The Top Pair Threshold: Status & New Developments

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thanks to T. Teubner, A. Manohar, I. Stewart, P. Ruiz-Femenia

C. Reisser, C. Farrell, M. Stahlhofen

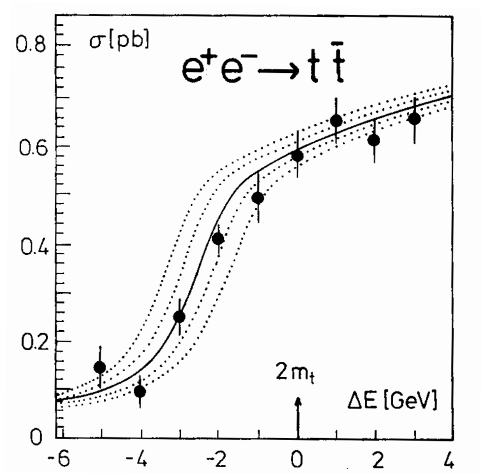
Outline

- Brief introductions
  - measurements & requirements
  - theory issues
- Status of QCD corrections
- Electroweak effects
  - hard electroweak corrections
  - finite lifetime effects & phase space matching
- Other Application:
  - $e^+e^- \to t\bar{t}H$
  - $e^+e^-, \gamma\gamma \to \tilde{t}\bar{\tilde{t}}$
  - m_b from $\sigma(e^+e^- \to$ hadrons$)$

Threshold Measurements

Threshold Scan: $\sqrt{s} \simeq 350$ GeV (Phase I)

- count number of $t\bar{t}$ events
- color singlet state
- background is non-resonant
- physics well understood
  (renormalons, summations)



$\longrightarrow \delta m_t^{\rm exp} \simeq 50$ MeV

$\longrightarrow \delta m_t^{\rm th} \simeq 100$ MeV

What mass?

$\sqrt{s}_{\rm rise} \sim 2m_t^{\rm thr} +$ pert.series

(short distance mass: 1S $\leftrightarrow$ $\overline{\rm MS}$)

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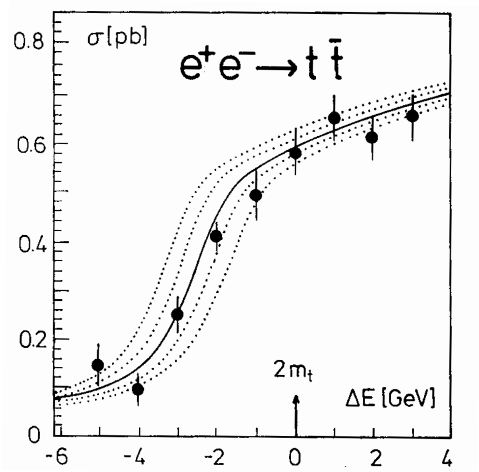

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Simulations $\mathcal{L} = 300\,{\rm fb}^{-1}$, $9+1$ scan points Peralta, Martinez, Miquel

$(\delta m_t)^{\rm stat} \sim 20$ MeV

$(\delta\lambda_t/\lambda_t)^{\rm stat} = 15 - 50\%$

$(\delta\alpha_s(M_Z))^{\rm stat} = 0.001$

$(\delta\Gamma_t)^{\rm stat} = 50$ MeV

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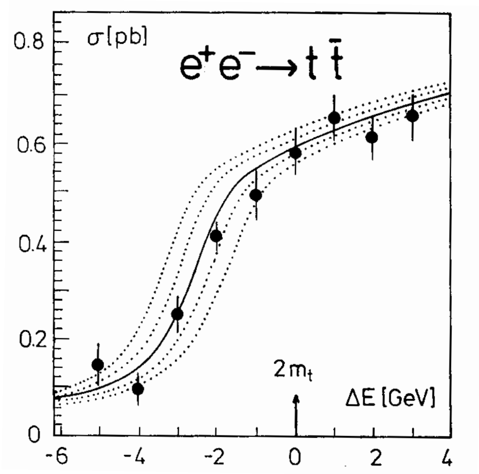

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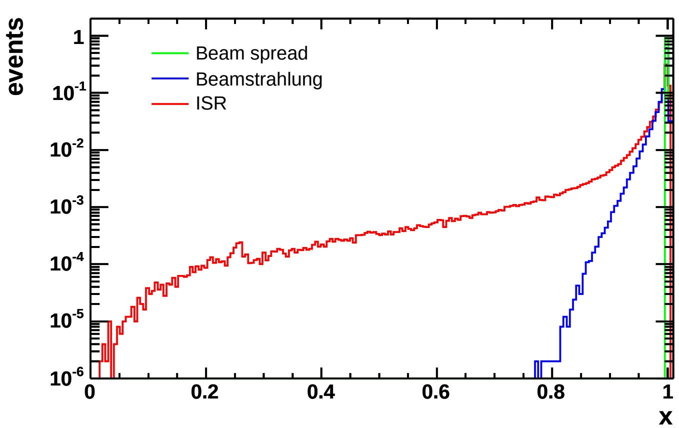
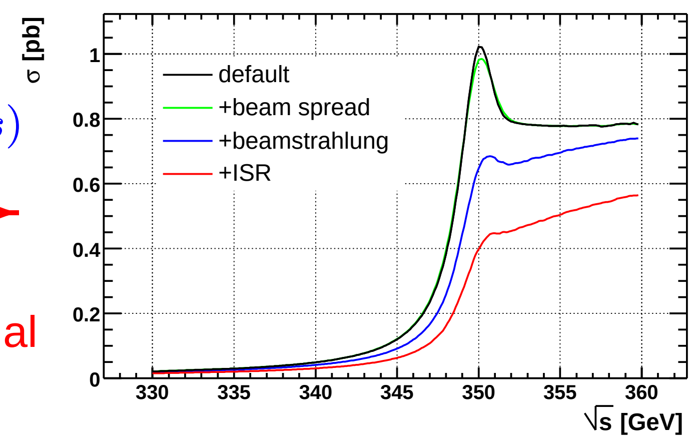
(short distance mass: 1S $\leftrightarrow$ $\overline{\rm MS}$)

Simulations Influence of Luminosity spectrum (LO QED effect!) Boogert



$$\sigma(s) = \int_0^1 dx L(x) \sigma^0(x^2 s)$$

- shape of $\mathcal{L}(x)$ essential
- babbah accolinearity



Threshold Measurements

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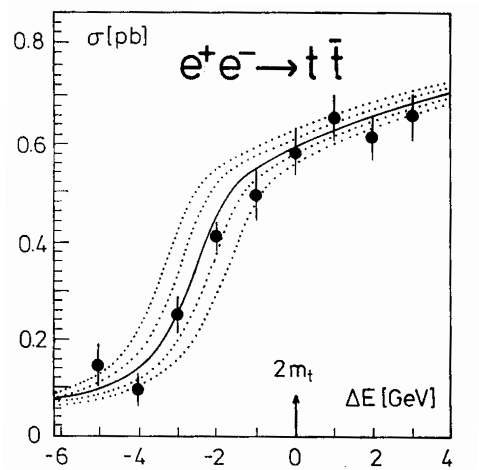


$\longrightarrow \delta m_t^{\rm exp} \simeq 50$ MeV

$\longrightarrow \delta m_t^{\rm th} \simeq 100$ MeV

What mass?

$\sqrt{s}_{\rm rise} \sim 2 m_t^{\rm thr} +$ pert.series

(short distance mass: 1S $\leftrightarrow$ $\overline{\rm MS}$)

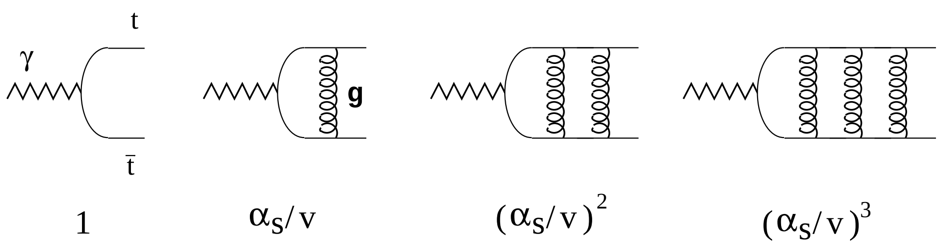
Simulations & theory goal: $(\delta\sigma/\sigma)^{\rm theo} \leq 3\,\%$

$(\delta m_t)^{\rm stat} \sim 20$ MeV	$(\delta m_t)^{\rm syst} \simeq 50$ MeV	$(\delta m_t)^{\rm theo} \simeq 100$ MeV
$(\delta\lambda_t/\lambda_t)^{\rm stat} = 15 - 50\%$	$(\delta\lambda_t/\lambda_t)^{\rm syst} = ?$	$(\delta\lambda_t/\lambda_t)^{\rm theo} \sim\ ?$
$(\delta\alpha_s(M_Z))^{\rm stat} = 0.001$	$(\delta\alpha_s(M_Z))^{\rm syst} = 0.002$	$(\delta\alpha_s(M_Z))^{\rm theo} \sim\ ?$
$(\delta\Gamma_t)^{\rm stat} = 50$ MeV	$(\delta\Gamma_t)^{\rm syst} = 15$ MeV	$(\delta\Gamma_t)^{\rm theo} \sim\ ?$

Theory Issues

m_t (hard) $\gg$ $\mathbf{p} \sim m_t v$ (soft) $\gg$ $E \sim m_t v^2$ (ultrasoft)

- perturbation theory in α_s breaks down $(\alpha_s/v)^n$



"Coulombsingularities"

$\longrightarrow$ Schrödinger Equation

- perturbation theory in α_s breaks down $\longrightarrow$ large logs $(\alpha_s \ln v)^n$

$m_t = 175$ GeV, $\mathbf{p} \sim 25$ GeV, $E \sim 4$ GeV $\Rightarrow \ln\left(\frac{m_t^2}{E^2}\right) = 8$ $\longrightarrow$ RGE's

$\Rightarrow$ "multi-scale problem"

vNRQCD: 1-step matching

pNRQCD $\longrightarrow$ A. Siegner's talk

Theory Issues

- $\boxed{\Gamma_t \approx 1.5\,\text{GeV} \gg \Lambda_{\rm QCD}}$ $\quad\Rightarrow\quad v = \sqrt{\frac{E}{m}} \;\longrightarrow\; v_{\rm eff} = \sqrt{\frac{E+i\Gamma_t}{m}}$

  (Fadin,Khoze)

  $\Rightarrow\; m_t \gg p = m v_{\rm eff} \gg E = m v_{\rm eff}^2 \gtrsim \Lambda_{\rm QCD}$ always true !

