Proceedings of the VIIIth International Workshop on Heavy Quarks and Leptons

HQL06

October 2006

Deutsches Museum, Munich

Editors
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Organized by the Physics Department of the Technical University of Munich
and the Max-Planck Institute for Physics, Munich

Lattice QCD, Flavor Physics and the Unitarity Triangle Analysis

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Lattice QCD has always played a relevant role in the studies of flavor physics and, in particular, in the Unitarity Triangle (UT) analysis. Before the starting of the B factories, this analysis relied on the results of lattice QCD simulations to relate the experimental determinations of semileptonic B decays, $K-\overline{K}$ and $B_{d,s}-\overline{B}_{d,s}$ mixing to the CKM parameters. In the last years much more information has been obtained from the direct determination of the UT angles from non-leptonic B decays. In this talk, after a presentation of recent averages of lattice QCD results, we compare the outcome of the "classical" UT analysis (**UT***lattice*) with the analysis based on the angles determinations (**UT***angles*). We discuss the role of the different determinations of V_{ub}, and show that current data do not favour the value measured in inclusive decays. Finally we show that the recent measurement of Δm_s, combined with Δm_d and ε_K, allows a quite accurate extraction of the values of the hadronic parameters, $\hat{B}_K$, $f_{Bs}\,\hat{B}_{Bs}^{1/2}$ and ξ. These values, obtained "experimentally" by assuming the validity of the Standard Model, are compared with the theoretical predictions from lattice QCD.

1 Introduction

Lattice QCD has always played a relevant role in the history of the UT fit since the very beginning. At the time when the B factories had not started yet, the "classical" UT analysis relied on the results of lattice QCD simulations to relate the experimental studies of semileptonic B decays, $B_{d,s}-\overline{B}_{d,s}$ and $K-\overline{K}$ mixing to the CKM parameters. Despite the lattice results were mostly obtained in the quenched approximation at that time, and some of the experimental determinations were still rather rough, these analyses allowed to reach at least three important results for flavor physics in the Standard Model (SM): i) the amount of indirect CP violation observed in kaon mixing (ε_K) was shown to be fully consistent with the expectation based on the CKM mechanism of CP violation; ii) and iii) quite accurate predictions for $\sin 2\beta$ and Δm_s were obtained.

Predictions of $\sin 2\beta$ exist since more than 15 years, see Fig. 1 (left): the first indication of a large value of this parameter, namely $\sin 2\beta > 0.55$, dates back to 1992 [1]. In 1995, the prediction $\sin 2\beta = 0.65 \pm 0.12$ was derived [2]. Five years later, when direct measurements were not available yet, we obtained the more accurate estimate $\sin 2\beta = 0.698 \pm 0.066$ [3]. These results are in remarkable agreement with the present experimental average, $\sin 2\beta = 0.675 \pm 0.026$ [4].

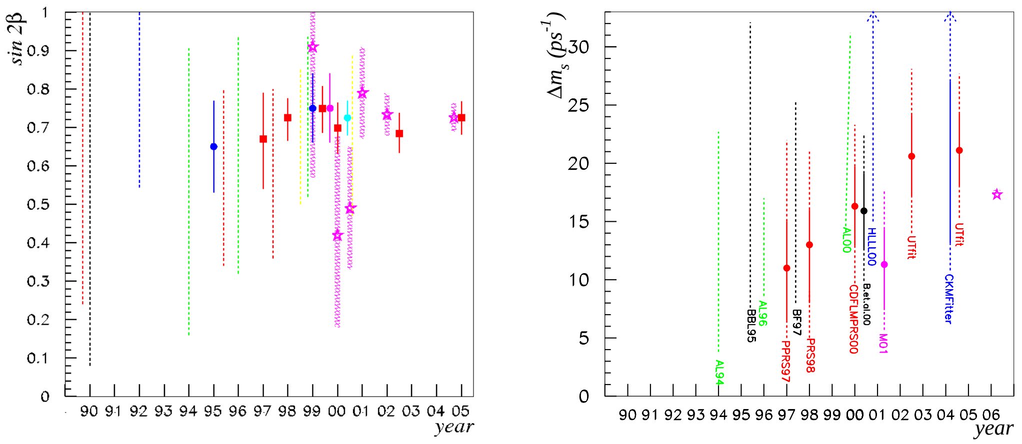

Figure 1: *Evolution of the predictions for* $\sin 2\beta$ *(left) and* Δm_s *(right) over the years. The corresponding experimental determinations are indicated by larger bands and stars. See [5, 6] for the full list of original references*

