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ALPGEN

LOOPFESTV, SLAC ,19-21 July 2006

ALPGEN is

hep-ph 0206293 M.Mangano, M.Moretti, F.Piccinini, R.Pittau and A.Polosa

- A (tree level) ME generator (up to 10 partons, can be increased if needed)
- parton level description interfaced to PS (`HERWIG`, `PITHYA`) $\Rightarrow$ full showering and hadronization of the event
- fermions are massive
- for processes with heavy particle in the final state spin correlation is fully taken into account (in the narrow width approximation)
- ME PS matching option avaliable
- to provide a (better) description of process dependent features (partonic ME contributing, phase space optimization, heavy object decay, matching prescription, ...) ALPGEN is actually a collection of packages devoted to various (SM) processes.

Shopping list

- $W^{*}Q\bar{Q} + n\text{-}jets,$ — $W^{*} \equiv l\nu_{l}$ and $Q = b, t, (c)$.
- $W^{*} + n\text{-}jets$ — $jets \equiv$ "light" quarks, gluons
- $Z^{*}/\gamma^{*}Q\bar{Q} + n\text{-}jets$
- $Z^{*}/\gamma^{*} + n\text{-}jets$
- $Q\bar{Q} + n\text{-}jets$
- $Q\bar{Q}Q\bar{Q} + n\text{-}jets$
- $Q\bar{Q} + H + n\text{-}jets$
- $n\text{-}W + m\text{-}Z + l\text{-}H + n\text{-}jets$
- $n\text{-}jets$
- $m\text{-}\gamma + n\text{-}jets$
- $t(+W,\ +b,\ +Wb) + n\text{-}jets$
- $H + n\text{-}jets$ — ggH effective coupling $(m_t \to \infty)$
- $W^{*}(Z^{*}/\gamma^{*}) + m\text{-}\gamma + n\text{-}jets$ — in progress
- $Q\bar{Q} + m\text{-}\gamma + n\text{-}jets$ — in progress

The ALPHA Algorithm

F. Caravagliosand M. Moretti PLB 358 (1995) 332

F. Caravaglios, M. L. Mangano, M. Moretti and R. Pittau NPB 539 (1999) 215

The Idea: The Matrix Element 'is' the Legendre Transform Z of the (effective) lagrangian Γ (one-particle-irreducible Green Functions generator) $\rightarrow$ the problem can be recasted as a *minimum problem*, more suitable for a *numerical approach*

$$Z(J^\alpha) = -\Gamma(\phi^\alpha) + J^\alpha(x)\phi^\alpha(x)$$

where ϕ^α are the classical fields defined as the solutions of

$$J^\alpha = \frac{\delta\Gamma}{\delta\phi^\alpha} \; ,$$

and the J^α play the role of classical sources.

$$J^\alpha(p) = \sum_{j=1}^{n} \epsilon_\mu^\alpha \delta(p - p_j)$$

notice *finite number* of degrees of freedom

$$\mathcal{A} \sim \frac{\partial Z}{\partial J_1^\alpha \ldots \partial J_n^\gamma}\Big|_{J_1^\alpha=0,\ldots,J_n^\gamma=0}$$

($J_m^\alpha = J^\alpha(p_m)$, $\sim \rightarrow =$ after truncation)

Working Case: Pure YM theory

- At tree level $\Gamma = \mathcal{L}$

$$\mathcal{L}_{YM}(A) = -1/2 F^a_{\mu\nu} F^a_{\mu\nu} + J_\mu A_\mu$$

introducing *auxiliary fields* the lagrangian in *momentum space* is

$$\begin{aligned}\mathcal{L}_{YM}(A) &= -1/2(p_\mu A^a_\nu - p_\nu A^a_\mu)^2 + g f_{abc}(p_\mu A^a_\nu - p_\nu A^a_\mu) A^b_\mu A^c_\nu \\ &\quad -(B^a_{\mu\nu})^2 - 2g f_{abc} B^a_{\mu\nu} A^b_\mu A^c_\nu + J_\mu A_\mu\end{aligned}$$

1. J standard source terms: $J(p) = \sum_{j=1}^n \epsilon^a_\mu \delta(p - p_j)$, *i.e.* it contains the relevant excitation for the external particles
2. $A(p)$ are found as solutions of the equation of motion (Feynman Gauge)

$$\begin{aligned}A_\mu(p) &= \frac{g}{p^2} f_{abc} \left[2(p-k)\cdot A^b(q) A^c_\mu(k) - 2q_\mu A^b(q)\cdot A^c_\mu(k) \right. \\ &\quad \left. - B^b_{\mu\nu}(q) A^c_\nu(k)\right] + \frac{1}{p^2} J_\mu(p) \\ B^a_{\mu\nu}(p) &= -g f_{abc} A^b_\mu(q) A^c_\nu(k)\end{aligned} \tag{1}$$

