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Theory of Semi-Leptonic B Decays: Exclusive and Inclusive

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1 Introduction

Semi-leptonic decays of B mesons play an important role in flavour physics. On one hand they are relatively simple as far as the effects of strong interactions are concerned (at least compared to non-leptonic decays), on the other hand they are an important ingredient for the determination of the unitarity triangle. The radius of the so-called "Unitarity Clock", the circle around the origin in the ρ-η plane, is determined by the ratio $|V_{ub}/V_{cb}|$ which is most cleanly determined from semi-leptonic decays.

The theoretical methods to evaluate the hadronic matrix elements have developed tremendously over the past fifteen years [1]. With the advent of the $1/m_b$ expansion a systematic, QCD based theory could be set up which resulted in a drastic reduction of model dependence in many theoretical calculations.

The $1/m_b$ expansion can be set up for both exclusive and inclusive decays. Any observable of a B meson decay can in general be written as

$$R = R_0 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right) R_1 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^2 R_2 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 R_3 + \cdots \tag{1}$$

where the coeffcients are expressed in terms of a set of non-perturbative matrix elements with computable prefactors. The strength of the method is that the leading term may in many cases be fixed by heavy quark symmetries and hence hadronic uncertainties enter the stage only at the level of corrections.

In the following I will give a short summary on the status of these methods for inclusive as well as for exclusive decays. In the next section I shall consider exclusive semi-leptonic decays for both heavy-to-heavy and heavy-to-light decays and discuss the impact on the CKM matrix elements V_{cb} and V_{ub}. In section 3 I will consider inclusive decays and discuss the methods to extract the necessary infomation for the $1/m_b$ expansion. Finally I give a few concluding remarks.

2 Exclusive Decays

The main ingredient for a description of exclusive decays are the form factors for the decays. In general, there are two independent form factors for a $0^- \to 0^-$ transition

$$\langle M(p')|\bar{q}\gamma_\mu(1-\gamma_5)b|B(p)\rangle = f_+(q^2)(p+p')_\mu + f_-(q^2)q_\mu\,, \quad q = p - p' \tag{2}$$

and another four independent form factors for the $0^- \to 1^-$ transition.

We will concentrate here on the decay modes $B \to D\ell\overline{\nu}_\ell$ and $B \to D^*\ell\overline{\nu}_\ell$ for the heavy-to-heavy ($b \to c$) case and on $B \to \pi\ell\overline{\nu}_\ell$ for the heavy-to-light ($b \to u$) case.

2.1 $B \to D\ell\overline{\nu}_\ell$ and $B \to D^*\ell\overline{\nu}_\ell$

In the heavy mass limit the relevant kinematic variable for a heavy meson is its four-velocity $v^\mu = P_H^\mu/M_H$. For a B meson the heavy b quark roughly moves with the same velocity, i.e. its momentum p is

$$p^\mu = m_b v^\mu + k^\mu \qquad \text{with} \qquad v^\mu = \frac{P_H^\mu}{M_H} \tag{3}$$

The residual momentum k of the b quark is assumed to be small compared to m_b and hence an expansion in powers of k/m_b is possible.

Using the four velocities of the initial and final state hadrons we may write the differential rates as ($\omega = v \cdot v'$)

$$\frac{d\Gamma}{d\omega}(B \to D^*\ell\overline{\nu}_\ell) = \frac{G_F^2}{48\pi^3}|V_{cb}|^2 m_{D^*}^3(\omega^2-1)^{1/2}P(\omega)(\mathcal{F}(\omega))^2 \tag{4}$$

$$\frac{d\Gamma}{d\omega}(B \to D\ell\overline{\nu}_\ell) = \frac{G_F^2}{48\pi^3}|V_{cb}|^2(m_B+m_D)^2 m_D^3(\omega^2-1)^{3/2}(\mathcal{G}(\omega))^2 \tag{5}$$

where we have introduced the form factors $\mathcal{F}$ and $\mathcal{G}$.

