Precision relations of masses to Lagrangian parameters in SUSY

Loopfest V

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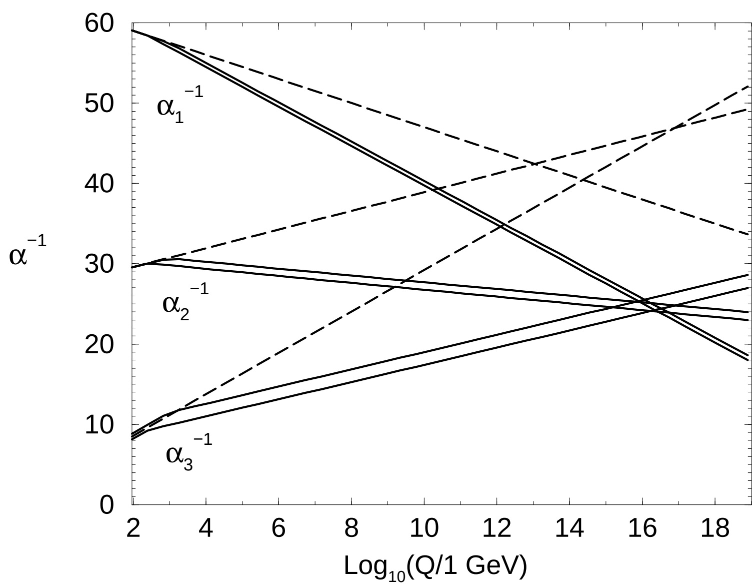
I will report on 2-loop and leading 3-loop contributions to gluino and Higgs masses in the MSSM. (Preprints to appear.)

Masses are the key observables in SUSY

Most of what we do not already know about supersymmetric extensions of the Standard Model involves the soft SUSY-breaking terms with positive mass dimension.

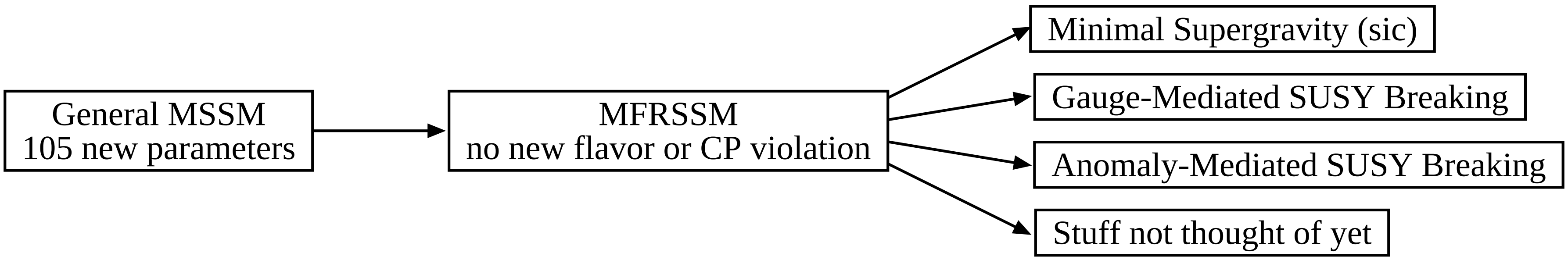
Predictions of specific models (Minimal Supergravity, Gauge Mediation, Anomaly Mediation, Extra-dimensional Mediation, ...) allow/require precise calculations.



The apparent unification of gauge couplings in the MSSM invites us to extrapolate the soft masses up to high scales, to see if they obey some Organizing Principle.

What is the Organizing Principle behind SUSY breaking?

A reasonable working hypothesis is the **Minimal Flavor-Respecting Supersymmetric Standard Model**. It is neither too painfully general, nor too naively specific:



MFRSSM parameter count:

3 gaugino masses	$M_1,\ M_2,\ M_3$
5 sfermion $(\text{mass})^2$	$m^2_{\tilde{Q}},\ m^2_{\tilde{u}},\ m^2_{\tilde{d}},\ m^2_{\tilde{L}},\ m^2_{\tilde{e}}$
3 $(\text{scalar})^3$ couplings	$A_{u0},\ A_{d0},\ A_{e0}$
3 Higgs mass parameters	$\mu,\ b,\ m^2_{H_u},\ m^2_{H_d}$ (but M_Z known)
1 input RG scale	Q_0

Total: 15 new parameters beyond the Standard Model

Gaugino Mass Unification is a popular and recurring theme.

$$M_1(Q) = M_2(Q) = M_3(Q) \equiv m_{1/2} \qquad \text{at } Q \approx 2 \times 10^{16} \text{ GeV},$$

resulting in

$$M_1 : M_2 : M_3 \approx 1 : 2 : 6$$

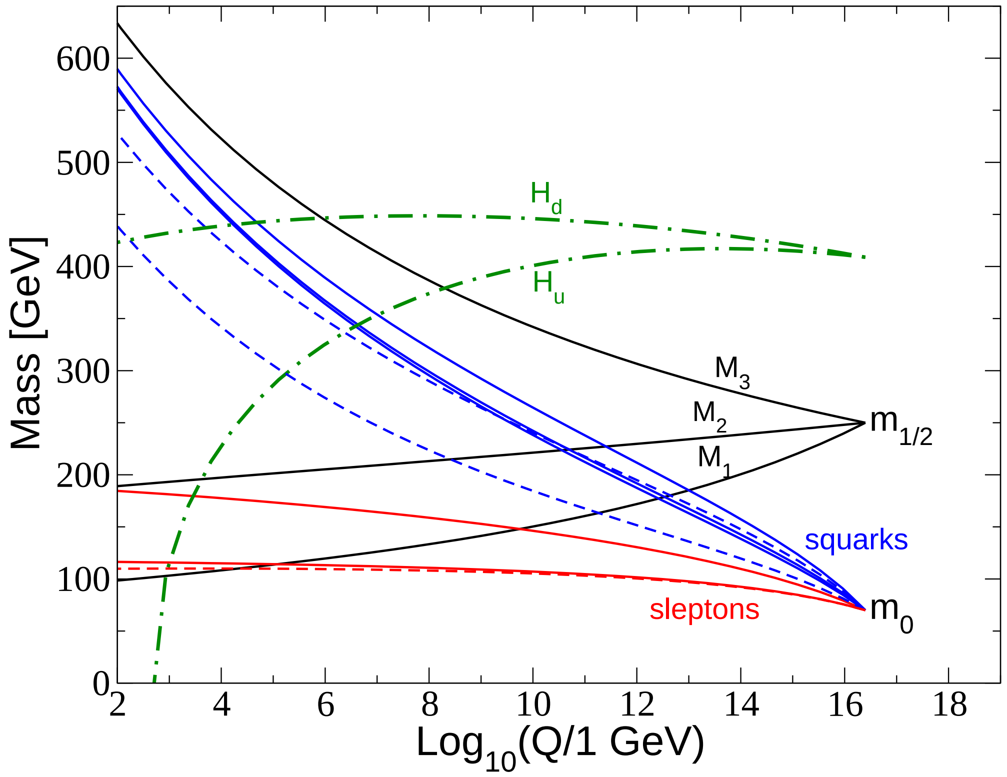
for Q near the TeV scale.