(Integration $\int \delta(p+q+k)\mathrm{d}k\mathrm{d}q$ is understood)

- The scattering amplitude is proportional to $\mathcal{L}_{YM}(A)$ with A as in (1)

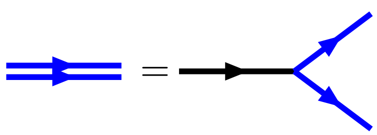
The (1) equations of motion are solved *iteratively* (expansion in g): the problem is solved with a *loop of matrices multiplication*

- $$J_\mu^a = \sum_{j=1}^{n} \epsilon_\mu^a(p)\delta(p - p_j)$$

- $$\begin{aligned} A_\mu^a(p_j) &= \epsilon_\mu^a(p_j) \\ B_{\mu\nu}^a(p_j) &= 0 \end{aligned}$$

($\textit{notice}$ truncation)

- $$\begin{aligned} A_\mu^a(p_j + p_k) &= \frac{g f_{abc}}{(p_j + p_k)^2} \left[2(p_j + p_k) \cdot A^b(p_j) A_\mu^c(p_k) \right. \\ &\quad \left. -2 p_j \cdot A^b(p_k) A_\mu^c(p_j) - B_{\mu\nu}^b(p_j) A_\mu^c(p_k) \right] \\ B_{\mu\nu}^a(p_j + p_k) &= -g f_{abc} A^b(p_k) A_\mu^c(p_j) \end{aligned}$$



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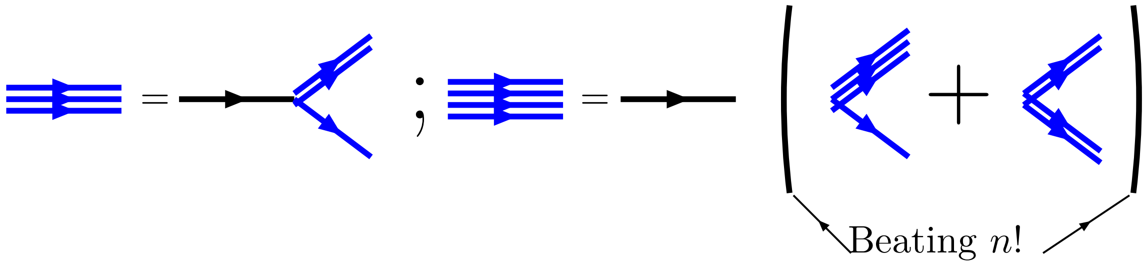
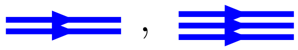


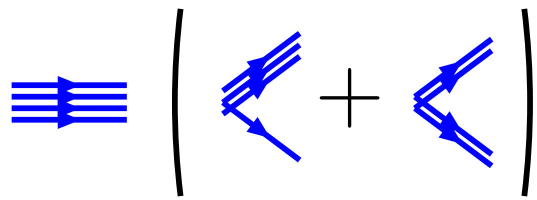
- . . .
- a few *remarks*
  1. .

      ··· Pure Numbers at each step !!!

     .
  2. At any stage terms like $A^a_\mu(\cdots + p_k + p_k + \dots)$ (two equal fourier frequencies) are dropped. This would come out from Pauli principle in the case of fermion and this terms do not affect the result.
  3. For a given process like n external gluon, for example, one should compute up to $A^a_\mu(p_{j_1} + \cdots + p_{j_n})$ terms, *i.e.* a *finite* number of terms. In practice, using the equation of motion, only terms up to $A^a_\mu(p_{j_1} + \cdots + p_{j_{n/2}})$ are required (for 6 and 7 external gluons the A^a_μ with up to 3 frequencies are required, for 8 and 9 external gluons up to 4 frequencies).
- The final result is obtained as

$$\mathcal{A}(p_{j_1}, \dots, p_{j_n}) = \mathcal{L}_{YM}(A)$$

where the A are given in the above steps.