It is well known that heavy quark symmetries allow normalization statements for the form factors in heavy-to-heavy transitions at the non-recoil point $v = v'$ or $\omega = v \cdot v' = 1$ [2–4]. In addition, effective-field-theory methods allow us to calculate corrections to these normalization statements. One finds

$$\mathcal{F}(\omega) = \eta_{\text{QED}}\eta_A\left[1 + \delta_{1/\mu^2} + \cdots\right] + (1-\omega)\rho^2 + \mathcal{O}((1-\omega)^2) \tag{6}$$

$$\mathcal{G}(1) = \eta_{\text{QED}}\eta_V\left[1 + \mathcal{O}\left(\frac{m_B - m_D}{m_B + m_D}\right)\right] \tag{7}$$

where $\mu = m_c m_b/(m_b + m_c)$ is the parameter of Heavy-Quark-Symmetry breaking, which governs the leading non-perturbative corrections δ_{1/μ^2}, ρ is a slope parameter and η_A and η_V are the perturbative corrections to the Axial-Vector and the Vector current due to QCD effects and $\eta_{\rm QED}$ are the QED corrections. The radiative corrections are known at the two-loop level [5], while the non-perturbative correction is estimated on the basis of a sum rule [6]; the currently best values are

$$\eta_A = 0.960 \pm 0.007\,, \quad \eta_V = 1.022 \pm 0.004\,,$$
$$\delta_{1/\mu^2} = -(8 \pm 4)\%\,, \quad \eta_{\rm QED} = 1.007 \tag{8}$$

Thus from heavy-quark symmetries one can obtain the form-factor normalization $\mathcal{F}(1)$ with an uncertainty of about 4%, while $\mathcal{G}(1)$ parametrically has a substantially larger uncertainty.

However, all the calculations based on the heavy quark limit might become obsolete, since unquenched lattice calculations become available which do not refer to the heavy mass limit [7]. These calculations compute the deviation of the two form factor from unity and the current results are

$$\mathcal{F}(1) = 0.91^{+0.03}_{-0.04} \qquad \mathcal{G}(1) = 1.074 \pm 0.018 \pm 0.016 \tag{9}$$

I is worth noticing that the uncertainty in $\mathcal{G}(1)$ is smaller than the one in $\mathcal{F}(1)$, which is currently at the same level as the the one obtained from heavy-quark considerations.

The results for the form factors may be used to obtain value for V_{cb} by extrapolating the data to the non-recoil point $v = v'$. Fig. 1 shows he current situation for this extrapolation.

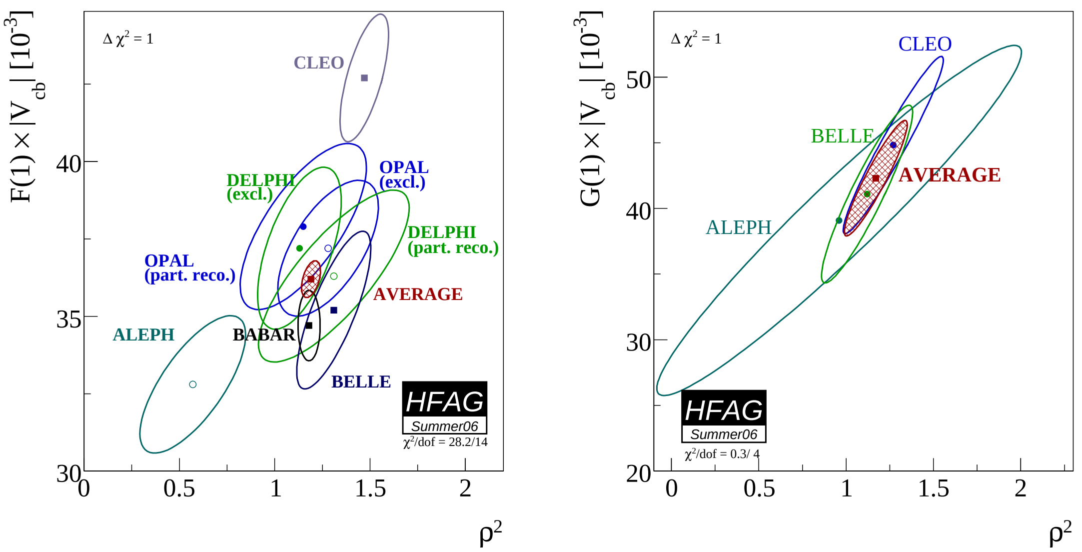

Figure 1: Measurements of $|V_{cb}|\mathcal{F}(1)$ (left) and $|V_{cb}|\mathcal{G}(1)$ versus the from factor slope. Plots are taken from [8].

