

WHiZard & O'Mega

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LoopFest V
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- **phase space** parametrization
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- **interfacing** to the rest of the world

$\mathcal{L}$, Feynman rules

parameters

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 $\sigma(s, \text{cuts})$

histograms

unweighted
events
(LHA#1)

$\mathcal{L}$, Feynman rules

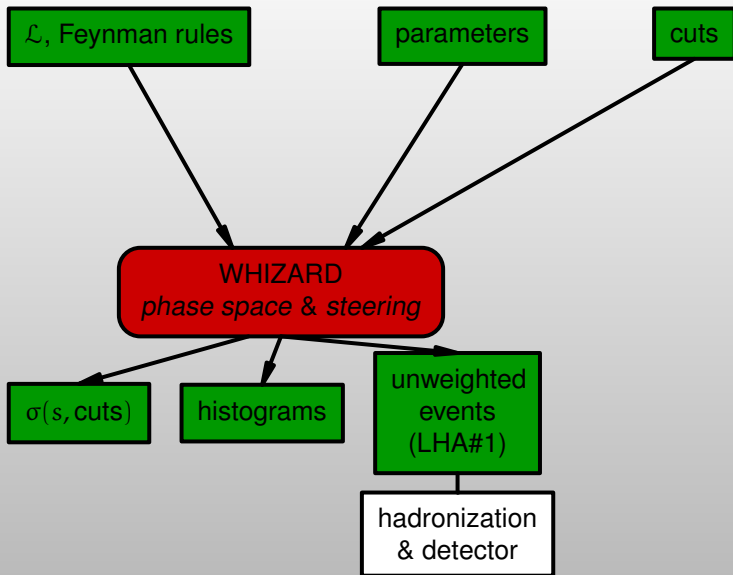
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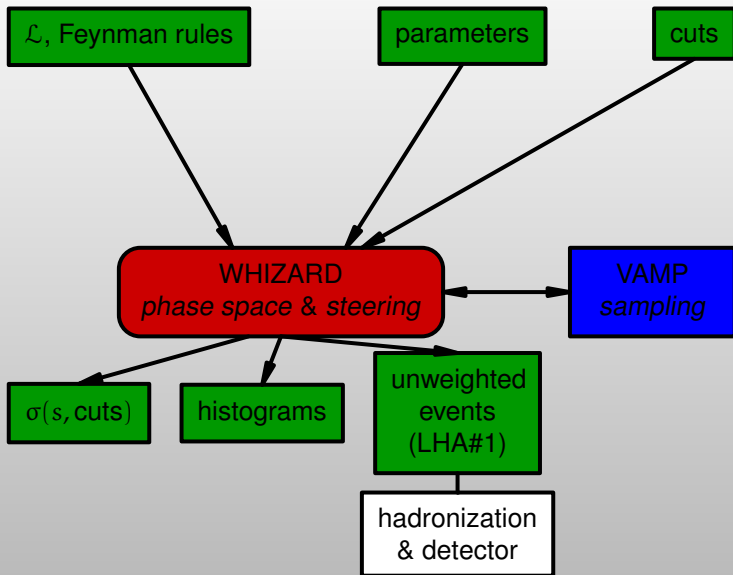
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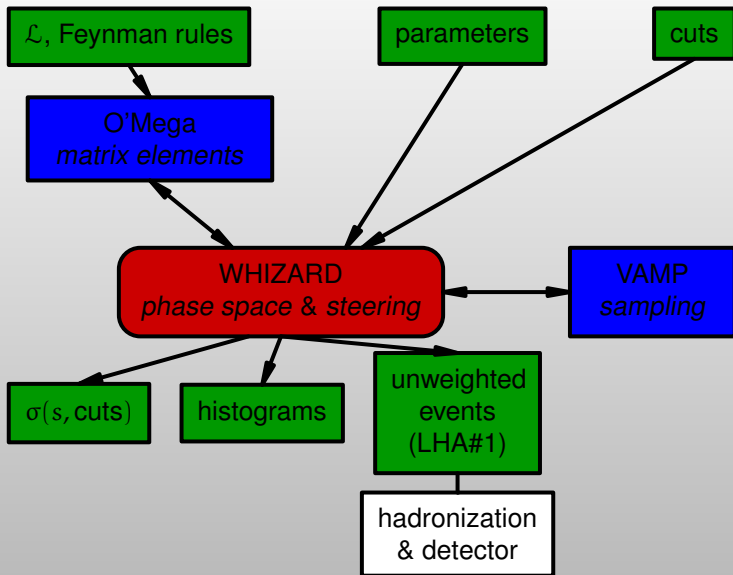


Kilian



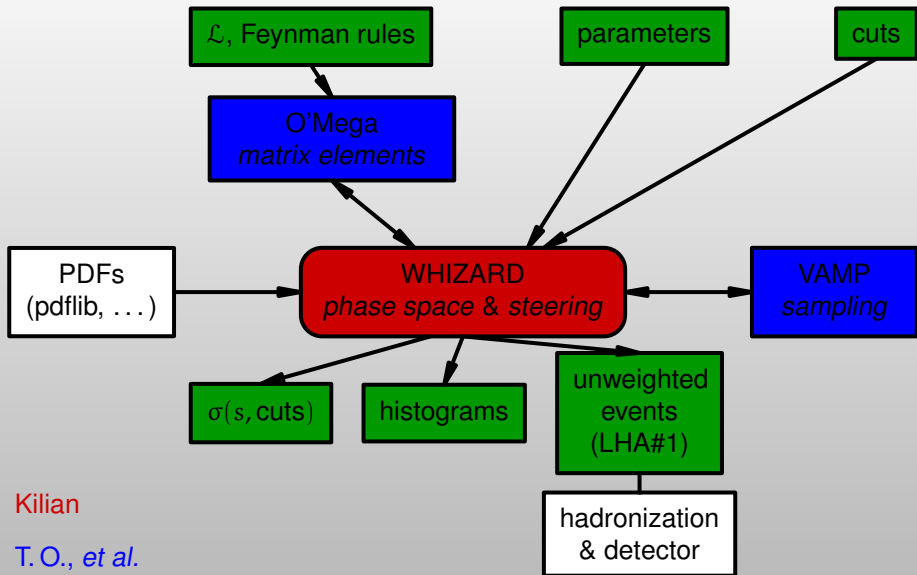
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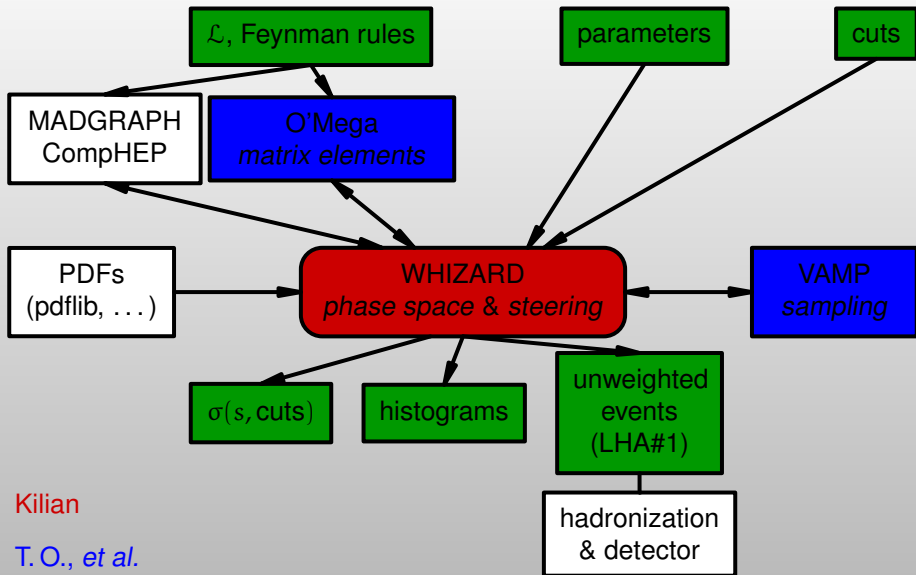
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The number of tree Feynman diagrams w/ n legs in vanilla ϕ^3 -theory grows like a **factorial**