$\mathcal{A} \sim$  $+ \ldots$

1. The prescription to drop terms with twice the same frequency is still kept $A(\dots + p_j + \dots)A(\dots + p_j + \dots) = 0$. Notice that because of this prescription *no functional derivative is required.*
2. Four momentum conservation is enforced dropping products which do not contains all external fourier frequencies (for $n = 4$, $A(p_1 + p_4)A(p_2)A(p_3)$ is ok $A(p_1)A(p_2)A(p_3)$ is not.
3. In this final step, again because of the equation of motion, only the interaction terms in $\mathcal{L}_{YM}(A)$ are computed and source and kinetic ones are dropped.

- loop of matrix multiplications
- number of building blocks $\sim$ *poles* of $\mathcal{S}$ matrix
- CPU cost $\sim K^n$ ($K \sim 2 \div 4$ depending on process and lagrangian

The ALPHA code

- The above *algorithm* has been implemented into a `FORTRAN` code `ALPHA` for the *authomatic* calculation of *fully massive, tree level* scattering amplitude, using the standard model lagrangian as input (and extensively tested using benchmark processes of interest at LEPII)

  1. authomatic calculation $\rightarrow$ *reliability*
  2. *compact* storing of the information (typical size of the *arrays* $A \sim 2^{next}$)
  3. fully massive matrix element
  4. reasonable CPU time performances: (slow growth, power law like, with number of external particle)

Challenges in QCD calculation

Process	$n = 7$	$n = 8$	$n = 9$	$n = 10$
$g\ g \to n\ g$	559,405	10,525,900	224,449,225	5,348,843,500
ME per minute	28000	9170	2870	870
$q\bar{q} \to n\ g$	231,280	4,016,775	79,603,720	1,773,172,275

Table 1:

Number of Feynman diagrams corresponding to amplitudes with different numbers of quarks and gluons. CPU performance on a pentium III 850MH

(*notice*: $n = 10$ expected in some R-parity breaking scenarios)

- One would like to be able to complement the calculation of parton-level matrix elements with the evaluation of the full hadronic structure of the final state.

  1. *Dual amplitudes* can be easily evaluated using the `ALPHA` algorithm, by taking N sufficiently large.

2. Dual amplitudes correspond to planar amplitudes in the $N \to \infty$ limit of QCD $\Longrightarrow$ identification of a specific colour flow $\Longrightarrow$ soft-gluon emission corrections to the hard process (Via PS, incoherent sum over the emission probabilities from each individual colour-string).

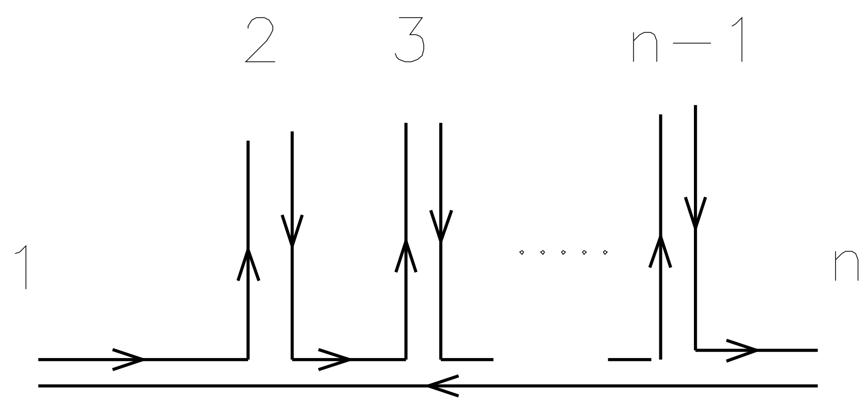


Figure 1:

Colour structure of the n-gluon amplitude in the large-N limit.

An example in the simpler and perhaps more familiar QED contest:

(a) $e^+e^- \to e^+e^-$;

(b) t and s channels interfere and are associated with a different radiation pattern.

(c) Go to the infinite (in practice two is enough) lepton flavour limit;

(d) compute $e^+e^- \to \mu^+\mu^-$ and $e^+\mu^- \to e^+\mu^-$ to obtain separately s and s channels contribution separately.

(e) compute the relative weight of the two channels and select one on a statystical basis.