To test this, or alternatives to it, we have to relate physical masses to running masses in the Lagrangian (with no superpartners decoupled).

Goal: reduce purely theoretical sources of uncertainty to a negligible level, if possible.

(Experimental sources of error are a big problem, but not MY problem.)

Predictions of a typical Organizing Principle:



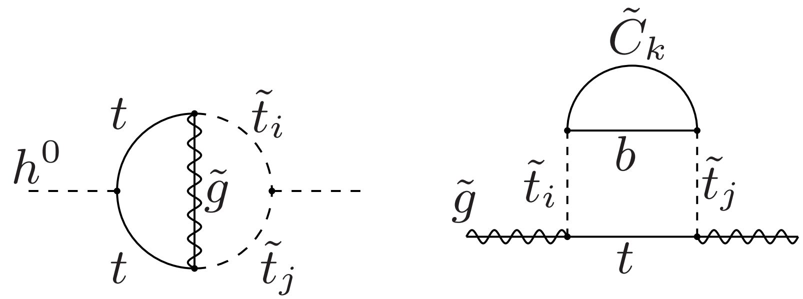
Determination of the running gluino mass parameter M_3 is crucial. It feeds "strongly" into any attempt to connect TeV scale physics with high-scale Organizing Principles in SUSY. The uncertainty in M_3 will likely dominate the errors in this effort, in the long run.

More generally, 2-loop (and some 3-loop) corrections to superpartner and Higgs masses will be mandatory if SUSY is correct, if we want experiment to be the dominant source of error in understanding Organizing Principles of SUSY breaking.

Some key features of the problem:

1) Two-loop diagrams involve many different mass scales simultaneously.

For example:



Large, diverse, and numerous hierarchies of ratios of squared masses will enter. Some of these hierarchies can be anticipated in advance, some can't.

This is a qualitative difficulty generally avoided in multiloop calculations in the Standard Model, where one knows in advance that

$$m_s^2 \ll m_c^2 \ll m_b^2 \ll m_t^2,$$

and calculations are organized around exploiting these hierarchies when doing multi-loop integrals.

2) To explore Organizing Principles, work in non-decoupled SUSY with mass-independent renormalization scheme

On-shell schemes are useful, as are effective theories in which some heavier superpartners are integrated out. Some problems are easier in those schemes. However:

- For the goal of running up to higher renormalization scales, we will want to know the running parameters in the full theory.
- Global fits can relate the directly measured observables to running $\overline{DR}$ input parameters.
- It is not so clear in advance what the best on-shell scheme input parameters will be. (For example, in the Higgs sector, A^0 mass or $H^\pm$ mass? For neutralinos and charginos, should the input parameters be masses, or mass differences, or some even more complicated kinematic function?)

3) Methods should be generic, reuseable from start to finish.

To avoid wasted effort, do calculations for scalars, fermions, vectors in a general perturbative field theory. Then apply to Higgs, squarks, sleptons, and to quarks, gluino, charginos, neutralinos, etc., or, ???

After all, SUSY might not be the correct answer, or it might be an incomplete answer.

To calculate physical masses

Evaluate self-energy = sum of 1-particle irreducible Feynman diagrams:

$$\Pi(s) = \Pi^{(1)}(s) + \Pi^{(2)}(s) + \ldots$$

where $s =$ the external momentum invariant.

The complex pole mass

$$s_{\rm pole} = M^2 - i\Gamma M$$

is the solution for complex s of:

$$\begin{aligned} s_{\rm pole} &= m_{\rm tree}^2 + \Pi(s_{\rm pole}) \\ &= m_{\rm tree}^2 + \Pi^{(1)}(m_{\rm tree}^2)\left[1 + \Pi^{(1)\prime}(m_{\rm tree}^2)\right] + \Pi^{(2)}(m_{\rm tree}^2) + \ldots \end{aligned}$$

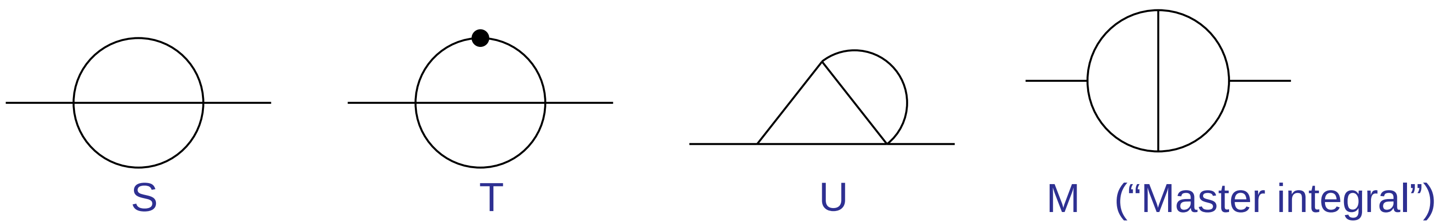
The pole mass is gauge invariant at each order in perturbation theory, can be related to kinematic masses as measured at colliders.

There are a large but finite number of 2-loop, two-point Feynman diagrams. Why not just do them once, for a general theory, and get it over with?

Method:

- Reduce all self-energies in general theory to a few basis integrals
- Basis integrals contain $\overline{\text{DR}}$ (or $\overline{\text{MS}}$) counter-terms, so finite.
- Numerically evaluate basis integrals quickly and reliably for arbitrary masses.