$$F(n) = (2n - 5)!! = (2n - 5) \cdot (2n - 7) \cdot \dots \cdot 3 \cdot 1$$

n		
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5		
6		
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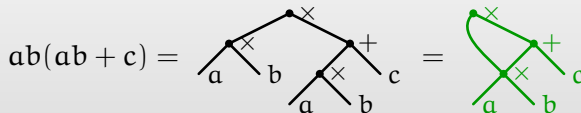
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∴ Feynman diagrams **redundant** for many external particles!

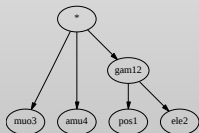
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$$ab(ab + c) = \begin{array}{c} \text{Diagram 1: A tree structure with root node } \times. \text{ Left child is node } \times, \text{ right child is node } +. \text{ Node } \times \text{ has children } a \text{ and } b. \text{ Node } + \text{ has children } a \times \text{ and } c. \text{ Node } a \times \text{ has child } b. \\ \text{Diagram 2: A DAG structure with root node } \times. \text{ Left child is node } \times, \text{ right child is node } +. \text{ Node } \times \text{ has children } a \text{ and } b. \text{ Node } + \text{ has children } a \times \text{ and } c. \text{ Node } a \times \text{ has child } b. \text{ The edges are highlighted in green.} \end{array} =$$

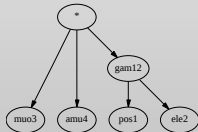
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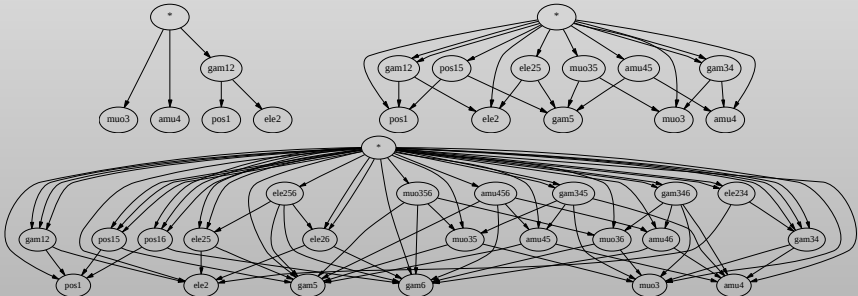
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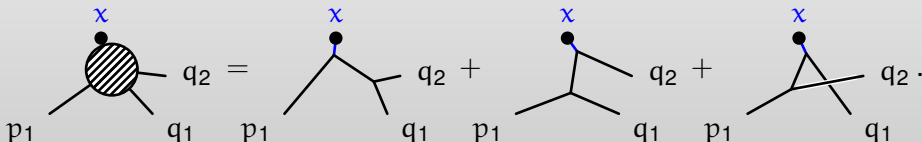
One particle off-shell wave functions (1POWs) are obtained from by applying the LSZ reduction formula to all but one line:

$$W(x; p_1, \dots, p_n; q_1, \dots, q_m) = \langle \phi(q_1), \dots, \phi(q_m); \text{out} | \Phi(x) | \phi(p_1), \dots, \phi(p_n); \text{in} \rangle .$$

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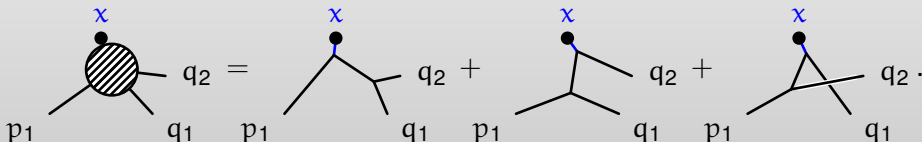
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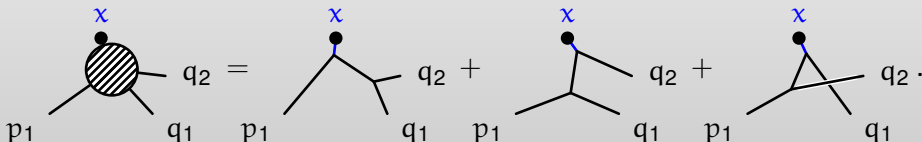


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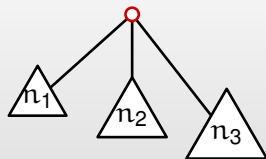
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There exists a well defined set of **keystones** K that allow to express the sum of Feynman diagrams through **1POWs**:

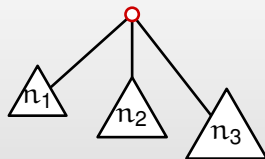
$$T = \sum_{i=1}^{F(n)} D_i = \sum_{k,l,m=1}^{P(n)} K_{f_k f_l f_m}^3(p_k, p_l, p_m) W_{f_k}(p_k) W_{f_l}(p_l) W_{f_m}(p_m)$$

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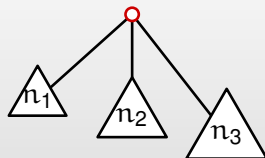


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n	$\sum$	$\sum$
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5	26	$1 \cdot (1, 1, 1, 1, 1) + 10 \cdot (1, 1, 1, 2) + 15 \cdot (1, 2, 2)$
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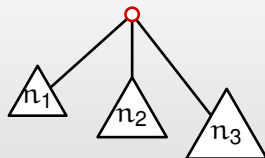
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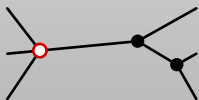
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Subtlety: some partitions for an **even** number of external lines are degenerate, e. g. $(1, 1, 1, 3)$ and $(1, 2, 3)$ contain the **same** diagram



and representatives must be chosen **consistently**.

Non trivial cross check from self-consistency of counting

$F(d_{\max}, n) = \#$ of Feynman diagrams with n external legs in unflavored

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2} \phi^2 - \sum_{d=3}^{d_{\max}} \frac{\lambda_d}{d!} \phi^d$$

theory. In a partition $N_{d,n} = \{n_1, n_2, \dots, n_d\}$ with $n = n_1 + n_2 + \dots + n_d$, there are

$$\tilde{F}(d_{\max}, N_{d,n}) = \frac{1}{(1 + \delta_{n_d, n_1 + n_2 + \dots + n_{d-1}})} \frac{n!}{|\mathcal{S}(N_{d,n})|} \prod_{i=1}^d \frac{F(d_{\max}, n_i + 1)}{n_i!}$$

Feynman diagrams ($|\mathcal{S}(N)|$ the size of the symmetric group of N).