3. Sudakov form factors to next-to-leading log (HERWIG)

The prescription to correctly generate the parton-shower associated to a given event in the large-N limit is therefore the following:

1. Calculate the $(n-1)!$ dual amplitudes corresponding to all possible planar colour configurations.
2. Extract the *most likely* colour configuration for this event on a statistical basis

Efficient event generation and $1/N$ corrections

- the number of dual amplitudes grows like $n!$
- size of non leading $1/N$ corrections?
- Solution:
  1. choose a standard $SU(3)$ orthonormal basis (Gell-Mann matrices for example)
  2. randomly select a non-vanishing colour assignement for the exernal gluons
  3. *if the event is accepted* choose randomly among the *contributing* dual amplitudes a color flow on the basis of their relative weight

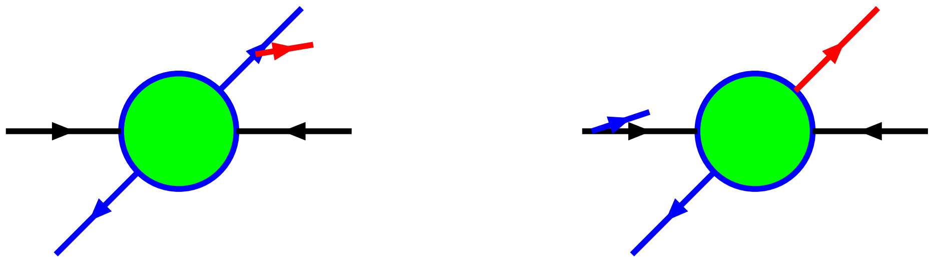
     Two advantages
     - dual amplitudes required only for a small number of phase space points
     - contributing dual amplitudes to a given external coulor assignment $\ll$ than total number.

From partons to jets: why do we need "matching?"

- we want to describe events with many hard jets in the final state: we need ME description AND parton evolution to hadrons (PS).
- Parton-level cuts should not be harder than jet cuts

  $p_{T parton} \geq p_{T cut} = E_{T jet}^{min}$ $\qquad$ $\Delta R(parton{-}parton) \geq \Delta R_{cut} = \Delta R_{jet}$

  One should start from softer parton-level cuts
- Due to softer cuts some events are obtained as:
  - two (or more) hard partons are clustered in the same jet
  - one (or more) jet is obtained from hard PS radiation
- double counting (suppressed by $O(\alpha_S)$)
- ME soft/collinear divergencies not dumped by Sudakov suppression.

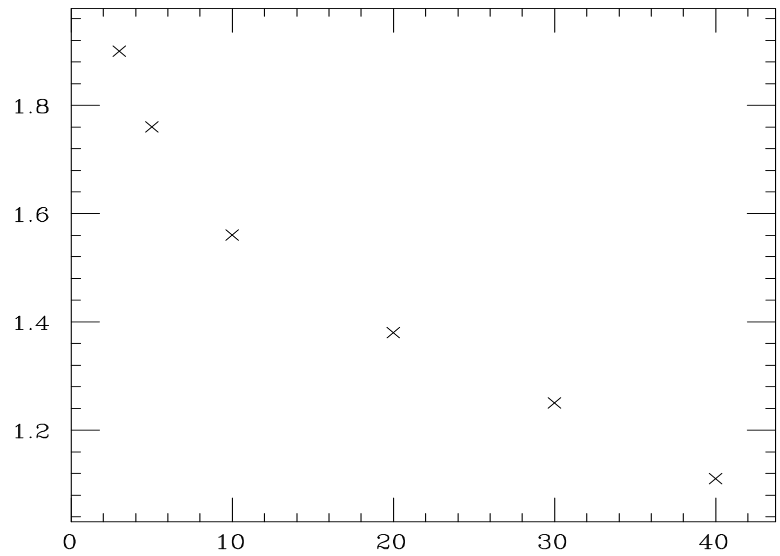


Black ME (initial partons); Blue ME (final partons); Red PS Two different emission leading to the same final state kinematics. In the left one the matrix element has no Sudakov damping for soft/collinear emission, leading to a divergent cross section.

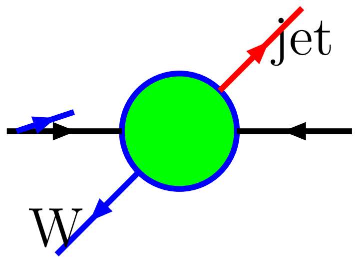
- Ideally the final jet cross-section should be independent of the parton-level generation cuts, even in the limiting case $p_{Tmin} \to 0$ and $\Delta R_{cut} \to 0$
- The double counting effect is suppressed by at least one power of α_S. Naively it should be small so why bother?
- A fixed order calculation accounting for the emission of coloured particles (of QCD origin) is *divergent* in the IR/Collinear limit.