Tarasov's basis and recurrence relations:



Can always reduce 2-loop self-energies to a linear combination of these, with coefficients rational functions of:

$s = p^2 =$ external momentum invariant

$x, y, z, \ldots$ = internal propagator masses

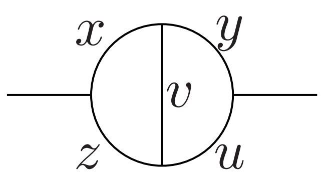
To evaluate basis integrals:

Values at $s = 0$ are known analytically, in terms of logs, polylogs.

$$\frac{\partial}{\partial s}\text{(basis integral)} \quad = \quad \text{(another self-energy integral)}$$
$$= \quad \text{(linear combination of basis integrals)}$$

So, we have a set of coupled, first-order, linear differential equations.

Consider the Master integral $M(x, y, z, u, v)$:



and the 12 U, S, T basis integrals obtained from it by removing propagators.

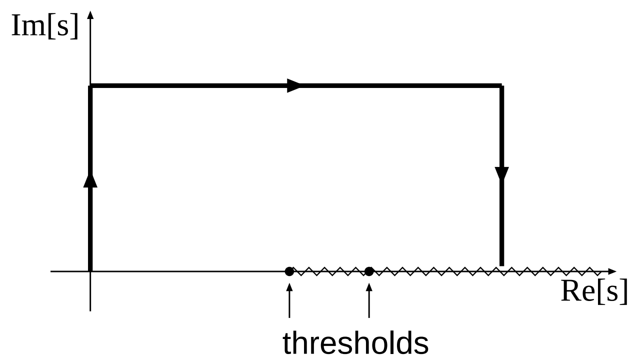
Call these 13 integrals I_n, $(n = 1, \ldots, 13)$.

Differential equations method for basis integrals

$$\frac{d}{ds} I_n = \sum_m K_{nm} I_m + C_n$$

Here K_{nm} are rational functions of s and $x, y, z \ldots$, and C_n are one-loop integrals. These are obtained by using Tarasov's recursion relations.

Solve for basis integrals I_n using Runge-Kutta integration in the complex s-plane, starting from known values at $s = 0$.



Method implemented for S, T, U type integrals by Caffo, Czyz, Laporta, Remiddi.

Dave Robertson and I have extended the method to also work for M:

TSIL = **T**wo-Loop **S**elf-energy **I**ntegral **L**ibrary

D.G. Robertson, SPM, hep-ph/0501132

Program written in C, callable from C++, Fortran

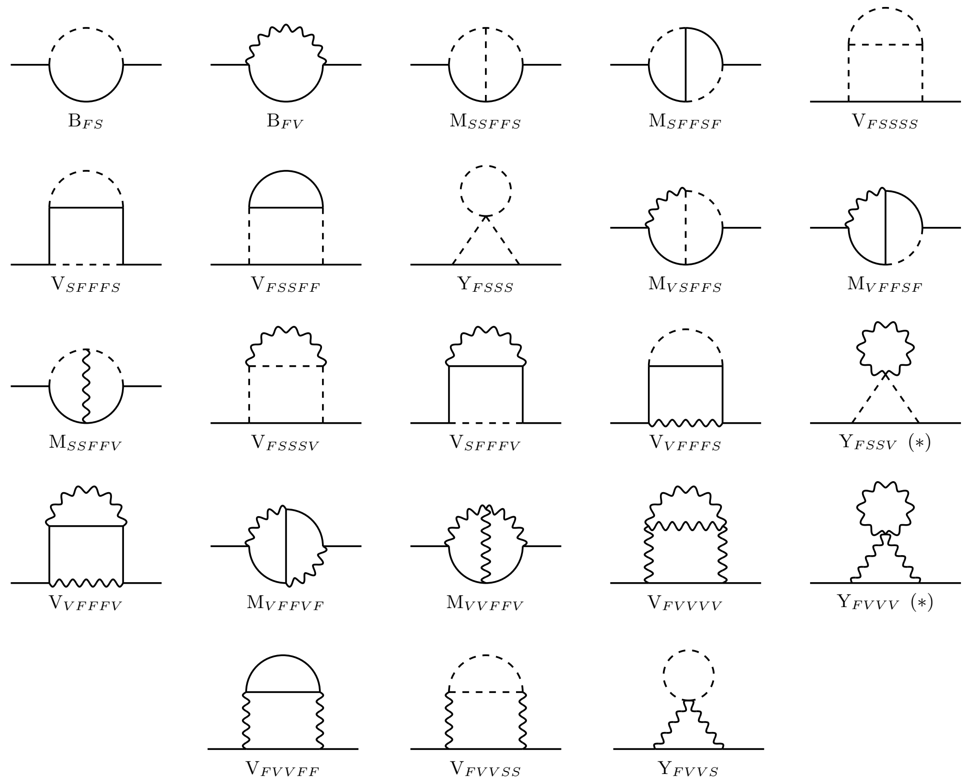
- Basis integrals computed for any values of all masses and s.
- All integrals from a given master integral obtained simultaneously in a single numerical computation.
- Checks on the numerical accuracy follow from changing choice of contour.
- Computation times generically $\ll$ 1 second on modern hardware.
- TSIL knows all special cases that have been done analytically in terms of polylogarithms

In the Hopi culture native to the American southwest, Tsil is the Chili Pepper Kachina. The Kachina are supernatural spirits, represented by masked figurines and impersonated by ceremonial dancers. They communicate between the tribe and their gods, who live in the San Francisco mountains and are never seen directly.

I have computed the 2-loop fermion pole masses in a general renormalizable theory with massless gauge bosons, in hep-ph/0509115.

Each diagram is reduced to a linear combination of basis integrals, ready to be computed numerically using the computer program TSIL (SPM, D.G. Robertson 2005).

Special case applications within the MSSM include the top quark mass, neutralino and chargino masses, and the gluino.



+ fermion mass insertions

+ ghost diagrams

+ counterterms

Checks on the calculation of 2-loop fermion pole masses:

- Independent of gauge-fixing parameter

  Individual diagrams depend on ξ; cancels in pole mass

- Pole mass is renormalization group invariant

  Checked analytically at 2-loop order; numerical check below

- Absence of divergent logs on shell

  Individual diagrams have $\log(1 - p^2/m^2)$, divergent as $p^2 \to m^2$; must and do cancel in pole mass

- Checks in (unphysical) supersymmetric limit

  Agrees with earlier calculation of scalar pole mass (SPM hep-ph/0502168)

Gluino pole mass at 2-loop order

(Y. Yamada, `hep-ph/0506262`; SPM, `hep-ph/0509115`)

The full formulas are a little too complicated to be presented in a talk, but are in the second paper. A C program based on TSIL can be obtained at:

`zippy.physics.niu.edu/gluinopole/`

Instead, I'll just show some simple special approximations.

