On the path towards complete 2-loop corrections for Standard Model precision observables

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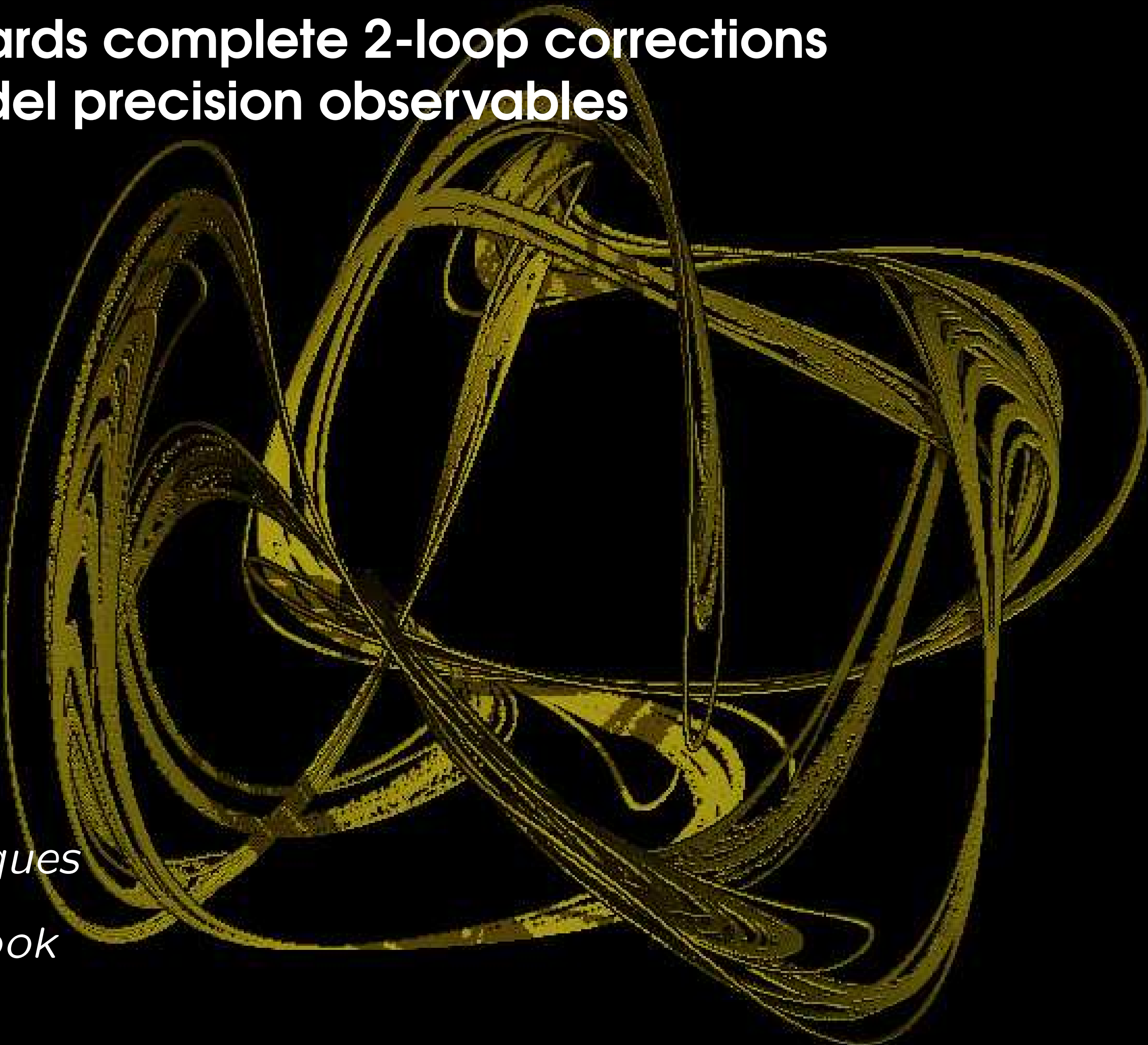


- *Introduction*
- *Two-loop techniques*
- *Results and outlook*

Introduction

Open questions of the Standard Model:

- Where is the Higgs boson?
- Is there a extended / unified symmetry group?
- How can gravity be described?
- What makes Dark Matter in the universe?
- Why is there more matter than anti-matter in the universe?

$\longrightarrow$ Physics beyond the Standard Model

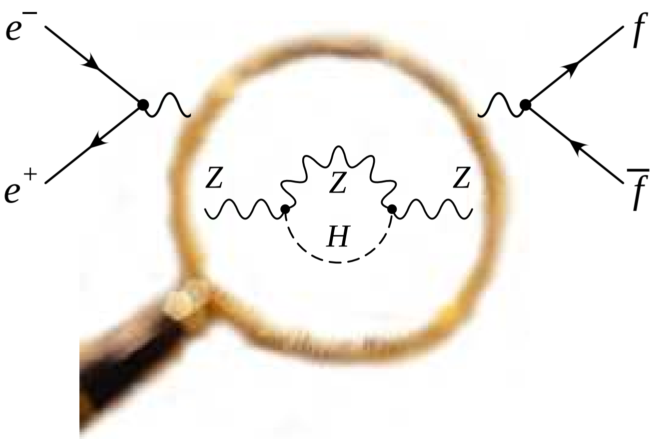
New particles and interactions beyond the Standard Model

Radiative effects

Virtual emission and re-absorption of **all** physical particles

→ Inference of information about Higgs boson and new physics from precision measurements even without direct observation

Example:



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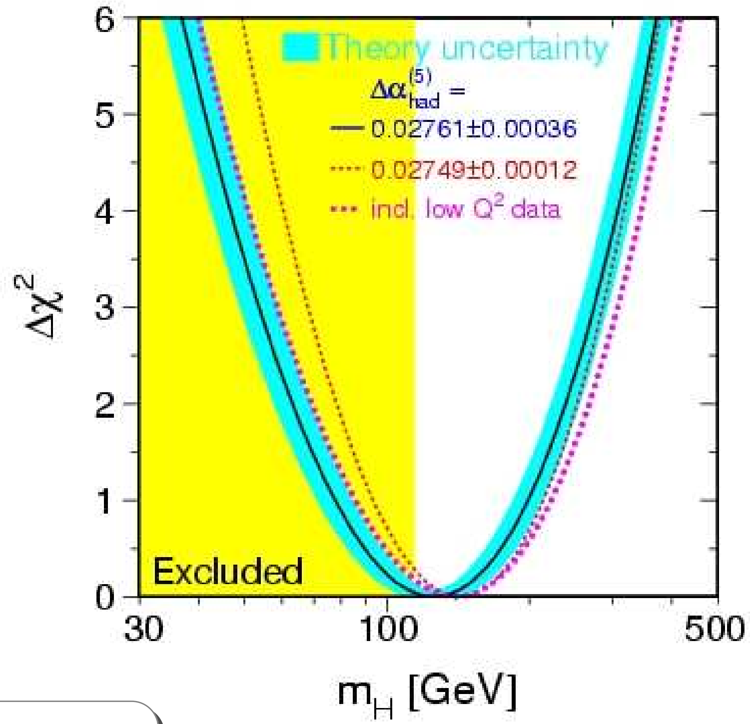
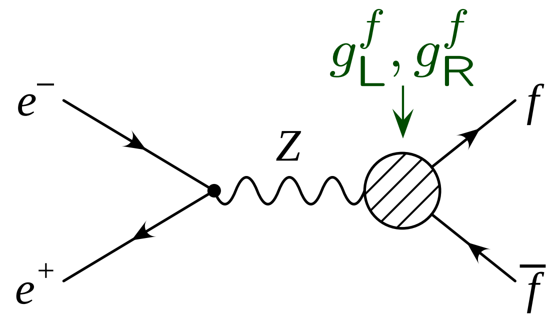


typical corrections of order $\mathcal{O}(\%)$

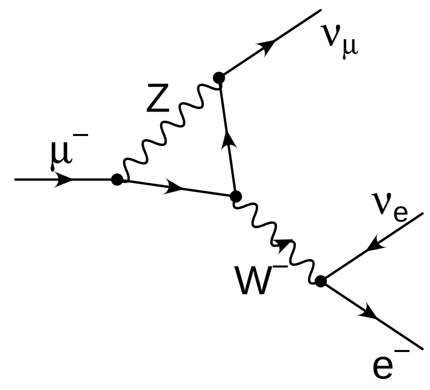
experimental precision up to $\sim 10^{-3}$