- This behaviour can be controlled adding toghether virtual and real contribution of the same order (not avaliable for large multiplicities), still the prediction in the soft/collinear region is unreliable: *resummation required*

- PS includes resummation, fixed order ME doesn't



Cross section (nb) for the production of a W ($\rightarrow e\nu_e$) + jets at the LHC. The hardest jet is required to have $p_T > 40$ GeV. The cross section is plotted against the cut on the parton p_T at the generation level (NO MATCHING). The soft/collinear sensitivity is clearly seen.



BLACK: ME initial state partons. BLUE: ME final state partons.
RED: PS partons

- The growth is due to this class of events: the ME weight grows up to ∞ for soft/collinear emission.
- Notice that this affects distributions as well
- That's why we want to describe these events with the PS and treat only hard emissions with ME

Towards matching of ME & PS

For e^+e^- physics a solution has been proposed

S. Catani et al., JHEP 0111 (2001) 063

L. Lönnblad, JHEP 0205 (2002) 046

which avoids double counting and shifts the dependence on the resolution parameter beyond NLL accuracy

The method consists in separating arbitrarily the phase-space regions covered by ME and PS, and use vetoed parton showers together with reweighted tree-level matrix elements for all parton multiplicities

Proposal to extend the procedure to hadronic collisions: no proof of NLL accuracy

F. Krauss, JHEP 0208 (2002) 015

The CKKW procedure has been successfully tested on LEP data

e.g. S. Catani et al., JHEP 0111 (2001) 063

R. Kuhn et al., hep-ph/0012025

F. Krauss, R. Kuhn and G. Soff, J. Phys. G26 (2000) L11

Recent work for hadronic collisions

- `Herwig` (P. Richardson)
- `Pythia` (S. Mrenna)

  S. Mrenna and P. Richardson, JHEP **0405** (2004) 040

- `SHERPA with APACIC++/AMEGIC++`

  F. Krauss, A. Schälicke, S. Schumann and G. Soff, Phys. Rev. D **70** (2004) 114009

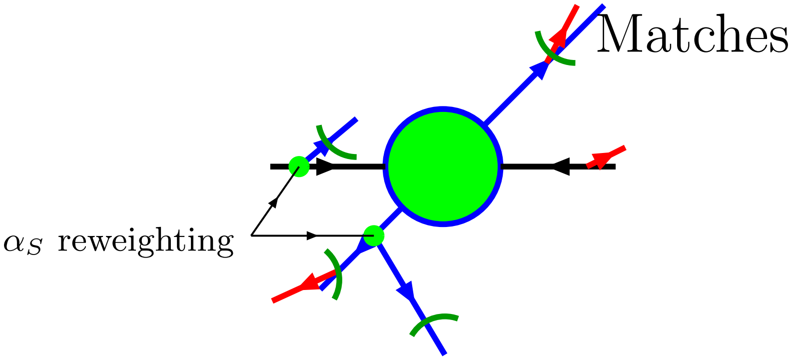
  F. Krauss, A. Schälicke, S. Schumann and G. Soff, Phys.Rev. D72 (054017) 2005.