In the following, squarks are always assumed to be degenerate and quarks to be massless, for simplicity. Also,

$$\alpha_s, \quad M_3, \quad \text{and} \quad m_{\text{squark}}$$

refer to running parameters in the $\overline{\text{DR}}$ scheme, evaluated at a renormalization scale $Q = M_3(Q)$.

The pole mass $M_{\tilde{g}}^{\rm pole}$ is computed in terms of these.

Example: In the special case of degenerate running masses, $M_3 = m_{\text{squark}}$, the result for the pole mass simplifies and can be written analytically:

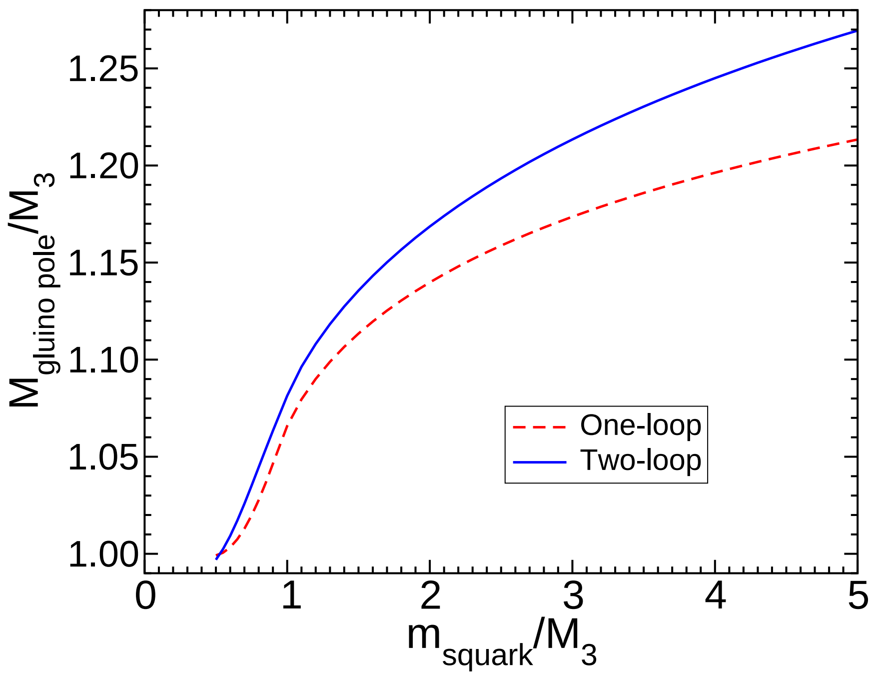
$$M_{\tilde{g}}^{\text{pole}} = M_3 \left[1 + \frac{\alpha_s}{4\pi} 9 + \left(\frac{\alpha_s}{4\pi} \right)^2 \left\{ 54\zeta(3) + \pi^2(53 - 36\ln 2) - 90 \right\} + \ldots \right]$$

$$= M_3 \left[1 \; + \; 0.716\, \alpha_s \; + \; 1.59\, \alpha_s^2 + \ldots \right]$$

(M_3 and α_s are running parameters evaluated at $Q = M_3$ in non-decoupled theory.)

However, the corrections for heavier squarks are quite large. . .

Dependence of gluino pole mass correction on the squark masses



For heavier squarks, part of the large corrections come from large logarithms that can be resummed using the renormalization group.

For $m_{\rm squark} \gg M_3$:

$$M_{\tilde{g}}^{\rm pole} \;=\; M_3\Big[1 \,+\, 0.955(L+1)\alpha_s \,+\, (0.46L^2 + 1.53L + 0.90)\alpha_s^2 + \ldots\Big]$$

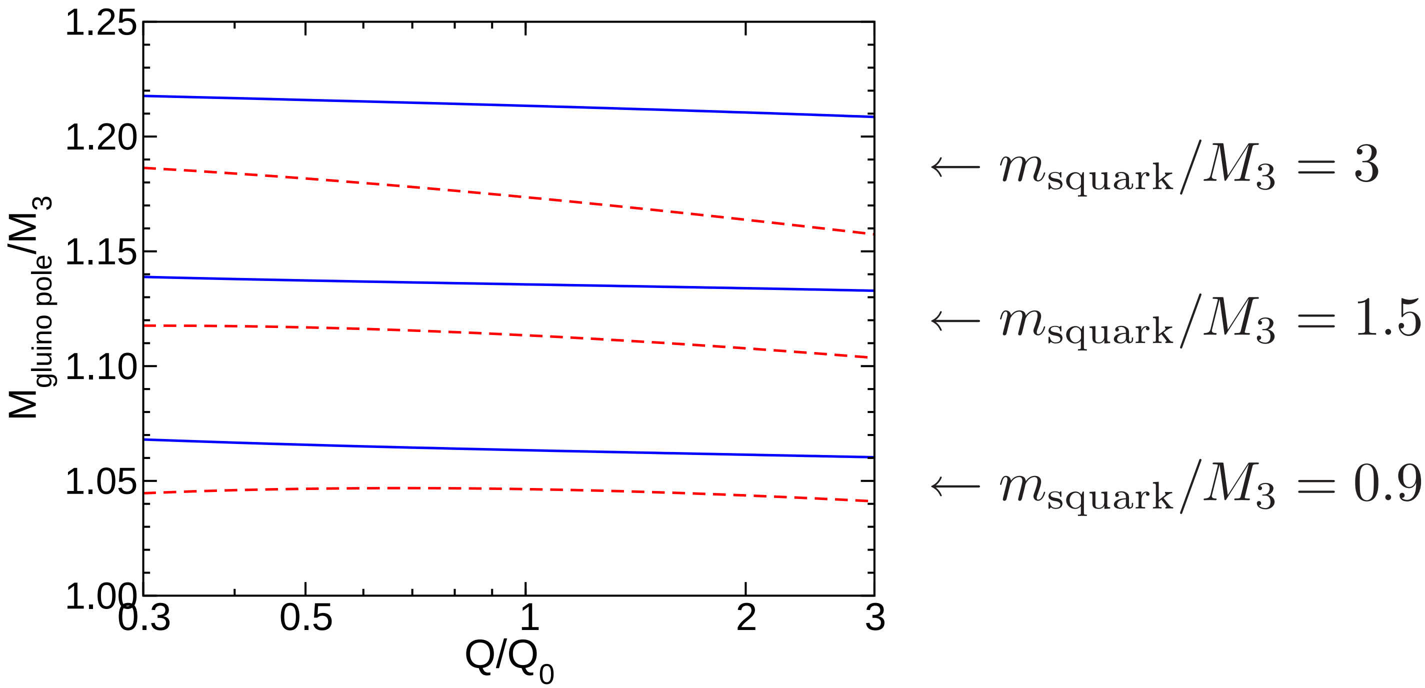
where $L \equiv \ln(m_{\rm squark}/M_3)$.

