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NRQCD and Quarkonia

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1 Introduction

Quarkonia play an important role in several high energy experiments. The diversity, quantity and accuracy of the data still under analysis and currently being collected is impressive and includes: data on quarkonium formation from BES at the Beijing Electron Positron Collider (BEPC), E835 at Fermilab, and CLEO at the Cornell Electron Storage Ring (CESR); clean samples of charmonia produced in B-decays, in photon-photon fusion and in initial state radiation, at the B-meson factories, BaBar at PEP-II and Belle at KEKB, including the unexpected observation of large amounts of associated $(c\overline{c})(c\overline{c})$ production and the observation of new and possibly exotics quarkonia states; the CDF and D0 experiments at Fermilab measuring heavy quarkonia production from gluon-gluon fusion in $p\overline{p}$ annihilations at 2 TeV; the Selex experiment at Fermilab with the preliminary observation of possible candidates for doubly charmed baryons; ZEUS and H1, at DESY, studying charmonia production in photon-gluon fusion; PHENIX and STAR, at RHIC, and NA60, at CERN, studying charmonia production, and suppression, in heavy-ion collisions. This has led to the discovery of new states, new production mechanisms, new decays and transitions, and in general to the collection of high statistics and precision data sample. In the near future, even larger data samples are expected from the BES-III upgraded experiment, while the B factories and the Fermilab Tevatron will continue to supply valuable data for few years. Later on, new experiments at new facilities will become operational (the LHC experiments at CERN, Panda at GSI, hopefully a Super-B factory, a Linear Collider, etc.) offering fantastic challenges and opportunities in this field. A comprehensive review of the experimental and theoretical status of heavy quarkonium physics may be found in the Cern Yellow Report prepared by the Quarkonium Working Group [1]. Many excellent reviews of the field have been presented at this meeting [2].

On the theory side, systems made by two heavy quarks are a rather unique laboratory. They are characterized by the existence of a hierarchy of energy scales in correspondence of which one can construct a hierarchy of nonrelativistic effective field theries (NR EFT), each one with less degrees of freedom left dynamical and thus simpler. Some of these physical scales are large and may be treated in perturbation theory. The occurrence of these two facts makes two heavy quark systems accessible in QCD. In particular, the factorization of high and low energy scales realized in the EFTs allows us to study low energy QCD effects in a systematic way. Today the remarkable progress in the construction of these nonrelativistic EFTs together with the advance in lattice QCD give us well based theory tools to investigate heavy quarkonia.

Therefore, on the one hand the progress in our understanding of EFTs makes it possible to move beyond phenomenological models and to provide a systematic description from QCD of several aspects of heavy-quarkonium physics. On the other hand, the recent progress in the measurement of several heavy-quarkonium observables makes it meaningful to address the problem of their precise theoretical determination. In this situation heavy quarkonium becomes a very special and relevant system to advance our understanding of strong interactions and our control of some parameters of the Standard Model.

Here I will briefly review some of the recent developments in the construction of NR EFTs with the main emphasis on the physical applications. For some reviews see [3–6].

2 Scales and Effective Field Theories

The description of hadrons containing two heavy quarks is a rather challenging problem, which adds to the complications of the bound state in field theory those coming from the nonperturbative QCD low-energy dynamics. A simplification is provided by the nonrelativistic nature suggested by the large mass of the heavy quarks and manifest in the spectrum pattern. As nonrelativistic systems, quarkonia are characterized by three energy scales, hierarchically ordered by the heavy quark velocity in the center of mass frame $v \ll 1$: the mass m (hard scale), the momentum transfer mv (soft scale), which is proportional to the inverse of the typical size of the system r, and the binding energy mv^2 (ultrasoft scale), which is proportional to the inverse of the typical time of the system. In bottomonium $v^2 \sim 0.1$, in charmonium $v^2 \sim 0.3$, in $t\bar{t}$ $v \sim 0.15$. In perturbation theory $v \sim \alpha_{\rm s}$. Feynman diagrams will get contributions from all momentum regions associated with these scales. Since these momentum regions depend on $\alpha_{\rm s}$, each Feynman diagram contributes to a given observable with a series in $\alpha_{\rm s}$ and a non trivial counting. Besides, the $\alpha_{\rm s}$ associated to different momentum region are evaluated at different scales. For energy scales close to $\Lambda_{\rm QCD}$, the scale at which nonperturbative effects become dominant, perturbation theory breaks down and one has to rely on nonperturbative methods. Regardless of this, the nonrelativistic hierarchy $m \gg mv \gg mv^2$, $m \gg \Lambda_{\rm QCD}$ will persist also below the $\Lambda_{\rm QCD}$ threshold.