  $\Rightarrow$ top threshold entirely perturbative ! $\longrightarrow$ “Schrödinger theory”

- $\boxed{E \sim \Gamma_t:}$ top quarks are always produced off-shell !

  $\longrightarrow$ methods for on-shell production do not apply for

  - theoretical computations
  - experimental analysis

“theory for unstable particles”

vNRQCD (stable quarks)

$\underline{\mathcal{L} = \mathcal{L}_{\text{usoft}} + \mathcal{L}_{\text{potential}} + \mathcal{L}_{\text{soft}}}$

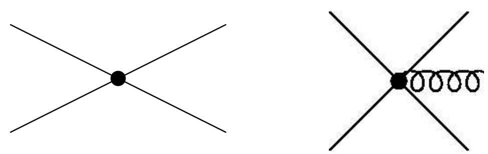
Luke, Manohar, Rothstein, Stewart, A.H.

$$D^\mu = \partial^\mu + i\mu_U^\epsilon g_s(m\nu^2)A^\mu$$

$\mathcal{L}_{\text{usoft}}$:

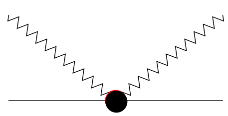
$$\psi_{\mathbf{p}}^\dagger(x)\left\{iD^0 - \frac{(\mathbf{p}-i\mathbf{D})^2}{2m_t} - \delta m_t\right\}\psi_{\mathbf{p}}(x)$$

$\mathcal{L}_{\text{potential}}$:



$$\mu_S^{2\epsilon}\, V(\nu)\, \psi_{\mathbf{p}'}^\dagger \psi_{\mathbf{p}} \chi_{-\mathbf{p}'}^\dagger \chi_{-\mathbf{p}}$$

$\mathcal{L}_{\text{soft}}$:



$$\mu_S^{2\epsilon}\, U_{\mu\nu}(\nu)\; \psi_{\mathbf{p}'}^\dagger A_q^\mu A_{q'}^\nu \psi_{\mathbf{p}}$$

$$\mu_U \;\propto\; \mu_S^2/m$$

$$V = \left[\frac{\mathcal{V}_c(\nu)}{\mathbf{k}^2} + \frac{\mathcal{V}_k(\nu)\pi^2}{m|\mathbf{k}|} + \frac{\mathcal{V}_r(\nu)(\mathbf{p}^2+\mathbf{p}'^2)}{2m^2\mathbf{k}^2} + \frac{\mathcal{V}_2(\nu)}{m^2} + \frac{\mathcal{V}_s(\nu)}{m^2}\mathbf{S}^2 + \frac{\mathcal{V}_\Lambda(\nu)}{m^2}\Lambda + \frac{\mathcal{V}_t(\nu)}{m^2}\,T\right]$$

$$\mathbf{k} \equiv \mathbf{p} - \mathbf{p}'$$

vNRQCD (stable quarks)

$\mathcal{L} = \mathcal{L}_{\rm usoft} + \mathcal{L}_{\rm potential} + \mathcal{L}_{\rm soft}$

Luke, Manohar, Rothstein, Stewart, A.H.

$$D^\mu = \partial^\mu + i\mu_U^\epsilon g_s(m\nu^2)A^\mu$$

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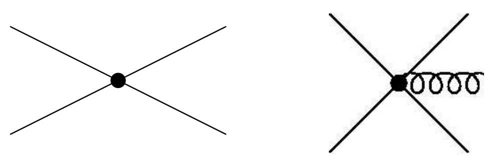
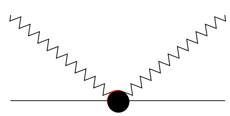
$\mathcal{L}_{\rm potential}$:



$$\mu_S^{2\epsilon}\,V(\nu)\,\psi_{\mathbf{p}'}^\dagger\psi_{\mathbf{p}}\chi_{-\mathbf{p}'}^\dagger\chi_{-\mathbf{p}}$$

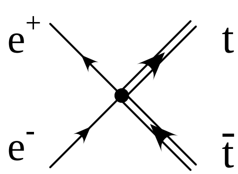
$\mathcal{L}_{\rm soft}$:



$$\mu_S^{2\epsilon}\,U_{\mu\nu}(\nu)\;\psi_{\mathbf{p}'}^\dagger A_q^\mu A_{q'}^\nu\psi_{\mathbf{p}}$$

$\mu_U \;\propto\; \mu_S^2/m$

external currents: (production & annihilation)



$$\mathbf{O}_{\mathbf{p}} = C_{V,A}(\nu)\cdot[\bar{e}\gamma^i(\gamma_5)e][\psi_{\mathbf{p}}^\dagger\,\boldsymbol{\sigma}^i\,\tilde{\chi}_{-\mathbf{p}}^*] + \cdots \qquad t\bar{t}\;(^3S_1)$$

Cross Section at NNLL Order

Schematic:

$$\sigma_{\rm tot} \quad \propto \quad {\rm Im}\left[\int d^4x\, e^{-i\hat{q}x} \left\langle\, 0 \,\middle|\, T\, \mathbf{O}^\dagger_{\mathbf{p}}(0)\, \mathbf{O}_{\mathbf{p}'}(x) \middle|\, 0 \,\right\rangle\right]$$

$$\propto \quad {\rm Im}\left[\left(C_A(\nu)^2 + C_V(\nu)^2\right) G(0,0,\sqrt{s})\right]$$

$$\left(-\frac{\nabla^2}{m} - \frac{\nabla^4}{4m^3} + V(\mathbf{r}) - (\sqrt{s} - 2m - 2\delta m) - i\Gamma_t\right) G(\mathbf{r},\mathbf{r}') = \delta^{(3)}(\mathbf{r}-\mathbf{r}')$$

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fully known
at NNLL order ✓

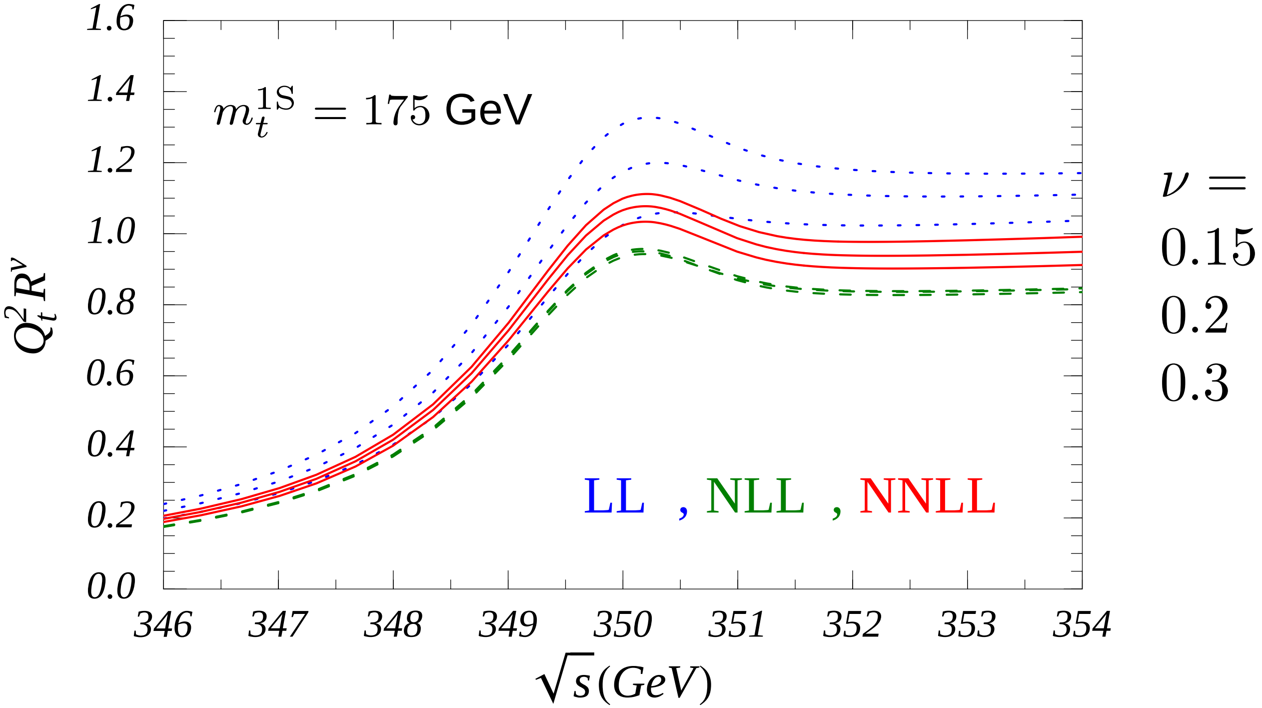
Manohar,Stewart; AH ’99-’03
Pineda,Soto ’00-’01
Peter ’94, Schröder ’98

NLL ✓ Luke etal. ’99
NNLL (matching) ✓ Benke etal; Czarnecki etal ’99
NNLL (non-mixing) ✓ AH ’03
NNLL (mixing) ess. unknown
spin-dependent (soft) Penin etal. ’04
usoft n_f Stahlhofen, AH ’05