Precision observables

- Couplings of Z boson
  to fermions with left-/right-spin
  - effective weak mixing angle
    $$\sin\theta^f_{\mathrm{W,eff}} = \frac{1}{2}\,\frac{g^f_{\mathrm{R}}}{g^f_{\mathrm{L}} - g^f_{\mathrm{R}}}$$
  - total decay rate
    $$\Gamma_{\mathrm{Z}} = C\left((g^f_{\mathrm{L}})^2 + (g^f_{\mathrm{R}})^2\right)$$



- Mass of W boson, muon decay rate
  $$\Gamma_\mu \propto 1/M^4_{\mathrm{W}}$$



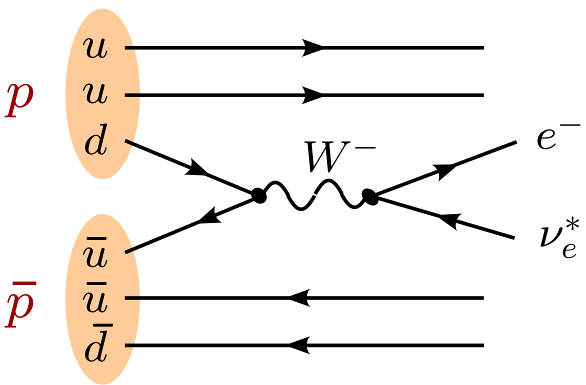
- R_{b}, R_{c}, R_{l}, σ^0_{had}, . . .

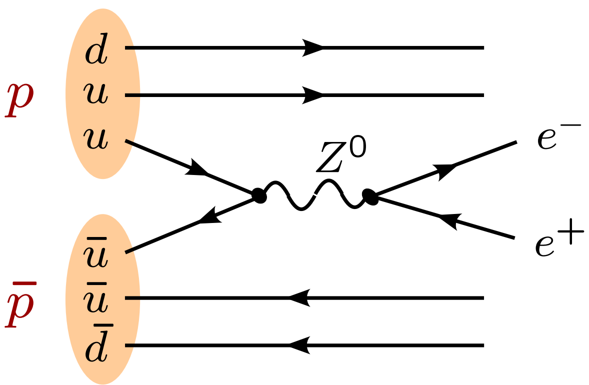
Precision measurements

	W mass [GeV]	$\sin\theta^{\text{lept}}_{\text{W,eff}}$
now	80.410 ± 0.032	0.23153 ± 0.00016
Tevatron	± 0.027	± 0.00016
LHC	± 0.015	± 0.00015
ILC/GigaZ	± 0.007	± 0.000013

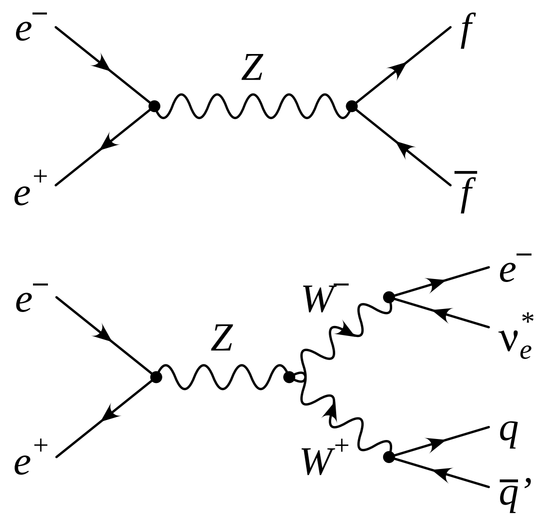
Tevatron

Large **H**adron-**C**ollider ($\gtrsim$ 2007)





International **L**inear **C**ollider (2015?)



Status of loop corrections

1980's

Observable	W mass	$\sin\theta^{\text{lept}}_{\text{W,eff}}$	Z width
α	$\surd$	$\surd$	$\surd$

Sirlin, Marciano '80
G. Degrassi, A. Sirlin '93   P. Gambino, A. Sirlin '94

Status of loop corrections

1991

Observable	W mass	$\sin\theta^{\text{lept}}_{\text{W,eff}}$	Z width
α	$\surd$	$\surd$	$\surd$
$\alpha\alpha_{\text{s}}$	$\surd$	$\surd$	$\surd$

Djouadi '88
Halzen, Kniehl '91

Status of loop corrections

1995

Observable	W mass	$\sin\theta^{\text{lept}}_{\text{W,eff}}$	Z width
α	$\surd$	$\surd$	$\surd$
$\alpha\alpha_s$	$\surd$	$\surd$	$\surd$
$\alpha\alpha_s^2$	$\surd$	$\surd$	$\surd$

Avdeev et al. ’94
Chetyrkin, Kühn, Steinhauser ’95

Status of loop corrections

1998

Observable	W mass	$\sin\theta^{\text{lept}}_{\text{W,eff}}$	Z width
α	$\surd$	$\surd$	$\surd$
$\alpha\alpha_{\text{s}}$	$\surd$	$\surd$	$\surd$
$\alpha\alpha_{\text{s}}^2$	$\surd$	$\surd$	$\surd$
$\alpha^2 m_{\text{t}}^4$, $\alpha^2 m_{\text{t}}^2$	$\surd$	$\surd$	$\surd$

R. Barbieri et al. '93   J. Fleischer, O.V. Tarasov, F. Jegerlehner '95
Degrassi, Gambino, Vicini '96   Degrassi, Gambino, Sirlin '97,98

Status of loop corrections

2003

Observable	W mass	$\sin\theta^{\text{lept}}_{\text{W,eff}}$	Z width
α	$\surd$	$\surd$	$\surd$
$\alpha\alpha_s$	$\surd$	$\surd$	$\surd$
$\alpha\alpha_s^2$	$\surd$	$\surd$	$\surd$
$\alpha^2 m_t^4,\ \alpha^2 m_t^2$	$\surd$	$\surd$	$\surd$
$\alpha^3 m_t^6,\ \alpha^2\alpha_s m_t^4$	$\surd$	$\surd$	$\surd$

v.d.Bij, Chetyrkin, Faisst, Jikia, Seidensticker '01
Faisst, Kühn, Seidensticker, Veretin '03

Status of loop corrections

2006

Observable	W mass	$\sin\theta^{\text{lept}}_{\text{W,eff}}$	Z width
α	$\surd$	$\surd$	$\surd$
$\alpha\alpha_{\text{s}}$	$\surd$	$\surd$	$\surd$
$\alpha\alpha_{\text{s}}^2$	$\surd$	$\surd$	$\surd$
$\alpha^2 m_{\text{t}}^4,\ \alpha^2 m_{\text{t}}^2$	$\surd$	$\surd$	$\surd$
$\alpha^3 m_{\text{t}}^6,\ \alpha^2\alpha_{\text{s}}\, m_{\text{t}}^4$	$\surd$	$\surd$	$\surd$
α^2	$\surd$	$\surd$	