From this input one extracts a value for V_{cb} from exclusive decays:

$$V_{cb,excl} = (39.4 \pm 0.87^{+1.56}_{-1.24}) \times 10^{-3} \tag{10}$$

2.2 $B \to \pi \ell \overline{\nu}_\ell$

The rate for $B \to \pi \ell \overline{\nu}_\ell$ for vanishing lepton mass is given in terms of only one form factor

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 V_{ub}}{24\pi^3} |\vec{p}_\pi|^3 |f_+(q^2)|^2 \tag{11}$$

Heavy Quark Symmetries cannot be used as efficiently as in the heavy-to-heavy case and only relative normalization statements are possible. In this case it is more convenient to make us of other methods.

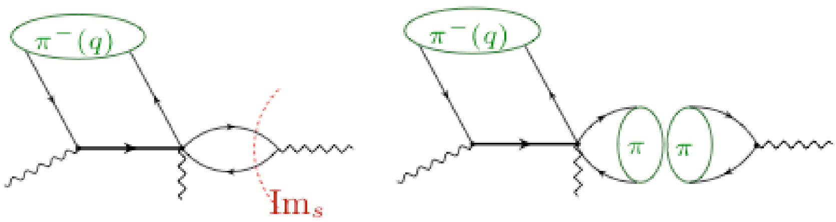

Figure 2: Feynman diagrams for the sum rule evaluation of the $B \to \pi$ form factor.

One possibility is to use QCD (light cone) sum rules [9, 10] which use dispersion relations and the light-cone expansion for the correlator

$$F_\lambda(p, q) = i \int d^4x \; e^{ipx} \langle \pi^+(q) | T\{\overline{u}\gamma_\lambda b(x) \; m_b \overline{b} i \gamma_5 d(0)\} | 0 \rangle \tag{12}$$

which is evaluated in the deep euclidean region using the Feynman diagrams shown in fig. 2. The imaginary part corresponding to the cut shown here is related to a sum of hadronic states which contains also the desired state.

Applying this to the $B \to \pi$ form factor one obtains an estomate for $f_B f_+(q^2)$ in the region of small q^2. The method has quite a few sources of uncertainties, which are from Higher Twists (≥ 4), from the b quark mass and renormalization scale, from the values of the condensates from the sum rule parameters (Threshold and Borel parameters) and finally from the Pion Distribution amplitude. Estimating the resulting uncertainties by varying the parameters we find [9, 10]

$$f_+(0) = 0.27 \times \left[1 \pm (5\%)_{tw>4} \pm (3\%)_{m_b,\mu} \pm (3\%)_{\langle \overline{q}q \rangle} \pm (3\%)_{s_0^B, M} \pm (8\%)_{a^\pi_{2,4}} \right] \tag{13}$$

which adds up to an uncertainty of about 15%.

Complementary information may be obtained from the lattice, since lattice simulations are restricted to large values of q^2 [11, 12]. Also for heavy-to-light decays first unquenched results become available. In fig. 3 the latice data points are shown together with a fit using a pole model [13].

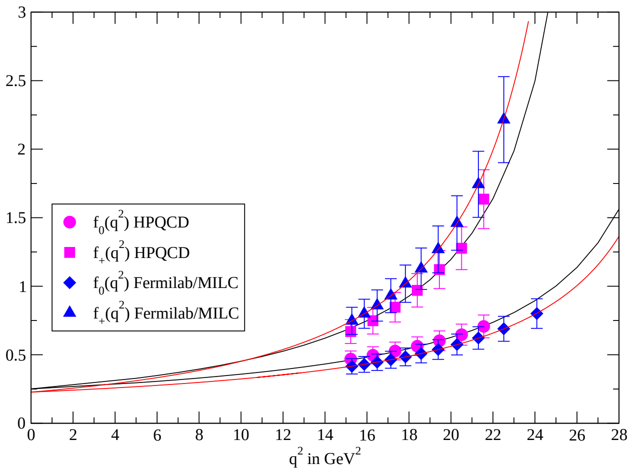

Figure 3: Lattice data for the two form factors involved in $B \to \pi$ transitions. The solid line is a fit to the data using a pole model [13].

