimentally
- The size of $\langle \bar{q}q \rangle_0$ determines the order at which mass terms appear in the perturbative expansion

HOW?

- $\boldsymbol{K_{e4}}$ ($K \to \pi\pi e\nu$): no other hadrons, pions close to threshold
  No theoretical uncertainty on the form factors, only on $a_0^2 = f(a_0^0)$
- $\boldsymbol{K^\pm \to \pi^0\pi^0\pi^\pm}$: "cusp" at $M_{00}^2 = 4m_{\pi^+}^2$ due to rescattering
  Theoretical uncertainty of the Cabibbo-Isidori model: 5%

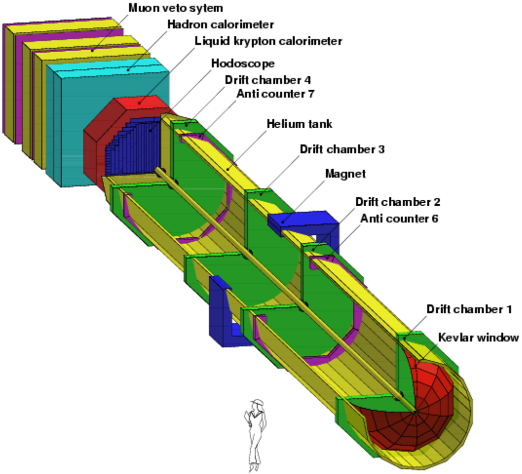
Simultaneous K^+ and K^- beams with $p_K = (60 \pm 3)$ GeV/c
to measure charge asymmetry in $K \to 3\pi$ decays



The NA48/2 detector

- **Magnetic spectrometer:**
  4 drift chambers
  + 1 dipole magnet
  $\boldsymbol{\sigma(p)/p \simeq 0.9\%}$ @ 20 GeV/
  ($p_{\perp}^{kick} \simeq$ 120 MeV/c)

- **Electromagnetic calorimeter:**
  Liquid krypton calorimeter
  $\boldsymbol{\sigma(E)/E \simeq 1\%}$ @ 20 GeV
  $\sigma(t) \simeq$ 265 ps for 50 GeV e^-



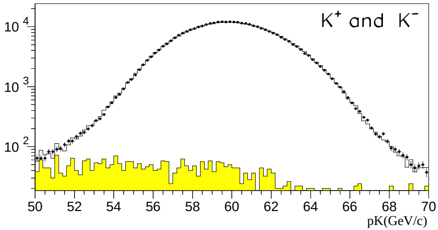
$K^{\pm} \to \pi^{+}\pi^{-}e^{\pm}\nu_e$: Selection criteria and reconstruction

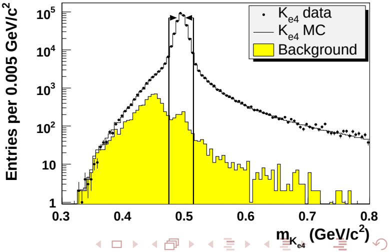
- 3 tracks building a good vertex
- 1 electron with the same charge as the vertex (E/p and Linear Discriminant Analysis)
- Missing p_T: elliptical cut in the (p_T,$M_{3\pi}$) plane

Two reconstruction strategies:

1. Assume kaon mass, extract kaon momentum (quadratic equation)
2. Assume kaon momentum, extract kaon mass (linear equation)

Final sample: $\sim$ 370,000 selected events





$K^\pm \to \pi^+\pi^- e^\pm \nu_e$: Background estimate

Expectation from MC simulation:

Channel	Rejection	Level
$K^\pm \to \pi^+\pi^-\pi^\pm$ with $\pi \to e\nu$	Vertex selection	$\sim 0.2\%$
$K^\pm \to \pi^+\pi^-\pi^\pm$ with π mis-ID	LDA, ellipse (p_T)	$\sim 0.2\%$
$K^\pm \to \pi^\pm\pi^0_D$ and $K^\pm \to \pi^0_D\pi^0\pi^\pm$	E/p for pion-ID	$< 0.01\%$

Estimate from DATA:

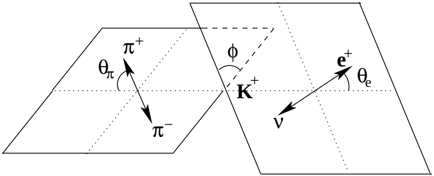
"Wrong sign" events ($\pi^\pm\pi^\pm e^\mp \nu_e$) can only be background ($\Delta S = \Delta Q$ rule)

Factor 2 scaling applied to background from $K^\pm \to \pi^+\pi^-\pi^\pm$

Estimated background: $\sim$ 0.5% of the signal, in agreement with the expectation

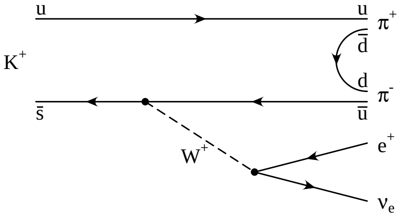
$K^{\pm} \rightarrow \pi^{+}\pi^{-}e^{\pm}\nu_e$: Form factor parametrisation

Kinematic variables



- $M_{\pi\pi}$
- $M_{e\nu}$
- $\cos\theta_\pi$ (in the $\pi\pi$ cm system)
- $\cos\theta_e$ (in the $e\nu$ cm system)
- ϕ (angle between the $\pi\pi$ and $e\nu$ planes in the K cm system)

Matrix element



$$T = \frac{G_F}{\sqrt{2}} V_{us}^{*} \bar{u}(p_\nu)\gamma_\mu(1-\gamma_5)v(p_e)(V^\mu - A^\mu)$$

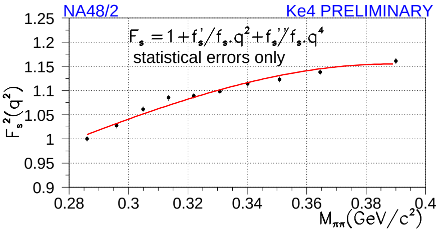
Form factors

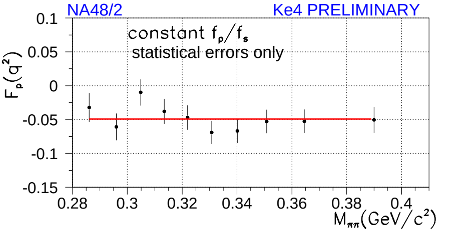
$$F = F_s e^{i\delta_0^0} + F_p e^{i\delta_1^1}\cos\theta_\pi$$
$$G = G_p e^{i\delta_1^1}$$
$$H = H_p e^{i\delta_1^1}$$

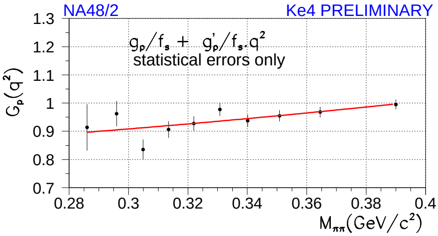
$K^{\pm} \to \pi^+\pi^- e^{\pm}\nu_e$: Fit strategy

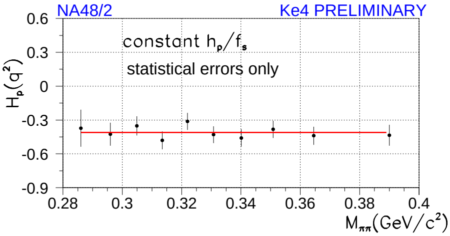
- A grid of **$10 \times 5 \times 5 \times 5 \times 12 = 15000$ equal population bins** was defined in the 5-dimensional space of the kinematic variables, separately for K^+ and K^-
- In the first step **10 independent 5 parameter** (F_s,F_p,G_p,H_p,δ) **fits** were performed for each bin in $M_{\pi\pi}$, minimising a log-likelihood estimator that takes into account also the limited MC statistics
- In the second step the 5 sets of points were fitted with a **polynomial in powers of $q^2 = \frac{s}{4m_\pi^2} - 1$**, truncating the expansion according to the sensitivity
- The **dependence on $M_{e\nu}$** was found to be **negligible** within the total uncertainty and a possible f_e term was not included in the fit
- The ***D*-wave contribution** was also found to be **negligible** within the statistical uncertainty and was excluded from the fit