Cross Section at NNLL Order

1S mass - RG-improved, with NNLL non-mixing terms

Manohar,Stewart,Teubner,AH



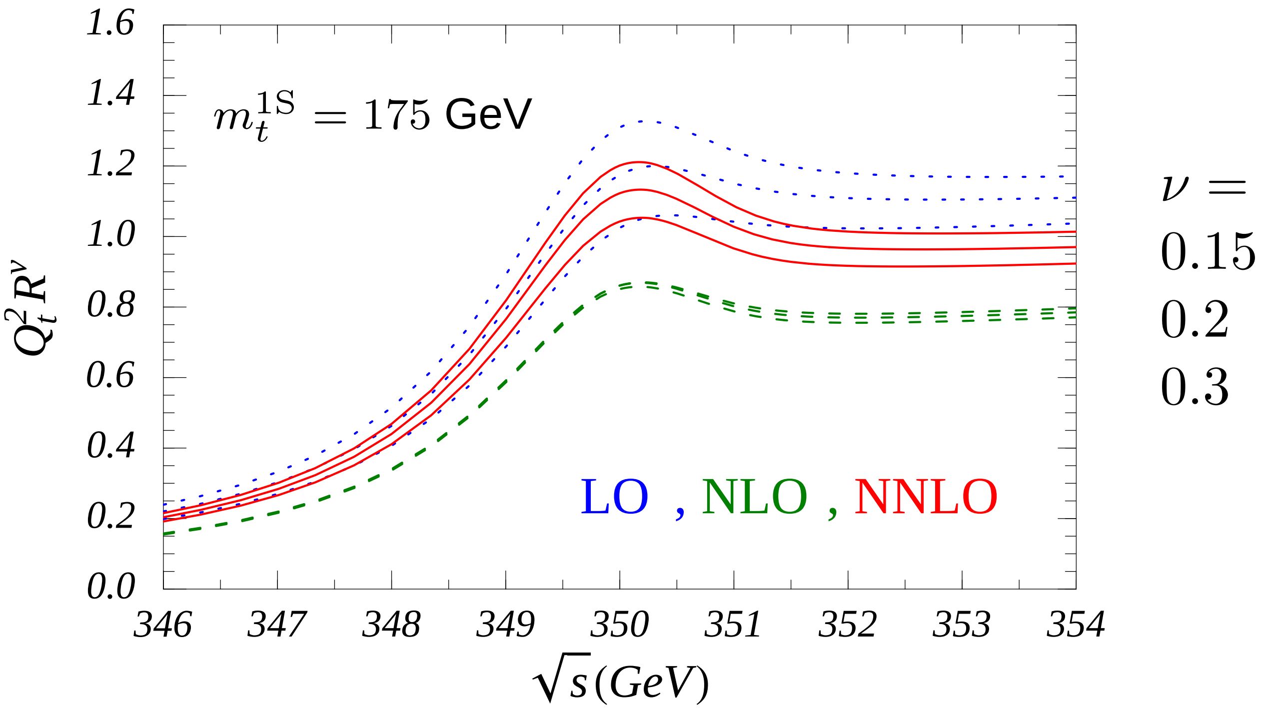
- RGI expansion shows better convergence
- theory error: $\delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} \sim \pm 6\%$ goal: 3%

$\longrightarrow$ full NNLL (mixing) running of $C(\nu)$ required $\longrightarrow$ Stahlhofen, AH (usoft w.i.p.)

Cross Section at NNLL Order

1S mass - fixed order approach

Teubner,AH; Melnikov, Yelkovski;Yakovlev; Beneke,Signer,Smirnov; Sumino, Kiyo



- peak position stable (threshold masses: 1S, PS, . . .)
- large sensitivity to factorization/renormalization scale setting
- NNNLO partial results: Penin etal. ’02 ’05, Beneke etal. ’05, Eiras etal. ’05

NRQCD (unstable quarks)

"inclusive treatment"

⇒ Optical Theory: effective complex indices of refraction for absorptive processes

⇒ NRQCD: contributions from real Wb final states included in EFT matching conditions to QCD+ew. theory (=SM)

- complex matching conditions & anomalous dimensions
- effective Lagrangian non-hermitian
- total rates through the optical theorem
- phase space matching
- power counting maintained
- expansion around mass-shell (Beneke etal. '04)
  $\longrightarrow$ automatic in NRQCD

Christoph Reisser, AH; Phys. Rev. D 71, 074022 (2005)
Christoph Reisser, AH; hep-ph/0604104

Electroweak Effects

3 classes:

- "Hard" electroweak
- Electromagnetic
- Finite lifetime

AHH, hep-ph/0604185

$\longrightarrow$ No general theory for all cases and observables !

$\longrightarrow$ EFT for certain observables and given powercounting.

$$\Gamma_t \sim m_t \alpha \sim m_t \alpha_s^2 \quad \Rightarrow \quad \boxed{v \sim \alpha_s \sim \alpha^{1/2}}$$

status for $\sigma_{\rm tot}$:

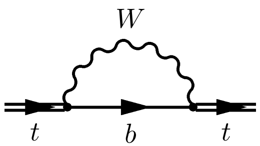
	LL	NLL	NNLL
"Hard" e.w.	✓	✓	✓
El. mag.	✓	(✓)	?
Fin. life.	✓	w.i.p.	w.i.p.

NRQCD (unstable quarks)

quark bilinears: $(\to \mathcal{L}_{\rm usoft})$

time dilatation correction

$\mathrm{Im}\Sigma_t = \frac{1}{2}\Gamma_t$



$$= i\Sigma_t \quad \Longrightarrow \quad \delta\mathcal{L} = \psi_{\mathbf{p}}^\dagger \left[i\frac{\Gamma_t}{2}\left(1 - \frac{\mathbf{p}^2}{2m_t^2}\right)\right]\psi_{\mathbf{p}}$$

- power counting: $\Gamma_t \propto m_t g^2 \sim m_t v^2 \quad \Rightarrow \quad \boxed{g \sim g' \sim v \sim \alpha_s}$
- finite lifetime is LL order, $E \to E + i\Gamma_t$ Fadin,Khoze
- NNLL time dilatiation effect
- $E \to E + i\Gamma_t$ prescription does not work beyond LL order

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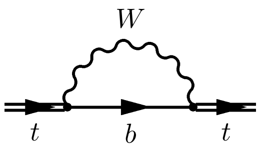

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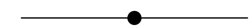
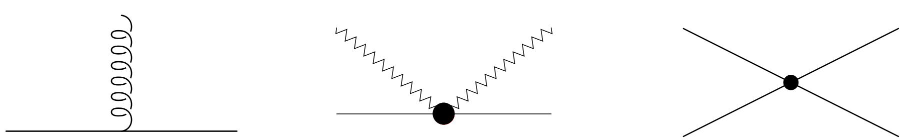
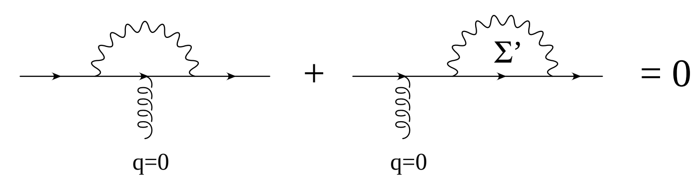


gluon interactions & potentials:





- electroweak corrections either beyond NNLL order or vanish due to gauge cancellations

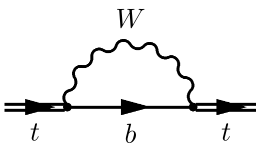
$\rightarrow$ ultrasoft gluon interference effects vanish at NLL and NNLL order (new !) [Khoze etal., Melnikov etal.]

NRQCD (unstable quarks)

quark bilinears: $(\to \mathcal{L}_{\text{usoft}})$

time dilatation correction

$$\text{Im}\Sigma_t = \frac{1}{2}\Gamma_t$$



$= i\Sigma_t \quad \Longrightarrow \quad \delta\mathcal{L} = \psi^\dagger_{\mathbf{p}} \left[i\frac{\Gamma_t}{2}\left(1 - \frac{\mathbf{p}^2}{2m_t^2}\right)\right]\psi_{\mathbf{p}}$

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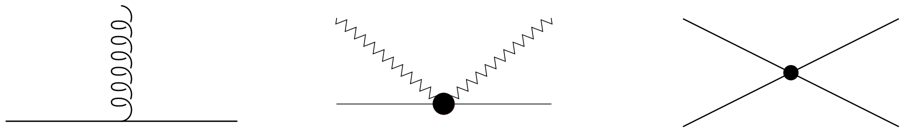
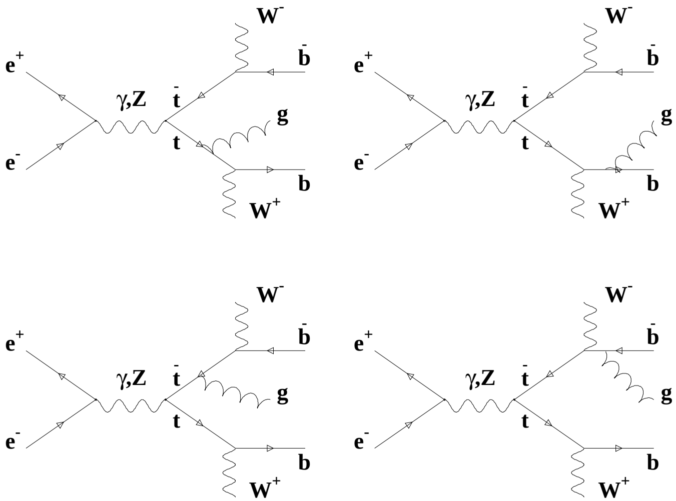




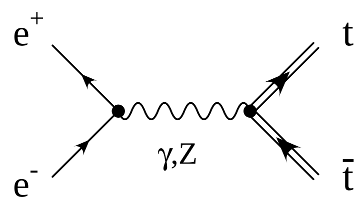
- electroweak corrections either beyond NNLL order or vanish due to gauge cancellations

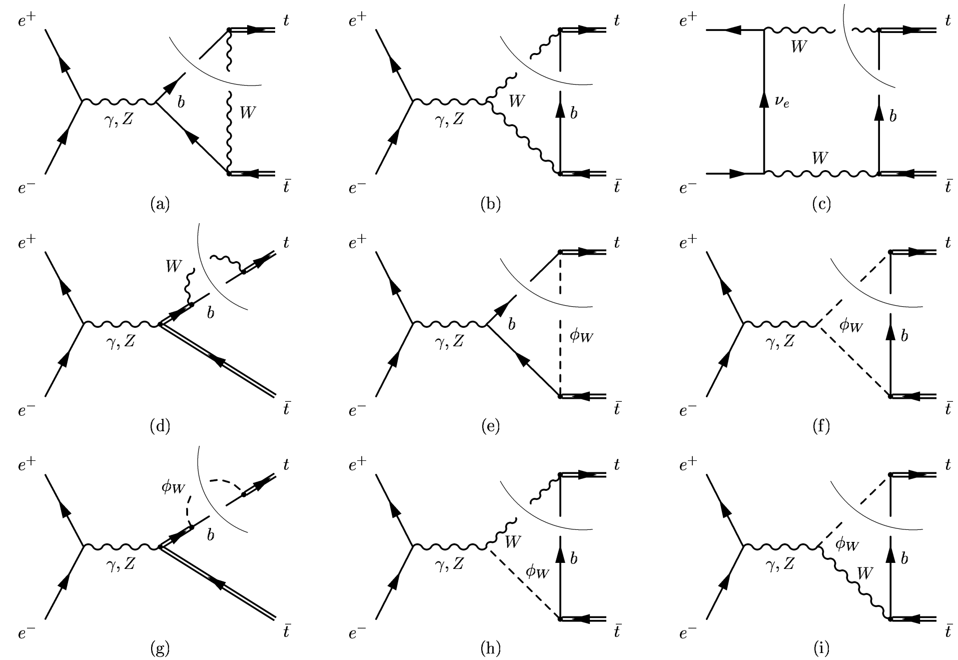
$\longrightarrow$ ultrasoft gluon interference effects vanish at NLL [Khoze etal., Melnikov etal.] and NNLL order (new !)

