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S. Recksiegel, A. Hoang, S. Paul

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Rare K Decays

Michael Arenton  
Physics Department  
University of Virginia  
P.O. Box 400714  
Charlottesville, VA 22904, USA

1 Introduction

We review recent results on rare K decays from KTeV and NA-48. By rare decays we mean both those modes where the experiments are pushing the branching fraction measurements and limits to lower and lower values and also small branching fraction modes where experimental advances now allow their study with relatively large statistics.

2 The KTeV Experiment

![Diagram of KTeV E-832 configuration](image)

Figure 1: Plan view of KTeV E-832 configuration as used for rare decay measurements (regenerator not shown).
Here we give a brief description of the KTeV experiment [1] [2]. There were two configurations, E832 and E799, shown respectively in Fig. 1 and Fig. 2. E832 was designed primarily to measure the direct CP violation parameter $\epsilon'/\epsilon$, while E799 was devoted to rare $K_L$ decays. Two $K_L$ beams were generated by 800 GeV/c protons on a target. In E832 a regenerator in one of the beams converted $K_L$ to $K_S$. In E799 the regenerator was removed to have two $K_L$ beams. The beams were run at higher intensity in E799. The decays took place in a large vacuum decay region.

In both configurations a magnetic spectrometer consisting of two sets of x and y drift chambers before and after an analysing magnet measured charged particles. Photons were measured in a 3100 element array of pure CsI blocks which had energy resolution of $\sigma(E)/E = 0.45\% + 2%/\sqrt{E})$. This electromagnetic calorimeter was followed by layers of steel and concrete absorber and scintillators for muon identification. Several arrays of counters vetoed on the presence of charged particles or photons outside the aperture of the spectrometer and calorimeter. The E799 configuration also included a set of transition radiation detectors for improved electron identification.

There were two data taking runs for each configuration, in 1997 and 1999. The 1999 E832 run repeated the 1997 run with somewhat better running conditions to check the systematics of the $\epsilon'/\epsilon$ measurement. The 1999 E799 run was devoted to increasing the sensitivity for rare decays. To this end the $p_t$ kick of the analysing magnet was reduced to 150 MeV/c in 1999 from 200 MeV/c in 1997 to increase acceptance, particularly for 4 body decays. Also in 1999 several triggers were prescaled
to increase the data acquisition bandwidth for other triggers. E799 was sensitive to $2.5 \times 10^{11}$ and $3.5 \times 10^{11} K_L$ decays in 1997 and 1999 respectively.

### 3 The decay $K_L \rightarrow \pi^+\pi^-\gamma$

The decay $K_L \rightarrow \pi^+\pi^-\gamma$ proceeds through the amplitudes shown in Fig. 3. The two main contributions, of about equal magnitude, are from the CP violating Inner Bremsstrahlung (IB) and the CP conserving Direct Emission terms. The Direct Emission term is mostly magnetic dipole (M1) radiation. This must be modified with a form factor that is usually expressed in a $\rho$ pole form. As well as the M1 term there might be an electric dipole (E1) term. The E1 term is especially interesting because it is CP violating. The experiments have now reached statistical levels where searching for the E1 term is possible.

The KT eV results on $K_L \rightarrow \pi^+\pi^-\gamma$ [3] are based on a sample of 112,100 events over a background of $671 \pm 41$ events recorded in the 1997 run of E832. The amplitudes are determined from a fit to the distribution of the $\gamma$ energy in the $K_L$ rest frame which shows a falling distribution at low $E_\gamma$ from IB and a broad peak at high $E_\gamma$ from M1 DE. Interference of the M1 and E1 DE amplitudes would show up in the intermediate energy region. The distribution and fit are shown in figure 4. The DE form factor parameters are found to be $g_{M1} = 1.198 \pm 0.035 \pm 0.086$ and $a_1/a_2 = -0.738 \pm 0.007 \pm 0.018(\text{GeV}^2)$. The ratio of the direct emission to total
decay rate is $0.0689 \pm 0.021$. An upper limit for the magnitude of an E1 term is found to be $|g_{E1}| < 0.21$ at the 90% confidence level.

