Using Nuisance Parameters To Model Centering Uncertainty

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Centering Uncertainty

- **The imprecision involved with aligning an instrument or target on/over a survey reference point.**
- **For instruments, may be largely avoided by free stationing and resection techniques.**
- **Still plays a role in the use of vertical sighting pipes.**

The traditional approach

- **Centering uncertainty has traditionally been handled by inflating the uncertainties of the associated observations.**
- **This approach is described in recent textbooks. Other approaches not found.**
- **This approach is applied (universally?) by commercially-available adjustment software.**

Example: two distances from station A

Traditional distance observation equations:

$$Dist_{A\_B} = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2} + e_{A\_B}$$
$$Dist_{A\_C} = \sqrt{(x_C - x_A)^2 + (y_C - y_A)^2 + (z_C - z_A)^2} + e_{A\_C}$$

Corresponding part of stochastic model:

$$\sigma^2_{DistA\_B} = \sum \sigma^2_{Dist-related\_A\_B} + \sigma^2_{Centering\_A} + \sigma^2_{Centering\_B}$$
$$\sigma^2_{DistA\_C} = \sum \sigma^2_{Dist-related\_A\_C} + \sigma^2_{Centering\_A} + \sigma^2_{Centering\_C}$$

Distances AB and AC treated as if uncorrelated.

Problem with traditional model

- **Mis-centering is NOT independent for each observation.**
- **Traditional stochastic model does not account for non-zero covariances.**
- **Traditional model makes propagated uncertainties too pessimistic.**

How to fix this?

- **Could try to add covariances to the traditional centering model. (complicated!)**

How to fix this?

- **Alternatively, treat each independent centering of an instrument / target as a new point to be estimated in the adjustment (nuisance parameters).**
- **Create a new class of observations ("centerings") to relate these nuisance parameters to the coordinates of the actual monument.**

Example: two distances from station A

New observation equations:

$$Dist_{A1\_B1} = \sqrt{(x_{B1} - x_{A1})^2 + (y_{B1} - y_{A1})^2 + (z_{B1} - z_{A1})^2} + e_{A1\_B1}$$

$$Dist_{A1\_C1} = \sqrt{(x_{C1} - x_{A1})^2 + (y_{C1} - y_{A1})^2 + (z_{C1} - z_{A1})^2} + e_{A1\_C1}$$

$$Centering\_x_{A1}\_x_A = 0 = x_{A1} - x_A + e_{xA1\_A}$$

$$Centering\_y_{A1}\_y_A = h_{A1\_A} = y_{A1} - y_A + e_{yA1\_A}$$

$$Centering\_z_{A1}\_z_A = 0 = z_{A1} - z_A + e_{zA1\_A}$$

$$Centering\_x_{B1}\_x_B = 0 = x_{B1} - x_B + e_{xB1\_B}$$

$$Centering\_y_{B1}\_y_B = h_{B1\_B} = y_{B1} - y_B + e_{yB1\_B}$$

$$Centering\_z_{B1}\_z_B = 0 = z_{B1} - z_B + e_{zB1\_B}$$

$$Centering\_x_{C1}\_x_C = 0 = x_{C1} - x_C + e_{xC1\_C}$$

$$Centering\_y_{C1}\_y_C = h_{C1\_C} = y_{C1} - y_C + e_{yC1\_C}$$

$$Centering\_z_{C1}\_z_C = 0 = z_{C1} - z_C + e_{zC1\_C}$$

Example: two distances from station A

Corresponding part of stochastic model:

$$\sigma^2_{DistA1\_B1} = \sum \sigma^2_{Dist-related\_A1\_B1}$$
$$\sigma^2_{DistA1\_C1} = \sum \sigma^2_{Dist-related\_A1\_C1}$$

$$\sigma^2\_Centering\_x_{A1}\_x_A = k_h{}^2$$
$$\sigma^2\_Centering\_y_{A1}\_y_A = k_y{}^2$$
$$\sigma^2\_Centering\_z_{A1}\_z_A = k_h{}^2$$

$$\sigma^2\_Centering\_x_{B1}\_x_B = k_h{}^2$$
$$\sigma^2\_Centering\_y_{B1}\_y_B = k_y{}^2$$
$$\sigma^2\_Centering\_z_{B1}\_z_B = k_h{}^2$$

$$\sigma^2\_Centering\_x_{C1}\_x_C = k_h{}^2$$
$$\sigma^2\_Centering\_y_{C1}\_y_C = k_y{}^2$$
$$\sigma^2\_Centering\_z_{C1}\_z_C = k_h{}^2$$

Conclusion

The traditional treatment of centering uncertainty does not account for correlations among the associated surveying observations.

These correlations may be handled implicitly by introducing nuisance parameters to represent each independent centering of an instrument or target, and by explicitly recognizing the centering operation as a set of observations.