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A similar situation holds for Δm_s. A precise indirect determination of Δm_s from the other constraints of the UT fit was available since 1997: $\Delta m_s \in [6.5, 15.0]$ ps^{-1} at 68% probability and $\Delta m_s < 22$ ps^{-1} at 95% probability [7]. A compilation of the predictions for Δm_s by various collaborations as a function of time is shown in Fig. 1 (right). As can be seen from the plot, even in recent years, and despite the improved measurements, in some approaches [8, 9] the predicted range was very large (or corresponds only to a lower bound [8]). An upgraded version of our SM "prediction" for Δm_s obtained from the full UT fit is $\Delta m_s = (18.4 \pm 2.4)$ ps^{-1} [5], in remarkable agreement with the direct measurement $\Delta m_s = (17.77 \pm 0.12)$ ps^{-1} [10]. In Fig. 2 we show the compatibility plot for Δm_s, which illustrates the agreement, at better than $1\,\sigma$ level, of the measured value with the SM expectation.

In the last years, we got much more information on the UT from the direct determinations of the angles α, β and γ, obtained at the B factories from the studies of CP asymmetries in non-leptonic B decays. In the following we will call the ensemble of these measurements **UT***angles*: they allow a determination of and independently of

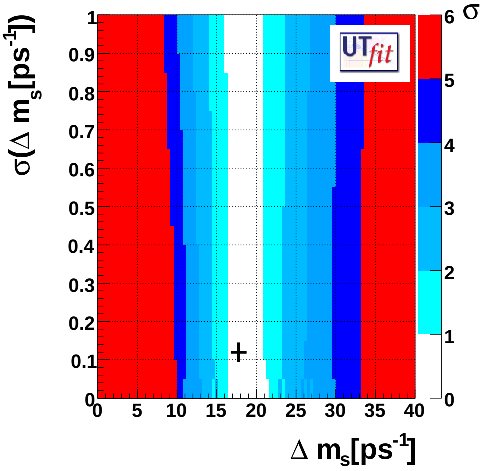

Figure 2: *Compatibility plot of the measured value* $\Delta m_s = (17.77 \pm 0.12)$ ps^{-1} *with the SM expectation from the other constraints of the UT fit.*

the hadronic parameters computed on the lattice. The precision in constraining and from the **UT***angles* is by now comparable to that obtained from the lattice-related constraints, denoted as **UT***lattice*. The latter include the determination of $|V_{ub}| / |V_{cb}|$ from semileptonic B decays, ϵ_K, Δm_d and Δm_s.

In this talk, after a presentation of recent averages of lattice QCD results, we will compare the outcome of the **UT***lattice* and **UT***angles* analyses. We will discuss the role played by the different determinations of V_{ub} and show that the value measured in inclusive decays is not favoured by the data. Finally, we will show that the recent measurement of the B_s-meson mixing amplitude Δm_s, combined with Δm_d and ϵ_K, allows a quite accurate extraction of the values of the hadronic matrix elements relevant for $K - \overline{K}$ and $B_{s,d} - \overline{B}_{s,d}$ mixing. Assuming the validity of the SM, we determine these hadronic quantities from the experimental data and compare them with recent lattice calculations. The content of this talk is mostly based on Ref. [6], but we take here the opportunity to update the results by taking into account the most recent experimental findings [5].

2 Averages of Lattice QCD results

Lattice QCD is the theoretical tool of choice to compute hadronic quantities. Being only based on first principles, it does not introduce additional free parameters besides the fundamental couplings of QCD, namely the strong coupling constant and the quark masses. In addition, the systematic uncertainties affecting the results of lattice calculations can be systematically reduced in time, with the continuously increasing availability of computing power and the development of new algorithms and improved theoretical techniques.