![Graph of E_γ distribution in K_L → π^+π^-γ with fit results.](image)

Figure 4: $E_\gamma$ distribution in $K_L \rightarrow \pi^+\pi^-\gamma$ with fit results.

![View of the NA48/2 detector.](image)

Figure 5: View of the NA48/2 detector. [5]
Figure 6: Variables describing the decay $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ [6]

KTeV has also searched for the E1 term in the related decay mode $K_L \rightarrow \pi^+ \pi^- e^+ e^-$ [4]. Along with amplitudes analogous to those of $K_L \rightarrow \pi^+ \pi^- \gamma$ this decay also has a term related to the K charge radius. The angular distributions of the $\pi\pi$ and ee pairs provide additional information making this decay more sensitive to E1 contributions. We find an upper limit of $|g_{E1}| / |g_{M1}| < 0.04$ at 90% confidence level.

4 NA48 results on $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$

The charged kaon extension of the NA48 experiment, NA48/2, has recently presented results on the related radiative decay $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$. Like $K_L \rightarrow \pi^+ \pi^- \gamma$ this decay has IB and DE amplitudes, but here the IB amplitude is CP conserving and much larger than DE.

The NA48/2 detector is shown in Fig. 5 [5] The NA48/2 experiment was primarily directed to searching for direct CP violation in $K^\pm \rightarrow 3\pi$ decays. It used simultaneous $K^\pm$ beams of 60 ± 3 GeV/c.

The Dalitz plot variables $W^2$ and $T^*_\pi$ used to describe the $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ decay are shown in Fig. 6. One searches for the interference term between the IB and the electric dipole direct emission. Previous experiments have not seen evidence of this interference. This analysis was based on 124,000 events, which is 30% of the available
data and 5 times the statistics of previous experiments.

In the analysis it was necessary to solve two problems that might cause distortions in the $W^2$ distribution that could mimic direct emission or interference terms. The first is misassignment of the $\gamma$'s, which was reduced by cuts on the $\pi^0$ and $K^\pm$ masses and requirement of agreement of the charged and neutral vertices. The second is backgrounds from the $K^\pm \rightarrow \pi^\pm \pi^0$ and $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ decays with coalesced $\gamma$'s, which were reduced using techniques to split coalesed $\gamma$'s. In the final analysis these backgrounds were reduced to less than 1% of the direct emission level.

**Figure 7: W distribution of NA48/2 data divided by Inner Bremsstrahlung shape. [6]**

Fig. 7 shows the measured W distribution divided by that expected for IB alone. Clearly this deviates from unity at high W. A fit yields a preliminary result for the fraction of direct emission of $(3.35 \pm 0.35_{\text{stat}} \pm 0.25_{\text{syst}})$% and of the interference $(-2.67 \pm 0.81_{\text{stat}} \pm 0.73_{\text{syst}})$%. These values are highly correlated, with a correlation coefficient of -0.92. This result is the first observation of the electric dipole interference term with high statistical certainty.

## 5 $K_L \rightarrow \pi^+\pi^-\pi^0\gamma$ and $K_L \rightarrow \pi^+\pi^-\pi^0e^+e^-$

First results on the radiative decays $K_L \rightarrow \pi^+\pi^-\pi^0\gamma$ and $K_L \rightarrow \pi^+\pi^-\pi^0e^+e^-$ have been obtained by KTeV. $K_L \rightarrow \pi^+\pi^-\pi^0\gamma$ is expected to be dominated by the inner...
bremmstrahlung process with a theoretical branching fraction of \((1.65 \pm 0.03) \times 10^{-4}\) for \(E_\gamma < 10\text{MeV}\) [7]. The direct emission contribution is expected to be very small: \(\text{BR}_{\text{direct}} = (8a_1 + a_2 - 10a_3)^2 \times 10^{-10}\) where the \(a_i\) are unknown parameters of order 1 [8]. For \(K_L \to \pi^+\pi^-\pi^0e^+e^-\) there are no published theories. There should be IB and DE terms similar to \(K_L \to \pi^+\pi^-\pi^0\gamma\) with virtual photon conversion to an \(e^+e^-\) pair. In addition there should be a charge radius amplitude.