Obvious Questions: How big is the theoretical error? Can we estimate the 3-loop corrections? Is perturbation theory under control?

How NOT to estimate theoretical error: RG scale dependence

Run α_S, M_3 from Q_0 to a new RG scale Q, recompute pole mass:

Red $=$ 1-loop, Blue $=$ 2-loop



Scale dependence of 2-loop result is < 1%.

But, the 2-loop correction is much larger than the 1-loop scale dependence!

Dependence of the computation on the choice of RG scale significantly underestimates the true theoretical error.

A more useful estimate of the error uses RG and effective field theory techniques to obtain the 3-loop contributions for large

$$L = \ln(m_{\rm squark}/M_3).$$

Crucial ingredients:

- **2-loop** threshold corrections for M_3 in MSSM
  (SPM 2006)

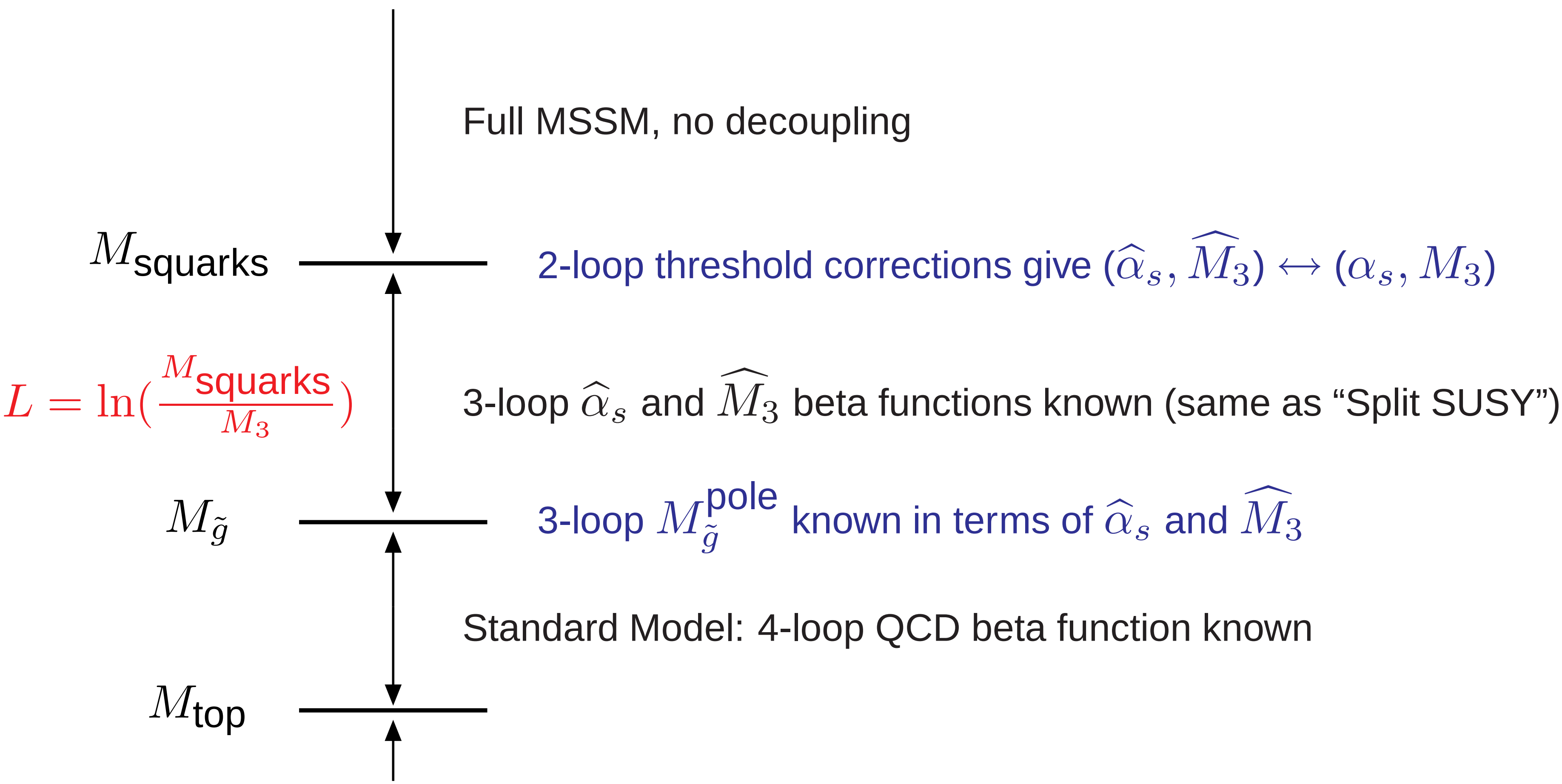
- **2-loop** threshold corrections for α_s in MSSM
  (Bern, DeFreitas, Dixon, Wong 2002; Harlander, Mihaila, Steinhauser 2005)

- **2-loop** pole mass in a theory with only fermions
  (Gray, Broadhurst, Grafe, Schilcher 1990)

- **3-loop** mass beta function in a theory with only fermions, but in different reps
  (Tarasov 1982, unpublished, available from KEK server, only in Russian!)

Three-loop gluino mass corrections for heavy squarks

Exploit the fact that beta functions are easier to compute, known to $\geq$3-loop order. Let the running parameters in the full MSSM be α_s, M_3, and in the effective theory with squarks decoupled, $\widehat{\alpha}_s, \widehat{M}_3$.



