Precision Standard Model Physics

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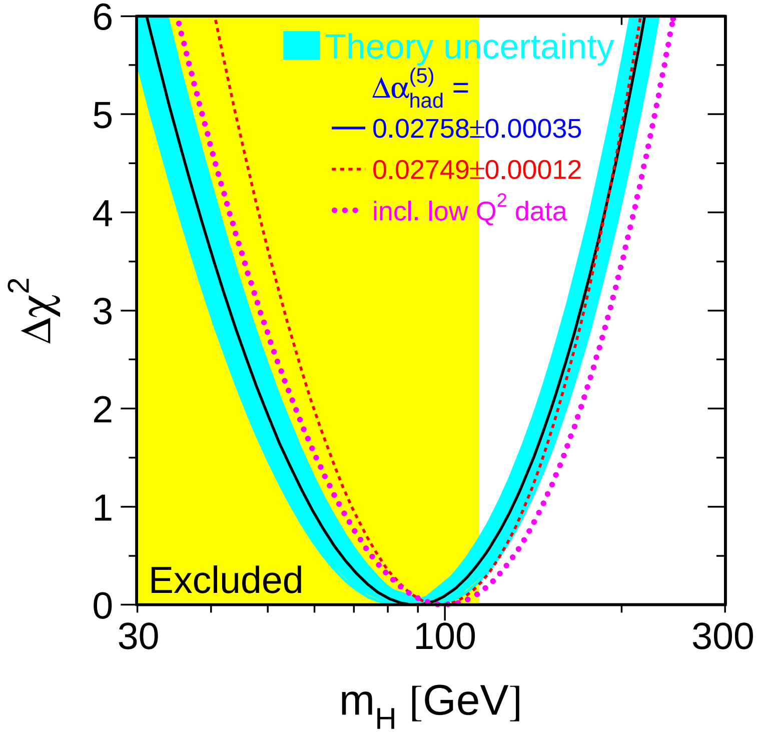
1 Introduction

Experiments at LEP/SLC/Tevatron

- confirmation of Standard Model as quantum field theory (quantum corrections significant)
- top mass $m_{\rm t}$ indirectly constrained by quantum corrections
  $\leftrightarrow$ in agreement with $m_{\rm t}$ measurement of Tevatron
- Higgs mass $M_{\rm H}$ indirectly constrained by quantum corrections
  $\hookrightarrow$ impact on Higgs searches

Great success of precision physics

- $M_{\rm H} > 114.4\,{\rm GeV}$ (LEPHIGGS '02)
  $\mathrm{e^+e^-} \not\to \mathrm{ZH}$ at LEP2
- $M_{\rm H} < 175\,{\rm GeV}$ (LEPEWWG '06)
  fit to precision data
  i.e. via quantum corrections



The role of precision at LHC and ILC

LHC: the discovery machine (Higgs & EWSB, SUSY, etc.?)

- QCD corrections (at least NLO) are substantial parts of predictions

  typical LO uncertainties $\sim$ several $10\%-100\%$

  corrections needed for signals and many background processes

- EW corrections also important for many observables

  (precision physics, searches at high scales, particle reconstruction, etc.)

ILC: the high-precision machine (precision $\to$ window to higher energy)

- old and new physics with high accuracy (typically $\delta\sigma/\sigma \lesssim 1\%$)

  $\hookrightarrow$ QCD and EW corrections required

- the ultimate precision at GigaZ/MegaW:

  precision increases by factor ~ 10 w.r.t. LEP/SLC

  EXP: $\Delta \sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}} \sim 0.00001, \qquad \Delta M_{\mathrm{W}} \sim 7\,\mathrm{MeV}$

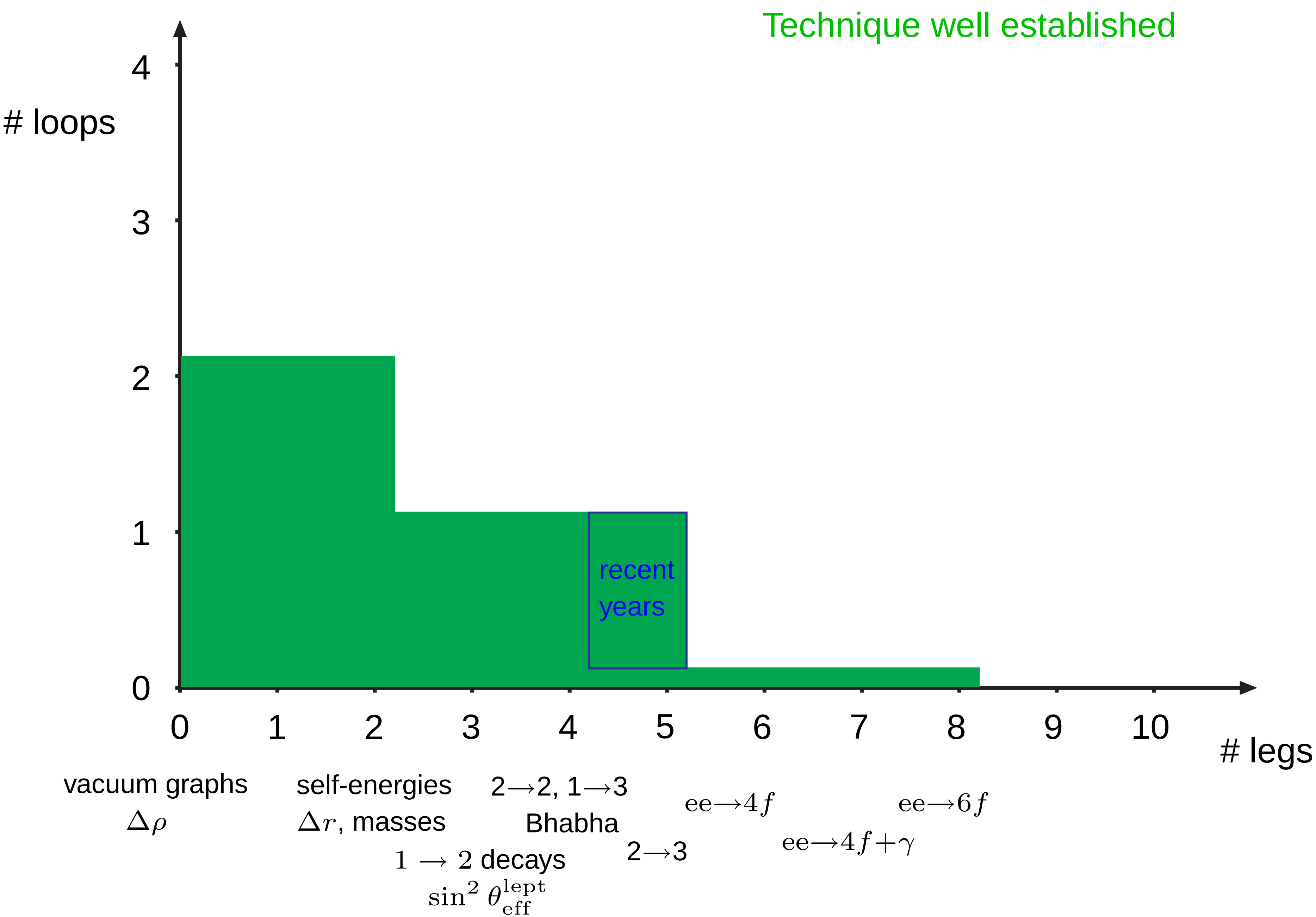
  TH: go from a few 10^2 to a few 10^4 (more complicated) diagrams

$\Rightarrow$ Precision calculations mandatory for LHC and ILC !