KTeV has observed \(K_L \to \pi^+\pi^-\pi^0\gamma\) both in data from E832 with \(\pi^0 \to \gamma\gamma\) and from E799 with \(\pi^0 \to e^+e^-\gamma\) yielding signals of 2853 and 2847 events respectively, as shown in Fig. 8. A preliminary result for the branching ratio with \(E_\gamma^\text{cm} > 10\text{MeV}\) is \(\text{BR} = (1.70 \pm 0.03_{\text{stat}} \pm 0.04_{\text{syst}} \pm 0.03_{\text{extsyst}}) \times 10^{-4}\) in good agreement with theory.

![Figure 8: \(\pi^+\pi^-\pi^0\gamma\) and \(\pi^+\pi^-\pi^0\gamma\) mass distributions from E832 and E799 data.](image)

KTeV has made a first observation of \(K_L \to \pi^+\pi^-\pi^0e^+e^-\) in the E799 data. In 40% of the data 132 candidates are observed with an estimated background level of \(1.2 \pm 0.9\) event. This is shown in Fig. 9. The preliminary result for \(E_{ee} > 20\text{MeV}\) is \(\text{BR} = (1.60 \pm 0.18_{\text{stat}}) \times 10^{-7}\).

6 \(K_L \to e^+e^-\gamma\)

Radiative decays of the type \(K_L \to \gamma^{(*)}\gamma^{(*)}\) are of interest largely because of their role in the measurement of the CKM matrix element \(|V_{td}|\) from the decay \(K_L \to \mu^+\mu^-\). This decay proceeds partly from a short distance coupling related to \(|V_{td}|\), but also has a long distance coupling related to \(K_L \to \gamma^{(*)}\gamma^{(*)}\) that must be subtracted.
The $K_L \rightarrow \gamma^{(*)}\gamma^{(*)}$ decays have been described by two form factor models. One is the vector dominance inspired model of Bergström, Masso and Singer (BMS) [9]. The other is the chiral perturbation theory model of D’Ambrosio, Isidori and Portoles (DIP) [10]. The parameters of these models can be determined by fits to the $m_{ee}$ distribution of the data. The BMS model contains a parameter $\alpha_{K^*}$. However experiments actually determine the quantity $C\alpha_{K^*}$ where $C = (8\pi\alpha_{em})^{1/2}G_{NL}\Gamma_{K^*K\gamma}m_p^2/(f_{K^*}f_{\rho}\Delta_{\gamma\gamma})$. A number of experiments using various decay modes have presented results for $\alpha_{K^*}$ but are inconsistent because the values of the parameters making up $C$ have changed over time. Therefore KTeV chooses to quote $C\alpha_{K^*}$ and compare it to $C\alpha_{K^*}$ from other experiments.

The KTeV form factor measurements are based on a sample of 83,000 $K_L \rightarrow e^+e^-\gamma$ decays. Of particular importance in the form factor fits is the handling of radiative corrections. KTeV has developed a Monte Carlo including the complete set of second order radiative diagrams.

The corrected preliminary KTeV results are a branching fraction of $(9.25\pm0.03_{stat}\pm0.07_{syst}\pm0.26_{extsyst}) \times 10^{-6}$, $C\alpha_{K^*} = -0.517\pm0.030_{fit}\pm0.022_{syst}$ for the BMS model and $\alpha_{DIP} = -1.729\pm0.043_{fit}\pm0.028_{syst}$ for the DIP model. A comparison of values of $C\alpha_{K^*}$ from various decay modes is shown in Fig.10.
Figure 10: Results on $C_{\alpha K^*}$ from various decay modes