Using the effective field theory matching and RG running technique, one obtains all terms of order

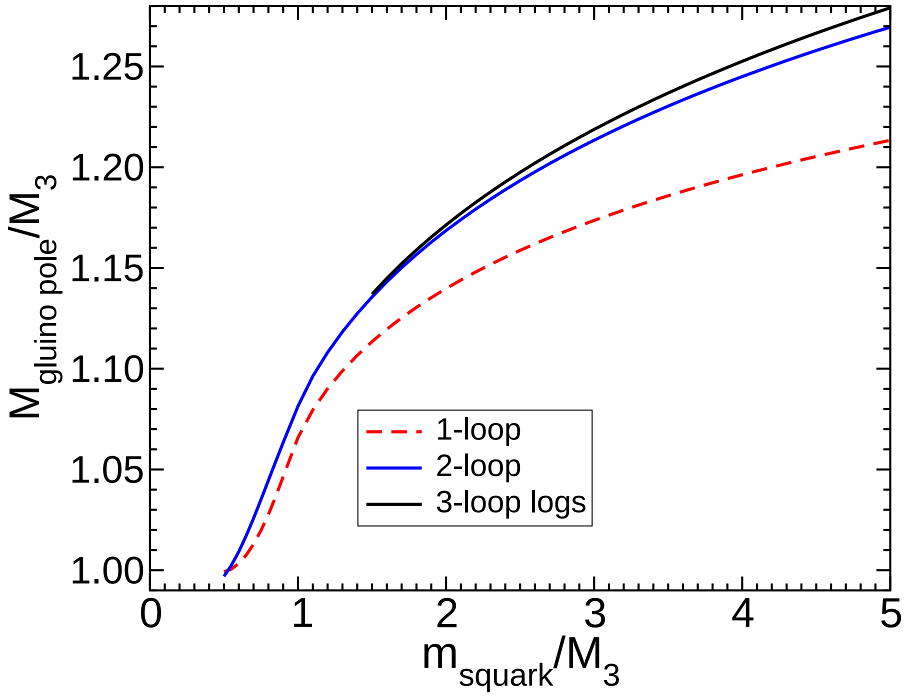
$\alpha_s^n L^n$	1-loop β functions, 0-loop threshold matching
$\alpha_s^n L^{n-1}$	2-loop β functions, 1-loop threshold matching
$\alpha_s^n L^{n-2}$	3-loop β functions, 2-loop threshold matching

for all n. The 3-loop pole mass for the gluino is:

$$M_{\tilde{g}}^{\text{pole}} = M_3 \Big[1 + 0.955\,(L+1)\,\alpha_s + (0.46L^2 + 1.53L + 0.90)\,\alpha_s^2 + (0.19L^3 + 0.32L^2 + 1.38L + \textbf{???})\,\alpha_s^3 + \mathcal{O}(M_3^2/m_{\tilde{Q}}^2) + \mathcal{O}(\alpha_s^4) \Big]$$

- The "leading log" does NOT dominate.
- Only a real 3-loop pole mass calculation can tell us what **???** is.

Three-loop "log-enhanced" effects on the gluino pole mass



The three-loop log corrections are only shown for $m_{\text{squarks}}/M_3 > 1.5$, where the approximation may start to become meaningful.

The actual 3-loop correction involves a non-log-enhanced piece, not captured in this analysis. However, circumstantially, this seems likely to be under 1%.

Another handle on the 3-loop contribution to the gluino pole mass.

The 3-loop pole mass for a heavy color octet fermion in the presence of 6 ordinary light quarks can be inferred from Melnikov and van Ritbergen (1999):

$$M_{\tilde{g}}^{\rm pole} = \widehat{M}_3 \Big[1 + 0.955\,\widehat{\alpha}_s + 1.69\,\widehat{\alpha}_s^2 + 3.4\,\widehat{\alpha}_s^3 + \mathcal{O}(\hat{\alpha}_s^4) \Big]$$

Note well: this is the result in an effective theory without squarks.

Equivalently, this is the result you would get in the MSSM if you “forgot” to compute all diagrams involving squarks, and worked in $\overline{\rm MS}$ instead of $\overline{\rm DR}$.

The α_S^3 contribution is agreeably small.

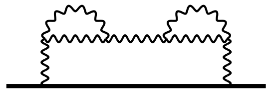
BUT WAIT! Maybe it is only small here because of accidental cancellation?

In fact, there **is** a fermion-boson loop cancellation (but **not** due to SUSY!)

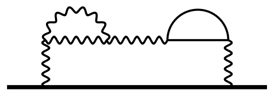
Divide the 3-loop contribution into eleven distinct group theory invariants:

$$M_{\tilde{g}}^{\text{pole}}|_{\text{3-loop}}/(\widehat{\alpha}_s^3 M_3) = 3.4$$

$$= \Big[+13.8$$

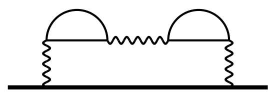


$$-11.4$$



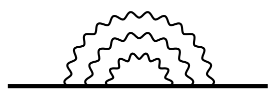
(massless quarks in loop)

$$+1.7$$



(massless quarks in loops)