The wide span of involved energy scales makes also a lattice calculation in full QCD extremely challenging. However, it is possible to exploit the existence of a hierarchy of scales by introducing a hierarchy of nonrelativistic effective field theories. Lower energy EFTs may be constructed by systematically integrating out modes associated to energy scales not relevant for the two quark system. Such integration is made in a matching procedure that enforces the equivalence between QCD and the EFT at any given order of the expansion in v. Any prediction of the EFT is therefore a prediction of QCD with an error of the size of the neglected order in v. By integrating out the hard modes, one obtains Nonrelativistic QCD [9, 10, 13]. In such EFT, soft and ultrasoft scales are left dynamical and still their mixing complicates calculations and power counting. In the last few years the problem of systematically treating the remaining dynamical scales in an EFT framework has been addressed by several groups [11] and has now reached a good level of understanding. So one can go down one step further and integrate out also the soft scale in a matching procedure to the lowest energy and simplest EFT that can be introduced for quarkonia, where only ultrasoft degrees of freedom remain dynamical. Here I will review potential NRQCD [7, 8], for an alternative and equivalent EFT (in the case in which $\Lambda_{\rm QCD}$ is the smallest scale) see [12]. In the case in which the soft scale is of the same order of $\Lambda_{\rm QCD}$, the matching to pNRQCD is still possible but it is nonperturbative.

3 NonRelativistic QCD (NRQCD)

NRQCD [9,10] is the EFT for two heavy quarks that follows from QCD by integrating out the hard scale m. Only the upper (lower) components of the Dirac fields remain relevant for dymanical quarks (antiquarks) at energies lower than m. Thus quark and antiquarks are described in terms of two-components Pauli spinor fields. The part of the NRQCD Lagrangian bilinear in the heavy quark fields is the same as Heavy Quark Effective Field Theory (HQET) (for a review see [14]) but for the case of two heavy quarks also four fermion operators have to be considered. The Lagrangian is organized as an expansion in v and $\alpha_{\rm s}(m)$:

$$\mathcal{L}_{\rm NRQCD} = \sum_n c_n(m,\mu) \times O_n(\mu, mv, mv^2, \Lambda_{\rm QCD})/m^n. \tag{1}$$

The NRQCD matching coefficients c_n are series in $\alpha_{\rm s}$ and encode the ultraviolet physics that has been integrated out from QCD. The low energy operators O_n are constructed out of two or four heavy quark/antiquark fields plus gluons. They are counted in powers of v. Since two scales, soft and the ultrasoft, are dynamical, the power counting in v is not unambiguous. The imaginary part of the coefficients of the 4-fermion operators contains the information on heavy quarkonium annihilations. The NRQCD heavy quarkonium Fock state is given by a series of terms, increasingly subleading, where the leading term is a $Q\overline{Q}$ in a color singlet state and the first correction, suppressed in v, comes from a $Q\overline{Q}$ in a color octet state plus a gluon. The NRQCD Lagrangian can be used for studies of spectroscopy (on the lattice), inclusive decays and electromagnetics threshold production of heavy quarkonia.

4 potential NonRelativistic QCD (pNRQCD)

pNRQCD [3,7,8] is the EFT for two heavy quark systems that follows from NRQCD by integrating out the soft scale mv. Here the role of the potentials and the quantum mechanical nature of the problem are realized in the fact that the Schrödinger equation appears as zero order problem for the two quark states. We may distinguish two situations: 1) weakly coupled pNRQCD when $mv \gg \Lambda_{\rm QCD}$, where the matching from NRQCD to pNRQCD may be performed in perturbation theory; 2) strongly coupled pNRQCD when $mv \sim \Lambda_{\rm QCD}$, where the matching has to be nonperturbative. Recalling that $r^{-1} \sim mv$, these two situations correspond to systems with inverse typical radius smaller than or of the same order as $\Lambda_{\rm QCD}$.

4.1 Weakly coupled pNRQCD

The effective degrees of freedom that remain dynamical are: low energy $Q\overline{Q}$ (Pauli spinor) states that can be decomposed into a singlet field S and an octet field O under colour transformations, have energy of order $\Lambda_{\rm QCD}$, mv^2 and momentum $\mathbf{p}$ of order mv; low energy (ultrasoft (US)) gluons $A_\mu(\mathbf{R}, t)$ with energy and momentum of order $\Lambda_{\rm QCD}$, mv^2. All the gluon fields are multipole expanded (i.e. expanded in the quark-antiquark distance r, R being the center of mass). The Lagrangian is then given by terms of the type

$$\frac{c_k(m,\mu)}{m^k} \times V_n(r\mu', r\mu) \times O_n(\mu', mv^2, \Lambda_{\rm QCD})\, r^n. \tag{2}$$

where the matching coefficients c_k are inherited from NRQCD and contain the logs in the quark masses, the pNRQCD potential matching coefficients V_n encode the non-analytic behaviour in r and the low energy operators O_n are constructed in terms of singlet, octet fields and ultrasoft gluons. At leading order in the multipole expansion, the singlet sector of the Lagrangian gives rise to equations of motion of the Schrödinger type. Each term in the pNRQCD Lagrangian has a definite power counting. The bulk of the interaction is carried by potential-like terms, but non-potential interactions, associated with the propagation of low energy degrees of freedom are present as well. Such retardation (or non-potential) effects start at the next-to-leading order (NLO) in the multipole expansion and are systematically encoded in the theory and typically related to nonperturbative effects [8]. There is a systematic procedure to calcolate corrections in v to physical observables: higher order perturbative (bound state) calculations in this framework become viable. In particular the EFT can be used for a very efficient resummation of large logs (typically logs of the ratio of energy and momentum scales) using the renormalization group (RG) adapted to the case of correlated scales [12, 15]; Poincaré invariance is not lost, but shows up in some exact relations among the matching coefficients [16]. The renormalon subtraction may be implemented systematically obtaining a perturbative series better behaved and allowing a factorization of the genuine QCD nonperturbative effects.