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$F(d_{\max}, n) = \#$ of Feynman diagrams with n external legs in unflavored

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2} \phi^2 - \sum_{d=3}^{d_{\max}} \frac{\lambda_d}{d!} \phi^d$$

theory. In a partition $N_{d,n} = \{n_1, n_2, \dots, n_d\}$ with $n = n_1 + n_2 + \dots + n_d$, there are

$$\tilde{F}(d_{\max}, N_{d,n}) = \frac{1}{(1 + \delta_{n_d, n_1 + n_2 + \dots + n_{d-1}})} \frac{n!}{|\mathcal{S}(N_{d,n})|} \prod_{i=1}^d \frac{F(d_{\max}, n_i + 1)}{n_i!}$$

Feynman diagrams ($|\mathcal{S}(N)|$ the size of the symmetric group of N).



$$F(d_{\max}, n) = \sum_{d=3}^{d_{\max}} \sum_{\substack{N=\{n_1, n_2, \dots, n_d\} \\ n_1 + n_2 + \dots + n_d = n \\ 1 \leq n_1 \leq n_2 \leq \dots \leq n_d \leq \lfloor n/2 \rfloor}} \tilde{F}(d_{\max}, N)$$

can be checked numerically up to $n = \mathcal{O}(100)$.

Even for vector particles, the 1POWs are 'almost' physical objects and satisfy simple **Ward Identities** in unbroken gauge theories

$$\frac{\partial}{\partial x_\mu} \langle \text{out} | A_\mu(x) | \text{in} \rangle_{\text{amp.}} = 0$$

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and in spontaneously gauge theories in R_ξ -gauge

$$\frac{\partial}{\partial x_\mu} \langle \text{out} | W_\mu(x) | \text{in} \rangle_{\text{amp.}} = \xi_W m_W \langle \text{out} | \phi_W(x) | \text{in} \rangle_{\text{amp.}} .$$

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☺ code for matrix elements can optionally be instrumented to check these Ward identities, testing the **consistency** a particular model and the **numerical stability** of expressions.

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😊 code for matrix elements can optionally be instrumented to check these Ward identities, testing the **consistency** a particular model and the **numerical stability** of expressions.

Amplitudes can be continued off-shell:

- **Slavnov-Taylor Identities** can be checked numerically by adding **operator insertions** implementing BRS transformations.

Slightly simplified `Model.T` signature that **all** models must implement:

```
module type Model.T =
  sig
    type flavor (* all quantum numbers *)
    val flavor_symbol : flavor -> string
    val conjugate : flavor -> flavor (* antiparticles *)
    val lorentz : flavor -> Coupling.lorentz (* spin *)
    val fermion : flavor -> int (* fermion, boson, antifermion *)
    val width : flavor -> Coupling.width (* scheme, not value! *)
    type gauge (* parametrized gauges *)
    val gauge_symbol : gauge -> string
    val propagator : flavor -> gauge Coupling.propagator
    type constant (* coupling constants *)
    val constant_symbol : constant -> string
    val fuse2 : flavor -> flavor ->
      (flavor * constant Coupling.t) list (*  $A_\mu(p_{12}) \leftarrow g\bar{\psi}(p_1)\gamma_\mu\psi(p_2)$  *)
    val fuse3 : flavor -> flavor -> flavor ->
      (flavor * constant Coupling.t) list (*  $\phi(p_{123}) \leftarrow g\phi(p_1)\phi(p_2)\phi(p_3)$  *)
    val fuse : flavor list -> (flavor * constant Coupling.t) list
  end
```

Supported models in WHiZard 1.51 / O'Mega 0.11

■ SM

😊 robust since many years

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- unitarity gauge and R_ξ gauges
- width treatments: constant, time-like, “fudge” scheme, complex mass [C. Schwinn], ...

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■ BSM models remain constantly evolving (as is the nature of BSM physics ...)

- “anomalous couplings”: robust since TESLA TDR
- various little higgs models available
- extra dimension models available
- feel free to DIY ...

credit for many recent implementations and applications: [J. Reuter]

- **comprehensive** comparison of three **independent** complete MSSM implementations
 - AMEGIC++ / SHERPA
 - MADGRAPH / HELAS
 - WHiZard / O'Mega

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 - 😊 contains comprehensive list of $2 \rightarrow 2$ cross sections for testing your own Feynman rules

$W^- Z \rightarrow X^-$				
Final state	MADGRAPH / HELAS		WHiZard / O'Mega	
	0.5 TeV	2 TeV	0.5 TeV	2 TeV
$\tilde{e}_L \tilde{\nu}_e^*$ $\tilde{\mu}_L \tilde{\nu}_\mu^*$ $\tilde{\tau}_1 \tilde{\nu}_\tau^*$ $\tilde{\tau}_2 \tilde{\nu}_\tau^*$				
$\tilde{d}_L \tilde{u}_L^*$ $\tilde{s}_L \tilde{c}_L^*$ $\tilde{b}_1 \tilde{t}_1^*$ $\tilde{b}_2 \tilde{t}_2^*$ $\tilde{b}_1 \tilde{t}_2^*$ $\tilde{b}_2 \tilde{t}_1^*$	—		—	
$\tilde{\chi}_1^0 \tilde{\chi}_1^-$ $\tilde{\chi}_2^0 \tilde{\chi}_1^-$ $\tilde{\chi}_3^0 \tilde{\chi}_1^-$ $\tilde{\chi}_4^0 \tilde{\chi}_1^-$ $\tilde{\chi}_1^0 \tilde{\chi}_2^-$ $\tilde{\chi}_2^0 \tilde{\chi}_2^-$ $\tilde{\chi}_3^0 \tilde{\chi}_2^-$ $\tilde{\chi}_4^0 \tilde{\chi}_2^-$	—		—	
$h^0 H^-$ $H^0 H^-$ $A^0 H^-$ $W^- h^0$ $W^- H^0$ $W^- A^0$ $Z H^-$	—		—	

$W^- Z \rightarrow X^-$				
Final state	MADGRAPH / HELAS		WHiZard / O'Mega	
	0.5 TeV	2 TeV	0.5 TeV	2 TeV
$\tilde{e}_L \tilde{\nu}_e^*$ $\tilde{\mu}_L \tilde{\nu}_\mu^*$ $\tilde{\tau}_1 \tilde{\nu}_\tau^*$ $\tilde{\tau}_2 \tilde{\nu}_\tau^*$			96.639(2) 96.638(2) 14.952(1) 85.875(2)	
$\tilde{d}_L \tilde{u}_L^*$ $\tilde{s}_L \tilde{c}_L^*$ $\tilde{b}_1 \tilde{t}_1^*$ $\tilde{b}_2 \tilde{t}_2^*$ $\tilde{b}_1 \tilde{t}_2^*$ $\tilde{b}_2 \tilde{t}_1^*$	— — — — — —		— — — — — —	— — — — — —
$\tilde{X}_1^0 \tilde{X}_1^-$ $\tilde{X}_2^0 \tilde{X}_1^-$ $\tilde{X}_3^0 \tilde{X}_1^-$ $\tilde{X}_4^0 \tilde{X}_1^-$ $\tilde{X}_1^0 \tilde{X}_2^-$ $\tilde{X}_2^0 \tilde{X}_2^-$ $\tilde{X}_3^0 \tilde{X}_2^-$ $\tilde{X}_4^0 \tilde{X}_2^-$			61.626(3) 2.8350(3)e3 — — 11.7619(7) — — —	
$h^0 H^-$ $H^0 H^-$ $A^0 H^-$ $W^- h^0$ $W^- H^0$ $W^- A^0$ $Z H^-$	— — — — — — —		— — — 7.6213(6)e4 4.2446(2) 1.07037(1) 0.17723(2)	— — — — — — —