Freitas,Hollik,Walter,Weiglein '00 Awramik,Czakon '02 Onishchenko,Veretin '02
Awramik, Czakon, Freitas '04,06 Meier,Hollik,Uccirati '05,06

Status of loop corrections

2006

Observable	W mass	$\sin\theta_{\mathrm{W,eff}}^{\mathrm{lept}}$	Z width
α	$\surd$	$\surd$	$\surd$
$\alpha\alpha_{\mathrm{s}}$	$\surd$	$\surd$	$\surd$
$\alpha\alpha_{\mathrm{s}}^2$	$\surd$	$\surd$	$\surd$
$\alpha^2 m_{\mathrm{t}}^4,\ \alpha^2 m_{\mathrm{t}}^2$	$\surd$	$\surd$	$\surd$
$\alpha^3 m_{\mathrm{t}}^6,\ \alpha^2\alpha_{\mathrm{s}}\, m_{\mathrm{t}}^4$	$\surd$	$\surd$	$\surd$
α^2	$\surd$	$\surd$	
$\alpha\alpha_{\mathrm{s}}^3 m_{\mathrm{t}}^2,\ \alpha^3 M_{\mathrm{H}}^4$	$\surd$	$\surd$	$\surd$

Boughezal, Tausk, v.d.Bij '05
Schröder, Steinhauser '05 Chetyrkin, Faisst, Kühn '06

Radiative loop corrections

	M_W [GeV]	$\sin\theta^{\text{lept}}_{W,\text{eff}}$
now	± 0.032	$\pm$ 16
Tevatron	± 0.027	$\pm$ 16
LHC	± 0.015	$\pm$ 15
ILC/GigaZ	± 0.007	$\pm$ 1.3
1-loop	± 0.450	$\pm$ 1000
2-/3-loop QCD	± 0.070	$\pm$ 45
ferm. 2-loop EW	± 0.050	$\pm$ 90
bos. 2-loop EW	± 0.002	$\pm$ 1
leading 3-loop	± 0.005	$\pm$ 25

Experimental precision sensitive to 2-/3-loop effects

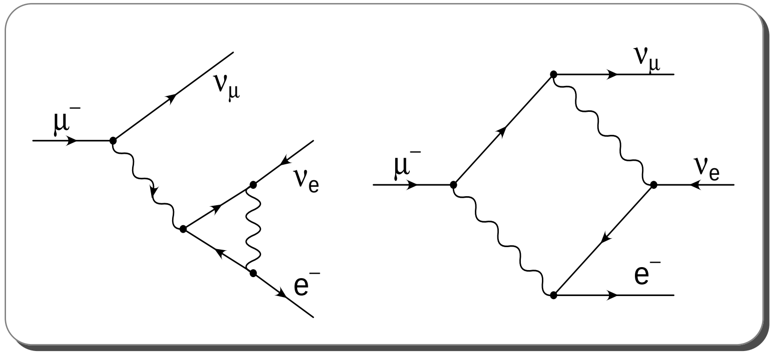
Marciano, Sirlin '80

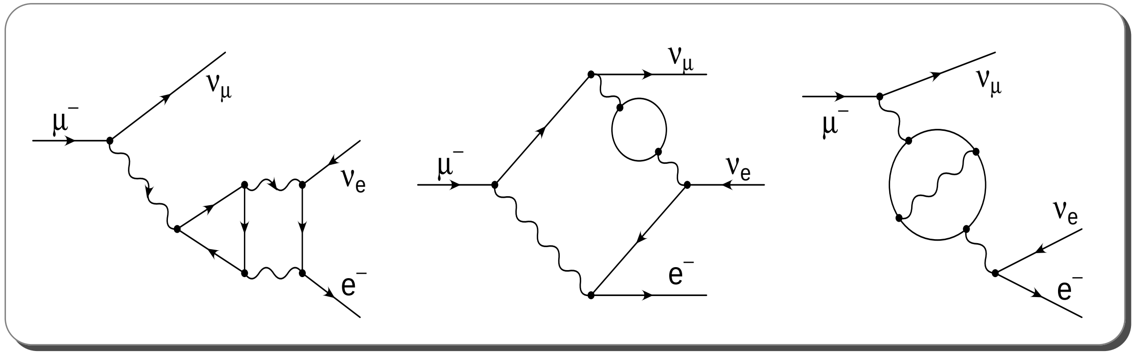
Djouadi et al. '88
Chetyrkin, Kühn, Steinhauser '95

Freitas et al. '00 Awramik, Czakon '03
Awramik, Czakon, Freitas, Weiglein '04

Awramik, Czakon, Freitas '06

Faisst, Kühn, Seidensticker, Veretin '03



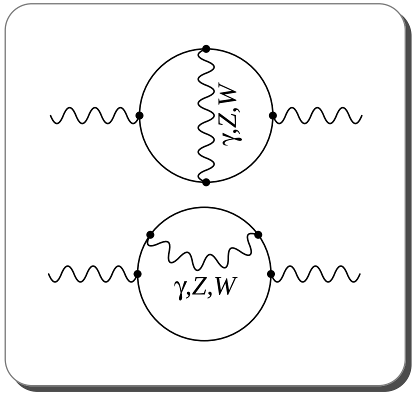


Two-loop techniques

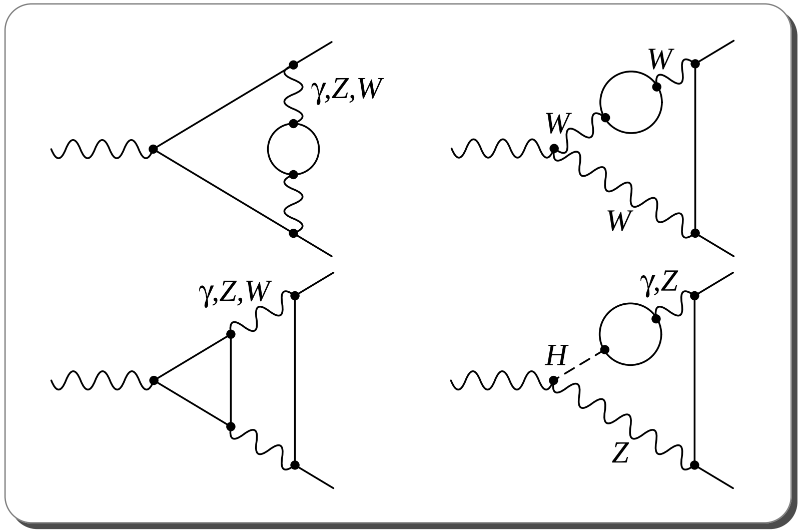
- **On-shell** renormalization of (W and Z) masses:

  Masses correspond to propagator poles

  Selfenergy corrections for mass renormalization



- Complication for corrections to $\sin^2\theta_{\text{eff}}^{\text{lept}}$:
  Two-loop vertex diagrams



- Divide into two classes
  - With closed fermion loops
  - No closed fermion loops