4.2 Strongly coupled pNRQCD

In this case the matching to pNRQCD is nonperturbative. Away from threshold (precisely when heavy-light meson pair and heavy hybrids develop a mass gap of order $\Lambda_{\rm QCD}$ with respect to the energy of the $Q\overline{Q}$ pair), the quarkonium singlet field S remains as the only low energy dynamical degree of freedom in the pNRQCD Lagrangian (if no ultrasoft pions are considered), which reads [3, 19, 20]:

$$\mathcal{L}_{\rm pNRQCD} = {\rm S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{2m} - V_S(r) \right) {\rm S}. \tag{3}$$

The matching potential $V_S(r)$ is a series in the expansion in the inverse of the quark masses: static, $1/m$ and $1/m^2$ terms have been calculated, see [19, 20]. They involve NRQCD matching coefficients and low energy nonperturbative parts given in terms of Wilson loops and field strengths insertions in the Wilson loop. In this regime we recover the quark potential singlet model from pNRQCD. However the potentials are calculated from QCD in the formal nonperturbative matching procedure. An actual evaluation of the low energy part requires lattice evaluation [17] or QCD vacuum models calculations [18, 27].

5 Applications

The condition $m \ll \Lambda_{\rm QCD}$ always holds and thus the first matching from QCD to NRQCD is a perturbative matching. NRQCD describes in principle all heavy quarkonia states and physical processes. However, since still the soft and the ultrasoft scales are dynamical, the power counting is not unambiguous and in some cases may differ from the perturbative inspired BBL counting [10, 34, 35]. The number of nonperturbative operators tend to increase with the order of the expansion in v, and their expectation values depend both on the quarkonium states and the US gluons. The NRQCD lattice implementation still requires the calculation of the NRQCD matching coefficients in the lattice regularization, which is still missing in many cases. Being

NRQCD a nonrenormalizable theory at the leading order Lagrangian, NRQCD lattice calculations maybe tricky.

The lowest energy EFT, pNRQCD, is simpler and as such may be more predictive. However, in the present formulation, it is valid only for states away from threshold. Since we are now integrating out also the soft scale, it is important to establish when $\Lambda_{\rm QCD}$ sets in, i.e. when we have to resort to non-perturbative methods. For low-lying resonances, it is reasonable, although not proved, to assume $mv^2 \gtrsim \Lambda_{\rm QCD}$. The system is weakly coupled and we may rely on perturbation theory, for instance, to calculate the potential. The theoretical challenge here is performing higher-order calculations and the goal is precision physics. For high-lying resonances, we assume $mv \sim \Lambda_{\rm QCD}$. The system is strongly coupled and the potential must be determined non-perturbatively, for instance, on the lattice. The theoretical challenge here is providing a consistent framework where to perform lattice calculations and the progress is measured by the advance in lattice computations. The number of nonperturbative operators maybe be greatly reduced with respect to NRQCD, since a further factorization at the soft scale is realized and nonperturbative contributions become typically only a function of the US gluons. The pNRQCD leading order strongly coupled Lagrangian is renormalizable allowing in principle a straightforward lattice implementation.

Both in NRQCD and pNRQCD a source of concern may arise from the large v^2 corrections in the charmonium case. Large (renormalon-like) perturbative contributions in the matching coefficients need to be properly taken care, resummed and subtracted.

6 QCD potentials

The QCD potentials achieve a well defined status and definition only in pNRQCD: they are the matching coefficients of the EFT and as such there is a well defined procedure to calculate them. They depend on the scale of the matching. In weakly coupled pNRQCD the soft scale is bigger than $\Lambda_{\rm QCD}$ and so the singlet and octet potentials have to be calculated in the perturbative matching. In [21] a determination of the singlet potential at three loops leading log has been obtained inside the EFT which gives the way to deal with the well known infrared singularity arising in the potential at this order [36]. From this, $\alpha_{\rm s}$ in the V regularization can be obtained, showing a dependence on the infrared behaviour of the theory at this order and for this regularization. The finite terms in the singlet static potential at three loops are not yet known but has been estimated [24]. Recently, also the logarithmic contribution at four loops has been calculated [23]. The three loop renormalization group improved calculation of the static singlet potential has been compared to the lattice calculation and found in good agreement up to about 0.25 fm [22]. The static octet potential is known at two loops [25] and again agrees well with the lattice data [26].

At a scale μ such that $mv \sim \Lambda_{\rm QCD} \gg \mu \gg mv^2$, confinement sets in and the potentials become admixture of perturbative terms, inherited from NRQCD, which encode high-energy contributions, and non-perturbative objects. Strongly coupled pNRQCD gives us the general form of the potentials obtained in the nonperturbative matching to QCD in the form of Wilson loops and Wilson loop chromoelectric and chromomagnetic field strengths insertions [19, 20], very well suited for lattice calculations. These will be in general complex valued functions. The real part controls the spectrum and the imaginary part controls the decays.