In spite of the appealing features of the lattice approach, the accuracy currently reached in the determination of the hadronic matrix elements is typically still at the level of 10-15%. So far, the main limiting factor for achieving an improved precision has been the lack of sufficient computing power, which has often prevented the possibility of performing "full QCD" simulations and forced the introduction of the quenched approximation. In this approximation an error is introduced which, besides being process dependent, is also difficult to reliably estimate.

Most of the lattice calculations relevant to B-physics used so far the quenched approximation. There are few exception in which $N_f = 2$ dynamical quarks are included in the QCD vacuum fluctuations [11]- [14] and only a single calculation with $N_f = 2 + 1$ [15] [1]. A similar situation holds for the lattice studies of $K - \overline{K}$ mixing. Only three unquenched calculations of the kaon parameter B_K have been produced so far, one including $N_f = 2$ dynamical quarks [16] and two with $N_f = 2 + 1$ [17,18].

It is important to emphasise that the quenched results, in spite of being unsatisfactory for having been obtained with unrealistic $N_f = 0$, have the advantage that the whole methodology of extracting the desired information from the simulation has been developed and understood, the procedure of non-perturbative renormalization has been implemented, and the whole plethora of results have been checked by many different groups, using various versions of the gauge and fermionic lattice actions. In a number of cases even the continuum extrapolation has been shown to be smooth. The unquenched studies, on the other hand, are sound for being unquenched (although the dynamical quarks are still much heavier than the physical up and down quarks). However, the consequences of the so called fourth-root trick implemented with the staggered fermion formulation are not clear, the non-perturbative renormalization in most of the cases is not carried out, and the results have not been checked yet by different groups. In this respect, unquenching is still a work in progress. At the same time, it is should be noted that the capability of decreasing significantly the values of the simulated light quark masses in recent unquenched calculations has allowed to largely reduce the uncertainty associated with the chiral extrapolation. Particularly relevant cases, in this respect, are the determinations of the pseudoscalar decay constant f_B and of the ratio ξ [15]. For all these reasons it is important to take into account, when producing averages of lattice QCD results, the outcome of the several more recent lattice calculations. The average lattice values that are being used in the

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[1] The "+1" indicates that an heavier strange quark is included in the sea, besides the two degenerate up and down quarks

UT analysis, for the quantities relevant to K- and B-physics, are:

$$\begin{aligned}
&\hat{B}_K = 0.79 \pm 0.04 \pm 0.081001[19], \\
&f_{Bs}\hat{B}_{Bs}^{1/2} = 262 \pm 35 \quad , \quad \xi = 1.23 \pm 0.06, \\
&f_{Bd} = 189 \pm 27 \quad , \quad f_{Bs} = 230 \pm 301001[20].
\end{aligned} \tag{1}$$

3 **UT***lattice*, **UT***angles* and role of $|V_{ub}|$

In Fig. 3 we show the results of the UT fit as obtained from the lattice-related constraints, **UT***lattice*, the direct determinations of the UT angles, **UT***angles*, and the full analysis [5, 6]. The corresponding determinations of and , as derived independently

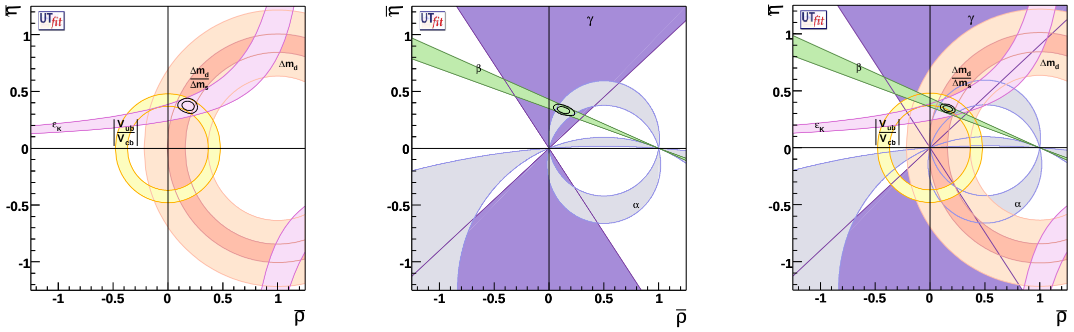

Figure 3: *Determination of and from* **UT***lattice (left),* **UT***angles (center) and the full UT (right) analyses. The* 68% *and* 95% *total probability contours are shown, together with the* 95% *probability regions from the individual constraints.*

from the **UT***lattice* and **UT***angles* analyses, are

$$\begin{array}{cc}
\textbf{UT}\textit{lattice} & \textbf{UT}\textit{angles} \\
= 0.188 \pm 0.036 \quad , & = 0.134 \pm 0.039 \\
= 0.371 \pm 0.027 \quad , & = 0.335 \pm 0.020\ .
\end{array} \tag{2}$$