$$-0.9$$

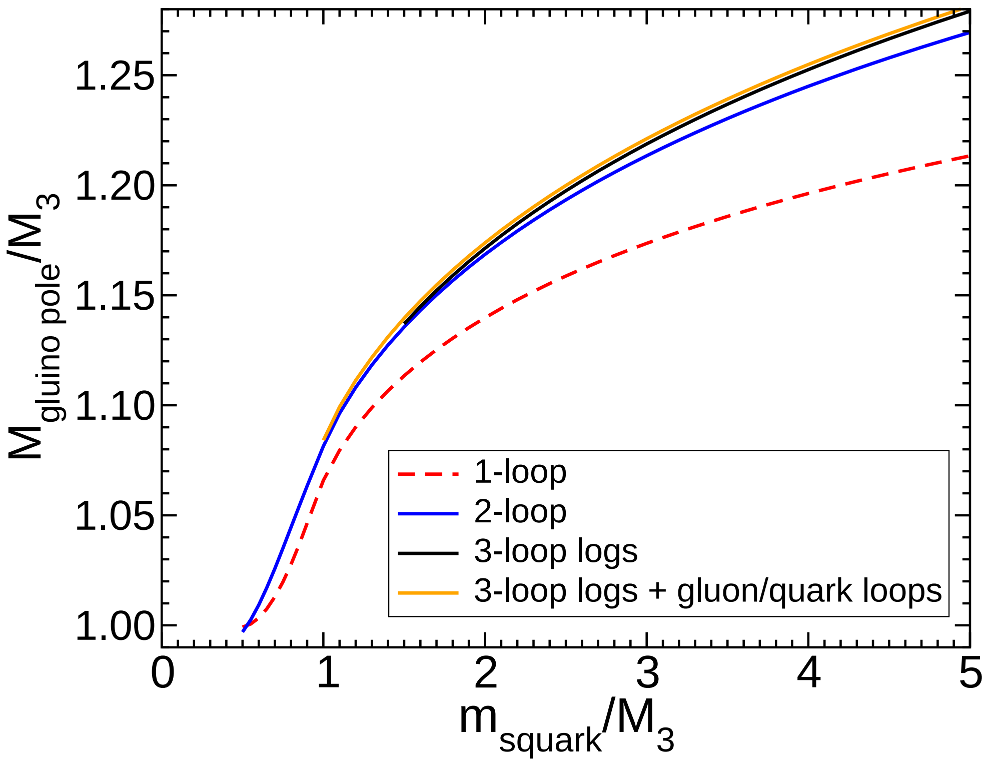


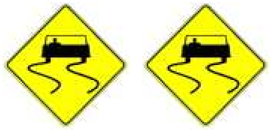
$$+\text{(seven smaller terms)}\Big]$$

The big contributions all come from diagrams without heavy particle loops.

 So maybe it is roughly numerically correct to just add this to the existing 2-loop contribution?

Including the contribution of gluons and quarks:



 Neglects, in the 3-loop part:

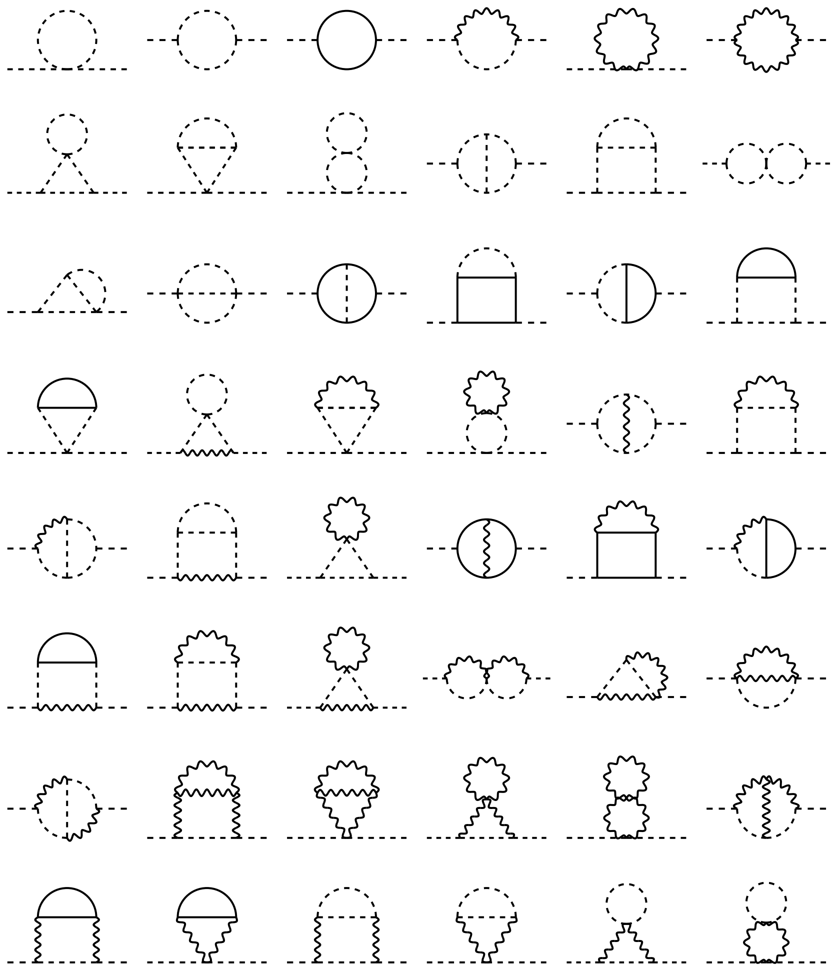
- squark loop effects not enhanced by logs
- epsilon scalars in $\overline{\text{DR}}$

2-loop corrections to scalar self-energies and pole masses in a general renormalizable theory

(hep-ph/0502168)

(Approximation: vector boson masses neglected in diagrams with two or more vector propagators.)

Applications to Higgs masses, slepton masses and squark masses in the MSSM.



+ fermion mass insertions + ghosts

+ counterterms

Many different groups have attacked the problem of the h^0 mass using different schemes (On-shell, $\overline{\text{DR}}$) and methods (diagrammatic, effective potential, effective field theory + RG), and combinations of these.

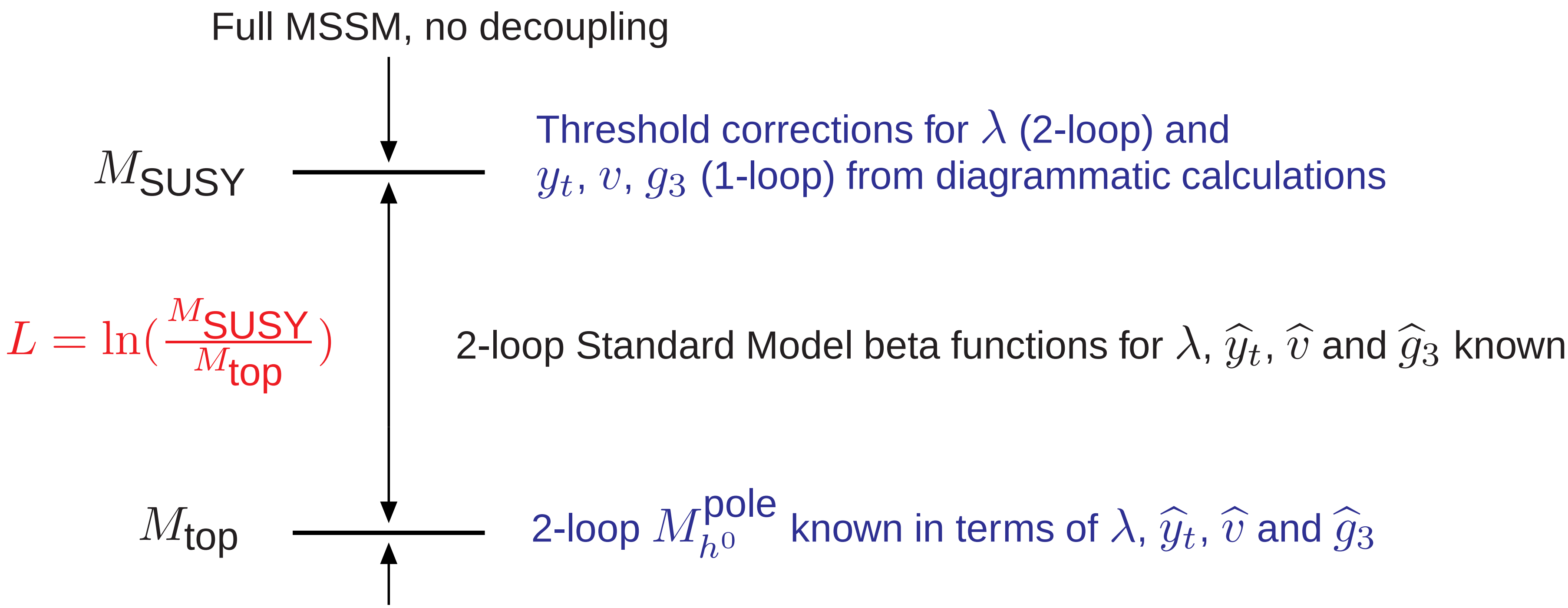
The most important 2-loop corrections are now known in all approaches.
But, what about 3 loops?