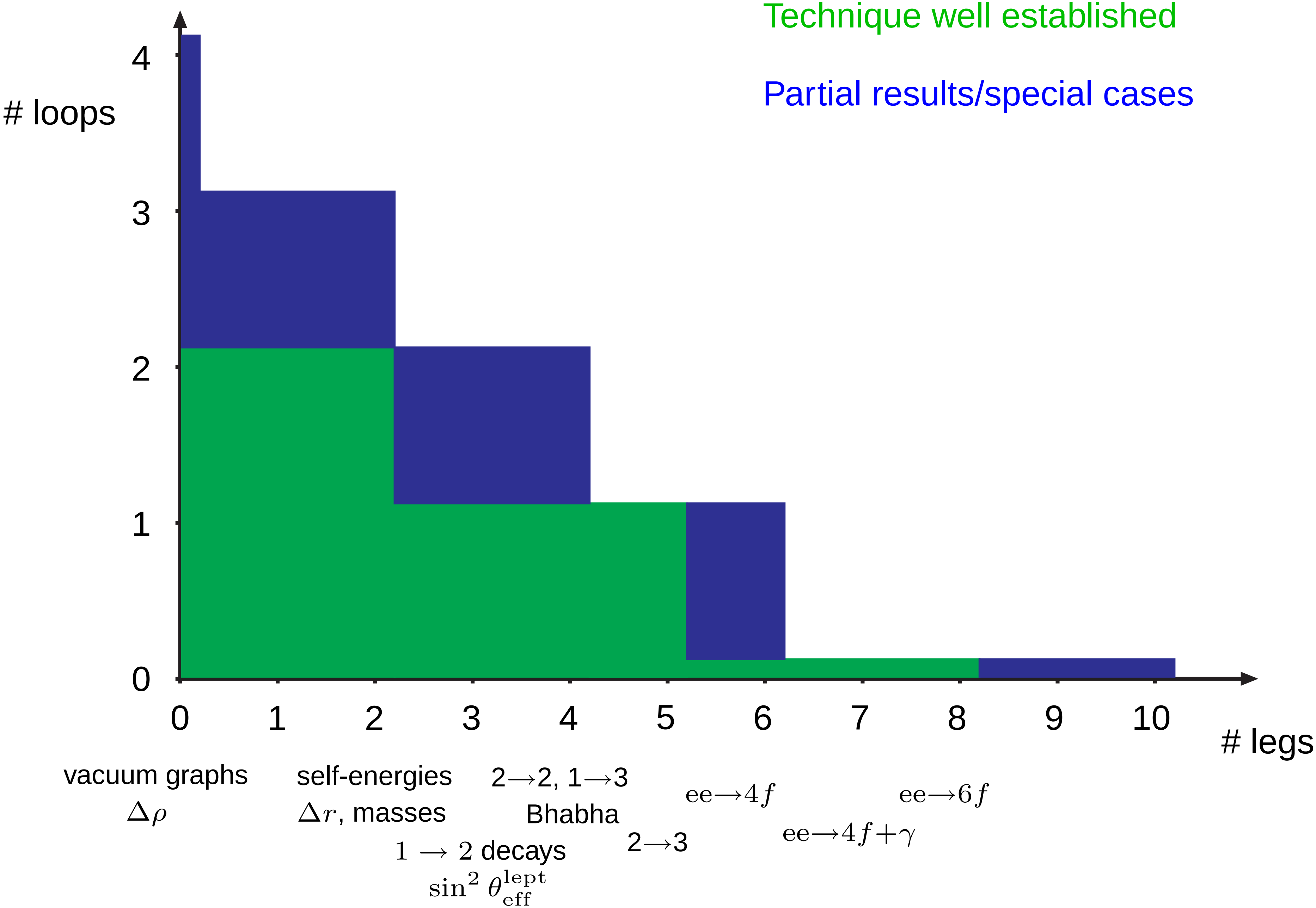
This talk: summary of recent developments (more topical than comprehensive)

- NNLO calculations to $2 \to 2$ scattering
- NLO corrections to many-particle processes
- precision calculations for LHC
- not or barely covered:
  physics beyond SM, automatization, MC and simulation tools,
  twistor-inspired methods, resummation,
  topics presented in dedicated talks

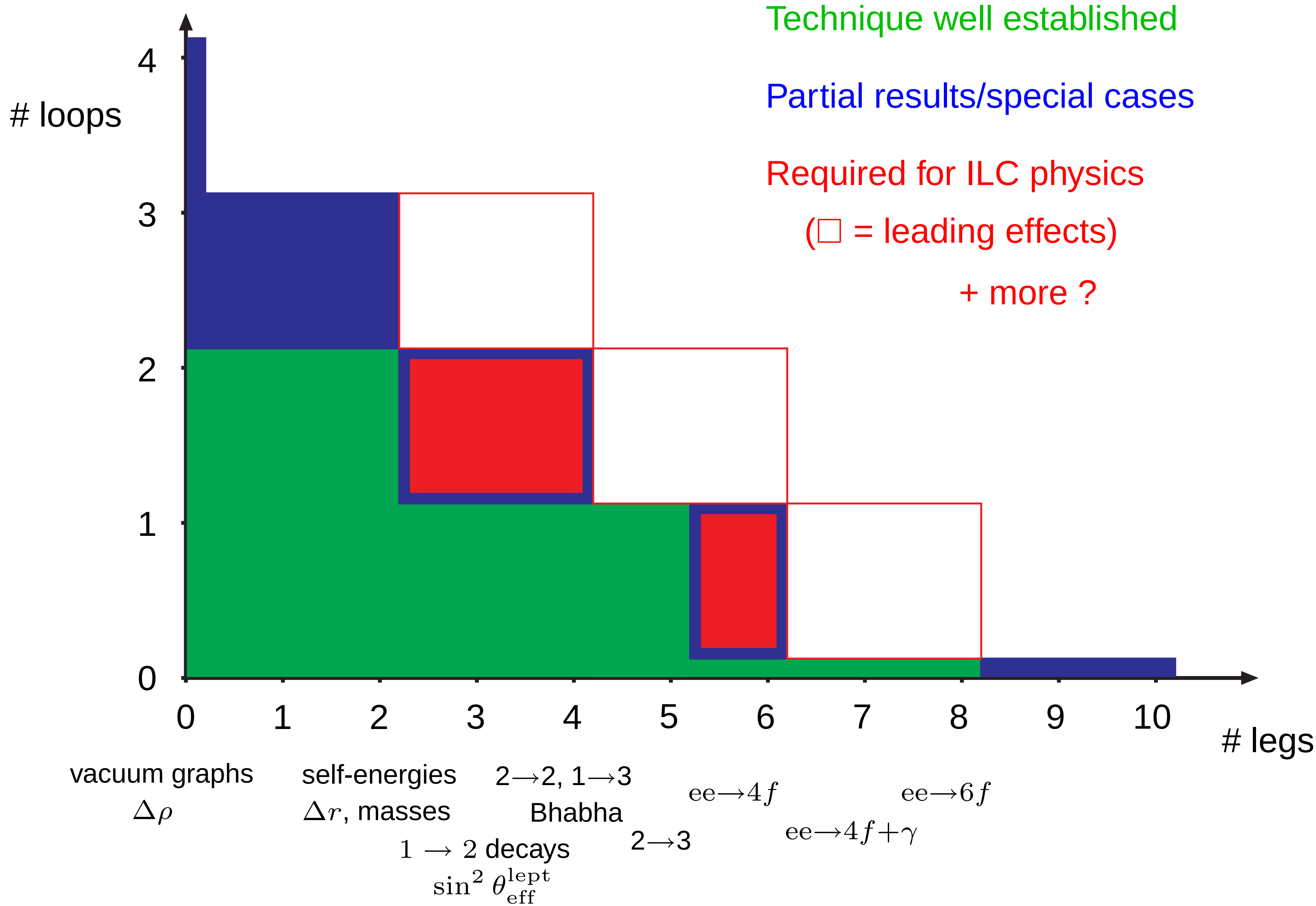
State-of-the-art in precision calculations



State-of-the-art in precision calculations



State-of-the-art in precision calculations



2 Multi-loop and NNLO calculations

2.1 EW precision observables

Most important precision observables:

- M_W (direct measurement vs. muon decay)
  - mixed QCD/EW 2-loop corrections known — Djouadi, Verzegnassi '87; Djouadi '88; Kniehl, Kühn, Stuart '88; Kniehl, Sirlin '93; Djouadi, Gambino '94
  - complete EW 2-loop corrections known — Freitas, Hollik, Walter, Weiglein '00; Awramik, Czakon '02; Onishchenko, Veretin '02
  - improvements by 3-loop $\Delta\rho$ — Avdeev et al. '94; Chetyrkin, Kühn, Steinhauser '95; v.d.Bij et al. '00; Faisst et al. '03; Boughezal, Tausk, v.d.Bij '05

    and 4-loop QCD $\Delta\rho$ — Schröder, Steinhauser '05; Chetyrkin et al. '06

  $\hookrightarrow$ theoretical uncertainty $\Delta M_W \sim 4\,\mathrm{MeV}$

- $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ (from various asymmetries)
  - mixed QCD/EW 2-loop and 3-loop $\Delta\rho$ corrections as for M_W
  - EW 2-loop corrections completed recently — Awramik, Czakon, Freitas, Weiglein '04; Hollik, Meier, Uccirati '05; Awramik, Czakon, Freitas '06

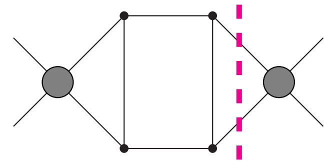
  $\hookrightarrow$ theoretical uncertainty $\Delta\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}} \sim 5\times 10^{-5}$

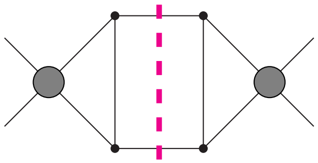
$\hookrightarrow$ Theoretical predictions in good shape for LHC

2.2 NNLO calculations for $2 \to 2$ processes

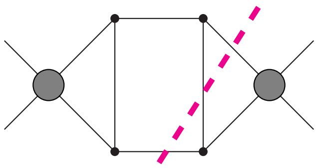
General structure of NNLO predictions:

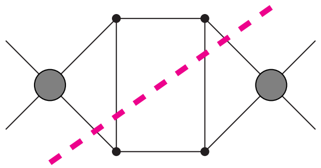
$$\Delta\sigma_{\mathrm{NNLO}} = F_{\mathrm{flux}} \int \mathrm{d}\Phi_2 \left[2\,\mathrm{Re}\left\{ \mathcal{M}^{(2\to 2)}_{\text{2-loop}} \, \mathcal{M}^{(2\to 2)*}_{\text{tree}} \right\} + \left| \mathcal{M}^{(2\to 2)}_{\text{1-loop}} \right|^2 \right]$$





$$+ \; F_{\mathrm{flux}} \int \mathrm{d}\Phi_3 \, 2\,\mathrm{Re}\left\{ \mathcal{M}^{(2\to 3)}_{\text{1-loop}} \, \mathcal{M}^{(2\to 3)*}_{\text{tree}} \right\} \; + \; F_{\mathrm{flux}} \int \mathrm{d}\Phi_4 \left| \mathcal{M}^{(2\to 4)}_{\text{tree}} \right|^2$$