Diagrams: Asymptotic expansions

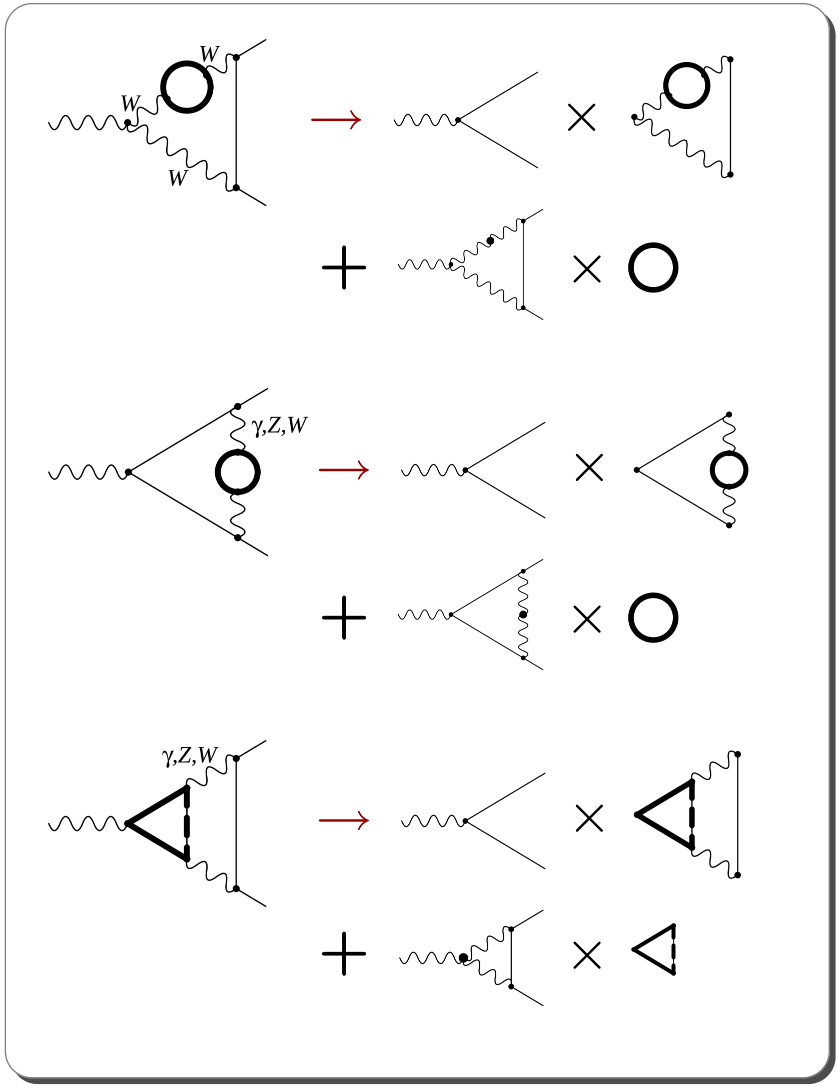
Top quark contributions

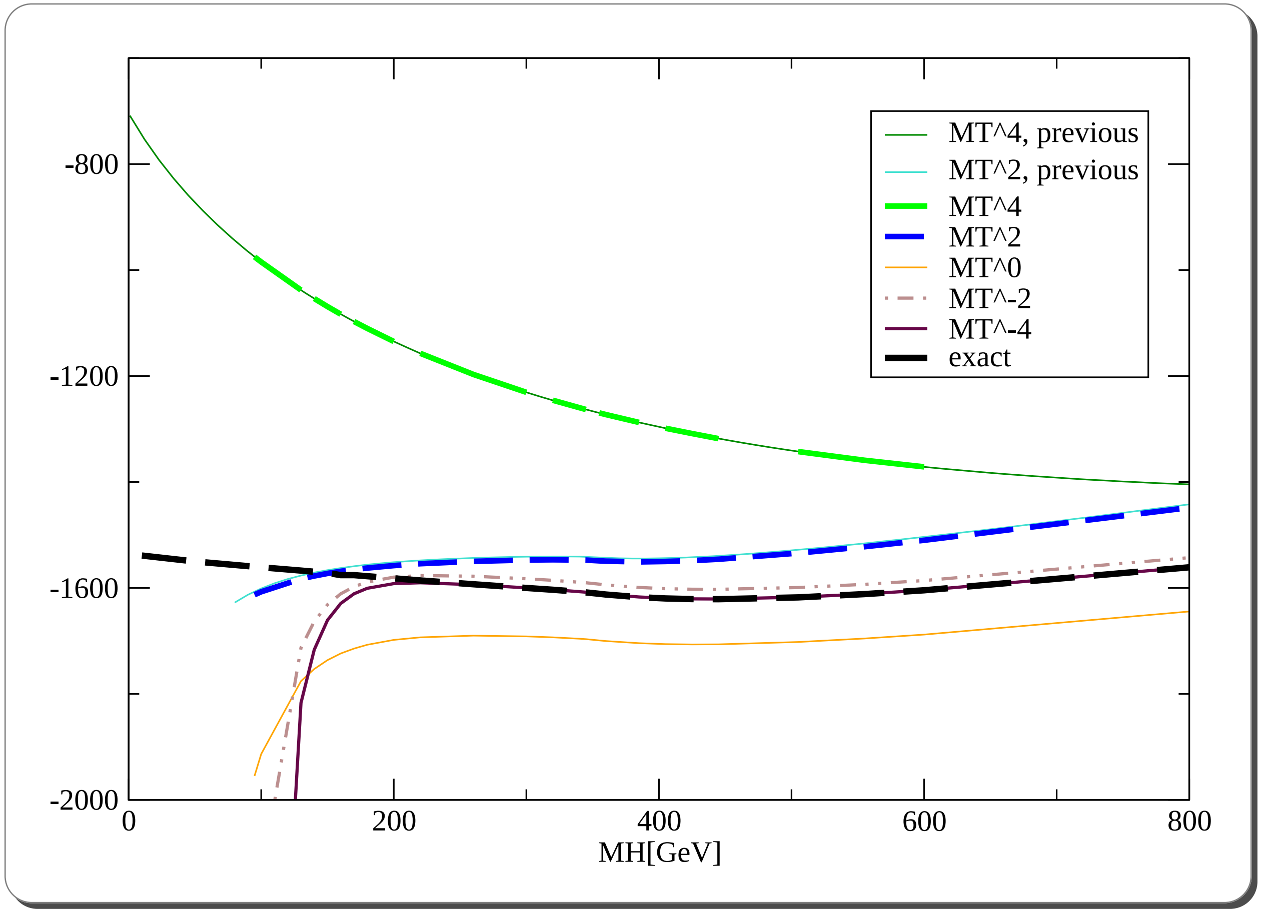
- Exploit large scale difference between top mass and other masses:

$$M_Z^2/m_t^2 \approx 1/4$$

- Simplifies diagrams to 2-loop tadpoles and 1-loop vertices
- Fast numerical evaluation
- Previously:
  leading $\alpha\, m_t^4$ and $\alpha^2 m_t^2$ contribution only

  G. Degrassi, P. Gambino, A. Sirlin '97





- Leading terms in agreement with previous result
  Degrassi, Gambino, Sirlin '97
- Expansion in ext. momentum as check

Total contribution of top-quark diagrams:

10th order expansion has relative error estimate: $\pm 1.3 \times 10^{-5}$

Bosonic corrections to $\sin^2\theta_{\text{eff}}^{\text{lept}}$

Awramik, Czakon, Freitas '06

Three scales M_Z, M_W, M_H

- Reduce number of scales by expansions and re-expansions

  $\longrightarrow$ Number of integrals increases to several 10,000

- Reduction to master integrals possible for sets of one- and two-scale integrals

- Expansion methods:
  - Expansion in $s_w^2 = \frac{M_Z - M_W}{M_Z} \sim 1/4$
  - Threshold expansion (diagrams with Z and W or Higgs boson)
    $\longrightarrow$ Method of regions
  - Large mass expansion (diagrams with Higgs boson)