The real part of the potential has been one of the first quantities to be calculated on the lattice (for a review see [17]). In the last year, there has been some remarkable progress. In [28], the $1/m$ potential has been calculated for the first time. The existence of this potential was first pointed out in the pNRQCD framework [19]. A $1/m$ potential is typically missing in potential model calculations. The lattice result shows that the potential has a $1/r$ behaviour, which, in the charmonium case, is of the same size as the $1/r$ Coulomb tail of the static potential and, in the bottomonium one, is about 25%. Therefore, if the $1/m$ potential has to be considered part of the leading-order quarkonium potential together with the static one, as the pNRQCD power counting suggests and the lattice seems to show, then the leading-order quarkonium potential would be, somewhat surprisingly, a flavor-dependent function. In [29], spin-dependent potentials have been calculated with unprecedented precision. In the long range, they show, for the first time, deviations from the flux-tube picture of chromoelectric confinement [27, 30]. The knowledge of the potentials in pNRQCD could provide an alternative to the direct determination of the spectrum in NRQCD lattice simulations: the quarkonium masses would be determined by solving the Schrödinger equation with the lattice potentials. The approach may present some advantages: the leading-order pNRQCD Lagrangian, differently from the NRQCD one, is renormalizable, the potentials are determined once for ever for all quarkonia, and the solution of the Schrödinger equation provides also the quarkonium wave functions, which enter in many quarkonium observables: decay widths, transitions, production cross-sections. The existence of a power counting inside the EFT selects the leading and the subleading terms in quantum-mechanical perturbation theory. Moreover, the quantum mechanical divergences (typically encountered in perturbative calculations involving iterations of the potentials, as in the case of the iterations of spin delta potentials) are absorbed by NRQCD matching coefficients. Since a factorization between the hard (in the NRQCD matching coefficients) and soft scales (in the Wilson loops or nonlocal gluon correlators) is realized and since the low energy objects are only glue dependent, confinement investigations, on the lattice and in QCD vacuum models become feasible [18, 27].

The potentials evaluated on the lattice once used in the Schrödinger equation produce the spectrum. The calculations involve only QCD parameters (at some scale and in some scheme).

7 Precision determination of Standard Model parameters

Given the advancement in the EFTs formulation and in the lattice calculations as well as the existence of several high order perturbative bound state calculations, quarkonia may become a very appropriate system for the extraction of precise determination of the Standard Model parameters like $\alpha_{\rm s}$ and the heavy quark masses. Such precise determinations are important for physics inside and beyond the Standard Model. Inside the QWG (www.qwg.to.infn.it) there is a topical group for such studies and in the QWG YR [1] there is a dedicated chapter.

7.1 c and b mass extraction

The lowest heavy quarkonium states are suitable systems to extract a precise determination of the mass of the heavy quarks b and c. Perturbative determinations of the $\Upsilon(1S)$ and J/ψ masses have been used to extract the b and c masses. The main uncertainty in these determinations comes from nonperturbative nonpotential contributions (local and nonlocal condensates) together with possible effects due to subleading renormalons. These determinations are competitive with those coming from different systems and different approaches (for the b mass see e.g. [84]). We report some recent determinations in Tab. 7.1.

A recent analysis performed by the QWG [1] and based on all the previous determinations indicates that at the moment the mass extraction from heavy quarkonium involves an error of about 50 MeV both for the bottom (1% error) and in the charm (4% error) mass. It would be very important to be able to further reduce the error on the heavy quark masses.

7.2 Determinations of $\alpha_{\rm s}$.

Heavy quarkonia leptonic and non-leptonic inclusive and radiative decays may provide means to extract α_s. The present PDG determination of α_s from bottomonium pulls down the global α_s average noticeably [1]. Recently, using the most recent CLEO data on radiative $\Upsilon(1S)$ decays and dealing with the octet contributions within weakly coupled pNRQCD, a new determination of $\alpha_{\rm s}(M_{\Upsilon(1S)}) = 0.184^{+0.014}_{-0.013}$ has been obtained [43], which corresponds to $\alpha_{\rm s}(M_Z) = 0.119^{+0.006}_{-0.005}$ in agreement with the central value of the PDG [71] and with competitive errors.

reference	order	$\overline{m}_b(\overline{m}_b)$ (GeV)
[85]	NNNLO*	$4.210 \pm 0.090 \pm 0.025$
[86]	NNLO +charm	$4.190 \pm 0.020 \pm 0.025$
[88]	NNLO	4.24 ± 0.10
[87]	NNNLO*	4.346 ± 0.070
[89]	NNNLO*	4.20 ± 0.04
[90]	NNNLO*	4.241 ± 0.070
[91]	NNLL*	4.19 ± 0.06
reference	order	$\overline{m}_c(\overline{m}_c)$ (GeV)
[79]	NNLO	1.24 ± 0.020
[88]	NNLO	1.19 ± 0.11

Table 1: Different recent determinations of $\overline{m}_b(\overline{m}_b)$ and $\overline{m}_c(\overline{m}_c)$ in the $\overline{\rm MS}$ scheme from the bottomonium and the charmonium systems. The displayed results either use direct determinations or non-relativistic sum rules. Here and in the text, the * indicates that the theoretical input is only partially complete at that order.