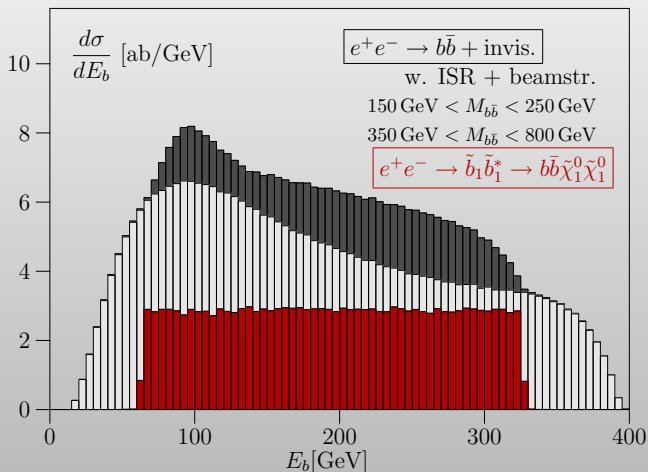
$W^- Z \rightarrow X^-$				
Final state	MADGRAPH / HELAS		WHiZard / O'Mega	
	0.5 TeV	2 TeV	0.5 TeV	2 TeV
$\tilde{e}_L \tilde{\nu}_e^*$	96.635(6)		96.639(2)	96.632(5)
$\tilde{\mu}_L \tilde{\nu}_\mu^*$	96.635(6)		96.638(2)	96.631(5)
$\tilde{\tau}_1 \tilde{\nu}_\tau^*$	14.9542(8)		14.952(1)	14.953(1)
$\tilde{\tau}_2 \tilde{\nu}_\tau^*$	85.875(5)		85.875(2)	85.870(4)
$\tilde{d}_L \tilde{u}_L^*$	—		—	—
$\tilde{s}_L \tilde{c}_L^*$	—		—	—
$\tilde{b}_1 \tilde{t}_1^*$	—		—	—
$\tilde{b}_2 \tilde{t}_2^*$	—		—	—
$\tilde{b}_1 \tilde{t}_2^*$	—		—	—
$\tilde{b}_2 \tilde{t}_1^*$	—		—	—
$\tilde{\chi}_1^0 \tilde{\chi}_1^-$	61.634(6)		61.626(3)	61.633(3)
$\tilde{\chi}_2^0 \tilde{\chi}_1^-$	2.8355(7)e3		2.8350(3)e3	2.8356(2)e3
$\tilde{\chi}_3^0 \tilde{\chi}_1^-$	—		—	—
$\tilde{\chi}_4^0 \tilde{\chi}_1^-$	—		—	—
$\tilde{\chi}_1^0 \tilde{\chi}_2^-$	11.7607(3)		11.7619(7)	11.7602(6)
$\tilde{\chi}_2^0 \tilde{\chi}_2^-$	—		—	—
$\tilde{\chi}_3^0 \tilde{\chi}_2^-$	—		—	—
$\tilde{\chi}_4^0 \tilde{\chi}_2^-$	—		—	—
$h^0 H^-$	—		—	—
$H^0 H^-$	—		—	—
$A^0 H^-$	—		—	—
$W^- h^0$	7.620(3)e4		7.6213(6)e4	7.6209(4)e4
$W^- H^0$	4.2446(2)		4.2446(2)	4.2445(2)
$W^- A^0$	1.07034(3)		1.07037(1)	1.07017(6)
$Z H^-$	0.177241(1)		0.17723(2)	0.17714(4)

$W^- Z \rightarrow X^-$						
Final state	MADGRAPH / HELAS		WHiZard / O'Mega		AMEGIC++ / SHERPA	
	0.5 TeV	2 TeV	0.5 TeV	2 TeV	0.5 TeV	2 TeV
$\tilde{e}_L \tilde{\nu}_e^*$	96.635(6)		96.639(2)	15.728(2)	96.632(5)	
$\tilde{\mu}_L \tilde{\nu}_\mu^*$	96.635(6)		96.638(2)	15.727(2)	96.631(5)	
$\tilde{\tau}_1 \tilde{\nu}_\tau^*$	14.9542(8)		14.952(1)	1.4268(2)	14.953(1)	
$\tilde{\tau}_2 \tilde{\nu}_\tau^*$	85.875(5)		85.875(2)	14.478(2)	85.870(4)	
$\tilde{d}_L \tilde{u}_L^*$	—		—	24.220(1)	—	
$\tilde{s}_L \tilde{c}_L^*$	—		—	24.221(1)	—	
$\tilde{b}_1 \tilde{t}_1^*$	—		—	40.676(4)	—	
$\tilde{b}_2 \tilde{t}_2^*$	—		—	8.3706(7)	—	
$\tilde{b}_1 \tilde{t}_2^*$	—		—	63.592(6)	—	
$\tilde{b}_2 \tilde{t}_1^*$	—		—	3.9236(5)	—	
$\tilde{\chi}_1^0 \tilde{\chi}_1^-$	61.634(6)		61.626(3)	16.389(1)	61.633(3)	
$\tilde{\chi}_2^0 \tilde{\chi}_1^-$	2.8355(7)e3		2.8350(3)e3	668.1(1)	2.8356(2)e3	
$\tilde{\chi}_3^0 \tilde{\chi}_1^-$	—		—	278.53(1)	—	
$\tilde{\chi}_4^0 \tilde{\chi}_1^-$	—		—	270.97(2)	—	
$\tilde{\chi}_1^0 \tilde{\chi}_2^-$	11.7607(3)		11.7619(7)	12.380(1)	11.7602(6)	
$\tilde{\chi}_2^0 \tilde{\chi}_2^-$	—		—	218.38(2)	—	
$\tilde{\chi}_3^0 \tilde{\chi}_2^-$	—		—	76.494(5)	—	
$\tilde{\chi}_4^0 \tilde{\chi}_2^-$	—		—	97.693(7)	—	
$h^0 H^-$	—		—	4.4399(5)e-3	—	
$H^0 H^-$	—		—	6.1592(2)	—	
$A^0 H^-$	—		—	5.9726(5)	—	
$W^- h^0$	7.620(3)e4		7.6213(6)e4	8.289(2)e4	7.6209(4)e4	
$W^- H^0$	4.2446(2)		4.2446(2)	15.783(3)	4.2445(2)	
$W^- A^0$	1.07034(3)		1.07037(1)	0.24815(7)	1.07017(6)	
$Z H^-$	0.177241(1)		0.17723(2)	0.25403(7)	0.17714(4)	