Major difficulties:

- 2-loop amplitudes $\mathcal{M}^{(2\to 2)}_{\text{2-loop}}$
- extraction and cancellation of IR (soft / collinear) singularities

  $\hookrightarrow$ in particular: single and double unresolved limits in real emission amplitudes

2-loop amplitudes for $2 \to 2$ and $1 \to 3$ processes

- Algebraic reduction to master integrals — Anastasiou, Gehrmann, Glover, Laporta, Lazopoulos, Oleari, Remiddi, Smirnov, Tausk, Veretin '00–'05

  by integration by parts, Lorentz invariance identities

  $\hookrightarrow$ calculation of master integrals by Mellin–Barnes technique, — Anastasiou, Czakon, Smirnov, Tausk, Tejeda-Yeomans '99–'05

  differential equations, — Gehrmann, Remiddi '00, '01 — numerical techniques (see below)

- Direct reduction of full 2-loop amplitudes — Moch, Uwer, Weinzierl '02–'05

  $\hookrightarrow$ higher transcendental functions $\to$ nested harmonic sums

- Upcoming alternative: fully numerical approach
  - ◇ via sector decomposition (box master integrals, etc.) — Binoth, Heinrich '00,'03
  - ◇ via Feynman parameter integrals (all 2-/3-point integrals) — Actis, Ferroglia, Passera, Passarino, Uccirati '02–'06
  - ◇ via Mellin–Barnes representation (box master integrals, etc.) — Anastasiou, Daleo '05

- Explicit algebraic results:
  - ◇ 2-loop amplitudes for massless $2 \to 2$ processes — Anastasiou, Bern, v.d.Bij, DeFreitas, Dixon, Ghinculov, Glover, Oleari, Schmidt, Tejeda-Yeomans, Wong '01–'04
  - ◇ 2-loop QCD amplitudes for $e^+e^- \to 3$ jets — Garland, Gehrmann, Glover, Koukoutsakis, Moch, Remiddi, Uwer, Weinzierl '02

Towards NNLO QED corrections to Bhabha scattering

Physics motivation:

- luminosity monitor at high-energy $\mathrm{e}^+\mathrm{e}^-$ colliders (LEP/ILC)
  $\hookrightarrow$ small-angle Bhabha scattering at LEP: BHLUMI (Jadach et al. –'97)
  (1-loop EW + higher-order QED log's)
- large cross-section $\to$ high-precision QED / EW test

Full NNLO QED prediction very important for running and coming $\mathrm{e}^+\mathrm{e}^-$ colliders

Status of 2-loop and (1-loop)^2 virtual corrections

- known:
  – $m_\mathrm{e} = 0$ Bern, Dixon, Ghinculov '00
  – closed fermion loops for $m_\mathrm{e} \neq 0$ Bonciani et al. '04
  – $m_\mathrm{e} \to 0$ (translated $m_\mathrm{e}=0$ result via known IR structure) Penin '05
- in progress: $m_\mathrm{e} \neq 0$ directly from massive master integrals (MI)
  – all but few MI for boxes exist Smirnov '01; Bonciani, Mastrolia, Remiddi '02 Heinrich, Smirnov '04; Czakon, Gluza, Riemann '04–'06
  – reduction of amplitudes to MI Czakon, Gluza, Riemann '04–'06 Bonciani, Ferroglia '05

Final steps to be made:

- some missing MI for massive 2-loop boxes
- combination of 2-loop virtual with (1-loop)$\otimes$(1γ real) and ($2\gamma/\mathrm{ee}$) real emission

Integration techniques for real radiation at NNLO

Soft/collinear singularities have very complicated overlapping structure !

$\hookrightarrow$ behaviour, e.g., described by "antenna functions" Kosower '03

Different approaches to singular integrations

- subtraction techniques
  - ◇ subtraction terms widely worked out and integrated for $e^+e^- \to n$ jets — Weinzierl '03; Kilgore '04; Frixione, Grazzini '04; Gehrmann-DeRidder, Gehrmann, Glover '04,'05; Del Duca, Somogyi, Trocsanyi '05
  - ◇ first applications:

    $e^+e^- \to 2$ jets — Gehrmann-DeRidder, Gehrmann, Glover '04; Frixione, Grazzini '04; Weinzierl '06

    $\mathcal{O}(\alpha_s^3/N_c^2)$ parts of $e^+e^- \to 3$ jets — Gehrmann-DeRidder, Gehrmann, Glover '05

- direct numerical integration via sector decomposition
  - ◇ technique described in detail — Heinrich '02,'06; Gehrmann-DeRidder, Gehrmann, Heinrich '03; Gehrmann-DeRidder, Gehrmann, Glover '03; Anastasiou, Melnikov, Petriello '04; Binoth, Heinrich '04
  - ◇ first applications:

    $e^+e^- \to 2$ jets, $pp \to H + X$, $W + X$ in NNLO QCD, $\mu \to e\bar{\nu}_e\nu_\mu$ in NNLO QED — Anastasiou, Melnikov, Petriello '04–'06

    first steps towards $e^+e^- \to 3$ jets in NNLO QCD — Heinrich '06

3 NLO corrections to multi-particle production

3.1 General considerations

Existing precision calculations for many-particle processes at LHC and ILC:

- with up to 5-point loop diagrams:

  $e^+e^- \to$ 4jets (QCD), $\nu\bar{\nu}H$, $t\bar{t}H$, $e\bar{e}H$, $\nu\bar{\nu}\gamma$, ZHH, ZZH, $\gamma\gamma \to t\bar{t}H$

  NLO EW/QCD: Glover/Miller, Campbell et al., Bern et al., Dixon/Signer, Nagy/Trocsanyi, Weinzierl/Kosower, GRACE-loop, Denner et al., You et al., Chen et al., Zhang et al., Zhou et al. '96–'06

  $pp \to$ 3jets, $\gamma\gamma$+jet, V+2jets, $t\bar{t}H$, $b\bar{b}H$, $t\bar{b}H^-$, $b\bar{b}V$, HHH

  NLO QCD: Bern et al., Kunszt et al., Kilgore/Giele, Campbell et al., Nagy, Del Duca et al., Campbell/Ellis, Beenakker et al., Dawson et al., Dittmaier et al., Peng et al., Plehn/Rauch, Febres Cordero et al. '96–'06

  $H \to 4$ leptons: NLO EW — Bredenstein et al. '06; NLO QED — Carloni-Calame et al. '06

- with up to 6-point loop diagrams (current technical frontier)

  $e^+e^- \to 4$ fermions (CC): NLO EW — Denner, Dittmaier, Roth, Wieders, '05

  $e^+e^- \to \nu\bar{\nu}HH$: NLO EW — GRACE-loop '05

  $gg \to gggg$: NLO QCD amplitude "only" — R.K.Ellis, Giele, Zanderighi '06

Complications in corrections to many-particle processes

- huge amount of algebra, long final expressions
  $\hookrightarrow$ computer algebra / automatization
- multi-dimensional phase-space integration
  $\hookrightarrow$ Monte Carlo techniques
- complicated structure of singularities and matching of virtual and real corrections
  $\hookrightarrow$ subtraction and slicing techniques
- numerically stable evaluation of one-loop integrals with up to 5,6,... external legs
  $\hookrightarrow$ techniques to solve problems with inverse kinematical (e.g. Gram) det's

  Stuart et al. '88/'90/'97; v.Oldenborgh/Vermaseren '90; Campbell et al. 96; Ferroglia et al. '02; del Aguila/Pittau '04; Binoth et al. '02/'05; Denner/Dittmaier '02/'05; v.Hameren et al. '05; R.K.Ellis et al. '05; Anastasiou/Daleo '05

  [But: most proposed methods not (yet?) used in complicated applications]
- treatment of unstable particles, issue of complex masses

Problem of unstable particles:

description of resonances requires resummation of propagator corrections

$\hookrightarrow$ mixing of perturbative orders potentially violates gauge invariance

Proposed solutions for loop calculations:

- naive fixed-width scheme
  $\hookrightarrow$ breaks gauge invariance only mildly (?),
  but partial inclusion of widths in loops screws up singularity structure
- pole expansions Stuart '91; Aeppli et al. '93, '94; etc.
  $\hookrightarrow$ consistent, gauge invariant,
  but not reliable at threshold or in off-shell tails of resonances
- effective field theory approach Beneke et al. '04; Hoang,Reisser '04
  $\hookrightarrow$ involves pole expansions,
  but can be combined with threshold expansions
- complex-mass scheme Denner, Dittmaier, Roth, Wieders '05
  $\hookrightarrow$ gauge invariant, simple, valid everywhere in phase space
  but needs complex masses everywhere (also in loops)

3.2 NLO EW corrections to $e^+e^- \to 4$ fermions

Denner, Dittmaier, Roth Wieders '05

Details of the calculation:

- final states: $\nu_\tau\tau^+\mu^-\bar{\nu}_\mu,\ \mathrm{u}\bar{\mathrm{d}}\mu^-\bar{\nu}_\mu,\ \mathrm{u}\bar{\mathrm{d}}\mathrm{s}\bar{\mathrm{c}}$ (charged current)
- helicity amplitudes via reduction of spinor chains
- complex-mass scheme proposed for unstable particles in loop calculations
- new tensor reduction methods for numerical stabilization Denner, Dittmaier '02,'05
- real corrections $e^+e^- \to 4f+\gamma$ from RacoonWW Denner et al. '99–'01
- checks:
  - – structure of UV, IR, and mass singularities
  - – gauge invariance with finite widths ('tHF versus background-field gauge)
  - – virtual $\oplus$ real corrections with slicing or subtraction
  - – two independent calculations

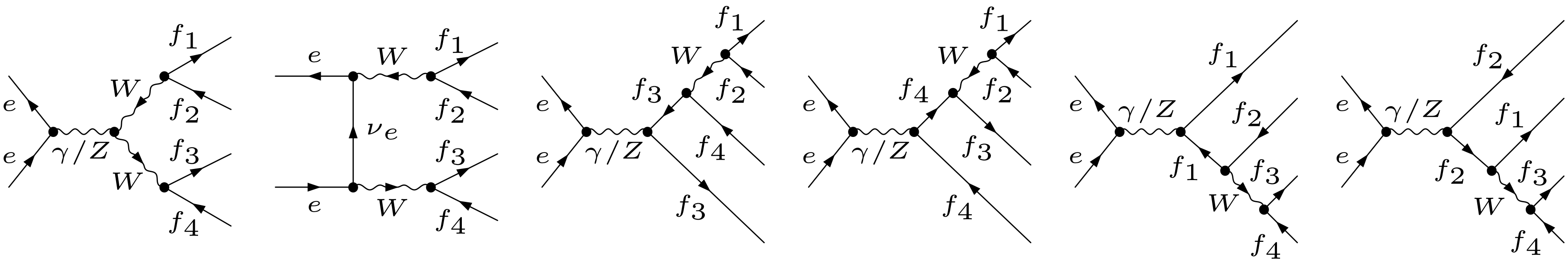
Physics motivation:

Improvement over "double-pole approximation" (DPA) for $e^+e^- \to \mathrm{WW} \to 4f$

needed for ILC:
- – M_W from WW threshold scan where DPA insufficient
- – TGC analysis at high energies

Some Feynman diagrams...

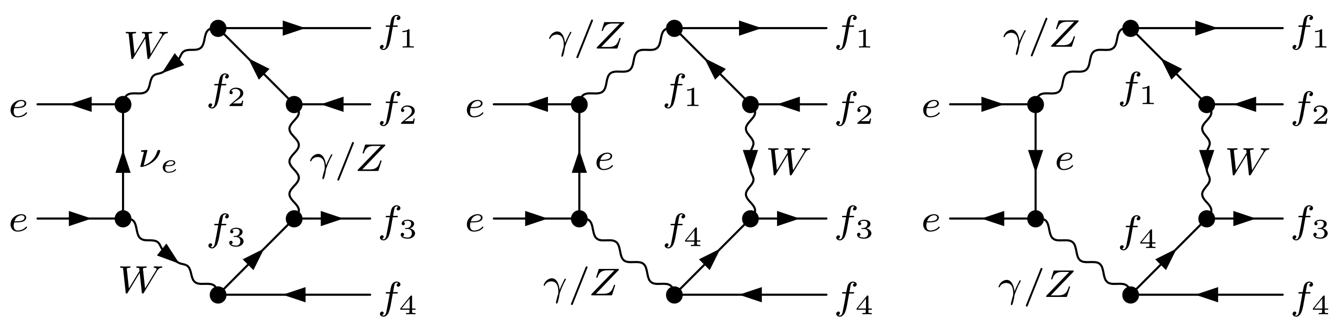
...for LO:



...for NLO:

total number $= \mathcal{O}(1200)$

40 hexagons



+ graphs with reversed fermion-number flow in final state

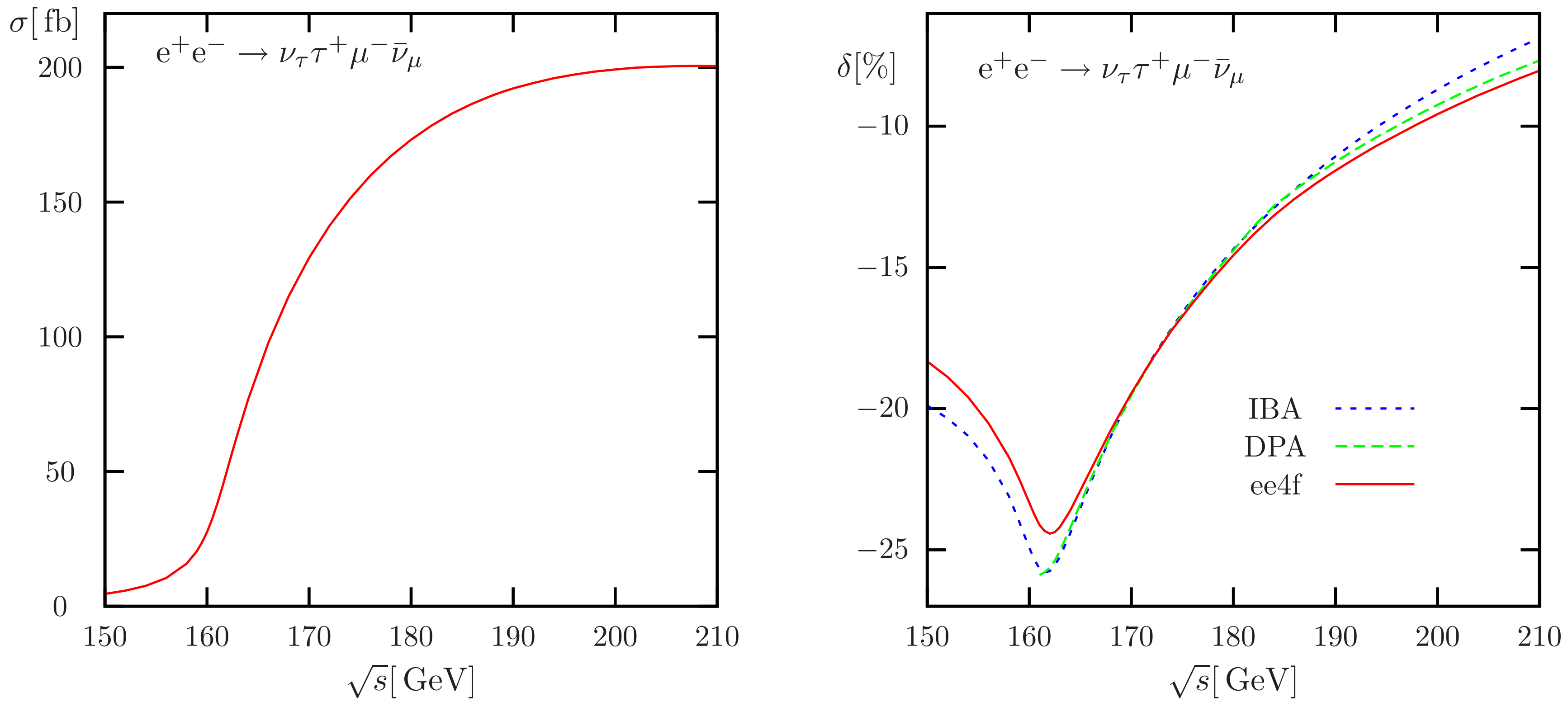
+ 112 pentagons

+ 227 boxes (‘tHF gauge) + many vertex and self-energy corrections

Numerical results for LEP2 energies

Complete $\mathcal{O}(\alpha)$ corrections to the total cross section

Denner, Dittmaier, Roth, Wieders '05



- $|\mathrm{ee4f} - \mathrm{DPA}| \quad \sim \quad 0.5\%$ for $170\,\mathrm{GeV} \lesssim \sqrt{s} \lesssim 210\,\mathrm{GeV}$
- $|\mathrm{ee4f} - \mathrm{IBA}| \quad \sim \quad 2\%$ for $\sqrt{s} \lesssim 170\,\mathrm{GeV}$

$\hookrightarrow$ agreement with error estimates of DPA and "Improved Born Approximation"

3.3 NLO EW corrections to $\mathrm{e^+e^-} \to \nu\bar{\nu}\mathrm{HH}$

Boudjema, Fujimoto, Ishikawa, Kaneko, Kato, Kurihara, Shimizu, Yasui '05

Full $2 \to 4$ calculation performed with GRACE-LOOP package — Belanger et al. hep-ph/0308080

- Number of loop diagrams (non-linear gauge, $m_\mathrm{e} \to 0$):
  $\#(\mathrm{e^+e^-}\to\nu_\mathrm{e}\bar{\nu}_\mathrm{e}\mathrm{HH}) \sim 3400$, $\#(\mathrm{e^+e^-}\to\nu_\mu\bar{\nu}_\mu\mathrm{HH}) \sim 1800$
- gauge-invariance check via non-linear gauge with gauge parameters
  (for vanishing particle widths)
- REDUCE and FORM used to process interference of LO and NLO amplitudes
  $\hookrightarrow$ 5- and 6-point integrals converted into 4-point integrals
- in-house library $\oplus$ FF for loop integrals
  v.Oldenborgh '91