NRQCD (unstable quarks)

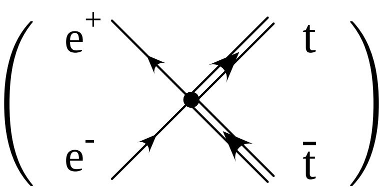
Currents:

- only bW-cuts included
- bW-cuts gauge invariant
- W treated as stable particle





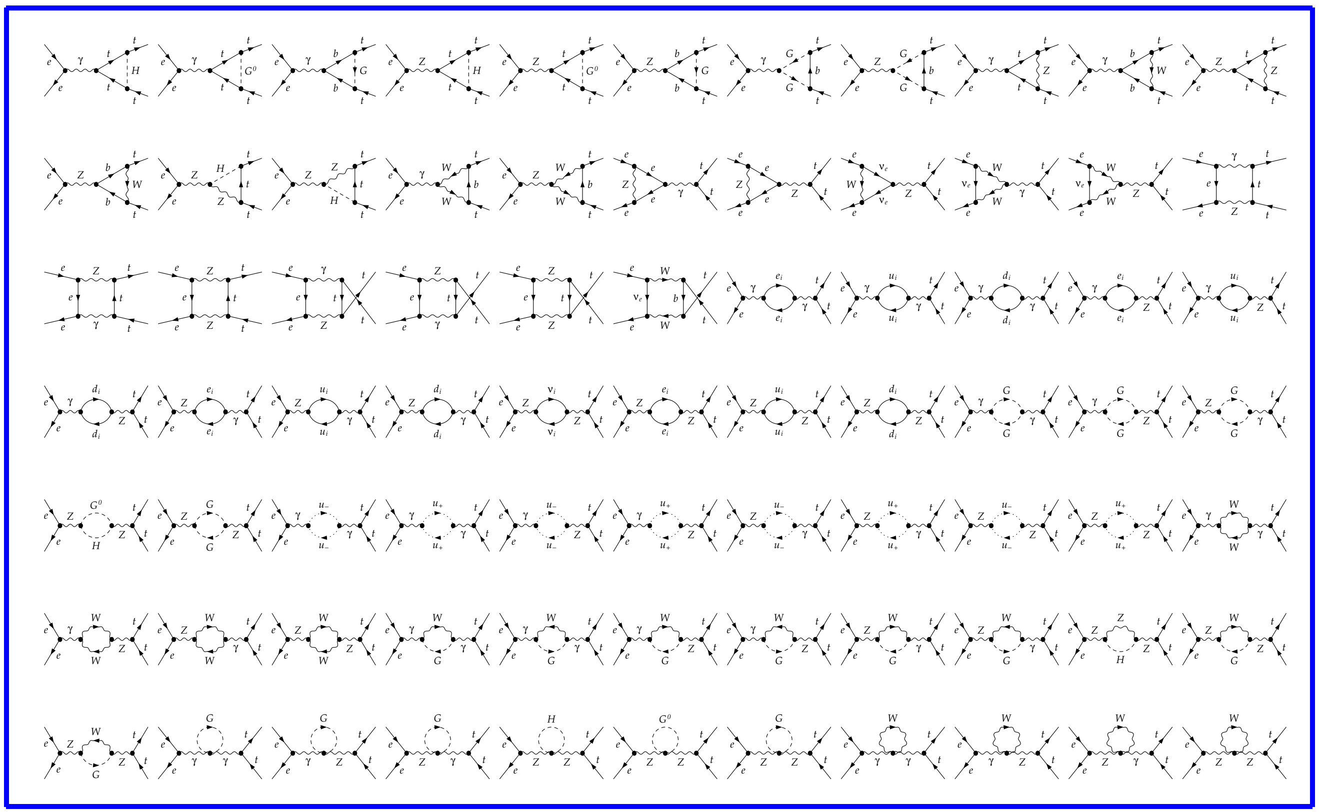
$$\mathbf{O_p} = \Big[\, C^{\rm LL} + C^{\rm NLL} + C^{\rm NNLL} + i C^{\rm NNLL}_{\rm abs} \cdots \Big] \cdot \Big(\quad \Big) + \cdots$$



Re$[C^{\rm NNLL}_{\rm ew}] \rightarrow$ weak contributions: C. Reisser, AH, hep-ph/0604104

Hard Electroweak Effects

- “real” short-distance electroweak corrections: $\mathcal{O}(\alpha_{\rm em}) \sim$ NNLL
- only matching conditions to $(e^+e^-)(t\bar{t})$ operators exist !

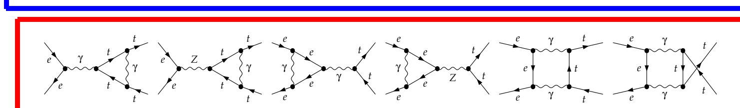


Grzadkowski, Kühn, etal. (1987)

Guth, Kühn (1992)

updated:

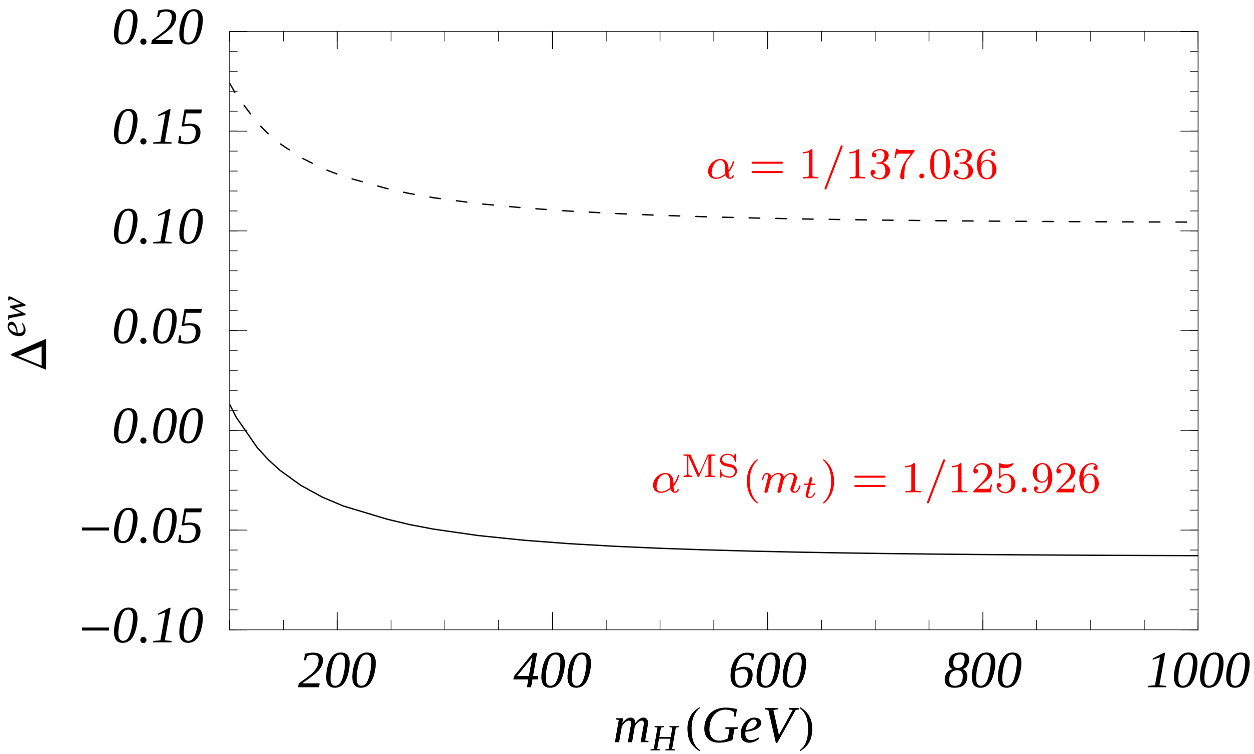
Reisser, AH hep-ph/0604104



QED corrections (+ real radiation, ISR) not yet determined $\longrightarrow$ w.i.p.