$W^- Z \rightarrow X^-$						
Final state	MADGRAPH / HELAS		WHiZard / O'Mega		AMEGIC++ / SHERPA	
	0.5 TeV	2 TeV	0.5 TeV	2 TeV	0.5 TeV	2 TeV
$\tilde{e}_L \tilde{\nu}_e^*$	96.635(6)	15.726(1)	96.639(2)	15.728(2)	96.632(5)	15.7249(8)
$\tilde{\mu}_L \tilde{\nu}_\mu^*$	96.635(6)	15.726(1)	96.638(2)	15.727(2)	96.631(5)	15.7264(8)
$\tilde{\tau}_1 \tilde{\nu}_\tau^*$	14.9542(8)	1.427(1)	14.952(1)	1.4268(2)	14.953(1)	1.42747(7)
$\tilde{\tau}_2 \tilde{\nu}_\tau^*$	85.875(5)	14.479(1)	85.875(2)	14.478(2)	85.870(4)	14.4780(7)
$\tilde{d}_L \tilde{u}_L^*$	—	24.220(3)	—	24.220(1)	—	24.219(1)
$\tilde{s}_L \tilde{c}_L^*$	—	24.220(3)	—	24.221(1)	—	24.220(1)
$\tilde{b}_1 \tilde{t}_1^*$	—	40.676(2)	—	40.676(4)	—	40.677(2)
$\tilde{b}_2 \tilde{t}_2^*$	—	8.3717(5)	—	8.3706(7)	—	8.3722(4)
$\tilde{b}_1 \tilde{t}_2^*$	—	63.596(3)	—	63.592(6)	—	63.591(3)
$\tilde{b}_2 \tilde{t}_1^*$	—	3.9242(2)	—	3.9236(5)	—	3.9244(2)
$\tilde{\chi}_1^0 \tilde{\chi}_1^-$	61.634(6)	16.389(5)	61.626(3)	16.389(1)	61.633(3)	16.391(1)
$\tilde{\chi}_2^0 \tilde{\chi}_1^-$	2.8355(7)e3	668.2(4)	2.8350(3)e3	668.1(1)	2.8356(2)e3	668.34(3)
$\tilde{\chi}_3^0 \tilde{\chi}_1^-$	—	278.5(1)	—	278.53(1)	—	278.58(2)
$\tilde{\chi}_4^0 \tilde{\chi}_1^-$	—	270.9(1)	—	270.97(2)	—	271.02(2)
$\tilde{\chi}_1^0 \tilde{\chi}_2^-$	11.7607(3)	12.379(4)	11.7619(7)	12.380(1)	11.7602(6)	12.380(1)
$\tilde{\chi}_2^0 \tilde{\chi}_2^-$	—	218.3(1)	—	218.38(2)	—	218.40(1)
$\tilde{\chi}_3^0 \tilde{\chi}_2^-$	—	76.50(3)	—	76.494(5)	—	76.497(4)
$\tilde{\chi}_4^0 \tilde{\chi}_2^-$	—	97.70(4)	—	97.693(7)	—	97.693(4)
$H^0 H^-$	—	4.439(6)e-3	—	4.4399(5)e-3	—	4.4395(2)e-3
$H^0 H^-$	—	6.1592(6)	—	6.1592(2)	—	6.1589(3)
$A^0 H^-$	—	5.9728(6)	—	5.9726(5)	—	5.9723(3)
$W^- h^0$	7.620(3)e4	8.29(1)e4	7.6213(6)e4	8.289(2)e4	7.6209(4)e4	8.2909(4)e4
$W^- H^0$	4.2446(2)	15.78(2)	4.2446(2)	15.783(3)	4.2445(2)	15.7848(8)
$W^- A^0$	1.07034(3)	0.24799(1)	1.07037(1)	0.24815(7)	1.07017(6)	0.24801(1)
$Z H^-$	0.177241(1)	0.25405(1)	0.17723(2)	0.25403(7)	0.17714(4)	0.25404(1)

- ILC example: sbottom pair production

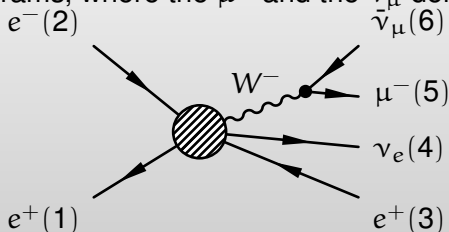
- ILC example: sbottom pair production
- complete irreducible background, ISR and beamstrahlung



Cascade decays are supported

■ \$ f90_SM -scatter "e+ e- -> e+ nue mu- numubar" \
 -cascade "3 + 4 ~ W-"

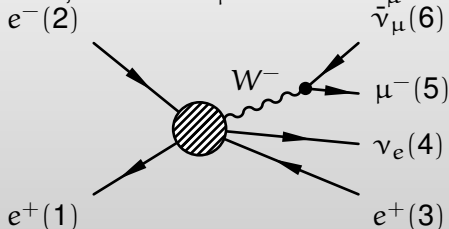
select only diagrams, where the μ^- and the $\bar{\nu}_\mu$ derive from a $W^\pm$



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■ \$ f90_SM -scatter "e+ e- -> e+ nue mu- numubar" \
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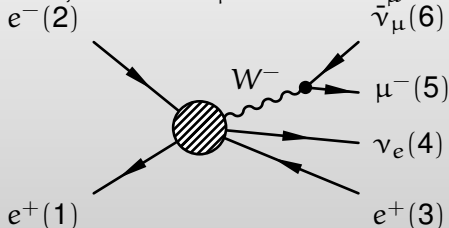


not necessarily fully gauge invariant, but correct spin correlations from compact expressions

Cascade decays are supported

- `$ f90_SM -scatter "e+ e- -> e+ nue mu- numubar" \`
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- ☹ not necessarily fully gauge invariant, but correct spin correlations from compact expressions

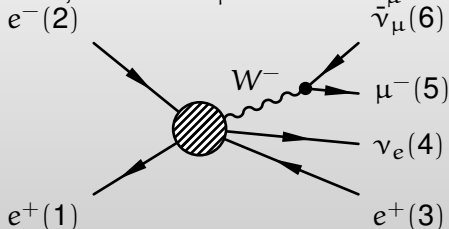
- `$ f90_SM -scatter "e+ e- -> e+ nue mu- numubar" \`
`-cascade "3 + 4 = W-"`

the same, but in addition **force the $W^\pm$ on-shell**

Cascade decays are supported

```
$ f90_SM -scatter "e+ e- -> e+ nue mu- numubar" \
-cascade "3 + 4 ~ W-"
```

select only diagrams, where the μ^- and the $\bar{\nu}_\mu$ derive from a $W^\pm$



☹ not necessarily fully gauge invariant, but correct spin correlations from compact expressions

```
$ f90_SM -scatter "e+ e- -> e+ nue mu- numubar" \
-cascade "3 + 4 = W-"
```

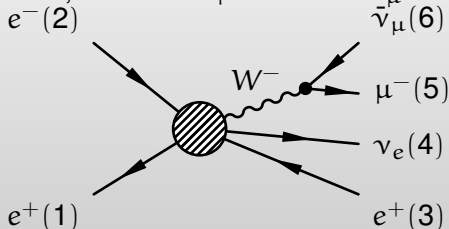
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select only diagrams, where the μ^- and the $\bar{\nu}_\mu$ derive from a $W^\pm$



☹ not necessarily fully gauge invariant, but correct spin correlations from compact expressions