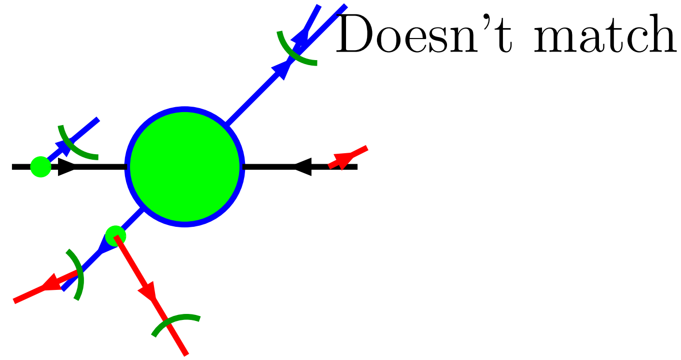
An alternative proposal

M.L. Mangano, FNAL MC Workshop, October 2002

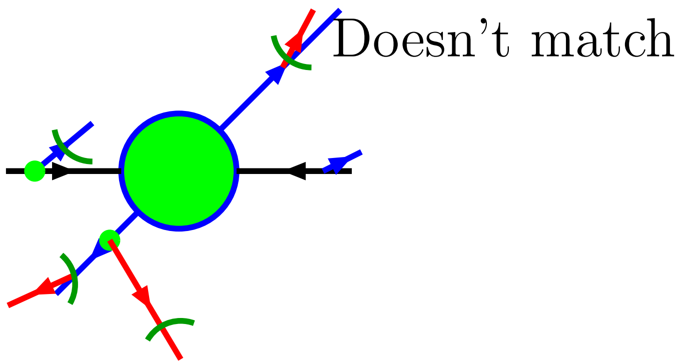
- generate event sample ($p_T > p_{Tmin}$ $\Delta R > \Delta R_{min}$
- shower the event and reconstruct particle clusters (jets) with a cone algorithm ($p_{Tjet} > p_{Tmin}$, $R_{jet} \geq R_{min}$)
  - Note: these clusters are just a computational device to define the sample. they don't need to coincide with "experimental" jet
- define the matching of a parton (LO matrix element) and a cluster as follows: a parton matches a cluster if the separation ΔR between the parton and the cluster is smaller than $\Delta \bar{R}$ (an arbitrary fixed quantity $\Delta \bar{R} \sim \Delta R_{min}$
- reject the event if more than one parton match the same cluster or if a parton doesn't match any cluster
- for *exclusive* samples also events with number of clusters different (larger) from number of partons are rejected

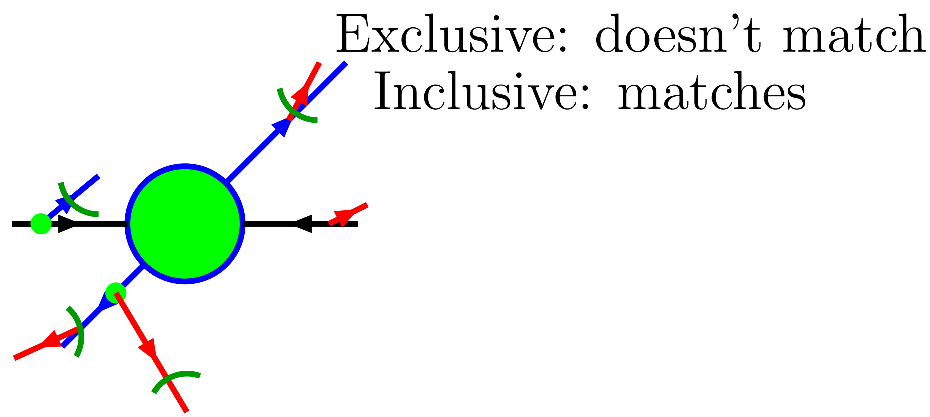
- reweight the ME reconstructing the branching tree and assigning to each branching $\alpha_S(Q_{branch})$ (to mimic the PS)





Left: all ME partons inside a distinct cone; Right two ME partons inside the same cone, one hard jets made from the shower

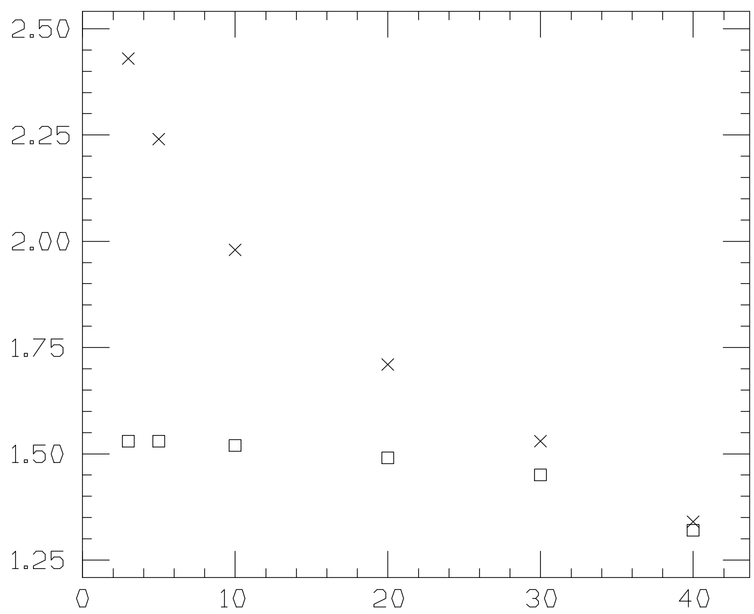




Left: a soft ME partons not inside a cone; Right all ME partons inside distinct cone, one extra hard jets made from the shower, since the number of jets is larger than matrix element partons, accepted only for inclusive samples