We firstly note that the errors have comparable sizes, i.e. the two analyses have reached at present a comparable level of accuracy. It is also interesting to observe that the **UT***angles* fit, based on the direct determination of the UT angles, does not rely at all on theoretical calculation of the hadronic matrix elements, for which there was a long debate about the treatment of values and error distributions [21]. In the **UT***angles* analysis, the treatment of theoretical errors is not an issue.

The results in eq. (2) also show the existence of a tension between the values of and obtained from the two analyses. This is also illustrated by the effects of the various constraints in the full UT fit, shown in the right plot of Fig. 3. It mainly appears to be a tension between the (presently quite accurate) measurement of $\sin 2\beta$ and the constraint coming from the determination of $|V_{ub}|$ from semileptonic B decays. The poor agreement is also evidenced by the comparison between the experimental value $\sin 2\beta = 0.675(26)$ and the value obtained by using all the other constraints in the UT fit, i.e. $\sin 2\beta = 0.759(37)$. The compatibility plot of $\sin 2\beta$, presented in Fig. 4, shows that this tension is indeed at the $2\,\sigma$ level.

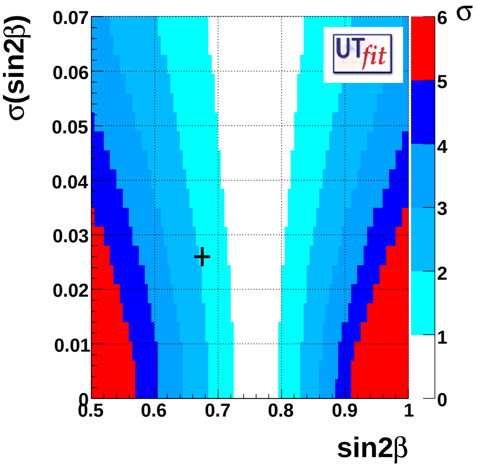

Figure 4: *Compatibility plot of the experimental value of* $\sin 2\beta$ *and the prediction from the rest of the UT fit.*

In order to further investigate where the tension comes from, it is worth recalling that there is a systematic difference between the exclusive and inclusive determination of $|V_{ub}|$ (the inclusive values are always larger than the exclusive ones). Current averages and errors are:

$$
\begin{aligned}
|V_{ub}|^{\text{excl.}} &= (35.0 \pm 4.0) \times 10^{-4} \\
|V_{ub}|^{\text{incl.}} &= (44.9 \pm 3.3) \times 10^{-4}
\end{aligned} \tag{3}
$$

These determinations also rely on non-perturbative hadronic quantities: the semileptonic form factors for exclusive semileptonic B decays and the HQET parameters $\overline{\Lambda}$, λ_1 and λ_2, for inclusive ones. While the form factors are determined from lattice QCD calculations and QCD sum rules, the HQET parameters are extracted, together with $|V_{ub}|$, directly the from the fits of the experimental data. In this latter case, however, a certain amount of model dependence has to be introduced, and various approaches (BLNP [22], DGE [23], BLL [24]) are currently considered. The systematic difference between the exclusive and inclusive determination of $|V_{ub}|$ might be explained by the uncertainties of the theoretical approaches.