Diagrams: Algebraic reduction

For example light fermion contributions

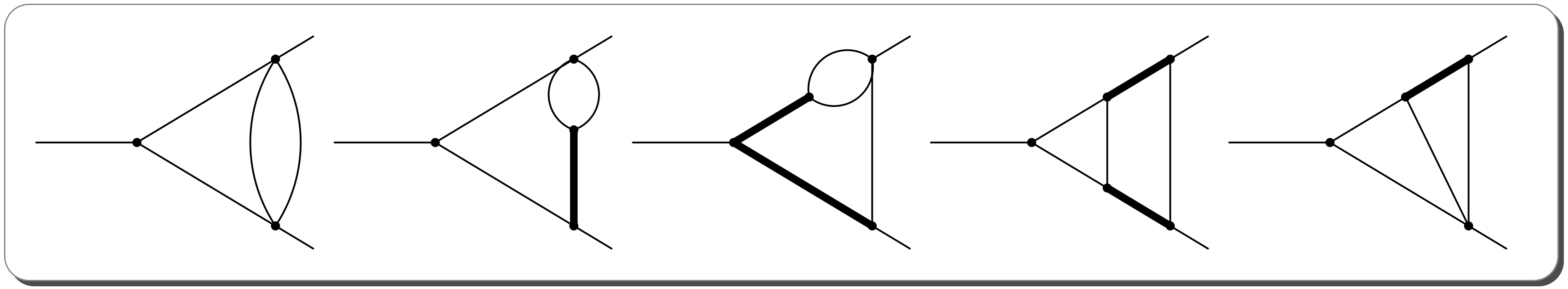
Take light fermions (all except top quark) massless
$\longrightarrow$ Only two scales M_W and M_Z

Integration-by-parts and **Lorentz-invariance identities** to reduce to master integrals

Chetyrkin, Tkachov '81 Gehrmann, Remiddi '00 Laporta '00

$\longrightarrow$ Symmetry relations to minimize number of independent integrals

Linear equation system with $\mathcal{O}(10^4)$ entries
$\longrightarrow$ Specialized computer tools, e.g. *IdSolver* Czakon '04



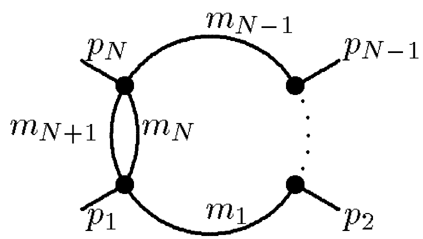
Scalar integrals: Semi-numerical integral evaluation

Topologies with **self-energy sub-loop** can easily be integrated by using dispersion relation for B_0 function: S. Bauberger et al. '95

$$B_0(p^2, m_1^2, m_2^2) = -\int_{(m_1+m_2)^2}^{\infty} \mathrm{d}s \, \frac{\Delta B_0(s, m_1^2, m_2^2)}{s - p^2}$$

with $$\Delta B_0(s, m_1^2, m_2^2) = (4\pi\mu^2)^{4-D} \frac{\Gamma(D/2-1)}{\Gamma(D-2)} \frac{\lambda^{(D-3)/2}(s, m_1^2, m_2^2)}{s^{D/2-1}},$$

$\lambda(a, b, c) = (a - b - c)^2 - 4bc$



$$T_{N+1}(p_i; m_i^2) = -\int_{s_0}^{\infty} \mathrm{d}s \, \Delta B_0(s, m_N^2, m_{N+1}^2)$$

$$\times \int \mathrm{d}^4 q \, \frac{1}{q^2 - s} \, \frac{1}{(q+p_1)^2 - m_1^2} \cdots \frac{1}{(q+p_1+\cdots+p_{N-1})^2 - m_{N-1}^2}$$

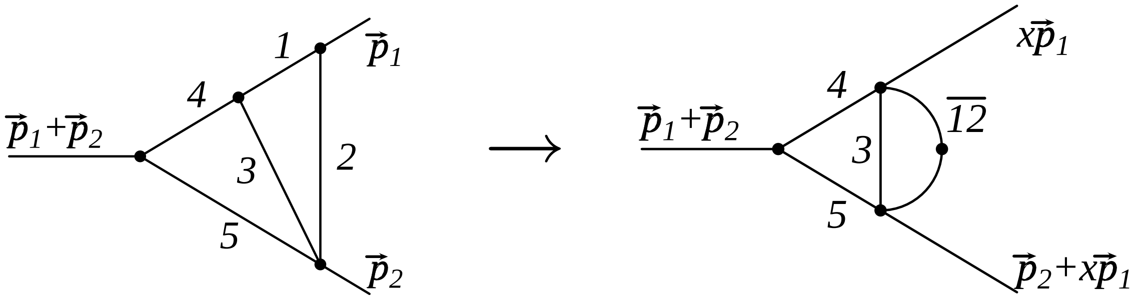
Dispersion relations for diagrams with **triangle subloop** difficult

→ Alternative: Use Feynman parameters J. v.d.Bij, A. Ghinculov '94

$$\frac{1}{(q+p_1)^2 - m_1^2}\,\frac{1}{(q+p_2)^2 - m_2^2} = \int_0^1 \mathrm{d}x\,\frac{1}{[(q+\bar{p})^2 - \overline{m}^2]^2}$$

$$\bar{p} = x\,p_1 + (1-x)p_2, \qquad \overline{m} = x\,m_1 + (1-x)m_2 - x(1-x)(p_1-p_2)^2$$

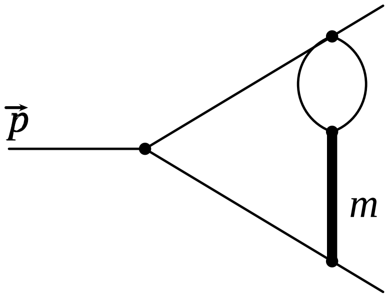
Reduces triangle to self-energy sub-loops:

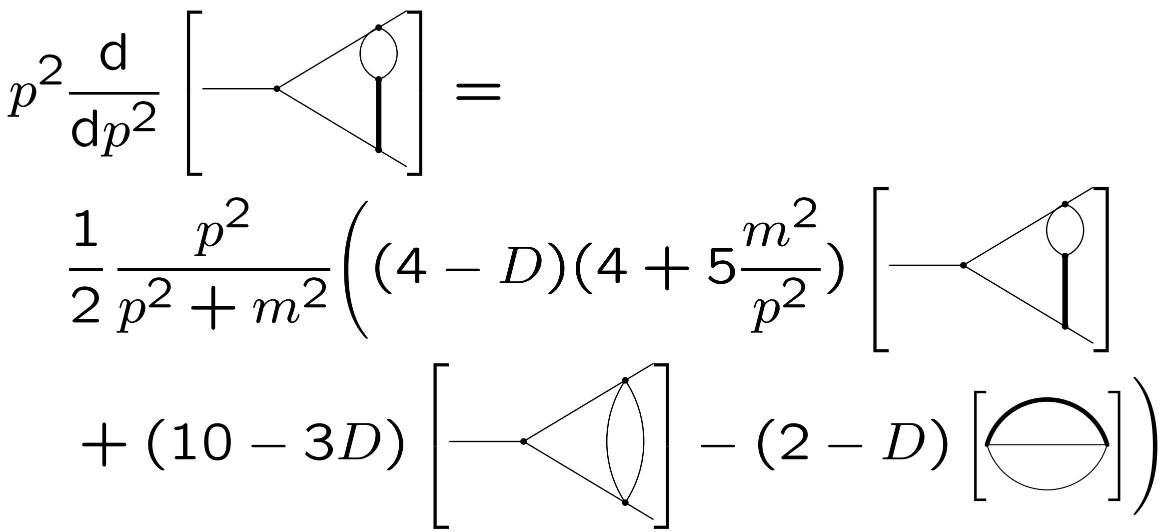


Integration over Feynman parameters and dispersion integral numerically with Gauss-Kronrod algorithm