7.3 Top-antitop production near threshold at ILC.

In [91, 92] the total cross section for top quark pair production close to threshold in e+e- annihilation is investigated at NNLL in the weakly coupled EFT. The summation of the large logarithms in the ratio of the energy scales is achieved with the renormalization group (for correlated scales) and significantly reduces the scale dependence of the results. Studies like these will make feasible a precise extractions of the strong coupling, the top mass and the top width at a future ILC.

8 Spectra

The NRQCD Lagrangian is well suited for lattice calculations [31]. The quark propagators are the nonrelativistic ones and since the heavy-quark mass scale has been integrated out, for NRQCD on the lattice, it is sufficient to have a lattice spacing a as coarse as $m \gg 1/a \gg mv$. A price to pay is that, by construction, the continuum limit cannot be reached. Another one is that the NRQCD Lagrangian has to be supplemented by matching coefficients calculated in lattice perturbation theory, which encode the contributions from the heavy-mass energy modes that have been integrated out. A recent unquenched determination of the bottomonium spectrum with staggered sea quarks can be found in [32]. The fact that all matching coefficients of NRQCD on the lattice are taken at their tree-level value induces a systematic error of order $\alpha_s v^2$ for the radial splittings and of order α_s for the fine and hyperfine splittings.

Inside pNRQCD we have to consider separately systems with a small interquark radius (low-lying states) and systems with a radius comparable or bigger than the confinement scale $\Lambda_{\rm QCD}^{-1}$ (high-lying states). It is difficult to say to which group a specific resonance may belong, since there are no direct measurements of the interquark radius. Electric dipole transitions or quarkonium dissociation in a medium, once a well founded theory treatment of such processes will be given, may give a clear cut procedure. At the moment one uses the typical EFT approaches assuming that a particular scales hierarchy holds and checking then a posteriori that the prediction and the error estimated inside such framework are consistent with the data.

Low-lying $Q\overline{Q}$ states are assumed to realize the hierarchy: $m \gg mv \gg mv^2 \gtrsim \Lambda_{\rm QCD}$ and they may be described in weakly coupled pNRQCD.

B_c mass (MeV)				
[82] (expt)	[81] (lattice)	[80] (NNLO)	[79] (NNLO)	[86] (NNLO)
$6287 \pm 4.8 \pm 1.1$	$6304 \pm 12^{+12}_{-0}$	6326(29)	6324(22)	6307(17)

Table 2: Different perturbative determinations of the B_c mass compared with the experimental value and a recent lattice determination.

Once the heavy quark masses are known, one may use them to extract other quarkonium ground-state observables. The B_c mass has been calculated at NNLO in [79,80,86], see Table 2. These values agree well with the unquenched NRQCD/Fermilab method) lattice determination of [81], which shows that the B_c mass is not very sensitive to non-perturbative effects. This is confirmed by a recent measurement of the B_c in the channel $B_c \to J/\psi\,\pi$ by the CDF collaboration at the Tevatron; they obtain with 360 pb^{-1} of data $M_{B_c} = 6285.7 \pm 5.3 \pm 1.2$ MeV [82], while the latest available figure based on 1.1 fb^{-1} of data is $M_{B_c} = 6276.5 \pm 4.0 \pm 2.7$ MeV (see http://www-cdf.fnal.gov/physics/new/bottom/060525.blessed-bc-mass/).

The bottomonium and charmonium ground-state hyperfine splitting has been calculated at NLL in [93]. Combining it with the measured $\Upsilon(1S)$ mass, this determination provides a quite precise prediction for the η_b mass: $M_{\eta_b} = 9421 \pm 10^{+9}_{-8}$ MeV, where the first error is an estimate of the theoretical uncertainty and the second one reflects the uncertainty in $\alpha_{\rm s}$. Note that the discovery of the η_b may provide a very competitive source of $\alpha_{\rm s}$ at the bottom mass scale with a projected error at the M_Z scale of about 0.003. Similarly, in [94], the hyperfine splitting of the B_c was calculated at NLL accuracy: $M_{B_c^*} - M_{B_c} = 65 \pm 24^{+19}_{-16}$ MeV.

High-lying $Q\overline{Q}$ states are assumed to realize the hierarchy: $m \gg mv \sim \Lambda_{\rm QCD} \gg mv^2$. A first question is where the transition from low-lying to high-lying takes place. This is not obvious, because we cannot measure directly mv. Therefore, the answer can only be indirect and, so far, there is no clear agreement in the literature. A weak-coupling treatment for the lowest-lying bottomonium states ($n = 1$, $n = 2$ and also for the $\Upsilon(3S)$) appears to give positive results for the masses at NNLO in [79] and at N^{3}LO* in [37]. The result is more ambiguous for the fine splittings of the bottomonium $1P$ levels in the NLO analysis of [42] and positive only for the $\Upsilon(1S)$ state in the N^{3}LO* analysis of [38].