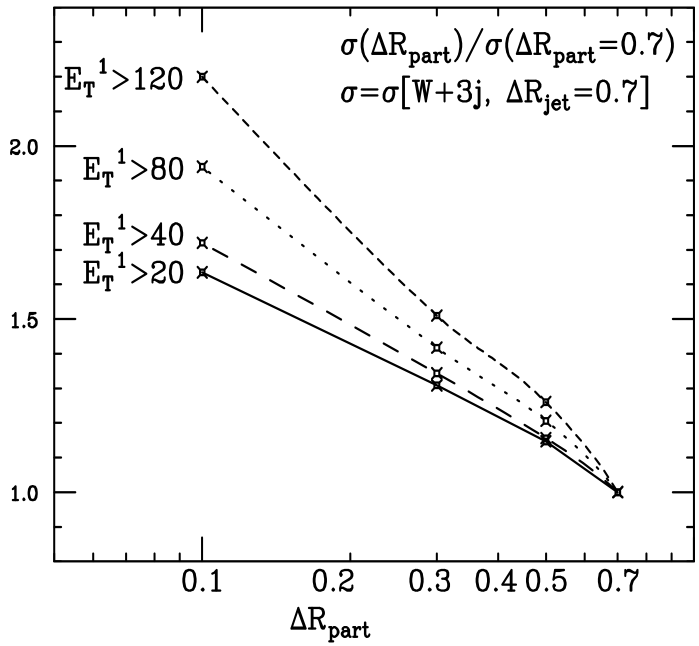
One still expects not better than LL (Sudakov) accuracy. However we expect a strongly reduced NLL sensitivity. From the practical point of view it is enough that these residual effects are smaller than the other systematics of the calculation

Ideally the whole prescription leads to samples independent from generation cuts. In practice the dependence from generation cuts is a measure of the success of the matching prescription



Cross section (nb) for the production of a W ($\to e\nu_e$) + jets. The hardest jet is required to have $p_T > 40$ GeV. The cross section is plotted against the cut on the parton p_T at the generation level. Crosses: no matching. Boxes: matching (one-jet inclusive sample)

Example: $W + 3$ jets at Tevatron



Cross section for $W + 3$ jets at Tevatron as a function of generation cuts (ΔR_{parton} $E_{T parton}$). The soft/collinear divergence is clearly seen. This feature is even more pronounced than in the $W + 1$ jet case: the larger the number of jets the larger the number of potentially "dangerous" *Logs*.