Lattice and QCD sum rules turn out to be nicely consistent, giving us some confidence that the form factor in $B \to \pi$ is under reasonable control. Using the lattice data one obtains for the rate above $q^2 = 16\,\mathrm{GeV}^2$

$$|V_{ub}|^2 \times (1.31 \pm 0.33)\,\mathrm{ps}^{-1} \qquad \text{HPQCD} \tag{14}$$

$$|V_{ub}|^2 \times (1.80 \pm 0.48)\,\mathrm{ps}^{-1} \qquad \text{Fermilab / MILC} \tag{15}$$

Gathering the information from the various methods we yield a consistent picture for the value for V_{ub} from exclusive decays. We quote the value from QCD sum rules taken from [10]

$$|V_{ub}| = (3.41 \pm 0.12^{+0.56}_{-0.38}) \times 10^{-3} \tag{16}$$

where the first uncertainty is experimental and the second one is theoretical.

3 Inclusive Decays

The $1/m_b$ expansion for inclusive decays [14–16] is set up in a similar way as one discusses deep inelastic scattering. The total rate is proportional to

$$\begin{aligned}
\Gamma &\propto \sum_X (2\pi)^4 \delta^4(P_B - P_X) |\langle X|\mathcal{H}_{eff}|B(v)\rangle|^2 = \int d^4x\, \langle B(v)|\mathcal{H}_{eff}(x)\mathcal{H}^\dagger_{eff}(0)|B(v)\rangle \\
&= 2\,\mathrm{Im} \int d^4x\, \langle B(v)|T\{\mathcal{H}_{eff}(x)\mathcal{H}^\dagger_{eff}(0)\}|B(v)\rangle \\
&= 2\,\mathrm{Im} \int d^4x\, e^{-im_b v\cdot x} \langle B(v)|T\{\widetilde{\mathcal{H}}_{eff}(x)\widetilde{\mathcal{H}}^\dagger_{eff}(0)\}|B(v)\rangle
\end{aligned} \tag{17}$$

where in the last step we have replaced the b quark field operator by $b \to \widetilde{b} = \exp(-im_b v \cdot v)b$, which corresponds to the replacement $p_b = m_b v + k$.

The next step is to perform an operator product expansion (OPE), which is in the case at hand not in the euclidean region, but rather in the minkowskian. One obtains for the operator product

$$\int d^4x e^{im_b vx} T\{\widetilde{\mathcal{H}}_{eff}(x)\widetilde{\mathcal{H}}_{eff}^\dagger(0)\} = \sum_{n=0}^{\infty} \left(\frac{1}{2m_Q}\right)^n C_{n+3}(\mu)\mathcal{O}_{n+3} \tag{18}$$

where $\mathcal{O}_j$ represents a set of local operators of dimension j and C_j are (perturbatively computable) Wilson coefficients, encoding the short distance contributions.

Taking the forward matrix element of (18) yields an expansion for the total rate of the form

$$\Gamma = \Gamma_0 + \frac{1}{m_Q}\Gamma_1 + \frac{1}{m_Q^2}\Gamma_2 + \frac{1}{m_Q^3}\Gamma_3 + \cdots \tag{19}$$

where the non-perturbative contributions are encoded in the forward matrix elements of the local operators. The general structure of such an expansion is

- Γ_0 is the decay of a free quark ("Parton Model")
- Γ_1 vanishes due to "Luke's theorem" [17]
- Γ_2 is expressed in terms of two parameters

$$\begin{aligned} 2M_H\mu_\pi^2 &= -\langle H(v)|\overline{Q}_v(iD)^2Q_v|H(v)\rangle & (20)\\ 2M_H\mu_G^2 &= \langle H(v)|\overline{Q}_v(-i\sigma_{\mu\nu})(iD^\mu)(iD^\nu)Q_v|H(v)\rangle & (21) \end{aligned}$$

  where μ_π is the kinetic energy parameter and μ_G is the chromomagnetic moment.