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$ f90_SM -scatter "e+ e- -> e+ nue mu- numubar" \
-cascade "3 + 4 = W-"
```

the same, but in addition **force the $W^\pm$ on-shell**

😊 gauge invariant

■ both can be combined arbitrarily with **boolean operators** `&&` and `||`

😊 O'Mega / WHiZard does general **colored amplitudes**

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■ **color flow** representation

$$\psi \rightarrow \psi^i$$

$$\psi \rightarrow \psi^{ij}, \psi^0$$

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😬 finally ...

■ **color flow** representation

$$\psi \rightarrow \psi^i$$

$$\psi \rightarrow \psi^{ij}, \psi^0$$

😊 can be implemented by enlarging the flavor quantum number and using special Feynman rules [\[à la Stelzer/Willenbrock\]](#)

$$|\text{flavor}\rangle \rightarrow |\text{flavor}\rangle \otimes |\text{color}\rangle$$

😊 O'Mega / WHiZard does general **colored amplitudes**

😞 finally ...

- **color flow** representation

$$\psi \rightarrow \psi^i$$

$$\psi \rightarrow \psi^{ij}, \psi^0$$

😊 can be implemented by enlarging the flavor quantum number and using special Feynman rules [\[à la Stelzer/Willenbrock\]](#)

$$|\text{flavor}\rangle \rightarrow |\text{flavor}\rangle \otimes |\text{color}\rangle$$

- currently complete “**proof of concept**” implementation in **Whizard**

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😞 generation of matrix elements can be time-consuming

- fast implementation in **O'Mega** on the way: **general “Colorize” functor** mapping of any uncolored model to a $SU(N)$ colored model

Forthcoming **native model file format**:■ **particle declarations**

```
particle e- e+ : spin=1/2, fermion, pdg=11, tex="e^-", tex.anti="e^+"
particle nue nuebar : spin=1/2, fermion, pdg=12, tex="\nu_e", tex.anti="\bar{\nu}_e"
particle A : spin=1, boson, pdg=22, tex="\gamma"
particle Z : spin=1, boson, pdg=23, tex="Z"
particle W+ W- : spin=1, boson, pdg=24, tex="W^+", tex.anti="W^-"
```



```
coupling e
coupling g
coupling gv
coupling ga
```

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particle Z : spin=1, boson, pdg=23, tex="Z"
particle W+ W- : spin=1, boson, pdg=24, tex="W^+", tex.anti="W^-"
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```
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```

■ **gauge currents $e\bar{e}\gamma e$, $\bar{e}Z(g_V - g_A\gamma_5)e$ and $g\bar{e}W(1 - \gamma_5)\nu_e$**

```
vertex e+, A, e- : { e * <1|V.e2|3> }
vertex e+, Z, e- : { gv * <1|V.e2|3> - ga * <1|A.e2|3> }
vertex e+, W-, nue : { g * <1|(V-A).e2|3> }
```

Forthcoming **native model file format**:

■ particle declarations

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particle e- e+ : spin=1/2, fermion, pdg=11, tex="e^-", tex.anti="e^+"
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particle A : spin=1, boson, pdg=22, tex="\gamma"
particle Z : spin=1, boson, pdg=23, tex="Z"
particle W+ W- : spin=1, boson, pdg=24, tex="W^+", tex.anti="W^-"
```

```
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coupling g
coupling gv
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```

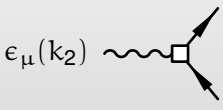
■ gauge currents $e\bar{e}\gamma e$, $e\bar{e}Z(g_V - g_A\gamma_5)e$ and $g\bar{e}W(1 - \gamma_5)\nu_e$

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vertex e+, A, e- : { e * <1|V.e2|3> }
vertex e+, Z, e- : { gv * <1|V.e2|3> - ga * <1|A.e2|3> }
vertex e+, W-, nue : { g * <1|(V-A).e2|3> }
```

■ triple gauge couplings

```
vertex W+, A, W- : { e * ((k1 - k2).e3*e1.e2
                        + (k2 - k3).e1*e2.e3 + (k3 - k1).e2*e3.e1) }
vertex W+, Z, W- : { g * ((k1 - k2).e3*e1.e2
                        + (k2 - k3).e1*e2.e3 + (k3 - k1).e2*e3.e1) }
```

■ noncommutative QED [à la Seiberg/Witten, Witten]



$$= ig \cdot \frac{i}{2} [(k\theta)_\mu \not{p} + (\theta p)_\mu \not{k} - (k\theta p) \gamma_\mu]$$

vertex e+, A, e- : { e * k2.[mu1]*[mu2].k3*<1|V.e2|3>
 - e * k2.[mu1]*[mu2].e2*<1|V.k3|3>
 - e * e2.[mu1]*[mu2].k3*<1|V.k2|3> }

■ noncommutative QED [à la Seiberg/Witten, Witten]

$$\begin{array}{c}
 \bar{u}(k_1) \\
 \nearrow \\
 \epsilon_\mu(k_2) \text{ --- wavy line --- } \square \text{ --- } \\
 \searrow \\
 u(k_3)
 \end{array}
 = ig \cdot \frac{i}{2} [(k\theta)_\mu \not{p} + (\theta p)_\mu \not{k} - (k\theta p)\gamma_\mu]$$

vertex e^+, A, e^- : $\{$

- $e * k_2.[\mu 1]*[\mu 2].k_3* \langle 1|V.e_2|3\rangle$
- $- e * k_2.[\mu 1]*[\mu 2].e_2* \langle 1|V.k_3|3\rangle$
- $- e * e_2.[\mu 1]*[\mu 2].k_3* \langle 1|V.k_2|3\rangle \}$

■ Tradeoff: convenience vs. complexity

- ☹️ full programming language harder to learn, but more appropriate than model file format for defining complicated models with **many** couplings, e. g. the MSSM, from scratch
- 😊 physics motivated **model file format** simplifies small extensions
 - additional particles
 - new vertices

$$e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-:$$

```
$ f90_MSSM -scatter "e+ e- -> ch1+ ch1-"
```

$$e^+ e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-:$$

```
$ f90_MSSM -scatter "e+ e- -> ch1+ ch1-"
```

```
pure function l1b1l1cp1cm1 (k, s) result (amp)
  real(kind=omega_prec), dimension(0:,:), intent(in) :: k
  integer, dimension(:), intent(in) :: s
  complex(kind=omega_prec) :: amp
  type(momentum) :: p1, p2, p3, p4
  type(bispinor) :: cp1_4, l1_2
  type(bispinor) :: cm1_3, l1b_1
  complex(kind=omega_prec) :: snc1_13
  type(vector) :: a_12, z_12
  type(momentum) :: p12, p13
  p1 = - k(:,1) ! incoming e+
  p2 = - k(:,2) ! incoming e-
  p3 =  k(:,3) ! outgoing ch1+
  p4 =  k(:,4) ! outgoing ch1-
  l1b_1 = u (mass(11), - p1, s(1))
  l1_2 = u (mass(11), - p2, s(2))
  cm1_3 = v (mass(69), p3, s(3))
  cp1_4 = v (mass(69), p4, s(4))
```

```

p12 = p1 + p2
a_12 = pr_feynman(p12, + v_ff(qlep,l1b_1,l1_2))
z_12 = pr_unitarity(p12,mass(23),wd_t1(p12,width(23)), &
    + va_ff(gnclep(1),gnclep(2),l1b_1,l1_2))
p13 = p1 + p3
snc1_13 = pr_phi(p13,mass(54),wd_t1(p13,width(54)), &
    + sr_ff(g_yuk_ch1_sn1_1_c,l1b_1,cm1_3))
amp = 0
amp = amp + snc1_13*( - sl_ff(g_yuk_ch1_sn1_1,l1_2,cp1_4))
amp = amp + z_12*( + va_ff(-gczc_1_1(1),-gczc_1_1(2),cm1_3,cp1_4))
amp = amp + a_12*( + v_ff(qchar,cm1_3,cp1_4))
amp = - amp ! 2 vertices, 1 propagators
end function l1b1lcp1cm1

