Simulation of the LiCAS Survey System for the ILC

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The LiCAS\(^1\) metrology system operating inside the Rapid Tunnel Reference Surveyor (RTRS) is a novel instrument dedicated to align and monitor the mechanical stability of linear accelerator tunnel. Two modern measurement techniques: Laser Straightness Monitors (LSM) and Frequency Scanning Interferometry (FSI) \([1, 2]\) for distance measurement will deliver accuracy unreachable for classical optical metrology. Detailed simulations of a LiCAS system including position reconstruction and analysis of the error propagation for the reference markers along the accelerator tunnel are presented. We demonstrate that the vertical accuracy of the order of \(O(200) \mu m\) over 600 m matching the TESLA \([3]\) specification is feasible.

1. PRINCIPLE OF THE LICAS-RTRS TRAIN OPERATION

Technical details of the LiCAS-RTRS train were introduced in separate presentations \([4, 5]\). In figure 1 the sensing parts of the LiCAS cars are presented. The train is composed of 6 cars, the distance between the centres of neighbouring cars is \(\sim 4.5\) m. Each car is equipped with 4 CCD cameras and two beam splitters (BS) constituting the straightness monitor. The straightness monitor measures the transverse translation \((T_x, T_y)\) and transverse rotation \((R_x, R_y)\) with respect to a \(z\) axis defined by the laser beam passing through all cars in a vacuum pipe. The laser beam is reflected back using the retro-reflector (RR) located in the last car, illuminating the upper CCD cameras of the straightness monitors. 6 FSI lines placed in the same vacuum pipe between each pair of cars are responsible for the distance measurement along the \(z\) axis \((T_z)\). In addition a clinometer located on each car provides a measurement of rotation around the \(z\) axis \((R_z)\). In figure 2 the schematic view of the LiCAS train operating in the accelerator tunnel was shown. When the train stops in front of the wall markers it firstly measures the relative position and rotation of all cars with respect to the first car. This defines the local reference frame of the train in which the location of the wall mounted reference markers are measured next. This procedure is repeated for each train stop. Each marker is measured up to 6 times. Finally the coordinates of each marker, expressed in the local train frames are transformed to the frame of the first train (the global frame) by fitting them to each other under the constraint that wall markers have not moved during the entire measurement.

2. SOFTWARE USED IN THE ANALYSIS

In order to model the LiCAS train operating inside the accelerator tunnel and to study the expected performance of this device a detailed numerical simulation has been performed. It consists of the description of the geometry of the system, ray tracer emulating the actual measurements, position reconstruction, LiCAS train Monte Carlo and error propagation package.

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Figure 1: Mechanical design of the measurement units for the LiCAS train. The sensing parts of each car are composed of 4 CCD cameras and 2 beam splitters (BS) constituting the straightness monitor and 6 internal FSI lines to measure the distance between the cars. The 6 external FSI lines pointing towards the tunnel wall measure the position of wall markers. In addition each car is equipped with a clinometer (not shown).

Figure 2: Principle of the LiCAS train operation. Top view of the two train stops along the accelerator tunnel are presented. For detailed description see text.
2.1. Definition of the Geometry and Ray Tracer

This part of the package is responsible for preparing input for further calculations by generating positions and angles of active objects inside the LiCAS train: CCD cameras, Beam Splitters (BS) and laser beam for straightness monitor, internal and external FSI lines for distance measurements (INTFSI, EXTFSI) including light sources and retroreflectors. The tunnel wall is also populated by equidistant wall markers. For each geometrical object its nominal position is defined together with the “deltas” – small random correction to make the model more realistic by introducing non-trivial calibration constants. The positions and rotation matrixes of various objects are subsequently transformed from local car frames to higher level frames of each train and the global tunnel frame. An additional randomisation at this stage is also possible accounting for the arbitrary position and orientation of the cars at particular train stops.