Hard Electroweak Effects

- “real” short-distance electroweak corrections: $\mathcal{O}(\alpha_{\rm em}) \sim$ NNLL
- global normalization correction: $\Delta^{\rm ew} \sim$ Re$[2c(1)^{\rm NNLL}_{\rm ew}]$

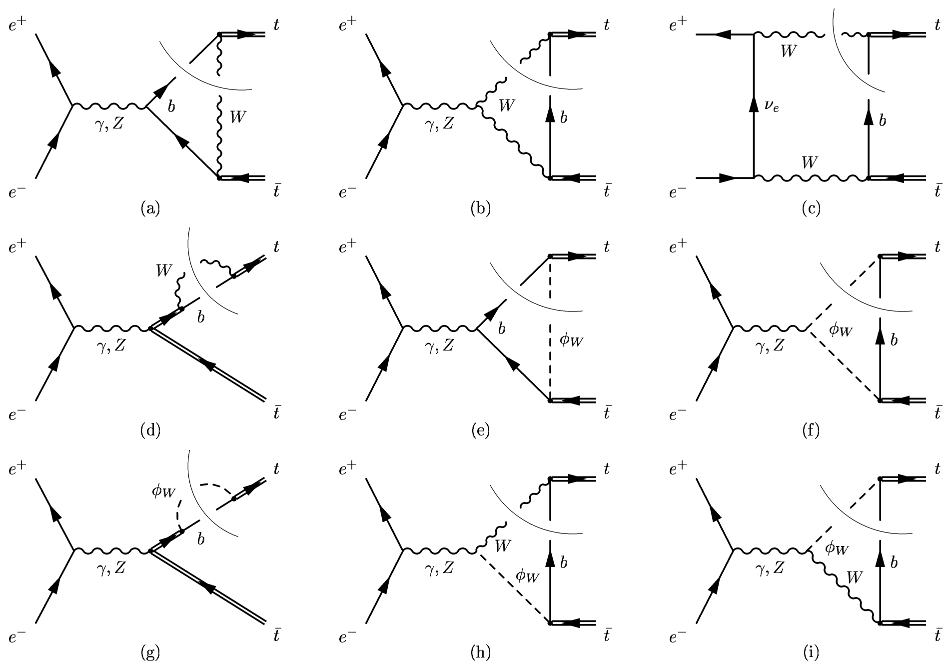


alt. scheme for $\alpha_{\rm QED}$:
$\alpha^{\overline{\rm MS}, n_f=8}(\mu = m_t)$

$m_t = 175$ GeV

Finite Lifetime Effects

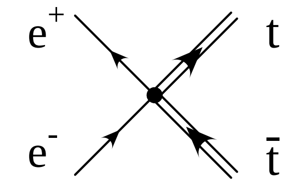
Currents:



Hard electroweak &
QCD matching corrections

bW^+ and $\bar{b}W^-$ cuts

$$\mathbf{O_p} = \left[C^{\mathrm{LL}} + C^{\mathrm{NLL}} + C^{\mathrm{NNLL}} + iC^{\mathrm{NNLL}}_{\mathrm{abs}} \cdots \right] \cdot \left(\quad \right) + \cdots$$



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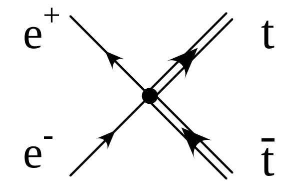
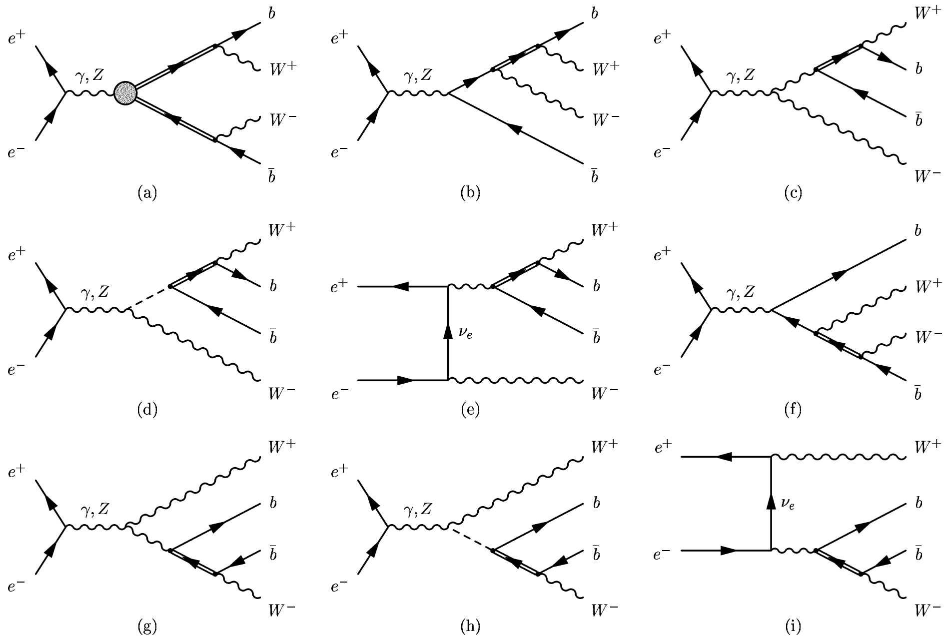
$$\mathbf{O_p} = \Big[C^{\rm LL} + C^{\rm NLL} + C^{\rm NNLL} + iC^{\rm NNLL}_{\rm abs} \ldots \Big] \cdot \Big(\quad \Big) + \cdots$$



$$\sigma_{\rm tot} \propto {\rm Im}\left[{\rm C}(\nu)^2\,{\rm G}(0,0,\sqrt{\rm s}+{\rm i}\Gamma_{\rm t})\right]$$

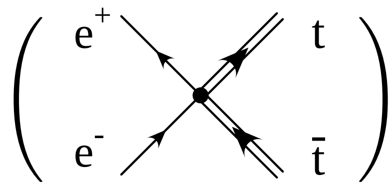
- accounts for irreducible interference contributions:

  resonant $\leftrightarrow$ non-resonant $W^+W^-b\bar{b}$ final states



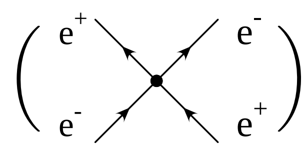
Finite Lifetime Effects

Currents:

$$\mathbf{O_p} = \Big[C^{\rm LL} + C^{\rm NLL} + C^{\rm NNLL} + iC^{\rm NNLL}_{\rm abs} \dots \Big] \cdot$$  $+ \cdots$

$$\sigma_{\rm tot} \propto {\rm Im}\left[{\rm C}(\nu)^2\, {\rm G}(0, 0, \sqrt{\rm s} + {\rm i}\Gamma_{\rm t})\right]$$

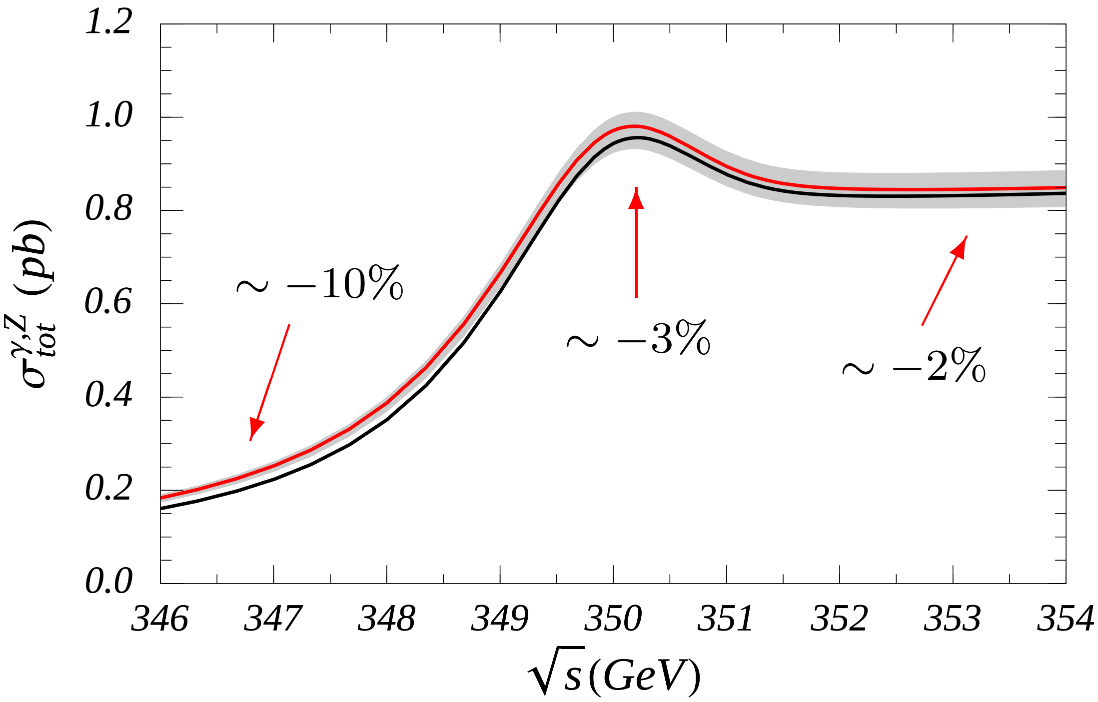
- $(\Delta\sigma^{\Gamma}_{\rm tot}) \sim \alpha_s \Gamma_t \frac{1}{\epsilon}$ $\Rightarrow$ NNLL logarithmic phase space UV divergences

$\longrightarrow$ NLL anom. dim. for $(e^+e^-)(e^+e^-)$ operator $\longrightarrow iC(\mu) \cdot$  ✓

$\longrightarrow$ matching for $iC(\mu)$:

- physical $W^+W^-b\bar{b}$ phase space $\longrightarrow$ “Phase Space Matching” w.i.p.
- $W^+W^-b\bar{b}$ fi nal state without tops input: Rieman,Kolodzej ’05

Total Cross Section



Reisser, AHH

- also included summation of phase space logs $\sim (\alpha_s \ln v)^n$
- finite lifetime corrections comparable to NNLL QCD corrections
- shift in the peak position: $30 - 50$ MeV ($\delta m_t^{\rm ex} \approx 50$ MeV)

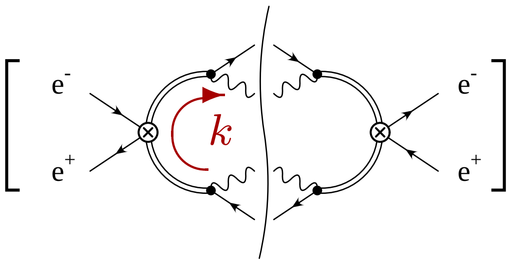
Phase Space Matching

Reisser, Ruiz-Femenia, AH

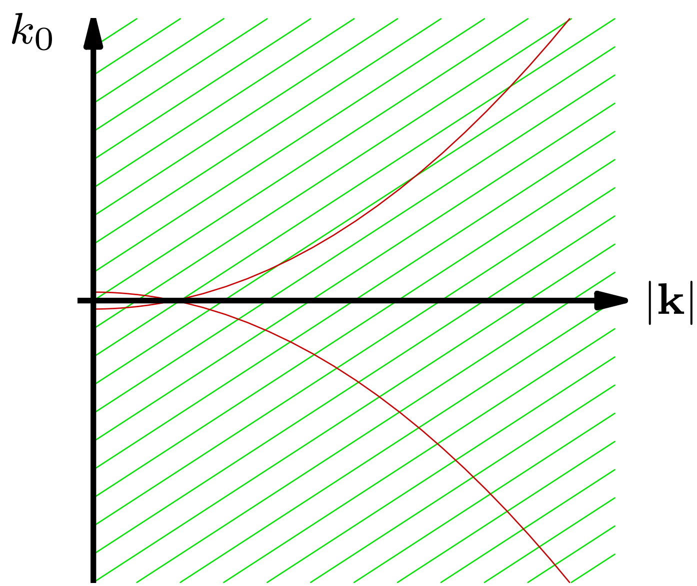
- required for $\sigma_{\rm tot}$ from optical theorem in EFT
- fi nite imaginary matching conditions for all EFT operators