Physics motivation:

Higgs self-coupling enters $\underbrace{\mathrm{e^+e^-}\to\mathrm{ZHH}}_{\sqrt{s}\lesssim 1\,\mathrm{TeV}}$ and $\underbrace{\mathrm{e^+e^-}\to\nu\bar{\nu}\mathrm{HH}}_{\sqrt{s}\gtrsim 1\,\mathrm{TeV}}$ in LO

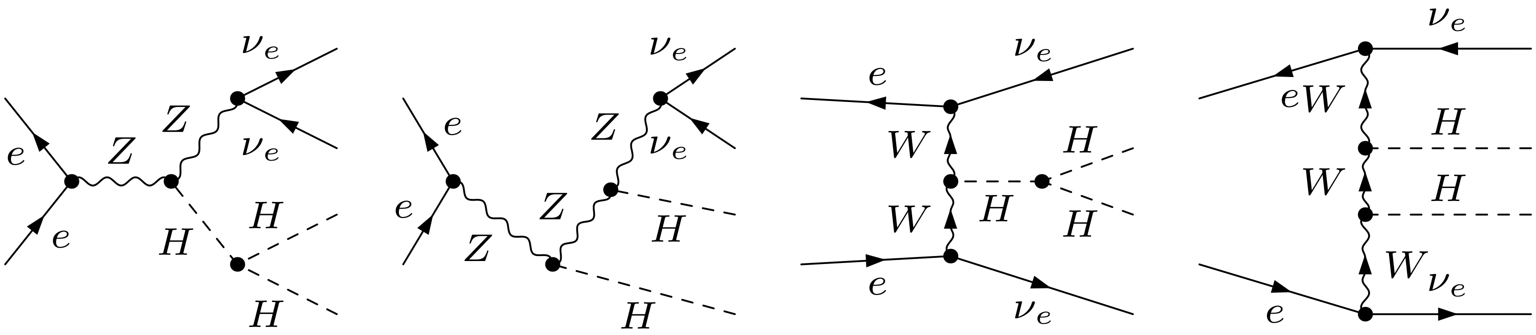
larger cross-section for $\sqrt{s}\lesssim 1\,\mathrm{TeV}$ / $\sqrt{s}\gtrsim 1\,\mathrm{TeV}$

$\hookrightarrow$ check of Higgs mechanism / information on EWSB

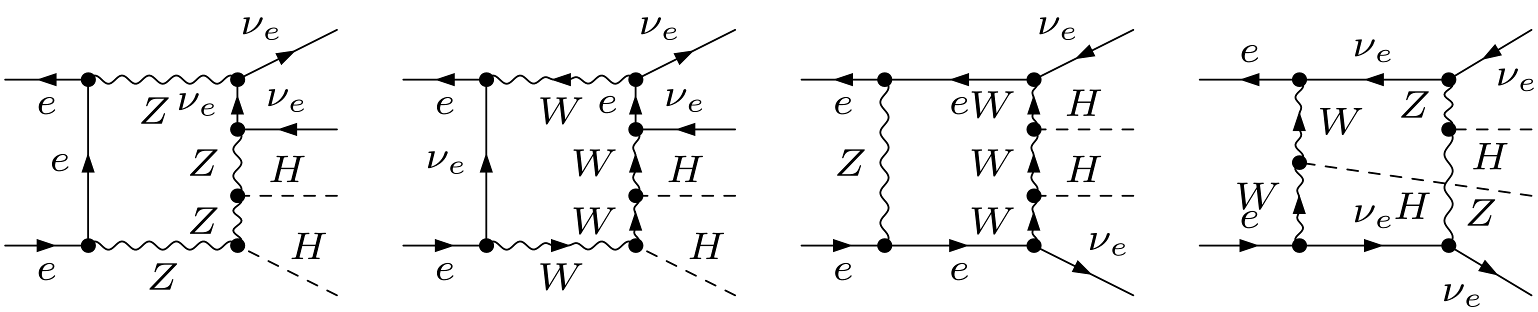
But: Both reactions have very small cross sections: $\sigma_{\mathrm{ZHH}+\nu\bar{\nu}\mathrm{HH}} \sim 0.1-1\,\mathrm{fb}$

Some Feynman diagrams...

...for LO: total number $= 18$



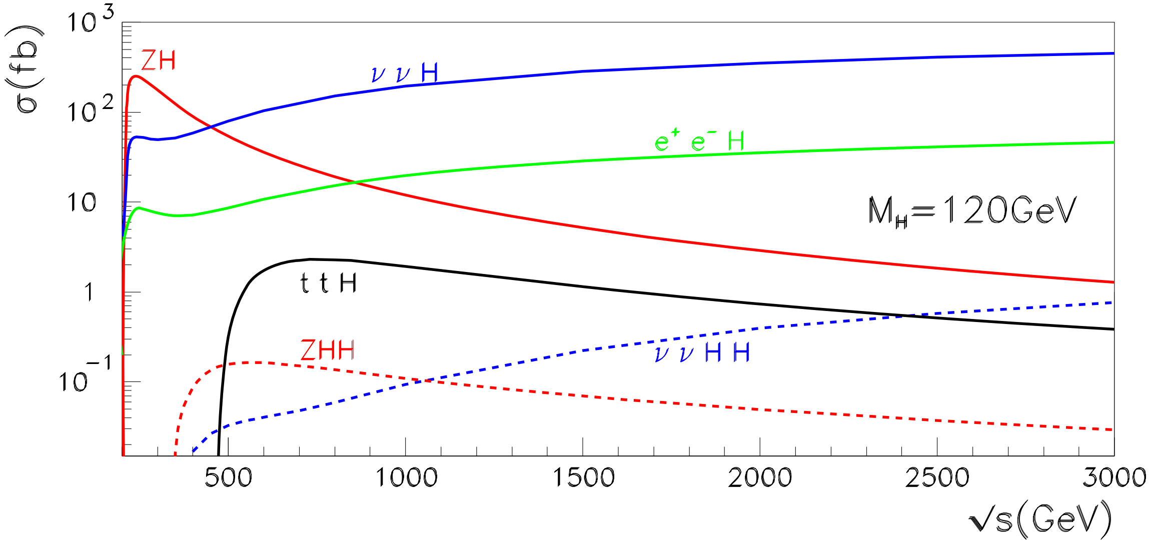
...for NLO: total number $= \mathcal{O}(4600)$ in ‘tHF gauge



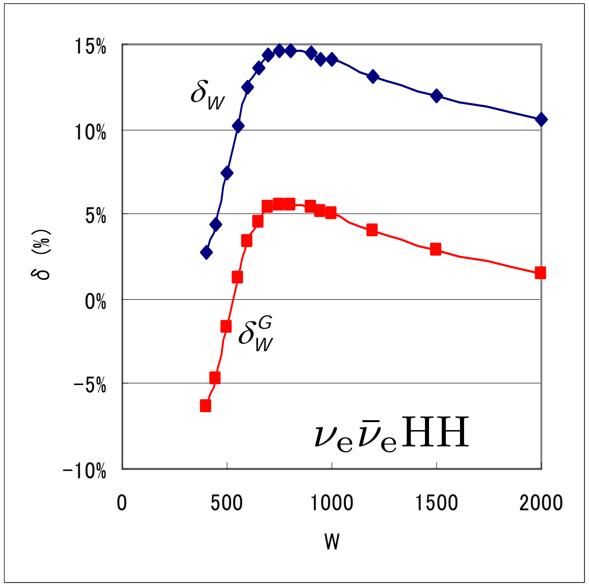
89 hexagons, 250 pentagons (‘tHF gauge), etc.

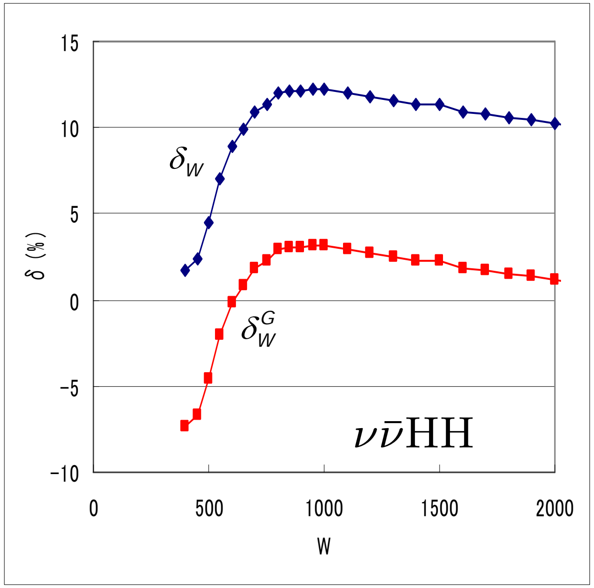
Numerical results: Boudjema et al. '05

Higgs production processes at the ILC in LO:



Weak (non-photonic) NLO corrections to $\mathrm{e^+e^-} \to \nu\bar{\nu}\mathrm{HH}$:





G_μ-scheme:

$$\delta_W^G = \delta_W - 4\Delta r$$

3.4 The 6-gluon amplitude at one loop

R.K.Ellis, Giele, Zanderighi ’06

Physics motivation:

$\mathrm{pp} \to 4\,$jets $+\, X$ in NLO QCD

$\hookrightarrow$ 6-gluon amplitude is most complicated ingredient

Details of the calculation:

- number of diagrams $= \mathcal{O}(12000)$
- QGRAF and FORM used for diagram generation and further processing
  Nogueira ’93
- colour-ordered helicity amplitudes
- semi-numerical evaluation of loop integrals — Giele,Glover ’04; R.K.Ellis, Giele, Zanderighi ’04,’05

  CPU time: $\mathcal{O}(9\mathrm{sec})$ per colour-ordered subamplitude per phase-space point
- numerical comparison with existing analytical results

  in $N = 4$ and $N = 1$ SUSY — Bern et al. ’93–’05; Bidder et al. ’04; Britto et al. ’05

  and specific helicity configurations in QCD — Bern et al. ’05,’06; Britto et al. ’06; Berger,Forde ’06

  $\hookrightarrow$ agreement for single phase-space points

4 Precision calculations for the LHC

4.1 Overview

Higher-order issues for LHC physics:

- Relevance of NNLO calculations ?
  - ◇ 2-jet production at NNLO QCD desirable
  - ◇ NNLO EW needed somewhere (e.g. Drell–Yan) ?
- Size of NLO EW corrections ? generically $\mathcal{O}(\alpha) \sim \mathcal{O}(\alpha_\mathrm{s}^2)$

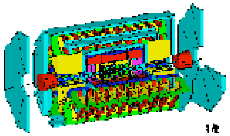
  But systematic enhancements of EW effects by
  - ◇ logarithms $\alpha \ln^n(M_\mathrm{W}/Q)$, $n = 2, 1$ (Sudakov and subleading) at high scales Q
  - ◇ kinematic effects from photon radiation off leptons (e.g. Drell–Yan)
- NLO QCD corrections ?

  $\hookrightarrow$ basically needed for all hard scattering processes

  Many $2 \to 3, 4$ background processes not yet known at NLO QCD !
- Higher-order-corrected PDFs:
  - ◇ NNLO splitting functions for quarks, gluons, photons completely known

    Moch, Vermaseren, Vogt '04,'05
  - ◇ NNLO QCD PDFs available (Alekhin02 NNLO, MRST2004nnlo)
  - ◇ NLO QCD $\oplus$ EW available (MRST2004qed)

Experimenters' wish list for the LHC: (J.Huston, Les Houches 2005)



Experimental priority list

- Note have to specify how inclusive final state is
  - what cuts will be made?
  - how important is b mass for the observables?
- How uncertain is the final state?
  - what does scale uncertainty look like at tree level?
  - new processes coming in at NLO?
- Some information may be available from current processes
  - pp->tT j may tell us something about pp->tTbB?
    - j=g->bB
  - CKKW may tell us something about higher multiplicity final states

1. pp->WW jet
2. pp->tT bB
   1. background to tTH
3. pp->tT + 2 jets
   1. background to tTH
4. pp->WWbB
5. pp->V V + 2 jets
   1. background to WW->H->WW
6. pp->V + 3 jets
   1. beneral background to new physics
7. pp->V V V
   1. background to SUSY trilepton

Beyond the SM Workshop at Columbia

Important process classes and discussed topics:

- jet physics
- heavy-quark production
- EW gauge-boson production $(\mathrm{V} = \gamma, \mathrm{Z}, \mathrm{W})$

V (Drell–Yan):	M_V, Γ_V, $\sin^2\theta_\mathrm{eff}^\mathrm{lept}$, V' searches, PDFs
VV:	TGCs
VVV and $\mathrm{VV}\to\mathrm{VV}$:	QGCs, EWSB

- Higgs production
- production of new-physics (e.g. SUSY) particles
- etc.

Important process classes and discussed topics:

- jet physics
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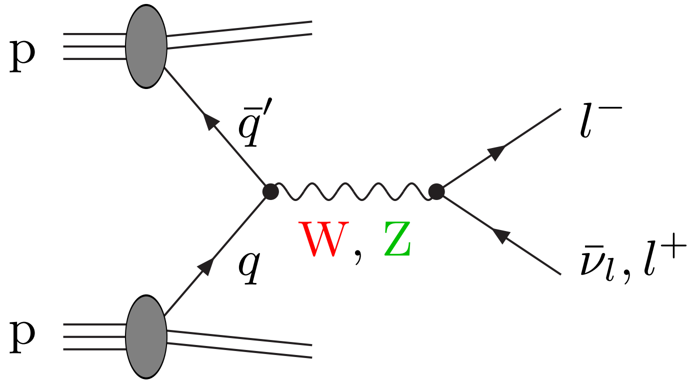
VV:	TGCs
VVV and $\mathrm{VV}\to\mathrm{VV}$:	QGCs, EWSB

briefly discussed in the following

- Higgs production
- production of new-physics (e.g. SUSY) particles
- etc.

4.2 Drell–Yan-like W and Z production

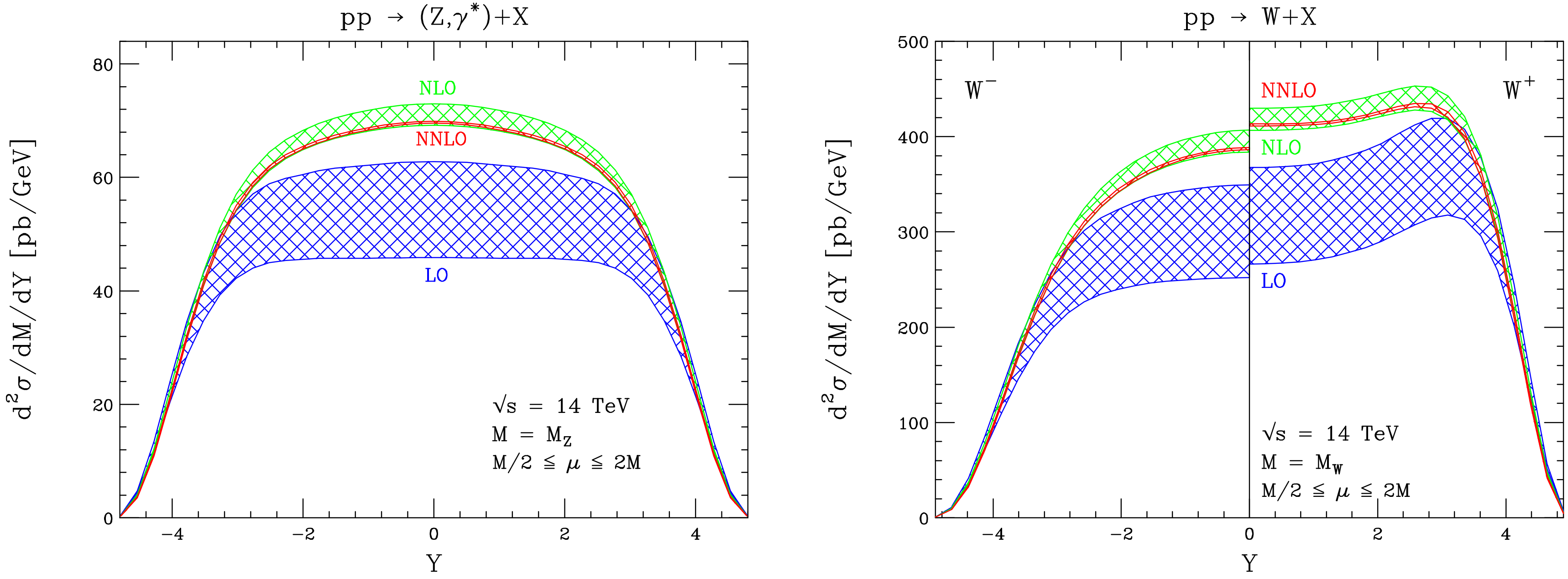


Physics goals:

- M_Z $\rightarrow$ detector calibration by comparing with LEP1 result
- $\sin^2\theta_{eff}^{lept}$ $\rightarrow$ comparison with results of LEP1 and SLC
- M_W $\rightarrow$ improvement to $\Delta M_W \sim 15\,\mathrm{MeV}$, strengthen EW precision tests
- decay widths Γ_Z and Γ_W from M_{ll} or $M_{T,l\nu_l}$ tails
- search for Z' and W' at high M_{ll} or $M_{T,l\nu_l}$
- information on PDFs

NNLO QCD corrections known for

- total cross section Hamberg, v.Neerven, Matsuura '91; v.Neerven, Zijlstra '92 Harlander, Kilgore '02
- W/Z rapidity distribution Anastasiou et al. '03



- fully differential cross section $\mathrm{pp}(\to \mathrm{W}) \to l\nu_l + X$ Melnikov, Petriello '06

Further improvements:

- Soft-gluon resummation (partially combined with γ emission) Balazs, Yuan '97; Landry et al. '02 Cao, Yuan '04
- NLO EW corrections

But: no proper combination of QCD $\oplus$ EW corrections yet !