Having defined the positions and angles of all participating objects the actual measurements are simulated using the straight line ray tracing (light scattering off the beam splitters and CCD measurements) as well as the distance measurements of the FSI lines. The measurements are smeared by the resolution of the corresponding sensors.

2.2. Reconstruction, Error Propagation and Train Monte Carlo

In order to study the expected precision on the position reconstruction of the tunnel reference markers the Simulgeo [6] package was used which allows for modelling of the opto-geometrical systems. This programme can be use in two different modes: (i) reconstruction and (ii) error propagation. In the first mode of operation it solves the unknown parameters of the system (positions and angles) having provided the geometry and measurements of sensing devices. This step is performed by minimising procedures. In the second mode Simulgeo is also capable of performing the full error propagation including correlations between various sub-components linked via common mechanical supports. This step is based on the matrix operation and the analytical approach to propagation of small errors.

Since the process of measurement generation and reconstruction is fully automated it is also possible to use Simulgeo in the train Monte Carlo by solving the unknown positions of markers in many repeated “train journeys” along the tunnel varying each time the measurements. Such an approach allows to explore the full distribution of the errors under study. It provides also a tool to investigate the systematics effects in the reconstruction trying to solve the system assuming different (miscalibrated) geometry with respect to the true geometry used in the ray tracer for the generation of measurements.

3. RESULTS OF THE TRAIN SIMULATIONS

Figure 3 (left column) contains the Simulgeo results obtained for the simulation of a single train stop. Expected precision on the car positions, car angles and wall markers positions are plotted as a function of car number. The errors on the $z$ position of the cars are growing approximately like $\sqrt{n}$ and the transverse translation errors are constant. This is because the precision of the instruments on all individual cars is identical and the transverse position is measured with respect to the same laser beam which defines the $z'$-axis of the local frame of the train. Car rotation angle around the $z$-axis (tunnel axis) also has constant precision as expected from the independent clinometers located on each car. The above results were obtained assuming the intrinsic resolution of the CCD cameras and FSI lines equal to $\sigma_{CCD} = \sigma_{FSI} = 1 \mu m$. The assumed precision of the clinometer was $\sigma_{\text{tilt}} = 1 \mu rad$. The above simulation was performed under the assumption that all calibration constants (positions and rotations of CCD cameras, FSI light sources and retro-reflectors) are known to the accuracy of $\sigma_{pos} = 1 \mu m$ for positions and $\sigma_{ang} = 1 \mu rad$ for angles.

The long-distance operation of the train inside the accelerator tunnel was simulated by a set of many identical trains displaced by 4.5 $m$ (distance between stops), each pair of them coupled via 5 overlapping wall markers. The results obtained for a distance of 90 $m$ (20 train stops) are presented in figure 3 (right column). The same assumptions
Figure 3: Results of the Simulgeo simulation for the single LiCAS train (left column) and for the 20 stops of overlapping trains (right column). The resolution on $X, Y, Z$ position and rotation angles $A, B, C$ around $x, y, z$ axis for as the function of car or train stop number are plotted. The last row presets the expected precision on the position reconstruction of the wall markers.
Figure 4: SIMULGEO error propagation results (open markers) compared to RMS values obtained from Monte Carlo simulation for 1000 trains (solid markers). Simplified model of LiCAS cars was used neglecting errors on calibration constants.

as for the single stop simulation were made about the intrinsic precision of CCD cameras, FSI measurements and about the accuracy of the calibration constants.

Figure 4 presents comparison of the results for matrix error propagation and the Monte Carlo approach for 1000 trains measuring the same set of wall markers. Because of the numerical complexity of the problem a simplified model of the LiCAS car was used neglecting errors on calibration constants (ie. assuming the only source of experimental uncertainties comes form the CCD and FSI distance measurements). Both methods provides results which are in good agreement within statistical errors of the Monte Carlo method. In addition in figure 5 full residua distribution of the wall markers are presented (reconstructed-generated). No systematic bias is visible and the errors are distributed according to Gaussian distribution as compared with the fitted curves.