  $\longrightarrow$ removal of unphysical EFT phase space contributions

$\sigma_{\rm tot} \sim$



$$\int_{-\infty}^{+\infty} dk_0 \int_{0}^{+\infty} d|\mathbf{k}| \, \frac{|\mathbf{k}|^2 \, \Gamma_t^2}{|k_0 - \frac{\mathbf{k}^2}{2m_t^2} + i\Gamma_t|^2 \, | - k_0 - \frac{\mathbf{k}^2}{2m_t^2} + i\Gamma_t|^2}$$



Phase Space Matching

Reisser, Ruiz-Femenia, AH

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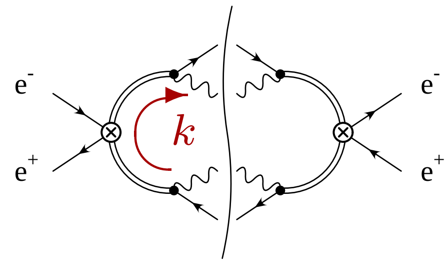
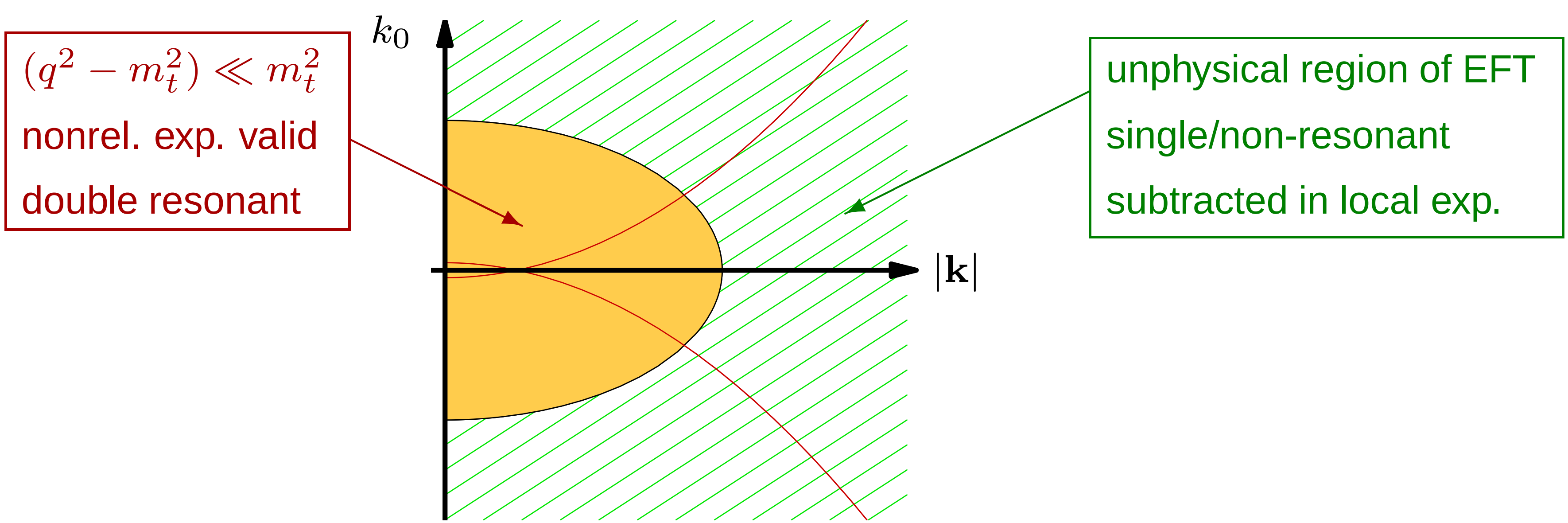


]

$$\int_{-\infty}^{+\infty} dk_0 \int_{0}^{+\infty} d|\mathbf{k}| \; \frac{|\mathbf{k}|^2\,\Gamma_t^2}{|k_0 - \frac{\mathbf{k}^2}{2m_t^2} + i\Gamma_t|^2\,|-k_0 - \frac{\mathbf{k}^2}{2m_t^2} + i\Gamma_t|^2}$$



Phase Space Matching

Reisser, Ruiz-Femenia, AH

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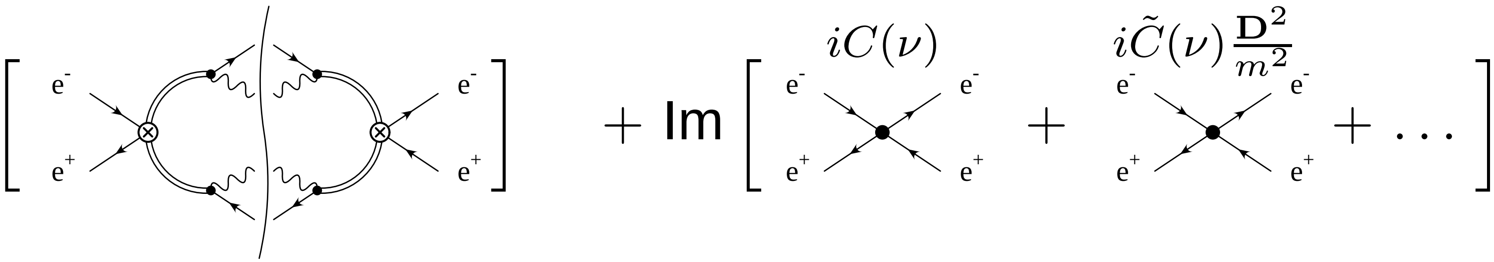
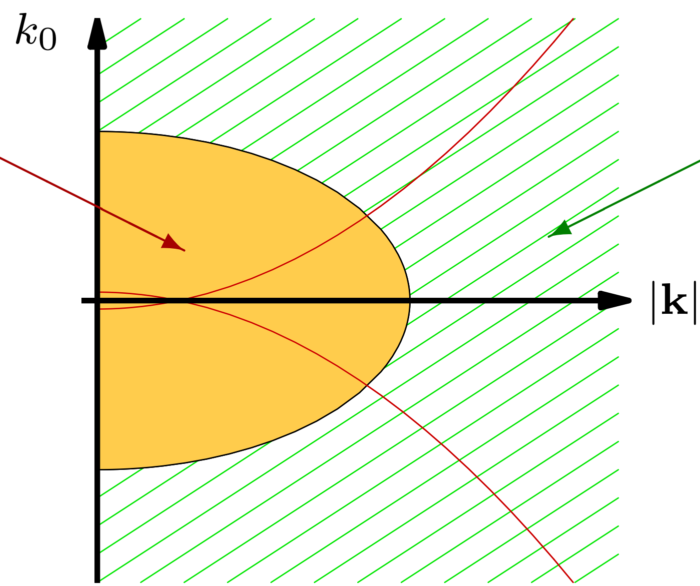


$(q^2 - m_t^2) \ll m_t^2$
nonrel. exp. valid
double resonant



unphysical region of EFT
single/non-resonant
subtracted in local exp.

$\Downarrow$

$C(1)\,,\ \tilde{C}(1)\,,\ldots$
(match. cond. for $4e$ ops.)

Phase Space Matching

Reisser, Ruiz-Femenia, AH

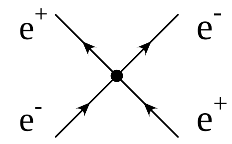
- required for $\sigma_{\rm tot}$ from optical theorem in EFT
- finite imaginary matching conditions (counterterms) for all EFT operators
  $\longrightarrow$ removal of unphysical EFT phase space contributions
- imaginary (finite) renormalization
- separation of hard and soft phase space contributions
  $\oplus$ makes summation of phase space logs meaningful
  $\ominus$ no fully differential predictions
- practical issue: “mild” power counting breaking effects
  $\longrightarrow$ dependence on experimental cuts
- S-wave production: contributes at NLL order
- P-, D-,. . . wave states: mandatory for consistent prediction
  at LL order (e.g. $e^+e^- \to \tilde{t}\bar{\tilde{t}}$)

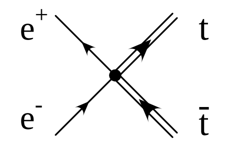
What's left to do . . . a lot!

NNLL Total Cross Section:

(A) electroweak corrections

- NLL + NNLL phase space matching $\longrightarrow \tilde{C}^{eeee}(\mu = m_t)$
- NNLL running of $\tilde{C}^{eeee}(\mu)$
- $\mathcal{O}(\alpha_s)$ corr.'s to $iC^{eett}_{\text{abs}}(\mu = m_t)$ for NNLL mixing effects
- QED effects: ISR, beamstrahlung, Coulomb singularities





(B) QCD corrections (RGE-improved)

- Current uncertainty: $(\delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}})^{\text{QCD}} \sim \pm 6\%$

  $\longrightarrow$ full NLL running of potentials

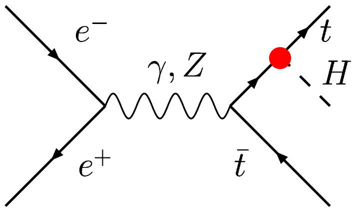
Fully Differential Cross Sections:

$\longrightarrow$ effi ciencies ? really needed ? interesting theoretically ?