- Γ_3 introduces two more parameters [18]

$$\begin{aligned} 2M_H\rho_D^3 &= -\langle H(v)|\overline{Q}_v(iD_\mu)(ivD)(iD^\mu)Q_v|H(v)\rangle & (22)\\ 2M_H\rho_{LS}^3 &= \langle H(v)|\overline{Q}_v(-i\sigma_{\mu\nu})(iD^\mu)(ivD)(iD^\nu)Q_v|H(v)\rangle & (23) \end{aligned}$$

  where ρ_D is the so-called Darwin Term and ρ_{LS} is the spin-orbit term

- Recently the $1/m_b^4$ contribution has been calculated at tree level for semileptonic decays [19]. This introduces five more parameters which have a simple intuitive interpretation:

  $\langle \vec{E}^2 \rangle$: Expectation value of the Chromoelectric Field squared
  $\langle \vec{B}^2 \rangle$: Expectation value of the Chromomagnetic Field squared
  $\langle (\vec{p}^2)^2 \rangle$: Fourth power of the residual b quark momentum
  $\langle (\vec{p}^2)(\vec{\sigma} \cdot \vec{B}) \rangle$: Mixed Chromomagnetic Moment and res. Momentum sqrd.
  $\langle (\vec{p} \cdot \vec{B})(\vec{\sigma} \cdot \vec{p}) \rangle$: Mixed Chromomagnetic field and res. helicity

3.1 $B \to X_c \ell \overline{\nu}_\ell$

The total rate becomes schematically [20]

$$\begin{aligned} \Gamma &= |V_{cb}|^2 \hat{\Gamma}_0 m_b^5(\mu)(1 + A_{ew}) A^{\mathrm{pert}}(r, \mu) \\ & \left[z_0(r) + z_2(r) \left(\frac{\mu_\pi^2}{m_b^2}, \frac{\mu_G^2}{m_b^2} \right) + z_3(r) \left(\frac{\rho_D^3}{m_b^2}, \frac{\rho_{\mathrm{LS}}^3}{m_b^2} \right) + \ldots \right] \end{aligned} \tag{24}$$

where A_{ew} and A^{pert} are the electroweak and the perturbative QCD corrections, $r = m_c^2/m_b^2$ and $z_i(r)$ are the phase space correction factors appearing in the different orders in $1/m_b$.

The state-of-the-art for this calculation includes the $1/m_b$ Expansion at tree level up to $1/m_b^4$, the complete $\mathcal{O}(\alpha_s)$ corrections for the partonic rate $(1/m_b^0)$ and the partial $\mathcal{O}(\alpha_s^2)$, while the $\mathcal{O}(\alpha_s)$ for $1/m_b^2$ terms under consideration.

The partonic rate has a significant scheme dependence, related to the strong dependence on the heavy quark mass. It is well known that the calculation in the pole mass scheme yields sizable QCD radiative corrections. However, switching to a scheme with a suitably chosen short-distance mass reduces the size of the QCD radiative corrections by absorbing them into the mass.

There are two schemes which are commonly used. The kinetic scheme [6] defines the kinetic mass $m_{\mathrm{kin}}(\mu)$ from a sum rule for the kinetic energy of a heavy quark, while the $1S$ scheme uses a mass definition derived from a perturbative calculation of the $\Upsilon(1S)$ mass [21]. Both schemes yield a comparable precision; for simplicity I will stick to the kinetic scheme in this talk.