Masses of high-lying quarkonia may be accessed either using the lattice nonperturbative potentials inside a Schrödinger equation [33] or via a direct lattice pNRQCD calculation.

9 Transitions and decays

9.1 Inclusive Decays

NRQCD gives a factorization formula for heavy quarkonium inclusive decay widths, precisely it factorizes four-fermion matching coefficients and matrix elements of four fermion operators [10]. Color singlet operator expectation values may be easily related to the square of the quarkonium wave functions (or derivatives of it) at the origin. Octet contributions remain as nonperturbative matrix elements of operators evaluated over the quarkonium states. In some situations the octet contributions may not be suppressed and become as relevant as the singlet contributions in the NRQCD power counting. In particular octet contributions may reabsorb the dependence on the infrared cut-off of the Wilson coefficients, solving the problem that arised originally in the color singlet potential model [61].

Systematic improvements are possible, either by calculating higher-order corrections in the coupling constant or by adding higher-order operators.

In order to describe electromagnetic and hadronic inclusive decay widths of heavy quarkonia, many NRQCD matrix elements are needed. The specific number depends on the order in v of the non-relativistic expansion to which the calculation is performed and on the power counting. At order mv^5 and within a conservative power counting, S- and P-wave electromagnetic and hadronic decay widths for bottomonia and charmonia below threshold depend on 46 matrix elements [34]. More are needed at order mv^7 [58–60]. Order mv^7 corrections are particularly relevant for P-wave quarkonium decays, since they are numerically as large as NLO corrections in α_s, which are known since long time [61] and to which the most recent data are sensitive [1, 62]. NRQCD matrix elements may be fitted to the experimental decay data [63, 64] or calculated on the lattice [65–67]. The matrix elements of color-singlet operators are related at leading order to the Schrödinger wave functions at the origin [10] and, hence, may be evaluated by means of potential models [68] or potentials calculated on the lattice [17]. However, a great part of the matrix elements remain poorly known or unknown.

In the matching coefficients large contributions in the perturbative series coming from bubble-chain diagrams may need to be resummed [69].

In lattice NRQCD in [32], the ratio $\Gamma(\Upsilon(2S) \to e^+e^-)/\Gamma(\Upsilon(1S) \to e^+e^-) \times M^2_{\Upsilon(2S)}/M^2_{\Upsilon(1S)}$ has been calculated. The result on the finest lattice compares well with the experimental one.

The imaginary part of the potential provides the NRQCD decay matrix elements in pNRQCD. For excited states, they typically factorize in a part, which is the wave function in the origin squared (or its derivatives), and in a part which contains gluon tensor-field correlators [34, 54–56]. This drastically reduces the number of non-perturbative parameters needed; in pNRQCD, these are wave functions at the origin and universal gluon tensor-field correlators, which can be calculated on the lattice. Another approach may consist in determining the correlators on one set of data (e.g. in the charmonium sector) and use them to make predictions for another (e.g. in the bottomonium sector). Following this line in [54,57], at NLO in α_s, but at leading order in the velocity expansion, it was predicted $\Gamma_{\rm had}(\chi_{b0}(2P))/\Gamma_{\rm had}(\chi_{b2}(2P)) \approx 4.0$ and $\Gamma_{\rm had}(\chi_{b1}(2P))/ \Gamma_{\rm had}(\chi_{b2}(2P)) \approx 0.50$. Both determinations turned out to be consistent, within large errors, with the CLEO III data [1]. One should notice that at some order of the expansion in v, the scale $\sqrt{m\Lambda_{\rm QCD}}$ start also to contribute in pNRQCD jeopardizing in some cases the effective reduction of the nonperturbative operators [55].

For the lowest resonances, inclusive decay widths are given in weakly coupled pNRQCD by a convolution of perturbative corrections and nonlocal nonperturbative correlators. The perturbative calculation embodies large contributions and requires large logs resummation. The ratio of electromagnetic decay widths was calculated for the ground state of charmonium and bottomonium at NNLL order in [75]. In particular, they report: $\Gamma(\eta_b \to \gamma\gamma)/\Gamma(\Upsilon(1S) \to e^+e^-) = 0.502 \pm 0.068 \pm 0.014$, which is a very stable result with respect to scale variation. A partial NNLL* order analysis of the absolute width of $\Upsilon(1S) \to e^+e^-$ can be found in [76].

9.2 Electromagnetic transitions

Allowed magnetic dipole transitions between charmonium and bottomonium ground states have been considered in pNRQCD at NNLO in [40, 70]. The results are: $\Gamma(J/\psi \to \gamma\,\eta_c) = (1.5 \pm 1.0)$ keV and $\Gamma(\Upsilon(1S) \to \gamma\,\eta_b) = (k_\gamma/39\ \text{MeV})^3\ (2.50 \pm 0.25)$ eV, where the errors account for uncertainties (which are large in the charmonium case) coming from higher-order corrections. The width $\Gamma(J/\psi \to \gamma\,\eta_c)$ is consistent with [71]. Concerning $\Gamma(\Upsilon(1S) \to \gamma\,\eta_b)$, a photon energy $k_\gamma = 39$ MeV corresponds to a η_b mass of 9421 MeV. The pNRQCD calculation features a small quarkonium magnetic moment (in agreement with a recent lattice calculation [78]) and the interesting fact, related to the Poincaré invariance of the nonrelativistic EFT [16], that M1 transition of the lowest quarkonium states at relative order v^2 are completely accessible in perturbation theory [40].