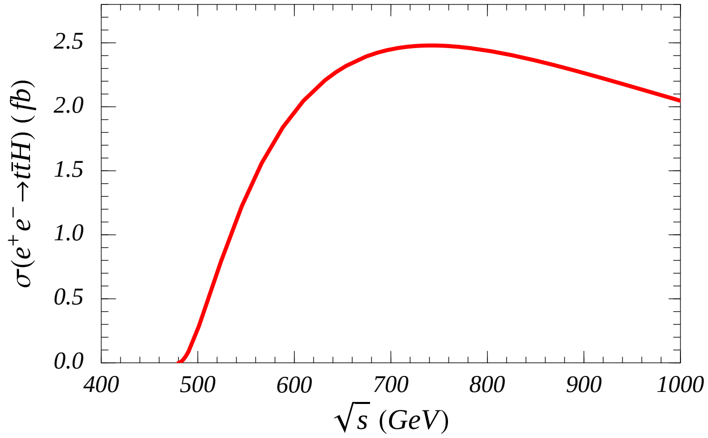
$\longrightarrow$ more involved treatment of unstable particles

Other Applications

$e^+e^- \to t\bar{t}H$ $\longrightarrow$ top-Yukawa coupling



- Theory Status: $\sigma(e^+e^- \to t\bar{t}H)$
  Born ✓ [Gaemers et al., Djouadi et al.]
  1-loop ew. ✓ [Denner et al., Belanger et al., You et al.]
  $\mathcal{O}(\alpha_s)$ fixed-order ✓ [Dittmaier et al., Dawson et al.]
  NLL large-E_H QCD endpoint corrections ✓ [C. Farrell, AHH]

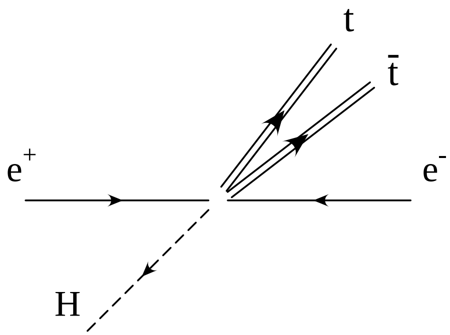


- tiny cross section for $\sqrt{s} = 500$ GeV
- measurement of Yukawa coupling difficult
- $\delta Y_t / Y_t \sim 30\%$ feasible at $\sqrt{s} = 500$ GeV [A. Juste, 2002]

Other Applications

$e^+e^- \to t\bar{t}H$

$\longrightarrow$ region of large Higgs energy



$\longrightarrow$ $t\bar{t}$ collinear

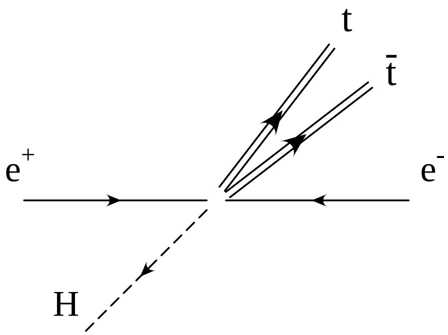
$\longrightarrow$ QCD effects localized in $t\bar{t}$ system

$\Rightarrow$ $t\bar{t}$ dynamics non-relativistic

Other Applications

$e^+e^- \longrightarrow t\bar{t}H$

$\longrightarrow$ region of large Higgs energy



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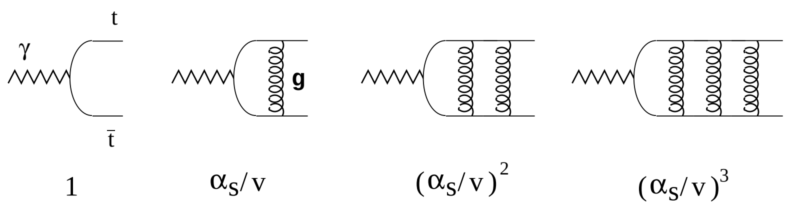
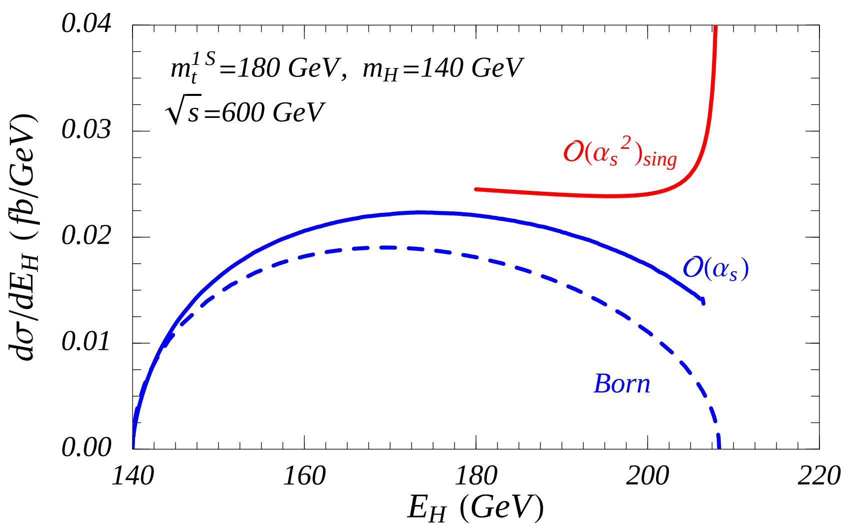




$\longrightarrow$ singularities: $\sim (\alpha_s/v)^n$,

$\sim (\alpha_s \ln v)^n$

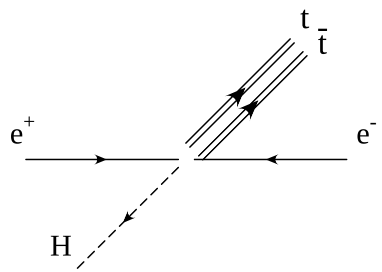
$\longrightarrow$ fi xed order expansion breaks down

$\Rightarrow$ summation of singular terms

Other Applications

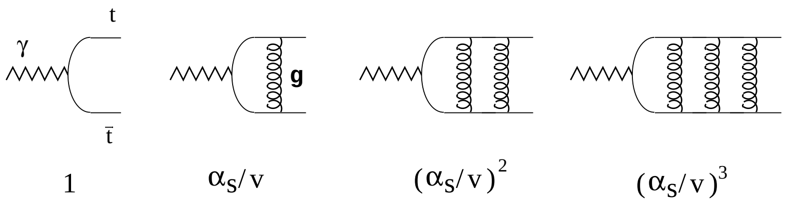
$e^+e^- \to t\bar{t}H$ $\quad\to$ factorization formula

$$\left(\frac{d\sigma}{dE_H}\right)_{\mathrm{E_H \approx E_H^{max}}} \sim C^2(\mu, \sqrt{s}, m_t, m_H) \times \mathrm{Im}[G(0,0,v,\mu)]$$



$+$ hard QCD corrections

soft & ultrasoft QCD corrections

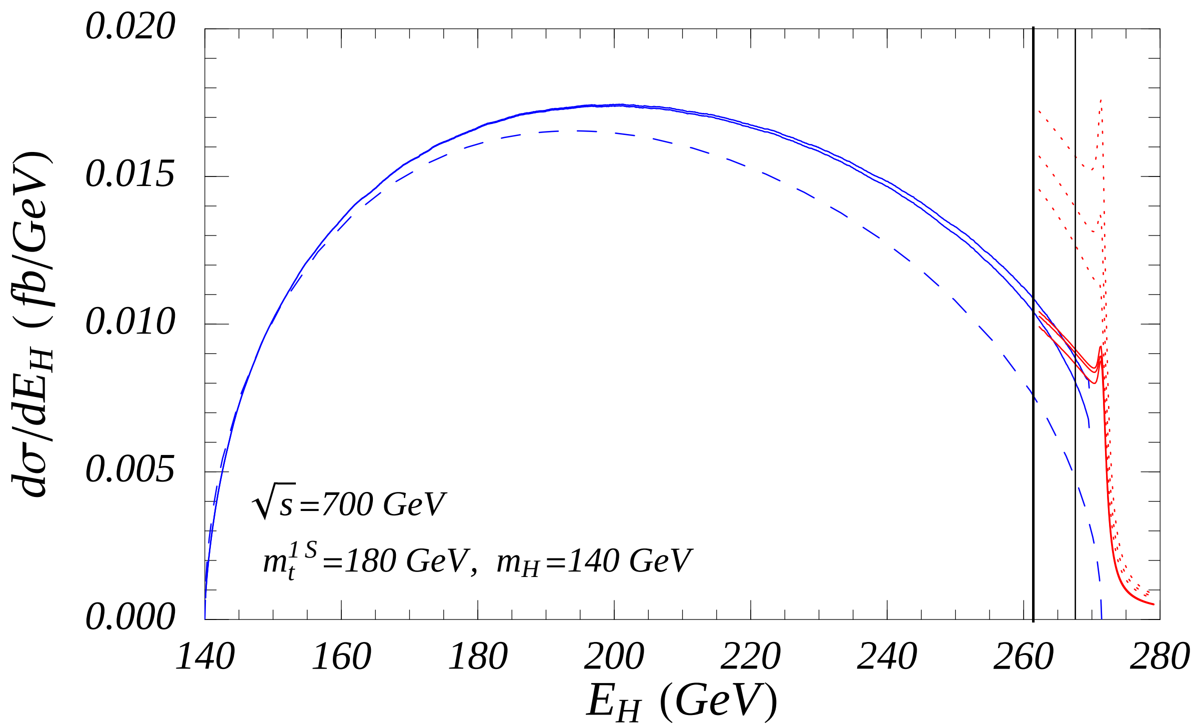


NLL formalism: Cailiin Farrell, AH; Phys.Rev.D72,014007 (2005)