The extraction of V_{cb} from (24) requires (aside from the quark masses m_b and m_c) the knowledge of the Heavy Quark Expansion (HQE) parameters μ_π, μ_G, ρ_D and ρ_{LS}. These parameters are obtained from the analysis of the leptonic energy and the hardonic invariant mass. It has been shown that the moments of these spectra can be computed reliably in HQE and hence one considers

$$\langle M_X^n \rangle = \frac{1}{\Gamma} \int dM_X \, M_X^n \int_{E_{\mathrm{cut}}} dE_\ell \, \frac{d^2\Gamma}{dM_x \, dE_\ell} \tag{25}$$

$$\langle E_\ell^n \rangle = \frac{1}{\Gamma} \int dM_X \int_{E_{\mathrm{cut}}} dE_\ell \, E_\ell^n \frac{d^2\Gamma}{dM_x \, dE_\ell} \tag{26}$$

Aside from extracting the HQE parameters in this way one may in addition study the dependence of the various moments on the cut energy E_{cut} which is the minimal lepton energy included in the integration. The fits show a very good agreement with the theoretical expectation [22], giving us some confidence that we do understand inclusive semi-leptonic decays at a precision level. In table 1 we show the fit results for the heavy quark parameters.

Using this method one can extract the value of V_{cb} to be

$$V_{cb} = 41.96 \pm 0.23_{\mathrm{exp}} \pm 0.35_{\mathrm{HQE}} \pm 0.59_{\Gamma_{sl}}) \times 10^{-3} \tag{27}$$

Quantity	Value	exp	HQE
m_b (GeV)	4.590	$\pm$ 0.025	$\pm$ 0.030
m_c (GeV)	1.142	$\pm$ 0.037	$\pm$ 0.045
μ_π^2 (GeV)^2	0.401	$\pm$ 0.019	$\pm$ 0.035
μ_G^2 (GeV)^2	0.297	$\pm$ 0.024	$\pm$ 0.046
ρ_D^3 (GeV)^3	0.174	$\pm$ 0.009	$\pm$ 0.022
$\rho_{\rm LS}^3$ (GeV)^2	-0.183	$\pm$ 0.054	$\pm$ 0.071

Table 1: Values of the HQE parameters [22]. The column "exp" contains the experimental uncertainty, while "HQE" contains the remaining uncertainy from the heavy quark expansion.

where the last uncertainty is from the experimental knowledge of the total semileptonic rate. Note that the relative theoretical uncertainty in V_{cb} is currently at the level of 2% and can possibly further reduced by including the newly calculated contribution of order $1/m_b^4$

3.2 $B \to X_u \ell \overline{\nu}_\ell$

The extaction of V_{ub} has to proceed along different lines due to the problem that in most of the phase space the $b \to u$ transitions are completely obscured by the much stronger $b \to c$ decays. Thus the analysis for V_{ub} has to make use of small corners of phase space in which the OPE described in the last section breaks down. For example, the lepton energy spectrum ($y = 2E_\ell/m_b$) close to the endpoint region $y \to 1$ becomes

$$\frac{d\Gamma}{dy} \stackrel{y\to 1}{=} \frac{G_F^2 |V_{ub}^2| m_b^5}{96\pi^3} \left[\Theta(1-y) + \frac{\mu_\pi^2 - \mu_G^2}{6m_b^2} \delta(1-y) + \frac{\mu_\pi^2}{6m_b^2} \delta'(1-y) + \cdots \right] \tag{28}$$

where the singular terms indicate a breakdown of the OPE close to $y = 1$, which yields in this case an expansion in terms of $1/[m_b(1-y)]$

It has been shown some time ago that thee singular terms can be resummed into a non-perturbative function, the so called shape function which is formally defined as [23–25]

$$2M_B f(\omega) = \langle B(v) | \bar{b}_v \delta(\omega + i(n \cdot D)) | B(v) \rangle \tag{29}$$

where the second and the third moments of this function may be related to the HQE parameters μ_π and ρ_D

$$f(\omega) = \delta(\omega) + \frac{\mu_\pi^2}{6m_b^2} \delta''(\omega) - \frac{\rho_D^3}{18m_b^3} \delta'''(\omega) + \cdots \tag{30}$$

In terms of the shape function one may write the resummed rate as

$$\frac{d\Gamma}{dy} = \frac{G_F^2 |V_{ub}^2| m_b^5}{96\pi^3} \int d\omega \, \Theta(m_b(1-y) - \omega) f(\omega) \tag{31}$$

and moment expansion of this expression yields (28).