EW corrections to W/Z production:

- NLO EW correction to W production — Baur, Keller, Wackeroth '98; Dittmaier, Krämer '02; Baur, Wackeroth '04; Arbuzov et al. '05; Carloni Calame et al. '06
- NLO EW correction to Z production — Baur, Keller, Sakumoto '97; Baur, Wackeroth '99; Brein, Hollik, Schappacher '99; Arbuzov et al. '06
- multi-photon radiation via leading logs — Baur, Stelzer '99; Carloni Calame et al. '03; Placzek, Jadach '04

Comparison of NLO EW corrections to W production:

$\mathrm{pp} \to \nu_l l^+(+\gamma)$ at $\sqrt{s} = 14\,\mathrm{TeV}$ — Les Houches SMH proceedings '06

$M_{\mathrm{T},\nu_l l}/\mathrm{GeV}$	50–∞	100–∞	200–∞	500–∞	1000–∞	2000–∞
σ_0/pb						
DK	$2112.2(1)$	$13.152(2)$	$0.9452(1)$	$0.057730(5)$	$0.0054816(3)$	$0.00026212(1)$
$\delta_{\mu^+\nu_\mu}/\%$						
DK	$-2.75(1)$	$-5.03(2)$	$-7.98(1)$	$-14.43(1)$	$-21.99(1)$	$-32.15(1)$
HORACE	$-2.77(1)$	$-5.08(1)$	$-8.01(1)$	$-14.44(1)$	$-21.99(1)$	$-32.16(1)$
SANC	$-2.76(2)$	$-5.06(2)$	$-7.96(2)$	$-14.41(2)$	$-21.94(2)$	$-32.12(2)$
WGRAD	$-2.69(1)$	$-4.84(1)$	$-7.96(1)$	$-14.48(1)$	$-22.03(1)$	$-32.3(1)$

$\hookrightarrow$ Large corrections at high transverse W mass $M_{\mathrm{T},\nu_l l}$!

4.3 EW corrections at hadron colliders

Electroweak effects in PDFs

Analogy to QCD-improved parton model:

Collinear splittings $q \to q\gamma$, $\gamma \to q\bar{q}$ lead to quark mass singularities

$\hookrightarrow$ absorb $\alpha \ln m_q$ singularities via factorization into redefined PDFs

Previous approach: no $\mathcal{O}(\alpha)$-corrected PDFs available

$\hookrightarrow$ factorization of collinear singularities in $\mathcal{O}(\alpha)$ in $\overline{\mathrm{MS}}$ scheme

but: neglect $\mathcal{O}(\alpha)$ effects in PDFs

Estimate of neglected $\mathcal{O}(\alpha)$ effects in PDFs: Spiesberger ’95, ’99; Roth, Weinzierl ’04

$$\Delta(\mathrm{PDF}) \lesssim 0.3\%\ (1\%) \quad \text{for} \quad x < 0.1\ (0.4), \quad \mu_{\mathrm{fact}} \sim M_{\mathrm{W}}$$

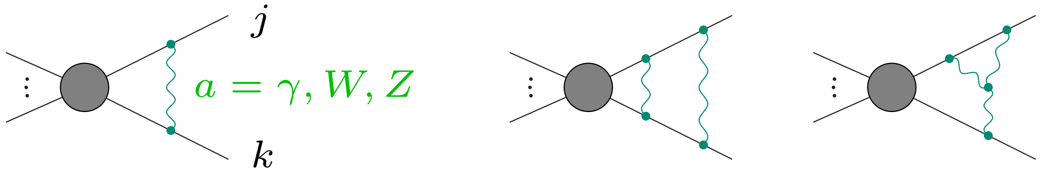
New situation: MRST2004QED set of $\mathcal{O}(\alpha)$-corrected PDFs Martin, Roberts, Stirling, Thorne ’0

$\hookrightarrow$ new PDFs should be used if EW $\mathcal{O}(\alpha)$ corrections are included

- use appropriate factorization scheme for $\mathcal{O}(\alpha)$ corrections (= DIS like)
- additional real corrections from photons in initial state
- find processes to measure $\mathcal{O}(\alpha)$ induced photon distribution
  MRST2004QED: start PDF from model assumption

Electroweak radiative corrections at high energies

Sudakov logarithms induced by soft gauge-boson exchange



etc.

+ sub-leading logarithms from collinear singularities

Typical impact on $2 \to 2$ reactions at $\sqrt{s} \sim 1\,\mathrm{TeV}$:

$$\delta_{\mathrm{LL}}^{1-\mathrm{loop}} \sim -\frac{\alpha}{\pi s_{\mathrm{W}}^2} \ln^2\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq -26\%, \qquad \delta_{\mathrm{NLL}}^{1-\mathrm{loop}} \sim +\frac{3\alpha}{\pi s_{\mathrm{W}}^2} \ln\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq 16\%$$

$$\delta_{\mathrm{LL}}^{2-\mathrm{loop}} \sim +\frac{\alpha^2}{2\pi^2 s_{\mathrm{W}}^4} \ln^4\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq 3.5\%, \qquad \delta_{\mathrm{NLL}}^{2-\mathrm{loop}} \sim -\frac{3\alpha^2}{\pi^2 s_{\mathrm{W}}^4} \ln^3\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq -4.2\%$$

$\Rightarrow$ Corrections still relevant at 2-loop level

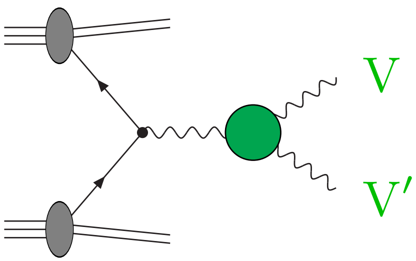
Note: differences to QED / QCD where Sudakov log's cancel

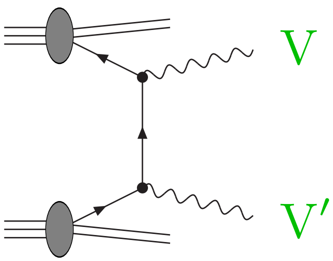
- massive gauge bosons W, Z can be reconstructed
  $\hookrightarrow$ no need to add "real W, Z radiation"
- non-Abelian charges of W, Z are "open" $\rightarrow$ Bloch–Nordsieck theorem not applicable

Extensive theoretical studies at fixed perturbative (1-/2-loop) order and suggested resummations via evolution equations

Beccaria et al.; Beenakker, Werthenbach; Ciafaloni, Comelli; Denner, Pozzorini; Fadin et al.; Hori et al.; Melles; Kühn et al. '00–'06

4.4 Gauge-boson pair production





$\mathrm{V}, \mathrm{V}' = \gamma, \mathrm{Z}, \mathrm{W}^{\pm}$

Physics issues:

- triple-gauge-boson couplings at high momentum transfer
- dynamics of longitudinal massive gauge bosons at high energies
  $\mathrm{W_L}, \mathrm{Z_L} \sim$ Goldstone bosons $\quad \to$ scalar sector

  strongly interacting longitudinal W/Z bosons if no Higgs exists
  $\hookrightarrow$ unitarity requires resonances
- important class of background processes to many searches (e.g. $\mathrm{H} \to \mathrm{VV} \to 4f$)

Requirements on adequate predictions:

- full LO matrix elements for $q\bar{q} \to 4f$ (spin correlations, off-shell effects)
  $\hookrightarrow$ respect gauge invariance
- NLO QCD and EW corrections

EW corrections to gauge-boson pair production

- $\mathrm{pp}(\to \mathrm{W}\gamma) \to l\bar{\nu}\gamma + X$ Accomando, Denner, Pozzorini '01; Accomando, Denner, Meier '05

  $\mathcal{O}(\alpha)$ correction in pole approximation

  $\hookrightarrow \quad \delta \sim -5\%\,(-24\%)$ for $p_{\mathrm{T},\gamma} \gtrsim 350\,\mathrm{GeV}\,(700\,\mathrm{GeV})$

- $\mathrm{pp} \to \mathrm{Z}\gamma + X$ Hollik, Meier '04 and $\mathrm{pp}(\to \mathrm{Z}\gamma) \to ll\gamma + X$ Accomando, Denner, Meier '05

  complete $\mathcal{O}(\alpha)$ correction for on-shell Z bosons / in pole approximation

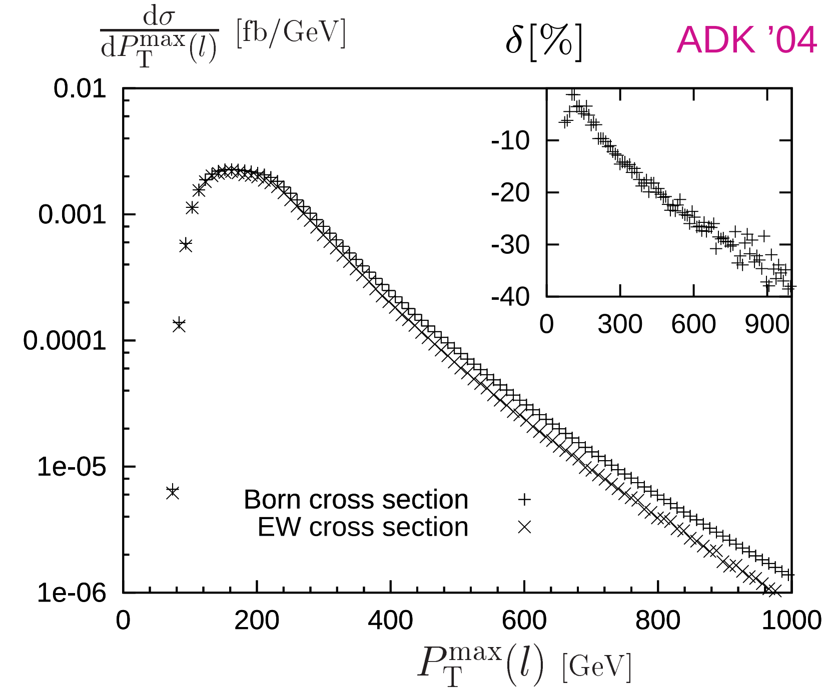
  $\hookrightarrow \quad \delta \sim -20\%$ for $M_{\gamma\mathrm{Z}} \lesssim 2\,\mathrm{TeV}$