$K^{\pm} \to \pi^{+}\pi^{-}e^{\pm}\nu_e$: F, G and H





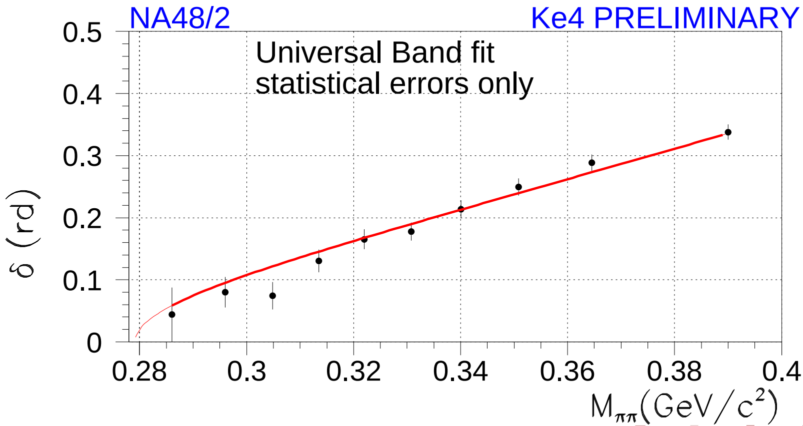




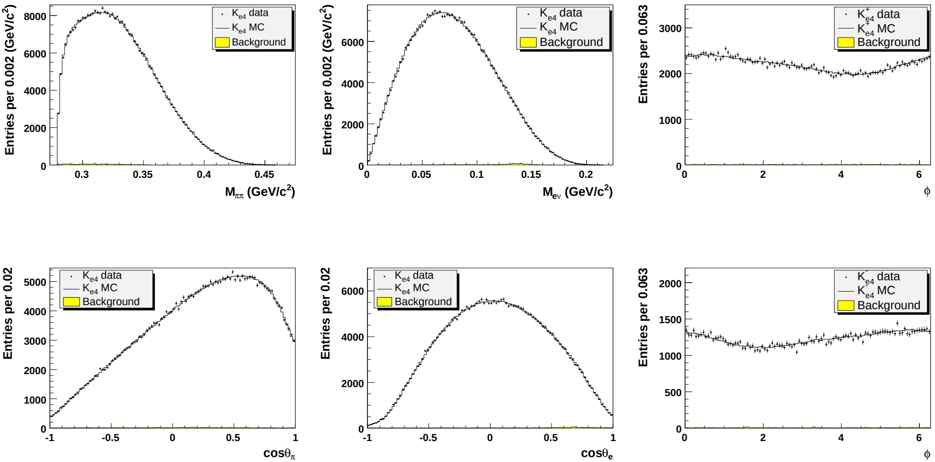
$K^{\pm} \rightarrow \pi^{+}\pi^{-}e^{\pm}\nu_e$: a_0^0 and a_0^2

The $\delta = \delta_0^0 - \delta_1^1$ distribution was fitted with a **1 parameter (a_0^0) function** given by the numerical solution of the Roy equations in Phys. Rept. 353, 207

a_0^0 and a_0^2 were constrained to lie on the **centre of the universal band**



$K^{\pm} \to \pi^+\pi^- e^{\pm}\nu_e$: Data/MC comparison after the fit



$K^{\pm} \to \pi^+\pi^- e^{\pm}\nu_e$: Systematics and preliminary result

Systematic checks

- Two independent analyses with different reconstruction methods, acceptance corrections, fit methods and MC parameters
- Stability of acceptance vs time (variation of the simulated beam conditions)
- Uncertainty on background estimate (checked with data and MC)
- Uncertainty on electron-ID efficiency (variation of LDA cut)
- Uncertainty on radiative corrections (fraction of total effect with or w/o PHOTOS)
- Bias from neglected s_e dependence (MC tests)