For example, Degrassi et al (`hep-ph/0212020`) estimate a 1-1.5 GeV contribution coming from leading log 3-loop effects, if $M_{\rm squark} = 1000$ GeV.

To address this, I combine:

- diagrammatic approach at 2 loops
- effective field theory + RG method for leading and next-to-leading log 3-loop corrections in the non-decoupled $\overline{\text{DR}}$ scheme.

Schematic picture of the strategery:



This gives all contributions at N loop order of the form:

$$g_3^{2j} y_t^{2N-2j} L^N \qquad \text{and} \qquad g_3^{2j} y_t^{2N-2j} L^{N-1}.$$

for each $j = 1, 2, \ldots N$.

For simplicity, in this talk I will present the result in the following limits:

- Heavy Higgs decoupling $m_{A^0} \gg m_{h^0}$.
- Large $\sin\beta \approx 1$
- Small top squark mixing
- y_t and g_3 3-loop corrections only
- Heavy, degenerate superpartners with mass $M_{\rm SUSY} \gg m_{\rm top}$.

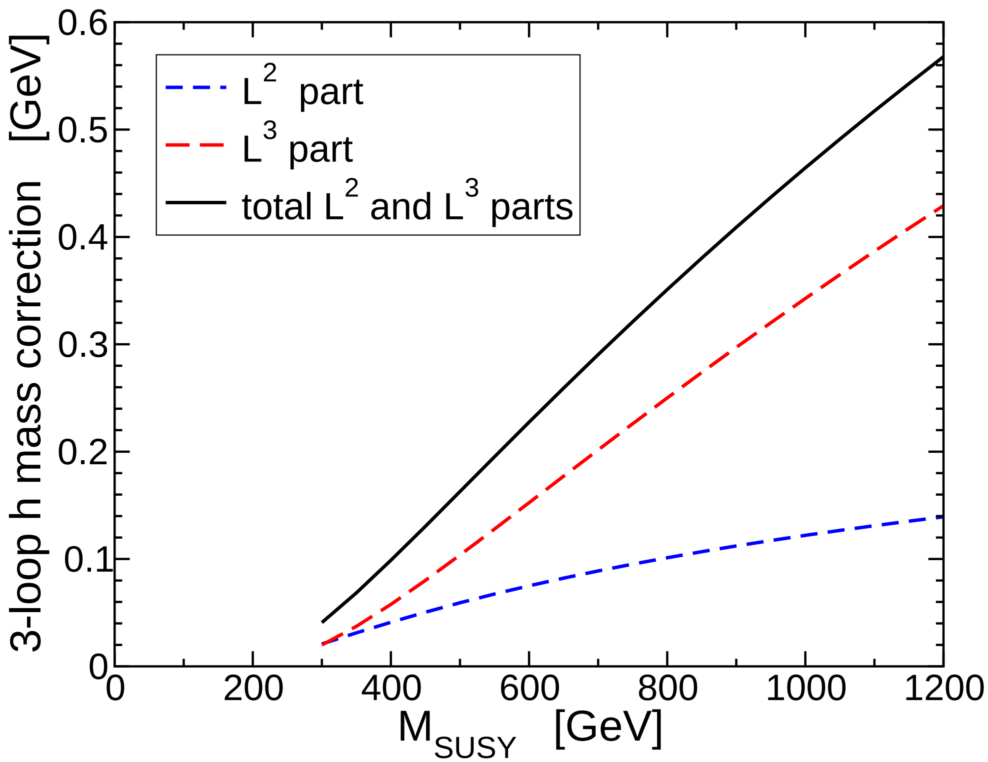
Define: $L \equiv \ln(M_{\mathsf{SUSY}}/m_{\rm top})$

Three-loop correction to the lightest Higgs squared mass in MSSM:

$$\Delta m_{h^0}^2 = \frac{1}{(16\pi^2)^3} y_t^2 m_t^2 \Big[(5888 g_3^4 - 5376 g_3^2 y_t^2 + 720 y_t^4) L^3 + (2304 g_3^4 - 1440 g_3^2 y_t^2 + 666 y_t^4) L^2 + (\textbf{???}) L + (\textbf{???}) \Big]$$

- All parameters are running $\overline{\text{DR}}$, evaluated at $Q = M_{\text{SUSY}}$
- Significant cancellation between strong and Yukawa effects (more fortuitous than in other schemes)
- Squark mixing and non-degeneracy is significant (to appear)
- To get **???**, need 3-loop Standard Model Higgs coupling beta function, and 2-loop threshold corrections
- To get **???**, need a real 3-loop calculation.

Numerically, these 3-loop L^3 and L^2 corrections for the h^0 mass look like:



(Assumes $m_{h^0} = 120$ GeV.)

Top squark mixing adds a significant correction to the L^2 piece (to appear).

Questions

- How, precisely, does the gluino pole mass relate to the gluino mass that will be reported by LHC experiments?
  Is the difference negligible?
- How, precisely, do the other sparticle pole masses relate to the masses that will be reported by the LHC and ILC?
  The differences seem unlikely to be negligible.
- What will be the best way(s) to organize input parameters vs. output parameters?
- What, if anything, can the ILC do to help pin down the gluino mass parameter?