Cailin Farrell, AH; hep-ph/0604166

Other Applications

$e^+e^- \to t\bar{t}H$ $\to$ NLL Higgs energy spectrum

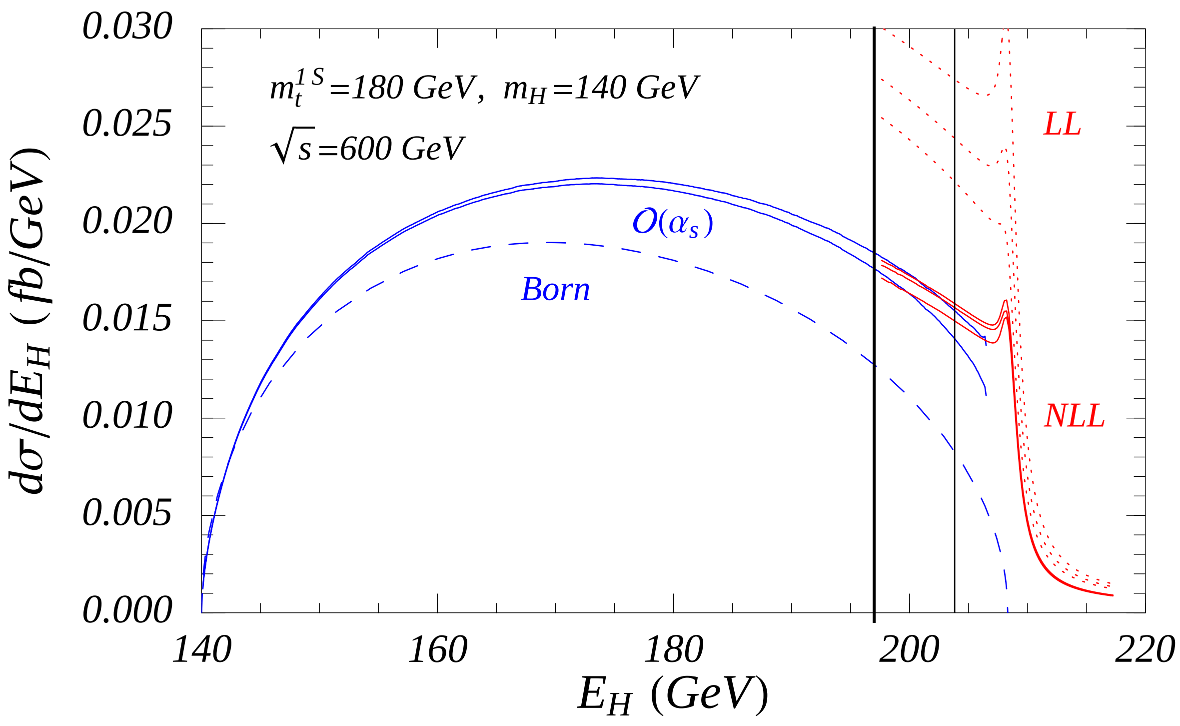


Farrell, AH '05

- large E_H endpoint regions increases for smaller $\sqrt{s}$ / larger m_H
- NLL matching: full theory results from Denner, Dittmaier, Roth, Weber '04

Other Applications

$e^+e^- \to t\bar{t}H$ $\to$ NLL Higgs energy spectrum



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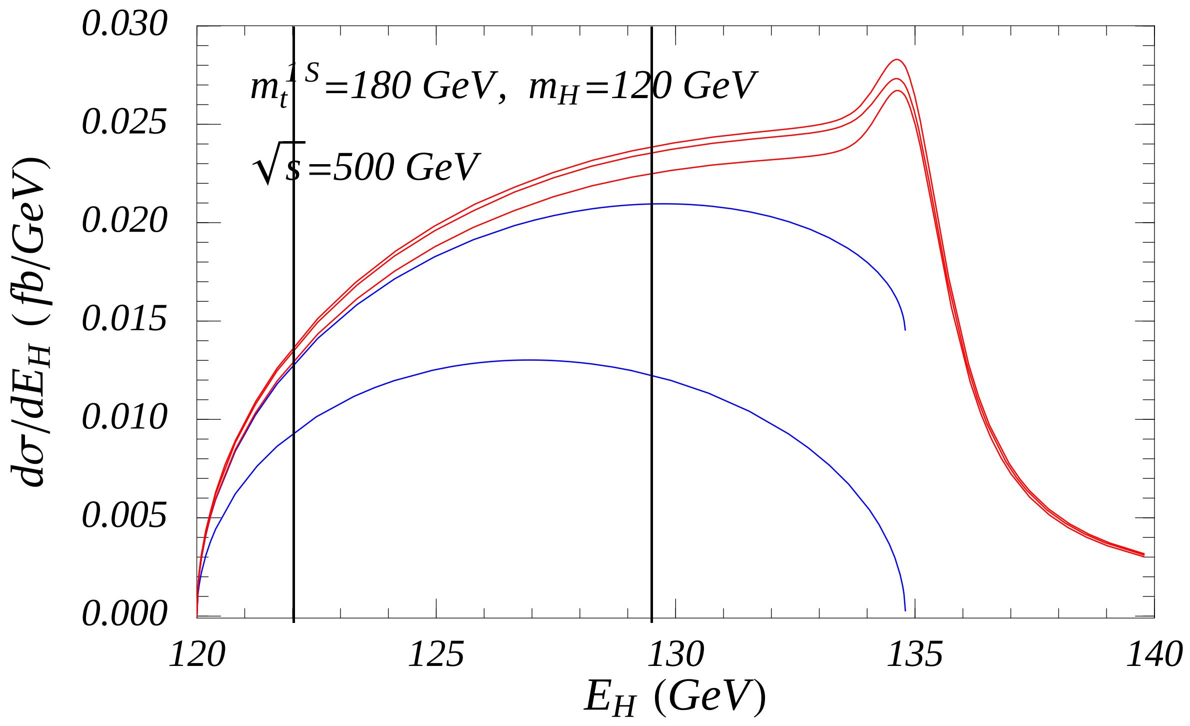
Other Applications

$e^+e^- \to t\bar{t}H$ $\to$ NLL Higgs energy spectrum

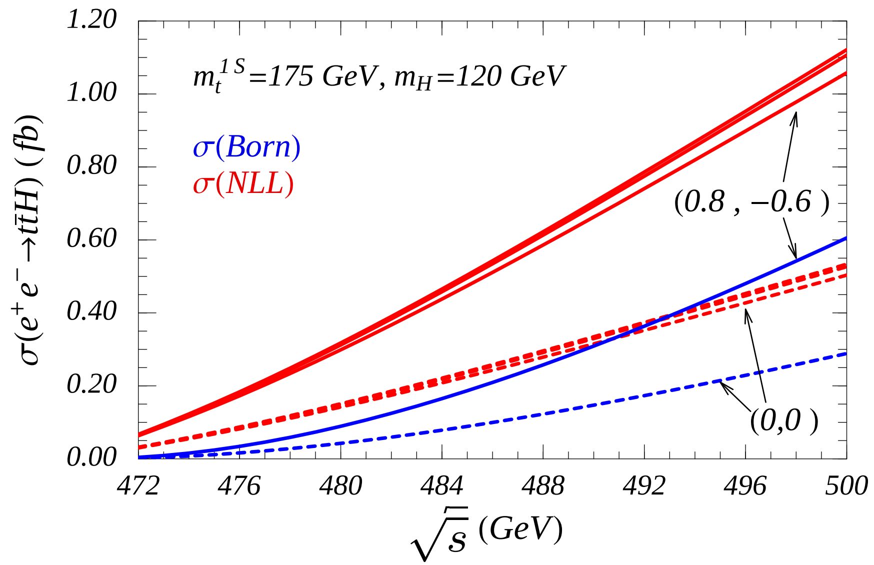


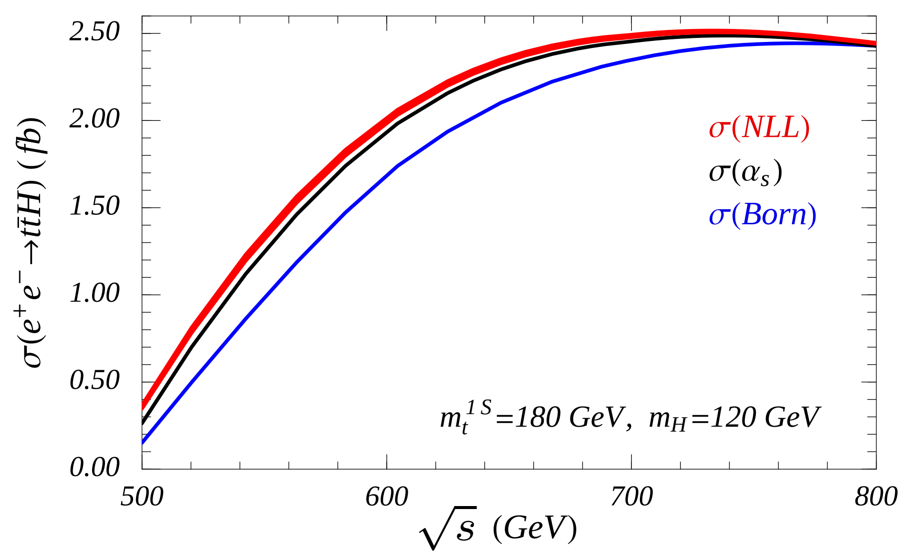
Farrell, AH '06

- large E_H endpoint regions increases for smaller $\sqrt{s}$ / larger m_H
- NLL matching: full theory results from Denner, Dittmaier, Roth, Weber '04
- $\sqrt{s} = 500$ GeV: extension of factorization formula for low-E_H endpoint

Other Application

$e^+e^- \to t\bar{t}H$ → total cross section





500 GeV:

- factor 2 enhancement over tree level from summation of $(\alpha_s/v)^n$, $(\alpha_s \ln v)^n$ terms
- another factor of 2 enhancement for $P_- = -80\%$, $P_+ = +60\%$
- essential for realistic studies for ILC (phase I) Juste '02, '06

$\Rightarrow$ $(\delta\lambda_t/\lambda_t)^{\rm ILC}_{\rm 500\,GeV} \sim 30\% \quad \overset{\sim}{\longrightarrow} \quad 10-15\%$

Conclusion

- Top pair total rate predictions become more and more realistic
  - missing NNLL QCD corrections w.i.p. $\rightarrow$ $d\sigma/\sigma < \pm 6\%$
  - full set of electroweak corrections (“hard” ✓, “QED” ?)
  - top quark finite lifetime: imaginary Wilson coefficients

    $\rightarrow bW$ final states

    $\rightarrow$ phase space matching

- reaching goals $\delta m_t \sim 100$ MeV is no totally free lunch

  still considerable work has to be invested

  to get final number

- Many interesting applications of threshold physics exist.

Colors

This is blue

This is red

This is brown

This is magenta

This is Dark Green

This is Dark Blue

This is Green

This is Cyan

Test how this color looks

Test how this color looks

Test how this color looks

Test how this color looks

Test how this color looks