7 $K_L \rightarrow \pi^\pm e^\mp \nu e^+ e^-$

KTeV has made the first measurements of the decay $K_L \rightarrow \pi^\pm e^\mp \nu e^+ e^-$ [11]. This is of course related to the radiative $K_{e3}$ decay mode $K_L \rightarrow \pi^\pm e^\mp \nu \gamma$. New tests of chiral perturbation theory are enabled by these measurements. Because of the missing $\nu$ there are less kinematic constraints available to use in signal selection. The worst backgrounds come from $K_L \rightarrow \pi^+ \pi^- \pi^0$ with $\pi^0$ decays to $e^+ e^- \gamma$ or $e^+ e^- e^+ e^-$, $K_L \rightarrow \pi^\pm e^\mp \nu \pi^0$ with $\pi^0 \rightarrow e^+ e^- \gamma$, and $K_L \rightarrow \pi^\pm e^\mp \nu \gamma$ where the $\gamma$ converts in material in the spectrometer. The analysis relies heavily on the full identification power of the CsI Calorimeter and the TRD’s.

A sample of 19466 candidate events is obtained with a background of about 5% from about 25% of the E799 data. A preliminary result for the branching fraction of $K_L \rightarrow \pi^\pm e^\mp \nu e^+ e^-$ with $m_{e^+ e^-} > 0.005\text{GeV}$ and $E_{e^+ e^-} > 0.03\text{GeV}$ is $(1.281 \pm 0.010_{\text{stat}} \pm 0.019_{\text{syst}} \pm 0.035_{\text{extsyst}}) \times 10^{-5}$.

A theoretical calculation in chiral perturbation theory [12] has been made of the quantity $R = \Gamma(K_L \rightarrow \pi^\pm e^\mp \nu e^+ e^-) / \Gamma(K_L \rightarrow \pi^\pm e^\mp \nu e^+ e^-)$. At leading order in the theory the predicted value of $R$ is $4.06 \times 10^{-5}$ whereas at next to leading order ($p^4$) it is $4.29 \times 10^{-5}$. The experimental result corresponds to $R = (4.54 \pm 0.15) \times 10^{-5}$ which is $3.2\sigma$ from the leading order and $1.7\sigma$ from the next to leading
order calculation.

Figure 11: Distribution of the minimum solution for $t$. Upper plot shows data (points) and leading order $\chi$PT calculation (lines). Lower plot show the ratio of data/calculation.

One may also examine agreement with theoretical predictions by looking at the distributions of the kinematics of the decay. One such variable is the momentum transfer $t$. Because of the missing $\nu$, experimentally there are two possible solutions for $t$ in each event. Figs. 11 and 12 show the distributions of the minimum solution for $t$. The points in both of these figures are the same. Fig. 11 shows a comparison to the distribution calculated with leading order chiral perturbation theory while Fig. 12 shows the comparison to next to leading order theory. One sees a better agreement with next to leading order, as shown in the ratio plots in the bottom parts of the figures, the ratio being flatter for NLO than for LO. The same conclusion is reached when the maximum solution for $t$ is examined (not shown).

8 \[ \pi^0 \rightarrow e^+e^- \]

$K_L$ decays are a copious source of “tagged” $\pi^0$'s. KTeV has used these to measure the rare decay $\pi^0 \rightarrow e^+e^-$. To lowest order this is described by the diagram shown in Fig.
Various calculations based on vector dominance or chiral perturbation theory predict branching fractions somewhat higher than this unitarity limit \[15\] \[16\] \[17\] \[18\] \[19\]. After appropriate analysis cuts the \(m_{ee}\) distribution shown in Fig.14 is obtained, for events of the final state \(\gamma\gamma\gamma\gamma e^+e^-\) with two \(\gamma\gamma\) pairs consistent with \(\pi^0\)'s and \(m_{\gamma\gamma\gamma\gamma ee}\) consistent with \(m_K\). The peak has 794 events with a background of 53.2 ± 9.5
Figure 14: $m_{ee}$ distribution after all other cuts.

Figure 15: Comparison of the measured $\pi^0 \rightarrow e^+e^-$ branching fraction to theories of refs [15] to [19].

The branching fraction for $\pi^0 \rightarrow e^+e^-$ with $x > 0.95$ is $(6.56 \pm 0.26_{stat} \pm 0.10_{syst} \pm 0.19_{extsyst}) \times 10^{-8}$, where $x = m_{ee}/m_{e^0}$. 
Fig. 15 shows this result in comparison with the unitarity limit and various theoretical calculations. This measurement is 7σ above the unitarity limit.