Scalar integrals: Other methods

- Differential equations to get analytical results for master integrals





- Analytical results through Mellin-Barnes representations (for one-scale master integrals) Czakon '05

- Sector decomposition (poor precision, but good for checks) T. Binoth, G. Heinrich '03

- Taylor expansions (in some cases)

Fermion loop triangle and treatment of γ_5

- Well-known problem in chiral quantum field theories:
  Non-existence of invariant regularization

- **Dimensional regularization** (DREG) preserves Lorentz- and gauge symmetries in **non-chiral** theories

  In chiral theories:

  $$\{\gamma_\mu, \gamma_5\}, \qquad \mathrm{Tr}(\gamma^\alpha \gamma^\beta \gamma^\gamma \gamma^\delta \gamma_5) = 4i\,\epsilon^{\alpha\beta\gamma\delta}$$

  cannot be simultaneously fulfilled in $D \neq 4$ dimensions

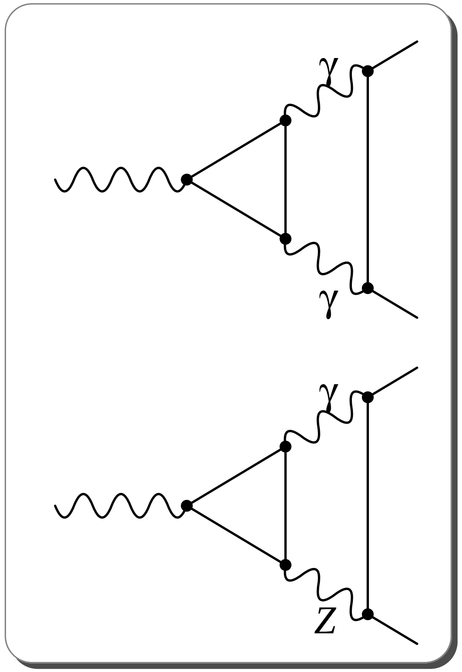
- Experience from muon decay:
  Terms arising from $\mathrm{Tr}(\gamma^\alpha \gamma^\beta \gamma^\gamma \gamma^\delta \gamma_5) = 4i\,\epsilon^{\alpha\beta\gamma\delta}$ are **UV-finite**
  $\longrightarrow$ Succesful use of 4-dim. Dirac algebra !

  Generates inconsistencies at $\mathcal{O}(D-4)$

- Contribution involving ϵ-tensors solely from top-quark diagrams

- Situation complicated by collinear divergencies

- Collinear divergencies cancel in complete result, but are present in single diagrams



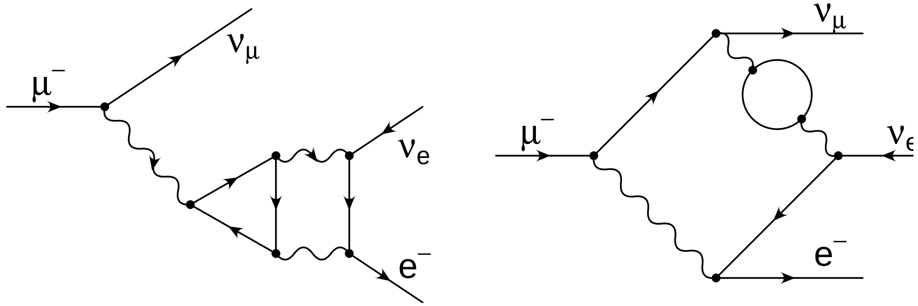
- With inconsistent treatment of γ_5: Only leading collinear poles cancel, but sub-leading divergencies and finite parts come out wrong

- Simplest solution: use photon mass as regulator

- IR divergence from anomaly cancels with one quark and lepton family

Results

Mass of W boson



$M_{W,exp} = (80.404 \pm 0.030)$ GeV

Computation from muon decay in Standard Model:

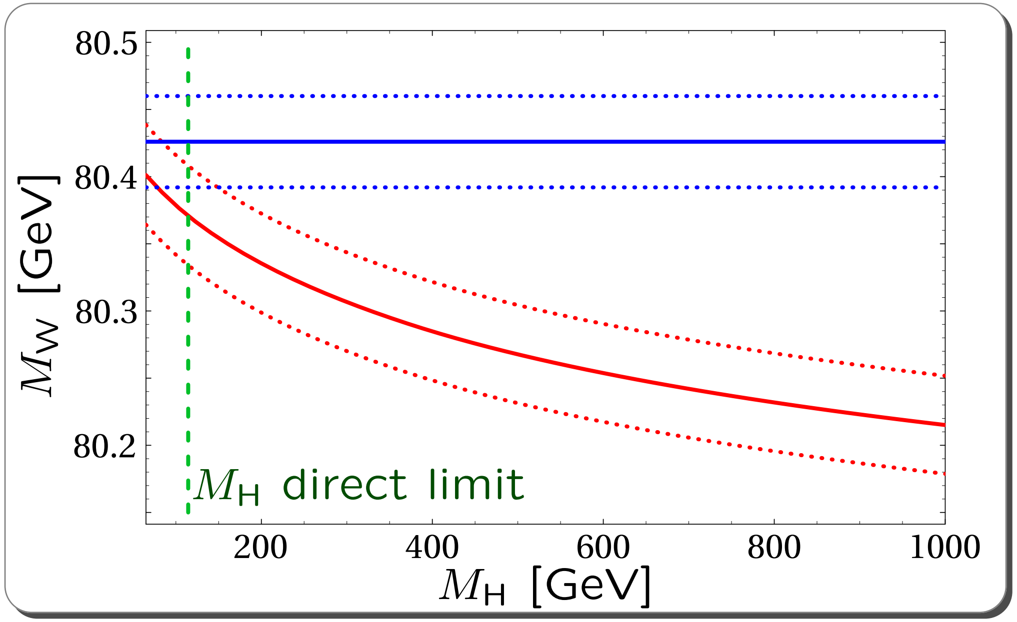
- complete 2-loop — Freitas, Hollik, Walter, Weiglein '00; Awramik, Czakon, Onishchenko, Veretin '02
- partial 3-loop, using expansion for large m_t — Faisst et al. '03; Boughezal, Tausk, v.d.Bij '05

Estimated theoretical error:

$\delta M_{W,th} \approx \pm 0.004$ GeV

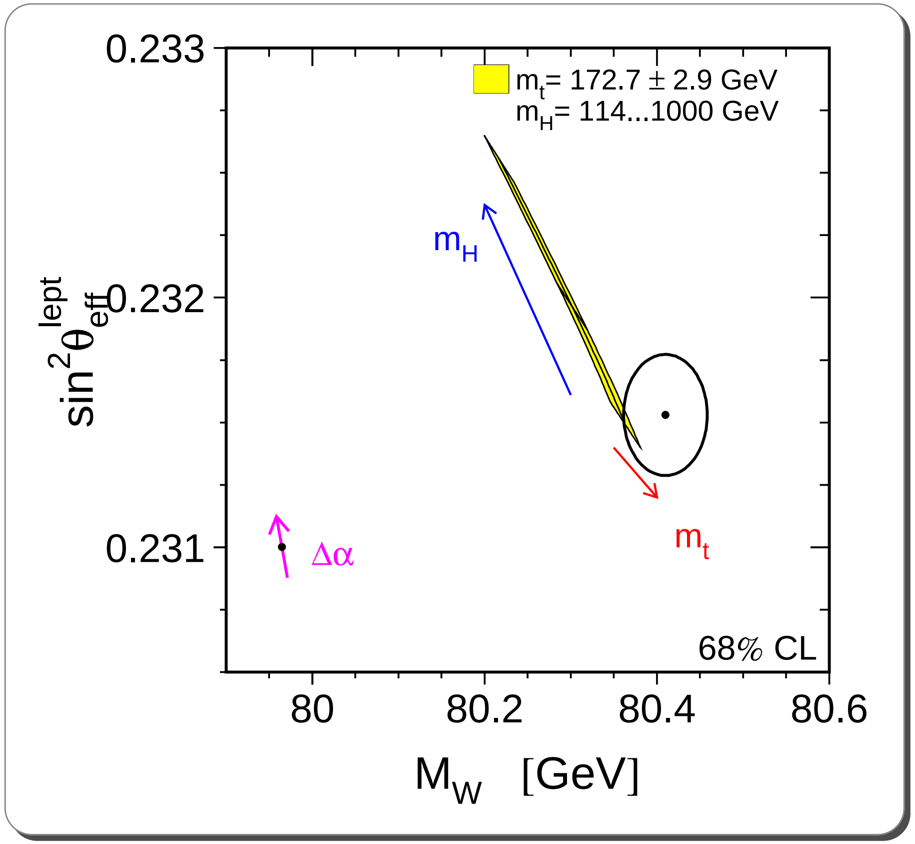
Impact of 2-loop corrections:

$\delta M_{W,2-loop} \approx 0.03$ GeV



Effective weak mixing angle

$\sin^2\theta_{\rm eff}^{\rm lept}$ is one of the most important quantities for testing the Standard Mode l and constraining $M_{\rm H}$.



LEPEWWG '05

Measurement from

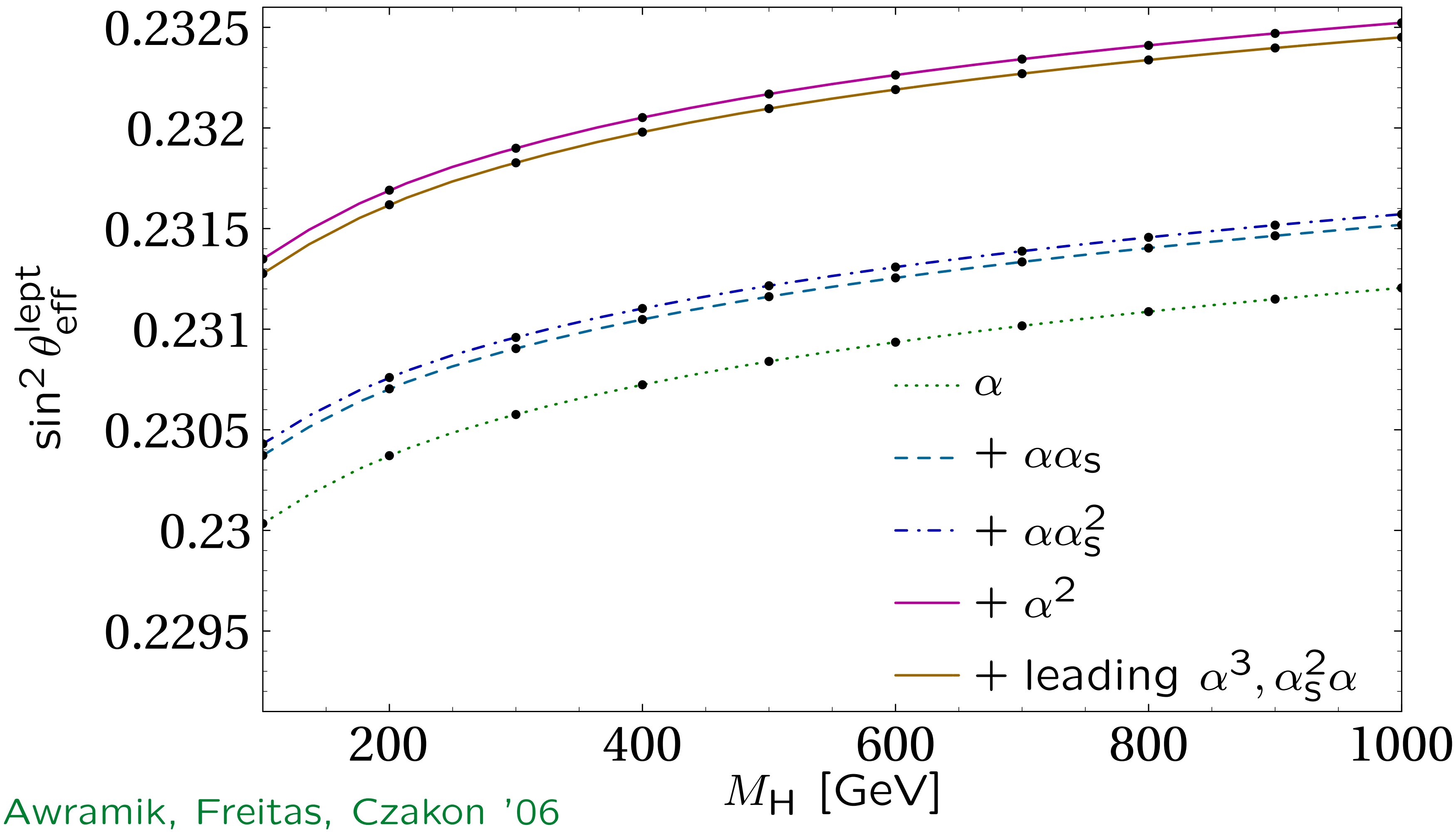
- left-right asymmetry (SLD)
- forward-backw. asymmetry (LEP+SLD)

on Z resonance

$\longrightarrow$ experimentally very clean

Final result for $\sin^2\theta_{\rm eff}^{\rm lept}$ uses G_μ as input
$\rightarrow$ include corrections to $M_{\rm W}$

Theoretical error:
$\delta_{\rm th}\sin^2\theta_{\rm eff}^{\rm lept} \approx 4.7\times 10^{-5}$



Awramik, Freitas, Czakon '06

Comparison to previous result with large-$m_{\rm t}$ expansion up to $O(\alpha^2 m_{\rm t}^2)$

G. Degrassi, P. Gambino, A. Sirlin '97

G. Degrassi, P. Gambino, M. Passera, A. Sirlin '98

M_H GeV	$\left(\Delta \sin^2\theta_{\rm eff}^{\rm lept}\right)_{\rm DGPS}$ $\times 10^{-4}$	$\left(\Delta \sin^2\theta_{\rm eff}^{\rm lept}\right)_{\rm ZFITTER}$ $\times 10^{-4}$
100	-0.45	-0.40
200	-0.69	-0.72
300	-0.85	-0.83
600	-1.17	-0.94
1000	-1.60	-1.28

Current experimental precision: $\sin^2\theta_{\rm eff}^{\rm lept} = 0.23150 \pm 0.00016$

Conclusions and outlook

- Precision observables test the Standard Model, give information about the **Higgs boson**, and tell a story about **new physics**
- Experimental precision at future colliders (LHC and ILC) requires calcualtion of **two-** and **three-loop** radiative corrections
- Complete electroweak 2-loop corrections to M_{W} and $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ and some leading higher-order corrections are available
- New results incorporated into ZFITTER 6.42 and used in experimental fits

More to be done...