In the weak-coupling regime, the magnetic-dipole hindered transition $\Upsilon(2S) \to$

$\gamma\,\eta_b$ at leading order [40] does not agree with the experimental upper bound [39]. It should be still clarified if this is related to the fact that $\Upsilon(2S)$ system belongs to the strong coupling regime or if it is due to large corrections (more relevant in the hindered case).

9.3 Semi-inclusive decays

The radiative transition $\Upsilon(1S) \to \gamma\, X$ has been considered in [72,73]. The agreement with the CLEO data of [74] is very satisfactory when one properly includes the octet contribution in pNRQCD [73]. In the same work it is found that the ratios for different n of the radiative decay widths $\Gamma(\Upsilon(nS) \to \gamma\, X)$ are better consistent with the data if $\Upsilon(1S)$ is assumed to be a weakly-coupled bound state and $\Upsilon(2S)$ and $\Upsilon(3S)$ strongly coupled ones [41].

In general in exclusive decays and for certain kinematical end points of semi-inclusive decays, NRQCD or pNRQCD should be supplemented by collinear degrees of freedom. This can be realized in the framework of Soft Collinear Effective Theory (SCET) [95].

10 Baryons with two or more heavy quarks

The SELEX collaboration at Fermilab reported evidence for five resonances that may be possibly identified with doubly charmed baryons, see the presentation of Jurgen Engelfried at this meeting [2] and [44]. Although these results have not been confirmed by other experiments (FOCUS, BELLE and BABAR) they have triggered a renewed theoretical interest in doubly heavy baryon systems.

Low-lying QQq states may be assumed to realize the hierarchy: $m \gg mv \gg \Lambda_{\rm QCD}$, where mv is the typical inverse distance between the two heavy quarks and $\Lambda_{\rm QCD}$ is the typical inverse distance between the centre-of-mass of the two heavy quarks and the light quark. At a scale μ such that $mv \gg \mu \gg \Lambda_{\rm QCD}$ the effective degrees of freedom are QQ states (in color antitriplet and sextet configurations), low-energy gluons and light quarks. The most suitable EFT at that scale is a combination of pNRQCD and HQET [45,46]. The hyperfine splittings of the doubly heavy baryon lowest states have been calculated at NLO in $\alpha_{\rm s}$ and at LO in $\Lambda_{\rm QCD}/m$ by relating them to the hyperfine splittings of the D and B mesons (this method was first proposed in [47]). In [45], the obtained values are: $M_{\Xi^*_{cc}} - M_{\Xi_{cc}} = 120 \pm 40$ MeV and $M_{\Xi^*_{bb}} - M_{\Xi_{bb}} = 34 \pm 4$ MeV, which are consistent with the quenched lattice determinations of [48, 49] and the quenched NRQCD lattice determinations of [50, 51]. Chiral corrections to the doubly heavy baryon masses, strong decay widths and electromagnetic decay widths have been considered in [52].

Also low-lying QQQ baryons can be studied in a weak coupling framework. Three quark states can combine in four color configurations: a singlet, two octets and a decuplet, which lead to a rather rich dynamics [45]. Masses of various QQQ ground states have been calculated with a variational method in [53]: since baryons made of three heavy quarks have not been discovered so far, it may be important for future searches to remark that the baryon masses turn our to be lower than those generally obtained in strong coupling analyses.

For QQQ baryons with a typical distance of the order $\Lambda_{\rm QCD}$ inverse, the form of the static, $1/m$ and spin dependent nonperturbative potentials have been obtained in pNRQCD [45]. Up to now only the static potential has been evaluated on the lattice [17, 77].

11 Gluelump spectrum and exotic states

Gluelumps are states formed by a gluon and two heavy quarks in a octet configuration at small interquark distance [97] . The mass of such nonperturbative objects are typically measured on the lattice. The tower of hybrids static energies [96] measured in lattice NRQCD reduces to the gluelump masses for small interquark distances. In pNRQCD [8, 26] the full structure of the gluelump spectrum has been studied, obtaining model independent predictions on the shape, the pattern, the degeneracy and the multiplet structure of the hybrid static energies for small $Q\overline{Q}$ distances that well match and interpret the existing lattice data. These studies may be important both to elucidate the confinement mechanism (the gluelump masses control the behaviour of the nonperturbative glue correlators appearing in the spectrum and in the decays) and in relation to the exotic states recently observed at the B-factories. The $Y(4260)$ in the charmonium sector may be identified with an hybrid state inside such picture. A complete pNRQCD description of heavy hybrids is still missing.