In Fig. 5 we show the compatibility plot between the direct determinations of $|V_{ub}|$ from exclusive and inclusive analyses and the rest of the fit. While the exclusive determination is in remarkable agreement with the expectation from the UT analysis in the SM, the inclusive value of $|V_{ub}|$ shows a deviation which is almost at the $3\,\sigma$ level. Our analysis suggests that, although both the exclusive and inclusive results are still compatible, there could be some problem with the theoretical calculations, and/or with the estimate of the uncertainties, of inclusive $b \to u$ semileptonic decays. In order to clearly solve this tension, an effort should be made to increase the precision of lattice QCD determinations of the form factors of $B \to \pi$ and $B \to \rho$ semileptonic decays, providing all of them in the unquenched case, with low light quark masses and studying the continuum limit of the relevant form factors. Note that this tension among exclusive and inclusive calculations is a peculiarity of $|V_{ub}|$, since the inclusive and exclusive determinations of $|V_{cb}|$ are in much better agreement.

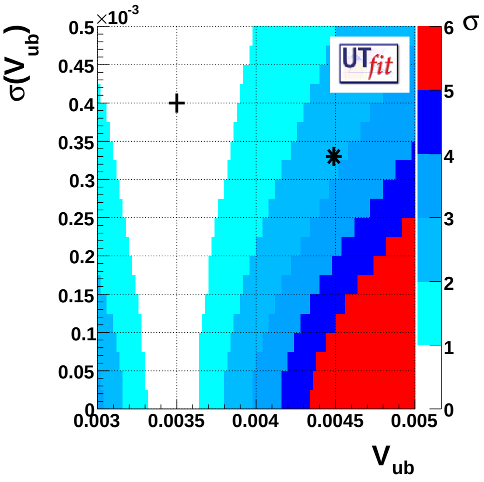

Figure 5: *Compatibility plot between the direct determinations of* $|V_{ub}|$ *from exclusive (cross) and inclusive (star) analyses and the rest of the UT fit.*

4 Constraints on lattice parameters

Assuming the validity of the SM, the constraints in the - plane from **UT***angles* and semileptonic B decays, combined with the measurements of Δm_d, Δm_s and ϵ_K, allow the "experimental" determination of several hadronic quantities which were previously taken from lattice QCD calculations. This approach has the advantage that the extracted values of $\hat{B}_K$ and of the B mixing parameters $f_{Bs,d}\,\hat{B}^{1/2}_{Bs,d}$ (or equivalently $f_{Bs}\,\hat{B}^{1/2}_{Bs}$ and ξ) can be compared directly with the theoretical predictions from lattice QCD.

Besides $\hat{B}_K$, $f_{Bs}\,\hat{B}^{1/2}_{Bs}$ and ξ, we can also extract the values of the leptonic decay constants f_B and f_{Bs} from the fit, using in addition the lattice values of the B mixing parameters, $\hat{B}_{Bd} = \hat{B}_{Bs} = 1.28 \pm 0.05 \pm 0.09$ [20]. With respect to the decay constants, the lattice calculations of the B-parameters are typically affected by smaller uncertainties, since some sources of systematic errors (like those related to the determination of the lattice scale and to chiral logs effects) are either absent or largely reduced. Moreover, quenched and unquenched estimates of the B parameters are found to be quite consistent within each others.

The results for $\hat{B}_K$, $f_{Bs}\,\hat{B}^{1/2}_{Bs}$, ξ, f_B and f_{Bs} extracted from the UT fit are presented in Tab. 1, together with the average values from lattice QCD calculations [19,20] also given in eq. (1). The agreement between the two determinations is remarkable. On the one hand, this comparison provides additional evidence of the spectacular success of the SM in describing flavor physics. On the other hand, the results given in Tab. 1 illustrate the accuracy and the reliability reached at present by lattice QCD calculations. The allowed probability regions in the B_K vs. ξ, $f_{Bs}\hat{B}^{1/2}_{Bs}$ vs. B_K and $f_{Bs}\hat{B}^{1/2}_{Bs}$ vs. ξ planes derived from the UT fit are shown in Fig. 6, together with the results from lattice QCD calculations.

The UTfit determination of $f_{Bs}\,\hat{B}^{1/2}_{Bs}$ has an accuracy of about 2%, which reflects the precision reached in the experimental determination of Δm_s. This uncertainty on $f_{Bs}\,\hat{B}^{1/2}_{Bs}$ is by far smaller than the one reached by lattice QCD calculations. On the other hand, the errors on the UTfit and lattice QCD determinations of $\hat{B}_K$ and ξ have comparable size, leaving in these cases the opportunity for further improvement of the theoretical calculations.