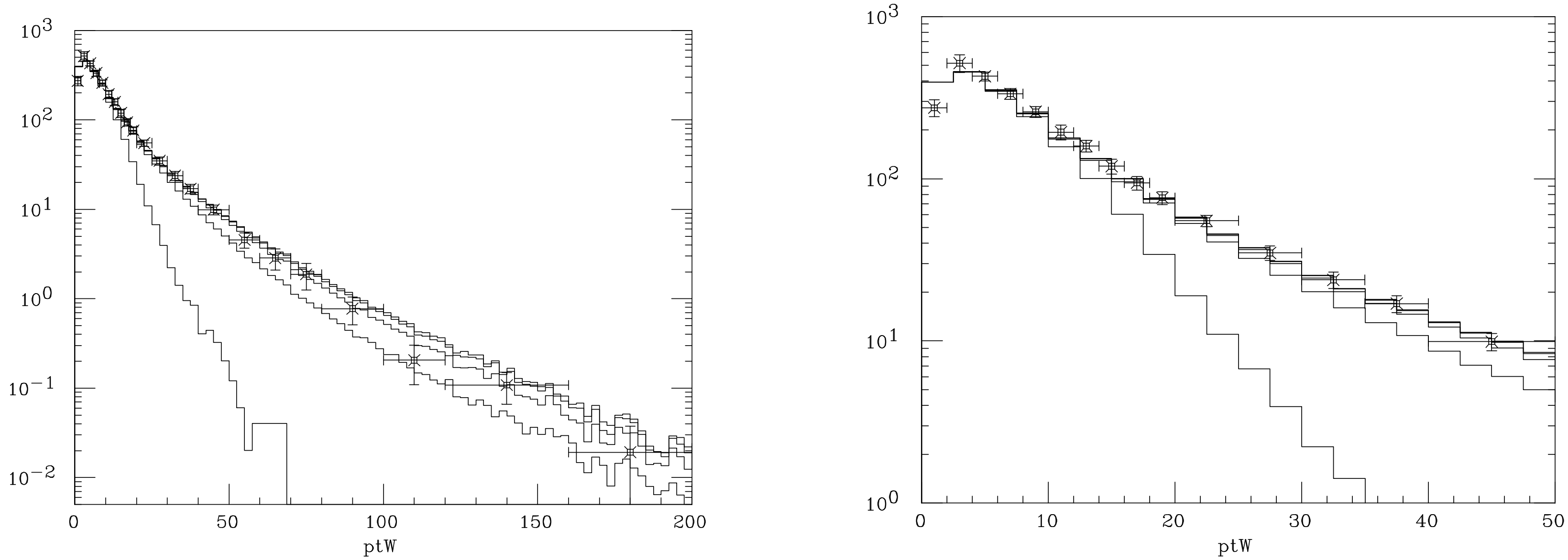


Figure 2: $p_{T,W}$ spectrum. The points represent run I CDF data. The curves correspond to the subsequent inclusion of samples with higher multiplicity, form the $W + 0$ jet, up to the $W + 4$ jets case. The right plot is the same as the left one, with an enhanced low-p_T scale.

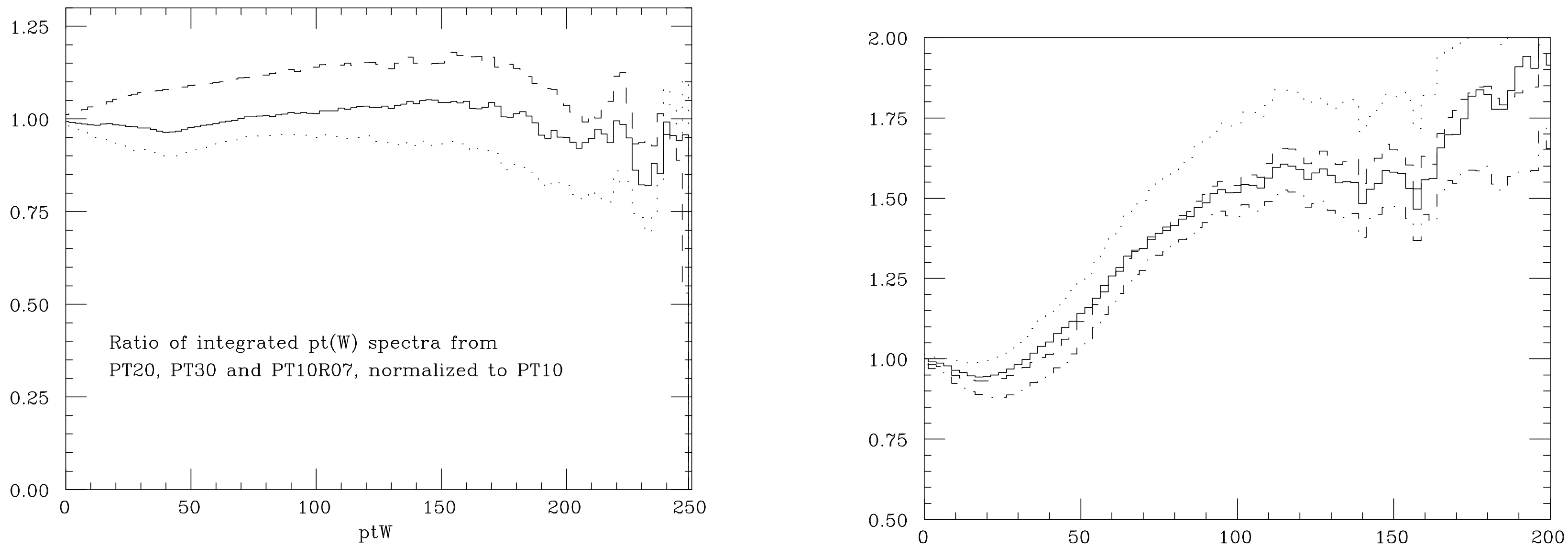


Figure 3: Effect of different genertaion cuts on the integrated $p_{T,W}$ spectrum. The right panel shows the ratios of the samples generated with PT20, PT30 and PT10R07, divided by PT10. The right panel shows all four samples divided by a plain (no ME correction) HERWIG W sample.

PT10, PT20, PT30 : $P_T > 10, 20, 30 GeV$, $\Delta R > 0.4$

PT30R07 : $P_T > 30$, $\Delta R > 0.7$

Work in progress

- Assesment of matching systematics: internal consistency, resolution parameters de
  (likely to be done on process by process basis)
- comparison against other codes and approaches (SHERPA, Madgraph, MC@NLO,
- study of alternative matching prescriptions
- inclusion of new processes and new effects (anomalous couplings, ...)