SIMULGEO calculations provide very precise results (taking into account correlations between subcomponents of the system) based on the exact opto-geometrical model of the survey procedure. However, from the numerical point of view, such an approach, manipulating large matrices, is very time and memory consuming. The 20 train stop results (90 m tunnel section) were obtained after 34 hours of CPU time using 1 GB RAM memory on a 2 GHz machine (the rank of the used matrix was of the order of 10 000). The numerical complexity of these calculations scale like \( N^2 \), where \( N \) is the number of involved coordinates. The simulation of the full 600 m tunnel section would require more then 7 weeks of CPU time.

### 3.1. Random Walk Model

To overcome the above mentioned limitations a simplified analytical formula inspired by a random walk model was derived to extrapolate the SIMULGEO predictions over long tunnel sections:

\[
\sigma_{xy,n} = \sqrt{l^2 \sigma_\alpha^2 \frac{n(n+1)(2n+1)}{6} + \sigma_{xy}^2 \frac{n(n+1)}{2}}, \quad \sigma_{z,n} = \sqrt{\sigma_z^2 \frac{n(n+1)}{2}}
\]

where \( n \) is the wall marker number, \( l \) is the effective length of the ruler (here: distance between cars), and the corresponding errors are the parameters of the random walk: \( \sigma_\alpha \) is the angular error, \( \sigma_{xy} \) are the transverse errors and \( \sigma_z \) is the longitudinal error. In this approach the procedure of accelerator alignment resembles the construction of a long straight line using short ruler. The overall error is a convolution of the precision of the ruler and the precision of the placement of the ruler with respect to the previous measurement. The asymptotic behaviour of the formulae from equation no. 1 is: \( \sigma_{xy,n} \sim n^{\frac{3}{2}} \), and \( \sigma_{z,n} \sim n \). This fast growth of errors (especially for transverse directions) is a consequence of the fact that the errors are highly correlated and the precision of the \( n^{th} \) element depends on the precision of all previous points.
Formulae 1 were fitted to the Simulgeo points determining $\sigma_x$, $\sigma_{xy}$, $\sigma_z$ and then extrapolated over a 600 m tunnel section (fig. 6). The obtained predictions refer to the precision of the placement of the $n^{th}$ accelerator component with respect to the first one. However, this is not the ultimate measure of the quality of the accelerator alignment. The relevant parameter is the mean deviation of each component from the ideal straight line which can be expected from the above procedure. To obtain the final prediction on the deviation of the alignment from the straight line a series of random walk trajectories was generated using the parameters fitted to the Simulgeo points (fig. 7 left column). A straight line was fitted to each trajectory and the corresponding residua were calculated. The extracted RMS values of the residua distributions for each marker along 600 m provide the measure of the accuracy of the whole procedure. Because of high correlation between errors for $n$ and $n+1$ marker the generated trajectories exhibit much smaller oscillations that would be expected from completely random process. Figure 7 (right column) summarises the results obtained in this analysis demonstrating that the vertical precision of the order of $O(100 \mu m)$ over 600 m is feasible. Interesting feature of our model is the fact that $X$ errors are smaller than $Y$ errors. This is because of the higher sensitivity of $Y$ errors to the car rotation around $Z$ axis which does not influence so much $X$ coordinate (where the precision is mainly dominated by the FSI distance measurements). It is worth to stress out that the precision for $X-Y$ position reconstruction can be swapped by changing the position of the wall marker from horizontal to vertical location or by placing two wall markers in front of each LiCAS car.
Figure 6: LiCAS train simulation: insertion contains the results of the exact simulation obtained with the Simulgeo for 20 train stops (90 m), extrapolated on the main plot to the 600 m tunnel section using the formula from the random walk model.

Figure 7: Examples of the random walk trajectories (left upper plot) and straight line fit to selected trajectory (left lower plot). RMS of the residua distribution from the straight line fit to the random walk trajectories: horizontal (right upper plot) and vertical direction (right lower plot).
References