12 Production

Before the advent of NRQCD, colour singlet production and colour singlet fragmentation underestimated the data on prompt quarkonium production at Fermilab by about an order of magnitude indicating that additional fragmentation contributions were missing [83]. The missing contribution was precisely the gluon fragmentation into colour-octet ${}^3S_1$ charm quark pairs. The probability to form a J/ψ particle from a pointlike $c\bar{c}$ pair in a colour octet ${}^3S_1$ state is given by a NRQCD nonperturbative matrix element which is suppressed by v^4 with respect to to the leading singlet term but is enhanced by two powers of $\alpha_{\rm s}$ in the short distance matching coefficient for producing colour-octet quark pairs. Introducing the leading colour-octet contributions, the data of CDF could be reproduced and this has been an important result of NRQCD [83]. NRQCD factorization has proved to be very successful to explain a large variety of quarkonium production processes (for a review see the production chapter in [1] and [5]). A formal proof of the NRQCD factorization formula for heavy quarkonium production has however not yet been obtained. Recently, there has been important work in the direction of an all order proof in [98, 99]. In particular, it has been shown that a necessary condition for factorization to hold at NNLO is that the conventional octet NRQCD production matrix elements have to be redefined with Wilson lines, acquiring manifest gauge invariance.

For production, a pNRQCD formulation is not yet existing. In principle to go through a further factorization also in production, if at all possible, may reduce the number of nonperturbative matrix elements and enhance the predictive power.

Two outstanding problems exist at the moment in quarkonium production: double charmonium production in e^+e^- collisions and charmonium polarization at the Tevatron.

In [100], BELLE reports $\sigma(e^+e^- \to J/\psi+\eta_c)\,\mathrm{Br}(c\bar{c} \to > 2 \text{ charged}) = 25.6 \pm 2.8 \pm 3.4$ fb and in [101], BABAR reports $\sigma(e^+e^- \to J/\psi + \eta_c)\,\mathrm{Br}(c\bar{c} \to > 2 \text{ charged}) = 17.6 \pm 2.8^{+1.5}_{-2.1}$ fb. Originally these data were about one order of magnitude above theoretical expectations. Recently, with some errors corrected in some of the theoretical determinations, NLO corrections in α_s calculated in [102] and higher-order v^2 corrections obtained in [103], the theoretical prediction has moved closer to the experimental one. In [104], a preliminary estimate of $\sigma(e^+e^- \to J/\psi + \eta_c)$ including the above corrections gives 16.7 ± 4.2 fb.

Still open is the problem of the BELLE measurement $\sigma(e^+e^- \to J/\psi+c\bar{c})/\sigma(e^+e^- \to J/\psi+X)$, about 80%, with respect to theory calculation that gives about 10% (see [1] for a detailed discussion).

Charmonium polarization has been measured at the Tevatron by the CDF collaboration at run I with 110 pb^{-1} [105] and recently at run II with 800 pb^{-1} [106]. The data of the two runs are not consistent with each other in the 7-12 GeV region of transverse momentum, p_T, and both disagree with the NRQCD expectation of an increased polarization with increased p_T. For large p_T, NRQCD predicts that the main mechanism of charmonium production is via color-octet gluon fragmentation, the gluon being transversely polarized and most of the gluon polarization being transferred to the charmonium. The CDF data do not show any sign of transverse polarization at large p_T.

A solution to such problem may be obtained in the case in which a nonperturbative power counting is valid [34, 35].

13 Challenges

For what concerns systems close or above the open flavor threshold, a complete and satisfactory understanding of the dynamics has not been achieved so far and a corresponding general NR EFT has not yet been constructed. Such systems are difficult to address also with a lattice calculation. Hence, the study of these systems is on a less secure ground than the study of states below threshold. Although in some cases one may develop an EFT owing to special dynamical conditions (as for the $X(3872)$ interpreted as a loosely bound $D^0\,\overline{D}^{*\,0} + \overline{D}^0\,D^{*\,0}$ molecule [107]), the study of these systems largely relies on phenomenological models [108, 109]. The major theoretical challenge here is to interpret the new states in the charmonium region discovered at the B-factories in the last few years.

Heavy ion experiments use quarkonium suppression as one of the smoking guns for quark-gluon plasma formation (cf. e.g. the media chapter in [1]). . To describe quarkonium suppression it would be important to formulate an EFT for heavy quarkonium in media and to obtain a clear definition of the heavy quark potential at finite T . Preliminary studies are ongoing with several approaches [110].

With a good control in theory and high statistic data sample available at present and future (Super-B factory) experiments, heavy quarkonia may also supply us with an alternative way of looking for new physics BSM, cf. [111] and the BSM chapter in [1].

14 Outlook

Today NR EFTs and lattice calculations allow us to investigate a wide range of heavy quarkonium observables in a controlled and systematic fashion and, therefore, learn about one of the most elusive sectors of the Standard Model: low-energy QCD.

Predictions based on non-relativistic EFTs are conceptually solid, and systematically improvable. EFTs have put quarkonium on the solid ground of QCD: quarkonium becomes a privileged window for precision measurements, new physics and confinement mechanism investigations.

Many new data on heavy-quark bound states are provided in these years by the B-factories, CLEO and the Tevatron experiments. Many more will come in the near future from BES-III, LHC and later Panda at GSI. They will show new (perhaps exotic) states, new production and decay mechanisms and they will be a great arena for new EFT tools.

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