```

9 fusions, 3 propagators, 3 diagrams

```

p12 = p1 + p2
a_12 = pr_feynman(p12, + v_ff(qlep,l1b_1,l1_2))
z_12 = pr_unitarity(p12,mass(23),wd_tl(p12,width(23)), &
    + va_ff(gnclep(1),gnclep(2),l1b_1,l1_2))
p13 = p1 + p3
snc1_13 = pr_phi(p13,mass(54),wd_tl(p13,width(54)), &
    + sr_ff(g_yuk_ch1_sn1_1_c,l1b_1,cm1_3))
amp = 0
amp = amp + snc1_13*( - sl_ff(g_yuk_ch1_sn1_1,l1_2,cp1_4))
amp = amp + z_12*( + va_ff(-gczc_1_1(1),-gczc_1_1(2),cm1_3,cp1_4))
amp = amp + a_12*( + v_ff(qchar,cm1_3,cp1_4))
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```

9 fusions, 3 propagators, 3 diagrams



readable code, can edited for **exotic models** (see below) or **NLO** vertex functions (see below)

Adding a photon $e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^- \gamma$:

```
$ f90_MSSM -scatter "e+ e- -> ch1+ ch1- A"
```

Adding a photon $e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^- \gamma$:

```
$ f90_MSSM -scatter "e+ e- -> ch1+ ch1- A"
```

```
pure function libl1cplcm1a (k, s) result (amp)
  real(kind=omega_prec), dimension(0:,:), intent(in) :: k
  integer, dimension(:), intent(in) :: s
  complex(kind=omega_prec) :: amp
  type(momentum) :: p1, p2, p3, p4, p5
  type(bispinor) :: cp1_4, ll_2
  type(bispinor) :: cm1_3, llb_1
  type(vector) :: a_5
  complex(kind=omega_prec) :: sn1_24, snc1_13
  type(bispinor) :: cp1_45, ll_25
  type(bispinor) :: cm1_35, llb_15
  type(vector) :: a_34, a_12, z_34, z_12
  type(momentum) :: p12, p13, p15, p24, p25, p34, p35, p45
  p1 = - k(:,1) ! incoming e+
  p2 = - k(:,2) ! incoming e-
  p3 =  k(:,3) ! outgoing ch1+
  p4 =  k(:,4) ! outgoing ch1-
  p5 =  k(:,5) ! outgoing A
  llb_1 = u (mass(11), - p1, s(1))
  ll_2 = u (mass(11), - p2, s(2))
  cm1_3 = v (mass(69), p3, s(3))
  cp1_4 = v (mass(69), p4, s(4))
  a_5 = conjg (eps (mass(22), p5, s(5)))
  p12 = p1 + p2
  a_12 = pr_feynman(p12, + v_ff(qlep,llb_1,ll_2))
  z_12 = pr_unitarity(p12,mass(23),wd_tl(p12,width(23)), &
    + va_ff(gnclep(1),gnclep(2),llb_1,ll_2))
```