It is worth recalling that the phenomenological extraction of the hadronic parameters and the comparison with lattice results assumes the validity of the SM and it is meaningful in this framework only. A similar strategy could be followed in any given extension of the SM when enough experimental information is available. In general, however, a model-independent UT analysis beyond the SM cannot be carried out without some “a priori” theoretical knowledge of the relevant hadronic parameters.

	$\hat{B}_K$	$f_{Bs}\,\hat{B}^{1/2}_{Bs}$ (MeV)	ξ	f_{Bd} (MeV)	f_{Bs} (MeV)
UT fit	0.75 ± 0.09	261 ± 6	1.24 ± 0.08	187 ± 13	231 ± 9
LQCD	$0.79 \pm 0.04 \pm 0.08$	262 ± 35	1.23 ± 0.06	189 ± 27	230 ± 30

Table 1: *Comparison between determinations of the hadronic parameters from the UT fit and the averages from lattice QCD calculations (eq. (1)).*

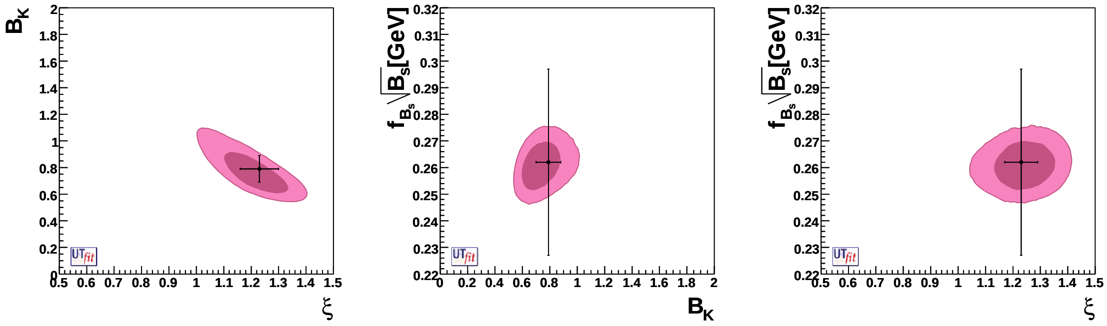

Figure 6: *Two-dimensional constraints in the* B_K *vs.* ξ, $f_{Bs}\sqrt{\hat{B}_s}$ *vs.* B_K *and* $f_{Bs}\sqrt{\hat{B}_s}$ *vs.* ξ *planes, using the* **UT***angles result for the CKM matrix and the experimental information on* ε_K, Δm_d *and* Δm_s. *The error bars show the results from lattice QCD calculations.*

For this reason, the error in the calculation of the hadronic matrix elements affects the uncertainties in the determination of the New Physics parameters [25, 26], which is at present one of the main motivations for the studies of flavor physics.

5 Conclusions

The recent precise determination of Δm_s by the CDF Collaboration allows a substantial improvement of the accuracy of the UT fit. Thanks to this measurement, it is possible to extract from the experiments the value of the relevant hadronic parameters, assuming the validity of the SM. The results of this fit turn out to be in remarkable agreement with the theoretical predictions of lattice QCD. It is also remarkable that the measurement of Δm_s, combined with all the information coming from the UT fit, allows the determination of $f_{Bs}\,\hat{B}_{Bs}^{1/2}$ with an accuracy of about 2% ($f_{Bs}\,\hat{B}_{Bs}^{1/2} = 261 \pm 6$ MeV).

The only exception to the general consistency of the UT fit is given by the inclusive semileptonic $b \to u$ decays, the analysis of which relies on the parameters of the shape function and other model-dependent assumptions. We observed that the present determination of $|V_{ub}|$ using inclusive methods is disfavoured by all other constraints almost at the $3\,\sigma$ level. This can come either from the fact that the central value of $|V_{ub}|$ from inclusive decays is too large, or from the smallness of the estimated error, or both. We think that it is worth investigating whether the theoretical uncertainty of the inclusive analysis has been realistically estimated. At the same time, an effort should be done for a substantial improvement of the theoretical and experimental accuracy in the extraction of $|V_{ub}|$ from exclusive decays. Indeed in the future a confirmation of these results, with smaller errors, might reveal the presence of New Physics in the generalised UT analysis [26].