NA48/2 preliminary result (2003 data)

$$f'_s/f_s = 0.169 \pm 0.009_{stat} \pm 0.034_{syst}$$

$$f''_s/f_s = -0.091 \pm 0.009_{stat} \pm 0.031_{syst}$$

$$f_p/f_s = -0.047 \pm 0.006_{stat} \pm 0.008_{syst}$$

$$g_p/f_s = 0.891 \pm 0.019_{stat} \pm 0.020_{syst}$$

$$g'_p/f_s = 0.111 \pm 0.031_{stat} \pm 0.032_{syst}$$

$$h_p/f_s = -0.411 \pm 0.027_{stat} \pm 0.038_{syst}$$

$$a_0^0 = 0.256 \pm 0.008_{stat} \pm 0.007_{syst} \pm 0.018_{theor}$$

$K^{\pm} \to \pi^0\pi^0 e^{\pm}\nu_e$: Selection and background estimate

- 4 photons compatible with two π^0 at the same vertex
- 1 electron track (E/p and shower width)
- Missing p_T: elliptical cut in the $(p_T, M_{3\pi})$ plane
- Using kaon momentum of 60 GeV/c along z, compute kaon mass (linear equation)

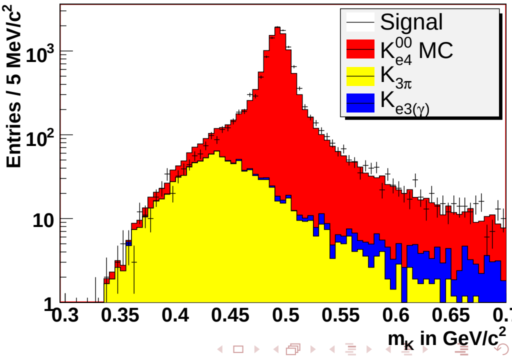
Background

Estimated from data (reversing cuts):

$K^{\pm} \to \pi^0\pi^0\pi^{\pm}$ with pion mis-ID

$K^{\pm} \to \pi^0 e^{\pm}\nu_e\gamma$ + accidental photon

Total contamination: $\sim 3\%$



$K^{\pm} \rightarrow \pi^0\pi^0 e^{\pm}\nu_e$: Branching ratio

Selected events

K_{e4}^{00} candidates: 9642, with a background contamination of 276 ± 94

Normalisation ($K^{\pm} \rightarrow \pi^0\pi^0\pi^{\pm}$) $\sim 8 \times 10^6$, negligible background

Systematic uncertainties

Acceptance, trigger efficiency and energy measurement of the calorimeter

NA48/2 preliminary result (2003 data)

$$BR(K_{e4}^{00}) = (2.587 \pm 0.026_{stat} \pm 0.019_{syst} \pm 0.029_{ext}) \times 10^{-5}$$

$K^\pm \to \pi^0\pi^0 e^\pm \nu_e$: Form factors

Same formalism as for K_{e4}^{+-}, but, for the symmetry of the $\pi^0\pi^0$ system,

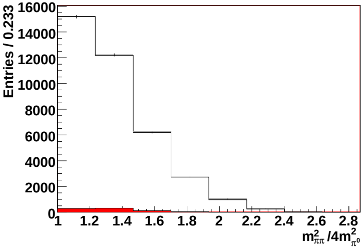
no *P*-wave!

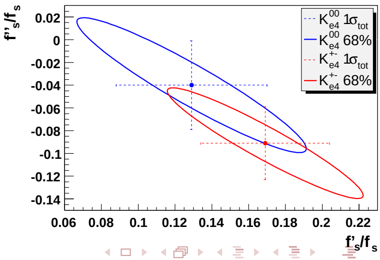
Effect of cusp at $M_{00}^2 = 4m_{\pi^+}^2$ taken into account, 37,700 events used

NA48/2 preliminary result (2003+2004 data)

$$f_s'/f_s = 0.129 \pm 0.036_{stat} \pm 0.020_{syst}$$

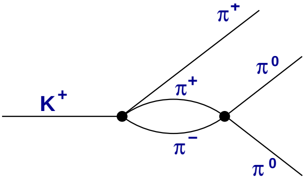
$$f_s''/f_s = -0.040 \pm 0.034_{stat} \pm 0.020_{syst}$$



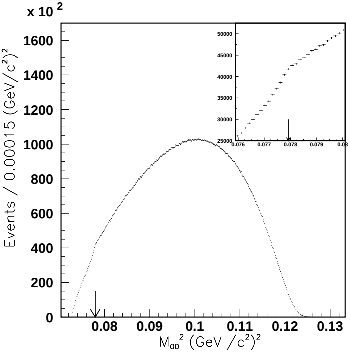


Selected events (2003): 23×10^6

M^2_{00} computed imposing the mean vertex of the π^0's (improved resolution close to threshold)