Backup slides

Proper definition of correction factors at two-loop

Define amplitude as expansion around complex pole:

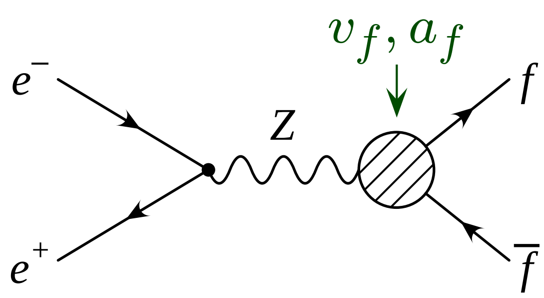
$$\mathcal{A}(e^+e^- \to f\bar{f}) = \frac{R}{s - \mathcal{M}_Z^2} + S + (s - \mathcal{M}_Z^2)\,S' + \cdots$$

$$\mathcal{M}_Z^2 = M_Z^2 - iM_Z\Gamma_Z$$

Expanding up to $\mathcal{O}(\alpha^2)$ and using $\mathcal{O}(\Gamma_Z/M_Z) = \mathcal{O}(\alpha)$ one can identify the electroweak form factor κ_f

$$\sin^2\theta_{\text{eff}}^{\text{lept}} = \Re e\{\kappa_l\}\left(1 - \frac{M_W^2}{M_Z^2}\right) \qquad \kappa_f = \frac{1 - v_f/a_f}{1 - v_f^{(0)}/a_f^{(0)}}$$

where v_f / a_f are the vector / axial-vector $Zf\bar{f}$ couplings

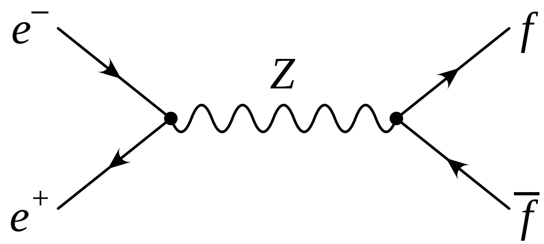


[(0) = tree-level]

Definition of Z exchange amplitude consistent with usual programs for SM fits (e.g. ZFITTER)

But:

Treatment of $\gamma - Z$ interference in ZFITTER **not** consisten t with complex pole scheme at $\mathcal{O}(\alpha^2)$.



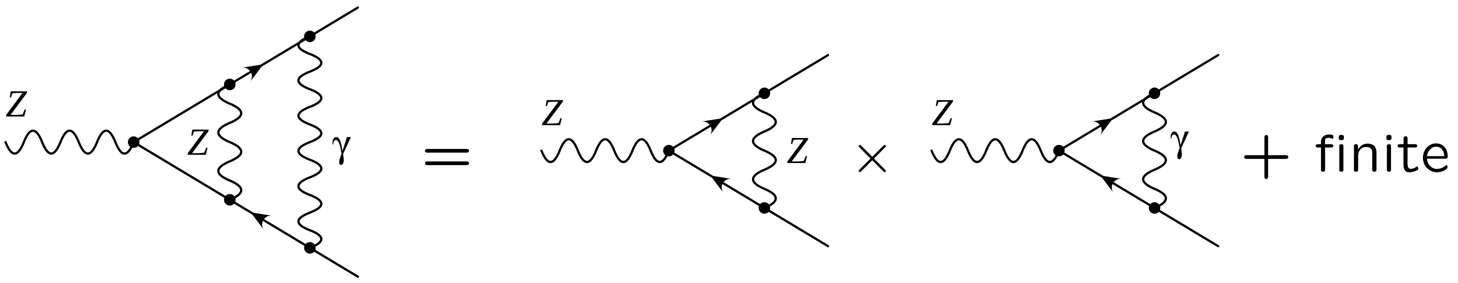
$\rightarrow$ Correction term for $\sin^2\theta_{\text{eff}}^{\text{lept}}$ (numerically small):

$$\sin^2\theta_{\text{eff}}^{\text{lept}} = \Re e\{\kappa_l\}\left(1 - \frac{M_{\text{W}}^2}{M_{\text{Z}}^2}\right) \; - \; \frac{\Gamma_{\text{Z}}}{M_{\text{Z}}} \frac{G_{\gamma ll,\text{v}}^{(0)}}{a_e^{(0)}(a_l^{(0)} - v_l^{(0)})} \; \Im m\{G_{\gamma ll,\text{a}}^{(0)}\}$$

Two-loop contribution:

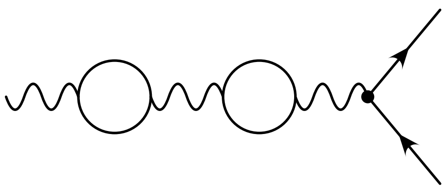
$$\kappa_l^{(2)} = \frac{a_l^{(2)} v_l^{(0)} a_l^{(0)} - v_l^{(2)} \left(a_l^{(0)}\right)^2 - \left(a_l^{(1)}\right)^2 v_l^{(0)} + a_l^{(1)} v_l^{(1)} a_l^{(0)}}{\left(a_l^{(0)}\right)^2 \left(a_l^{(0)} - v_l^{(0)}\right)} \Bigg|_{s=M_Z^2}$$

- Interplay between 2-loop terms and products of 1-loop terms to cancel IR-divergencies



$\longrightarrow$ $\sin^2\theta_{\text{eff}}^{\text{lept}}$ is IR-safe !

- Genuine 2-loop contributions contain products of imaginary parts of 1-loop terms



Error estimate

$$\mathcal{O}(\alpha^2\alpha_s) \approx \frac{\mathcal{O}(\alpha\alpha_s)}{\mathcal{O}(\alpha)}\,\mathcal{O}(\alpha^2)$$

Variation of $\overline{m}_t, \alpha_s$ for $\mu^2 = m_t^2/2 \ldots 2m_t^2$

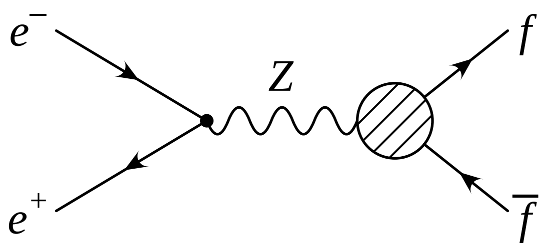
	geometric progression	scale/scheme dependence
$\mathcal{O}(\alpha^2\alpha_s)$ beyond leading m_t^4	$3.3 \ldots 2.8 \times 10^{-5}$	$0.1 \ldots 3.9 \times 10^{-5}$
$\mathcal{O}(\alpha\alpha_s^3)$	$1.5 \ldots 1.4$	$< 10^{-6}$
$\mathcal{O}(\alpha^3)$	$2.5 \ldots 3.5$	
Sum	$4.4 \ldots 4.7 \times 10^{-5}$	

Implementation in ZFITTER

Result coded in ZFITTER 6.42 via the fit formula
$\longrightarrow$ fast evaluation
error estimate also incorporated

Problem: new result only available for leptonic Zl^+l^- vertex

$\longrightarrow$ not usable for $Zb\bar{b}$ vertex,
which contains internal
massive top-quark propagators



until ZFITTER 6.40: Process $e^+e^- \longrightarrow (Z) \longrightarrow b\bar{b}$ computed without 2-loop corrections (not even partial 2-loop)

$\longrightarrow$ mismatch because 2-loop corrections to initial state Ze^+e^- known and taken into account for other final states correction in ZFITTER 6.42:

2-loop corrections to $\sin^2\theta^{\text{eff}}_{\text{lept}}$ in Ze^+e^- vertex for $e^+e^- \to (Z) \to b\bar{b}$,

but $Zb\bar{b}$ vertex still at 1-loop

$\to$ possible because initial and final state factorize on Z pole

$\to$ Shift in determination of pole asymmetry $A^{0,b}_{\text{FB}}$:

$\delta A^{0,b}_{\text{FB}} = 0.0006$ (compare to experimental error: 0.0017)

Freitas, Mönig '04

Arbuzov, Awramik, Czakon, Freitas, Grünewald, Mönig, Riemann, Riemann '06

Outlook: 2-loop corrections for $\sin^2\theta^{\text{eff}}_q$ for $b\bar{b}$ final states finished soon