I warmly thank the organisers for the very pleasant and stimulating atmosphere of the conference. I am indebted to all my friends of the **UTfit** Collaboration, with which most of the results presented in this talk have been obtained.

Bibliography

[1] M. Lusignoli, L. Maiani, G. Martinelli and L. Reina, Nucl. Phys. B **369** (1992) 139.

[2] M. Ciuchini, E. Franco, G. Martinelli, L. Reina and L. Silvestrini, Z. Phys. C **68** (1995) 239 [arXiv:hep-ph/9501265].

[3] M. Ciuchini *et al.*, JHEP **0107** (2001) 013 [arXiv:hep-ph/0012308].

[4] M. Hazumi, plenary talk at the ICHEP'06 conference, `http://ichep06.jinr.ru/reports/2_hazumi_ichep2006.ppt`

[5] The **UTfit** Collaboration, `http://www.utfit.org`

[6] M. Bona *et al.* [UTfit Collaboration], JHEP **0610** (2006) 081 [arXiv:hep-ph/0606167].

[7] P. Paganini, F. Parodi, P. Roudeau and A. Stocchi, Phys. Scripta **58** (1998) 556 [arXiv:hep-ph/9711261].

[8] A. Hocker, H. Lacker, S. Laplace and F. Le Diberder, Eur. Phys. J. C **21** (2001) 225 [arXiv:hep-ph/0104062].

[9] J. Charles *et al.* [CKMfitter Group], Eur. Phys. J. C **41** (2005) 1 [arXiv:hep-ph/0406184].

[10] A. Abulencia *et al.* [CDF Collaboration], Phys. Rev. Lett. **97** (2006) 242003 [arXiv:hep-ex/0609040].

[11] A. Ali Khan *et al.* [CP-PACS Collaboration], Phys. Rev. D **64** (2001) 054504 [arXiv:hep-lat/0103020].

[12] S. Aoki *et al.* [JLQCD Collaboration], Phys. Rev. Lett. **91** (2003) 212001 [arXiv:hep-ph/0307039].

[13] A. Ali Khan *et al.* [CP-PACS Collaboration], Phys. Rev. D **64** (2001) 034505 [arXiv:hep-lat/0010009].

[14] C. Bernard *et al.* [MILC Collaboration], Phys. Rev. D **66** (2002) 094501 [arXiv:hep-lat/0206016].

[15] A. Gray *et al.* [HPQCD Collaboration], Phys. Rev. Lett. **95** (2005) 212001 [arXiv:hep-lat/0507015].

[16] Y. Aoki *et al.*, Phys. Rev. D **72** (2005) 114505 [arXiv:hep-lat/0411006].

[17] E. Gamiz, S. Collins, C. T. H. Davies, G. P. Lepage, J. Shigemitsu and M. Wingate [HPQCD Collaboration], Phys. Rev. D **73** (2006) 114502 [arXiv:hep-lat/0603023].

[18] D. J. Antonio *et al.*, arXiv:hep-ph/0702042.

[19] C. Dawson, PoS **LAT2005** (2005) 007.

[20] S. Hashimoto, Int. J. Mod. Phys. A **20** (2005) 5133 [arXiv:hep-ph/0411126].

[21] M. Battaglia *et al.*, arXiv:hep-ph/0304132.

[22] B. O. Lange, M. Neubert and G. Paz, Phys. Rev. D **72**, 073006 (2005) [arXiv:hep-ph/0504071].

[23] J. R. Andersen and E. Gardi, JHEP **0601**, 097 (2006) [arXiv:hep-ph/0509360].

[24] C. W. Bauer, Z. Ligeti and M. E. Luke, Phys. Rev. D **64**, 113004 (2001) [arXiv:hep-ph/0107074].

[25] M. Bona *et al.* [UTfit Collaboration], JHEP **0603** (2006) 080 [arXiv:hep-ph/0509219].

[26] M. Bona *et al.* [UTfit Collaboration], Phys. Rev. Lett. **97** (2006) 151803 [arXiv:hep-ph/0605213].