Evidence for a cusp at M^2_{00} =4$m^2_{\pi^+}$ due to $\pi\pi$ rescattering



Fit to the Cabibbo-Isidori model (JHEP03 021):

$$M_0 = A_0\left(1 + \tfrac{1}{2}\, g_0 u + \tfrac{1}{2}\, h' u^2\right)$$

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$$g_0 = 0.645 \pm 0.004_{stat} \pm 0.009_{syst}$$

$$h' = -0.047 \pm 0.012_{stat} \pm 0.011_{syst}$$

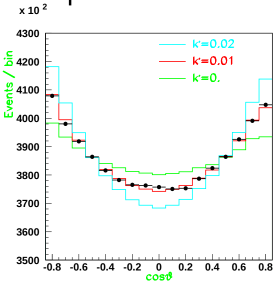
$$a_2 = -0.041 \pm 0.022_{stat} \pm 0.014_{syst}$$

$$a_0 - a_2 = 0.268 \pm 0.010_{stat} \pm 0.004_{syst} \pm 0.013_{theor}$$

$K^\pm \to \pi^0\pi^0\pi^\pm$: $k'v^2$ term

Fit in two steps

1. Fit the modified matrix element
$$M_0 = A_0(1 + \tfrac{1}{2}g_0 u + \tfrac{1}{2}h'u^2 + \tfrac{1}{2}k'v^2)$$
above the cusp in the plane **$\cos\theta$ vs M^2_{00}** $\cos\theta$ = angle between π^+ and π^0 in $\pi^0\pi^0$ cm
2. Reweight the MC with the obtained value of k' and fit the M^2_{00} distribution with the **Cabibbo-Isidori model** to obtain the cusp parameters



Systematic checks:

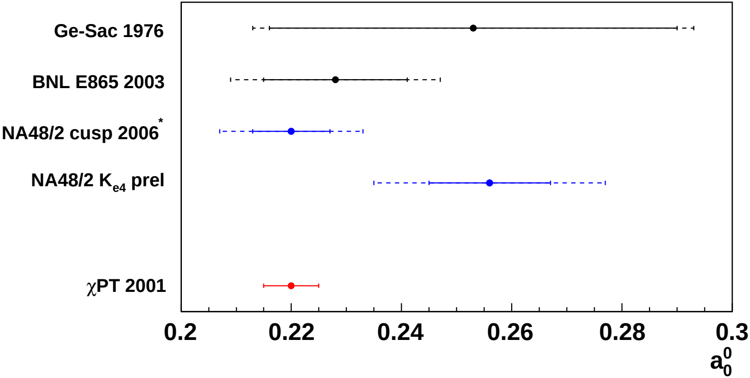
Acceptance, trigger efficiency

NA48/2 preliminary result (2003 data)

$$k' = 0.0097 \pm 0.0003_{stat} \pm 0.0008_{syst}$$

Note: h' and k' are not the same as the PDG parameters

Comparison of the scattering length results



* = different constraint for a_0^2

New results **compatible** with theory and previous measurements

K_{e4}: **higher sensitivity** than BNL E865 (higher statistics at high $M_{\pi\pi}$)

Conclusions

- Form factor and $\pi\pi$ scattering length measurement from K_{e4}^{+-}

$$a_0^0 = 0.256 \pm 0.008_{stat} \pm 0.007_{syst} \pm 0.018_{theor}$$

Compatible with χPT prediction and previous results, higher sensitivity

- Branching ratio and form factor measurements from K_{e4}^{00}

$$BR(K_{e4}^{00}) = (2.587 \pm 0.026_{stat} \pm 0.019_{syst} \pm 0.029_{ext}) \times 10^{-5}$$

BR: factor 8 better than latest measurement
Form factors: compatible with charged channel

- $\pi\pi$ scattering length and slope measurements from $K^{\pm} \to \pi^0\pi^0\pi^{\pm}$

$$a_0 - a_2 = 0.268 \pm 0.010_{stat} \pm 0.004_{syst} \pm 0.013_{theor}$$

- First evidence for a value of $k \neq 0$ in the $K^{\pm} \to \pi^0\pi^0\pi^{\pm}$ Dalitz plot

$$k' = 0.0097 \pm 0.0003_{stat} \pm 0.0008_{syst}$$