```

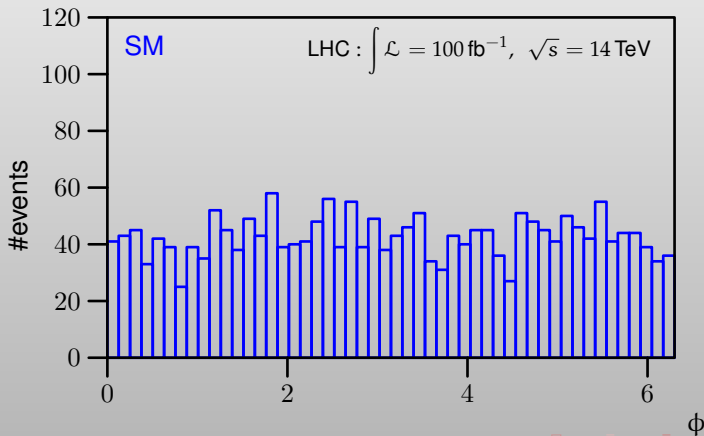
p13 = p1 + p3
snc1_13 = pr_phi(p13,mass(54),wd_tl(p13,width(54)), &
  + sr_ff(g_yuk_ch1_sn1_1_c,l1b_1,cm1_3))
p24 = p2 + p4
sn1_24 = pr_phi(p24,mass(54),wd_tl(p24,width(54)), &
  + sl_ff(g_yuk_ch1_sn1_1,l1_2,cp1_4))
p34 = p3 + p4
a_34 = pr_feynman(p34, + v_ff(qchar,cm1_3,cp1_4))
z_34 = pr_unitarity(p34,mass(23),wd_tl(p34,width(23)), &
  + va_ff(-gczc_1_1(1),-gczc_1_1(2),cm1_3,cp1_4))
p15 = p1 + p5
l1b_15 = pr_psi(p15,mass(11),wd_tl(p15,width(11)), + f_vf(-qllep,a_5,l1b_1))
p25 = p2 + p5
l1_25 = pr_psi(p25,mass(11),wd_tl(p25,width(11)), + f_vf(qllep,a_5,l1_2))
p35 = p3 + p5
cm1_35 = pr_psi(p35,mass(69),wd_tl(p35,width(69)), &
  + f_vf(-qchar,a_5,cm1_3))
p45 = p4 + p5
cp1_45 = pr_psi(p45,mass(69),wd_tl(p45,width(69)), + f_vf(qchar,a_5,cp1_4))
amp = 0
amp = amp + sn1_24*( + sr_ff(g_yuk_ch1_sn1_1_c,cm1_35,l1b_1))
amp = amp + snc1_13*( - sl_ff(g_yuk_ch1_sn1_1,l1_25,cp1_4) &
  + sl_ff(g_yuk_ch1_sn1_1,cp1_45,l1_2))
amp = amp + l1_25*( - f_vf(-qllep,a_34,l1b_1) &
  - f_vaf(-(gnclep(1)),gnclep(2),z_34,l1b_1))
amp = amp + l1b_15*( - f_srf(g_yuk_ch1_sn1_1_c,sn1_24,cm1_3) &
  + f_vf(qllep,a_34,l1_2) + f_vaf(gnclep(1),gnclep(2),z_34,l1_2))
amp = amp + z_12*( - va_ff(-(gczc_1_1(1)),gczc_1_1(2),cp1_45,cm1_3) &
  + va_ff(-gczc_1_1(1),-gczc_1_1(2),cm1_35,cp1_4))
amp = amp + a_12*( - v_ff(-qchar,cp1_45,cm1_3) + v_ff(qchar,cm1_35,cp1_4))
end function l1b1c1p1cm1a

```

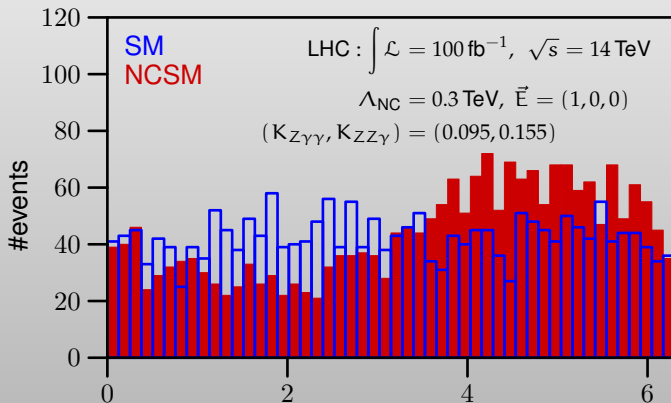
28 fusions, 10 propagators, 12 diagrams

- using omegalib and WHiZard [\[Alboteanu/T. O., Rückl, 2005\]](#)

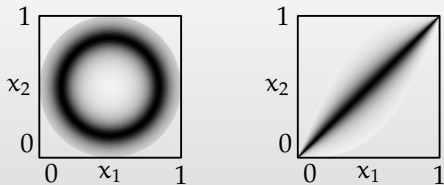
- using omegalib and WHiZard [Alboteanu/T. O., Rückl, 2005]
- standard acceptance cuts and $85 \text{ GeV} < m_{\ell+\ell^-} < 97 \text{ GeV}$,
 $200 \text{ GeV} < m_{\ell+\ell^- \gamma} < 1 \text{ TeV}$, $0 < \cos \theta_{\gamma}^* < 0.9$,
 $\cos \theta_Z > 0$ and $\cos \theta_{\gamma} > 0$ (favoring $\bar{q}q$ over $q\bar{q}$!)

 ϕ

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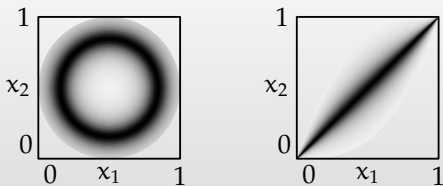


VEGAS' factorized ansatz handles



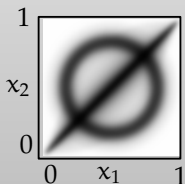
separately after mappings

VEGAS' factorized ansatz handles



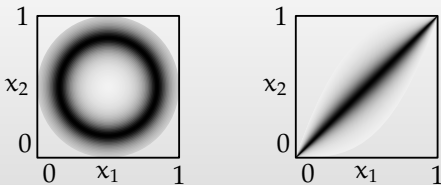
separately after mappings

☹️ fails for overlapping sing'ies



which is the common case
(for more than one Feynman
diagram)

VEGAS' factorized ansatz handles



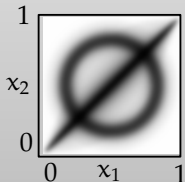
∴ adaptive multichannel approach

$$I(f) = \int_{\mathcal{M}} d\mu(p) f(p)$$

$$I(f) = \sum_{i=1}^{N_c} \alpha_i \int_0^1 g_i(x) d^n x \frac{f(\phi_i(x))}{g(\phi_i(x))}$$

separately after mappings

☹ fails for overlapping sing'ies

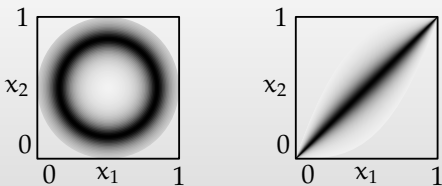


with

$$g = \sum_{i=1}^{N_c} \alpha_i \cdot (g_i \circ \phi_i^{-1}) \left| \frac{\partial \phi_i^{-1}}{\partial p} \right|$$

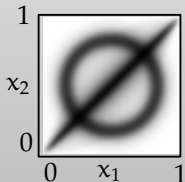
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which is the common case
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$$I(f) = \sum_{i=1}^{N_c} \alpha_i \int_0^1 g_i(x) d^n x \frac{f(\phi_i(x))}{g(\phi_i(x))}$$

with

$$g = \sum_{i=1}^{N_c} \alpha_i \cdot (g_i \circ \phi_i^{-1}) \left| \frac{\partial \phi_i^{-1}}{\partial p} \right|$$

😊 works with factorized g_i
adapted by VEGAS and α_i
adapted by variance reduction.

- WHiZard calls O'Mega to generate matrix elements according to `conf/whizard.prc`

```
model    SM
uuza     u,U                               Z,A               omega
ppza     u:d:s:U:D:S,u:d:s:U:D:S         Z,A               omega
uulla    u,U                               e1:e2,E1:E2,A        omega
pplla    u:d:s:U:D:S,u:d:s:U:D:S         e1:e2,E1:E2,A        omega
```

(CompHEP notation for historical reasons ...)

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model    SM
uuza     u,U                               Z,A               omega
ppza     u:d:s:U:D:S,u:d:s:U:D:S         Z,A               omega
uulla    u,U                               e1:e2,E1:E2,A        omega
pplla    u:d:s:U:D:S,u:d:s:U:D:S         e1:e2,E1:E2,A        omega
```

(CompHEP notation for historical reasons ...)

- generated Fortran90 code for matrix elements is compiled and linked with support libraries (PDFs, vamp, etc.)

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model    SM
uuza     u,U                                Z,A                                omega
ppza     u:d:s:U:D:S,u:d:s:U:D:S          Z,A                                omega
uulla    u,U                                e1:e2,E1:E2,A                          omega
pplla    u:d:s:U:D:S,u:d:s:U:D:S          e1:e2,E1:E2,A                          omega
```

(CompHEP notation for historical reasons ...)

- generated Fortran90 code for matrix elements is compiled and linked with support libraries (PDFs, vamp, etc.)
- generates unweighted events according to `results/whizard.in`

```
&process_input
process_id = "pplla"
sqrts = 14000
luminosity = 0
polarized_beams = F
structured_beams = T
/
```

etc.

- event samples are generated according to **phase space cuts**

```
process ppla
  cut E of 1 within 10 9999
  cut E of 2 within 10 9999
  cut E of 4 within 10 9999
  cut THETA(DEG) of 8 1 within 5 175
  cut THETA(DEG) of 8 2 within 5 175
  cut THETA(DEG) of 8 4 within 5 175
```

- event samples are generated according to **phase space cuts**

```
process plla
  cut E of 1 within 10 9999
  cut E of 2 within 10 9999
  cut E of 4 within 10 9999
  cut THETA(DEG) of 8 1 within 5 175
  cut THETA(DEG) of 8 2 within 5 175
  cut THETA(DEG) of 8 4 within 5 175
```

- simple **analysis** can be performed by filling histograms

```
process plla
  histogram M of 3 within 0 600 nbin 60
  histogram PT of 1 within 0 200 nbin 50
  histogram PT of 2 within 0 200 nbin 50
  histogram PT of 4 within 0 200 nbin 50
```

(will be generalized in the next version)

- no intention to become a full stand-alone replacement for Pythia and Herwig (unlike Sherpa)

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 - decay tables
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- semiautomatic support for loops. Proof of concept: exponentiated NLO results for $e^+e^- \rightarrow \tilde{\chi}^+ \tilde{\chi}^-$ [T. Robens, SUSY06]
- parallelization build into vamp, not enabled yet ...