9 Searches for Lepton Flavor Violating Decays

KTeV has searched for the lepton flavor violating decays $K_L \rightarrow \pi^0 \mu^\pm e^\mp$, $K_L \rightarrow \pi^0 \pi^0 \mu^\pm e^\mp$, and $\pi^0 \rightarrow \mu^\pm e^\mp$. There is nothing new to report on $K_L \rightarrow \pi^0 \mu^\pm e^\mp$ but recently new preliminary results have been obtained on the latter two modes. (The $\pi^0$ decay analysis is in effect a subset of the $K_L$ analysis.)

In the past, analyses of this sort have been done by defining a “box” in some kinematical space (usually in a 2 dimensional plot of the mass of all the particles making up the decay versus a transverse momentum that should be zero for a true decay), not looking at the data in the box while cuts to reduce backgrounds are established by studying their effects on the events near to, but outside the box, and finally opening the box and comparing the number of events found to what is expected from background evaluations. This is not in general the most sensitive procedure however, since the true signal would not usually be evenly distributed over a rectangular box.

Instead KTeV has based its analysis on a probability distribution function (PDF) formed from the distributions of mass and $p_t^2$. The PDF distribution for the decay mode $K_L \rightarrow \pi^0 \mu^\pm e^\mp$ is shown in Fig. 16 which illustrates the search regions. The PDF distribution for $K_L \rightarrow \pi^0 \pi^0 \mu^\pm e^\mp$ is very similar.

Determination of the background is a key to this analysis. Initial Monte Carlo calculations indicated that backgrounds originate from several common decay modes. Because of this and the very small branching fractions being probed it was not practical to calculate background levels by Monte Carlo simulations. Instead the data itself was used, loosening some cuts so that the PDF distributions of the background could be examined and then rescaling by the effects of the final cuts. Figure 17 shows this. The result of this is predicted backgrounds of $0.44 \pm 0.12$ events of $K_L \rightarrow \pi^0 \pi^0 \mu^\pm e^\mp$ and $0.03 \pm 0.02$ events for $\pi^0 \rightarrow \mu^\pm e^\mp$.

When the blind region in the PDF distribution was examined no events were found. Using the Feldman-Cousins method with the predicted backgrounds and no events seen yields preliminary 90% confidence level upper limits of $BR(K_L \rightarrow \pi^0 \pi^0 \mu^\pm e^\mp) < 1.58 \times 10^{-10}$ and $BR(\pi^0 \rightarrow \mu^\pm e^\mp) < 3.63 \times 10^{-10}$.

10 Conclusion

Sensitive results have recently been obtained on a number of rare K decays [20]. In $K_L \rightarrow \pi^+\pi^-\gamma$ and $K_L \rightarrow \pi^+\pi^- e^+e^-$ accurate measurement of the inner bremsstrahlung and M1 direct emission components have been made and searches done for the E1
amplitude. In the analogous charged decay $K^\pm \to \pi^\pm \pi^0 \gamma$ NA48/2 has found the first clear evidence for the E1 interference term. KTeV has observed the decays $K_L \to \pi^+ \pi^- \pi^0 \gamma$ and $K_L \to \pi^+ \pi^- \pi^0 e^+ e^-$. KTeV has made high statistics measurements of $K_L \to e^+ e^- \gamma$. KTeV has made the first measurements of $K_L \to \pi^\pm e^\mp \nu e^\pm e^-$. KTeV has made accurate measurements of $\pi^0 \to e^+ e^-$. Finally KTeV has new results on searches for the lepton flavor violating decays $K_L \to \pi^0 \pi^0 \mu^\mp e^\mp$ and $\pi^0 \to \mu^\pm e^\mp$.

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**Bibliography**


Figure 17: Signal probability distribution distributions for relaxed cut background samples. Note that the signal region is PDF > 10.


[20] KTeV results from other decay modes have been presented at this conference by E. Cheu.