In order to obtain a precise method for the extraction of V_{ub} one needs aside from some information on the shape function (this could be taken from the rare decay $B \to X_s \gamma$, which is governed by the same shape function) also to take into account the radiative and the $1/m_b$ corrections. In order to do this one has to use "Soft Collinear Effective Theory" (SCET), since in the endpoint region the light degrees of freedom become energetic [26, 27].

It has been shown the within SCET the inclusive rates in the endpoint can be factorized according to [28]

$$d\Gamma = H \otimes J \otimes S \tag{32}$$

where the symbol $\otimes$ means a convolution. The function H contains the hard contribution related to scales of order m_b, J is the "jet function" containing the scales $\sqrt{\Lambda_{\rm QCD} m_b}$ and S is the shape function with the soft pieces with scales $\Lambda_{\rm QCD}$. Note that both m_b and $\sqrt{\Lambda_{\rm QCD} m_b}$ are perturbative scales and hence H and J are computed in perturbation theory.

Without going into further details we only quote the state-of-the-art of this kind of calculation. The next to leading terms in the $1/m_b$ expansion have been investigated in [29, 30] and the QCD radiative corrections have been considered in [31]. Finally, in order to obtain quantitative predictions one needs a model for the shape functions. There are two approaches commonly used. One makes use of a model which is chosen such that the first few moments coincide with what is known from the local OPE [31] (BLNP). The second method [32] is based on the so called "dressed gluon exponentiation" (DGE) which is a QCD based model for the shape function. The results which are obtained from these two approaches are consistent [8]

$$\begin{array}{lcll} V_{ub} & = & (4.49 \pm 0.19_{\rm exp} \pm 0.27_{\rm theo}) \times 10^{-3} & \text{BLNP} \\ V_{ub} & = & (4.46 \pm 0.20_{\rm exp} \pm 0.20_{\rm theo}) \times 10^{-3} & \text{DGE} \end{array}$$

Aside from the shape function dependent methods there are also shape function insensitive methods [33]. However, although these methods have a smaller theoretical uncertainty, they are using a smaller part of the phase space and hence the experimental uncertainties are larger. From this method one obtains [8]

$$V_{ub} = (5.02 \pm 0.26_{\rm exp} \pm 0.37_{\rm theo}) \times 10^{-3} \tag{33}$$

4 Conclusion

From all the checks that have been made it is fair to conclude that the theory of semi-leptonic decays is in a mature state. Current methods allow us to extract V_{cb} with an overall relative accuracy at the level of 2%. This is a remarkble progress in view of the fact that the relative uncertainty of the Cabbibo angle is also not significantly better.

For the calculations of the inclusive rates for the determination of V_{cb} only small improvements are possible, e.g. by a calculation of the contributions of order α_s/m_b^2 and by the inclusion of the newly calculated $1/m_b^4$ terms. However, the progress in the lattice calculations of the exclusive form factors is very promising and the precision of these calculations is already in competition with the heavy quark expansion method. In the near future one may expect that exclusive methods using lattice data will become more precise than the heavy quark expansion method

The determination of V_{ub} has currently a relative theoretical uncertainty of about 8%, and possible improvements of the inclusive methods are still limited either by statistics or by model dependences for e.g. the subleading shape functions. Similar to the exclusive methods are catching up due to more precise and more reliable lattice data. On this basis a future improvement to a level of 5% relative uncertainty (or maybe even better) seems to be possible.

There seems to be a systematic tendency that the exclusive values of both V_{cb} and V_{ub} come out to be lower that the inclusive values, where this effect is more pronounced for the case of V_{ub}. It is interesting to note that the somewhat lower value of V_{ub} is more compatible with the time dependent CP asymmetry measured in $B \to J/\psi K_s$. However, all these effects are well within the uncertainties, which may have been estimated a bit too optimistic, since - in particular a theoretical uncertainty - is hard to estimate.

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