

# Eliminating Collimation Error in Area Leveling

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## Abstract

Performing differential leveling to a large number of points from a single level setup is sometimes called "area leveling." The horizontal distances from the instrument to the various points of interest may be grossly unequal; therefore the level's collimation error will not cancel during computation of the points' elevation differences. However, if two appropriately-positioned level setups are used to observe the same group of points, then the collimation errors of both levels can be estimated as parameters in a least squares adjustment. The resulting collimation corrections can have much better precision than those derived with the traditional "two-peg test."

## The Problem

In differential leveling, the effect of a level's collimation error can be almost entirely eliminated by carefully balancing the foresight length with the backsight length. However, in some settlement monitoring studies, the sheer number of points (or their close spacing) makes such balancing impractical. In this case, a large number of points may be observed from a single setup of the level ("area leveling"), but the resulting unbalanced sight lengths render the procedure vulnerable to collimation error.

Collimation error is the largest potential source of systematic error for area leveling with unbalanced sight lengths, even with geodetic-quality levels. The Leica NA3003 User Manual states the "the standard deviation of a collimation error determined once only under normal atmospheric conditions is about 2 seconds."

Even when the collimation correction is accurately determined by the traditional two-peg test prior to the start of work, the collimation error is subject to change from effects such as temperature shifts and mechanical shock. Repeated determination of collimation error throughout the day is time consuming.

## A Solution

If two levels are simultaneously used to observe the same group of monitoring points (or if one level is used from two positions), then the collimation errors of both levels may be treated as independent unknown parameters to be estimated in a least squares adjustment, along with the heights of the monitoring points.

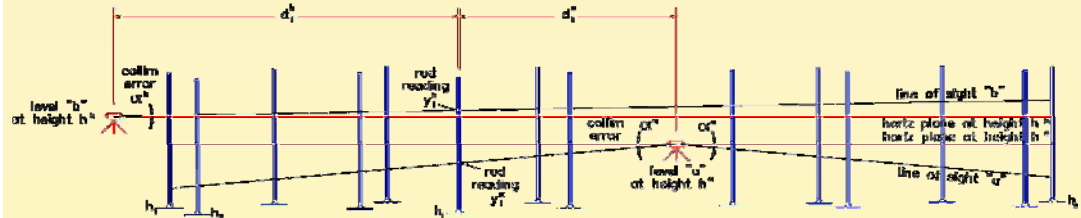
The resulting collimation corrections can have much better precision than those derived from the traditional two-peg test. In a recent settlement monitoring survey in the SNS RTBT tunnel, formal uncertainty propagation showed that collimation corrections were determined with an uncertainty of  $\pm 0.3$  to  $0.4$  arcseconds (1-sigma).

In addition to improved precision, these estimated collimation corrections are also likely to be more accurate than those derived by the traditional two-peg test. They are guaranteed to represent the same thermal state of the level as the observations to which they are being applied. And unlike the pre-work peg test, there exists no chance for mechanical shock to change the collimation during transport to the setup.

The positions of the levels must be chosen with some care. The best results are obtained by placing one level near the center of the points to be monitored, and the other level near the perimeter of the points. Avoid placing the two levels close to each other, or at opposite sides of the perimeter.

For settlement monitoring of long sections of tunnel, the procedure can be performed on successive segments of the tunnel, overlapping two points of the previous section each time.

If all rod readings are corrected for earth curvature, then the horizontal planes through each level may be represented as if they were parallel to each other. Despite the appearance of the sketch, there is no requirement that the points fall along a line; the procedure is just as valid for a square grid of points.



The rod readings observed from instrument "a" may be expressed as a set of  $n$  observation equations:

$$\begin{aligned} y_1^a &= h^a + d_1^a \tan \alpha^a - h_1 + e_1^a \\ y_2^a &= h^a + d_2^a \tan \alpha^a - h_2 + e_2^a \\ &\vdots \\ y_n^a &= h^a + d_n^a \tan \alpha^a - h_n + e_n^a \end{aligned}$$

Similarly, the rod readings observed from instrument "b" may be expressed as a set of  $n$  observation equations:

$$\begin{aligned} y_1^b &= h^b + d_1^b \tan \alpha^b - h_1 + e_1^b \\ y_2^b &= h^b + d_2^b \tan \alpha^b - h_2 + e_2^b \\ &\vdots \\ y_n^b &= h^b + d_n^b \tan \alpha^b - h_n + e_n^b \end{aligned}$$

With this pair of substitutions, the observation equations become linear in the parameters to be estimated:

$$\begin{aligned} c^a &= \tan \alpha^a \\ c^b &= \tan \alpha^b \end{aligned}$$

Combining the two sets of observation equations, and converting to matrix form, we have:

$$\begin{bmatrix} y^a \\ y^b \end{bmatrix}_{n \times 1} = \begin{bmatrix} -\mathbf{I}_{n \times n} & \mathbf{1}_{n \times 1} & \mathbf{0}_{n \times 1} & \mathbf{d}^a_{n \times 1} & \mathbf{0}_{n \times 1} \\ -\mathbf{I}_{n \times n} & \mathbf{0}_{n \times 1} & \mathbf{1}_{n \times 1} & \mathbf{0}_{n \times 1} & \mathbf{d}^b_{n \times 1} \end{bmatrix} \begin{bmatrix} h \\ h^a \\ h^b \\ c^a \\ c^b \end{bmatrix}_{n \times 1} + \begin{bmatrix} \mathbf{e}^a \\ \mathbf{e}^b \end{bmatrix}_{n \times 1}$$

The rank deficiency must be eliminated by defining a datum. An easy way to accomplish this is to select the column of the design matrix that corresponds to the desired benchmark, and delete that whole column. This has the effect of defining that point's elevation to be zero as a datum for the rest of the heights.

In order to find the weight matrix for the observations, the variances of the observations must be defined. The standard deviations of the rod readings can be reasonably modeled as a linear function of the sight distance. A conservative model for the Leica NA3003 is  $0.005 \text{ mm} + 2 \text{ ppm}$ .

Then weighted least squares is used to simultaneously solve for the heights of the monitoring points, the heights of the two levels, and the collimation error of the two levels.