- $\mathrm{pp}(\to \mathrm{WW}, \mathrm{WZ}, \mathrm{ZZ}) \to 4\,\mathrm{leptons} + X$

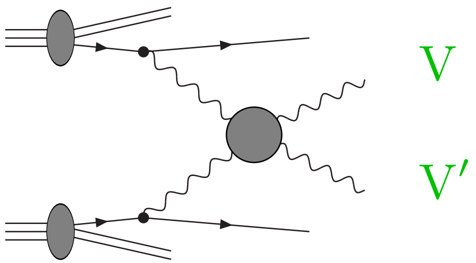
  Accomando, Denner, Pozzorini '01
  Accomando, Denner, Kaiser '04

  $\mathcal{O}(\alpha)$ correction in high-energy and pole approximations

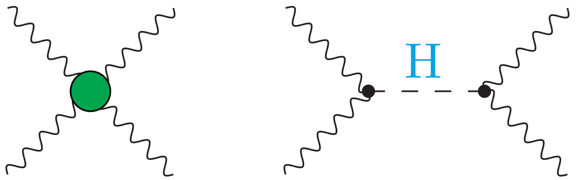
$\mathrm{pp} \to \mathrm{WZ} \to \mathrm{e}\nu_{\mathrm{e}}\mu^+\mu^-$ at $\sqrt{s} = 14\,\mathrm{TeV}$



4.5 Gauge-boson scattering



e.g.



Physics issues:

- link to Higgs production:
  vector-boson fusion with subsequent decay $\mathrm{H} \to \mathrm{WW}/\mathrm{ZZ} \to 4f$
- triple and quartic gauge-boson self-interaction
  $\hookrightarrow$ high sensitivity, but again ambiguities from formfactors
- $\mathrm{V_L V_L} \to \mathrm{V_L V_L}$: strong sensitivity to details of electroweak symmetry breaking
  if no Higgs exists $\to$ unitarity requires scalar and vector resonances

  However:

  - ◇ description of resonances is "ad hoc" (different "unitarization models")
    $\hookrightarrow$ large ambiguities
  - ◇ many (more qualitative) studies show that LHC could see the resonances

Comments and questions from a theorist

- Approximations made in many previous predictions

  1. no QCD corrections

  2. "effective vector-boson approximation" (EVA) ($\sim$ Weizsäcker–Williams)
     equivalence theorem (ET) (i.e. $\mathrm{V_L} \sim$ Goldstone boson)

  3. no EW corrections (some partial results on $\mathrm{VV} \to \mathrm{VV}$ known)

  Each of these approximations induces uncertainties of several 10% !

- Situation in SM-like scenario: (i.e. no resonances apart from Higgs)
  cross sections small; large background from $q\bar{q}$ annihilation
  $\hookrightarrow$ Can weak-coupling sector for $\mathrm{V_L}$ be experimentally verified ?

- Case with low background: like-sign W-pair production ($\to \mu^+\mu^+ +$ missing p_T)
  $\hookrightarrow$ How promising is this channel ?

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     $\hookrightarrow$ first NLO QCD results by Jäger, Oleari, Zeppenfeld '06

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     $\hookrightarrow$ results from full $2\to6$ matrix elements by Accomando et al.'05,'06

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  $\hookrightarrow$ How promising is this channel ?

New results with full $2 \to 6$ amplitudes (no EVA, no ET)

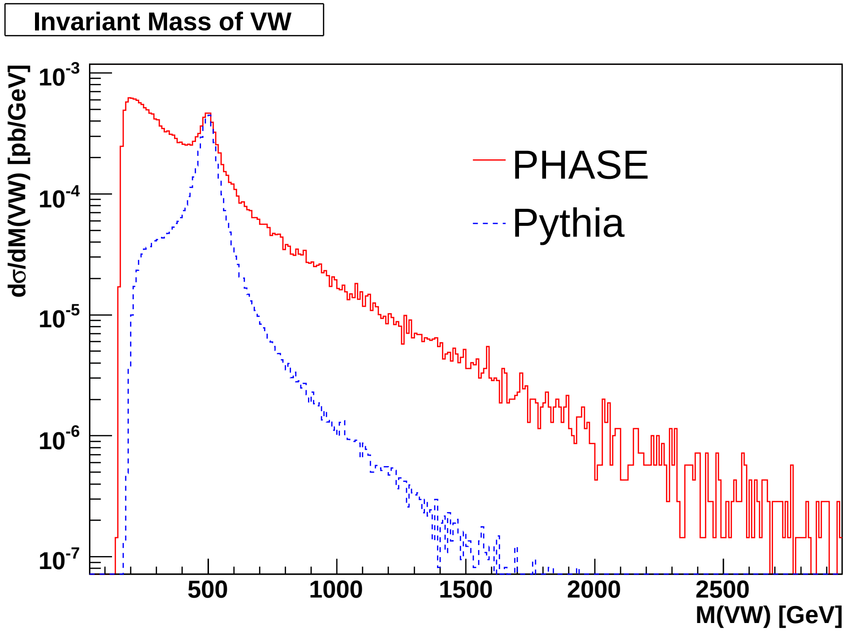
PHASE / PHANTOM = Monte Carlo generators employing full $2 \to 6$ matrix elements

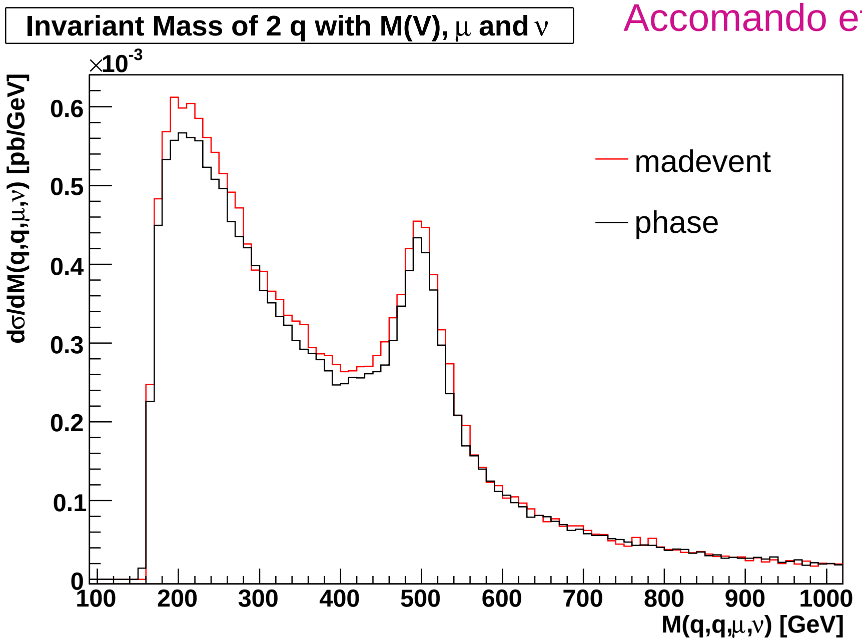
Accomando et al.'05 / Accomando et al.'06

But: no QCD and EW corrections

Comparison of different approaches:

example: processes containing $\mathrm{VW} \to \mathrm{VW}$ with $M_\mathrm{H} = 500\,\mathrm{GeV}$





Accomando et al. '05

Phase: all EW $2 \to 6$ diagrams, no EVA, no ET, but no QCD diagrams

Pythia: only EVA with longitudinal vector bosons

Madevent: no EVA, but on-shell approximation for produced VW pair

New results for $\mathrm{VV} \to \mathrm{VV}$ with NLO QCD corrections

Jäger, Oleari, Zeppenfeld '06

Specific processes: $\mathrm{pp} \to \mathrm{e}\nu_\mathrm{e}\mu\nu_\mu{+}2q,\ \mathrm{ee}\mu\mu{+}2q,\ \mathrm{ee}\nu_\mathrm{e}\nu_\mathrm{e}{+}2q$ in $\mathcal{O}(\alpha^6\alpha_\mathrm{s})$

Approximations: (inspired by "vector-boson fusion cuts")

gauge-invariant subset of t-channel diagrams,

i.e. no s-channel diagrams and neglect of some interferences

$\hookrightarrow$ no colour exchange between incoming partons

Scale dependence of the integrated cross section: $\mathrm{pp} \to \mathrm{e}^+\nu_\mathrm{e}\mu^-\bar{\nu}_\mu + 2q + X$

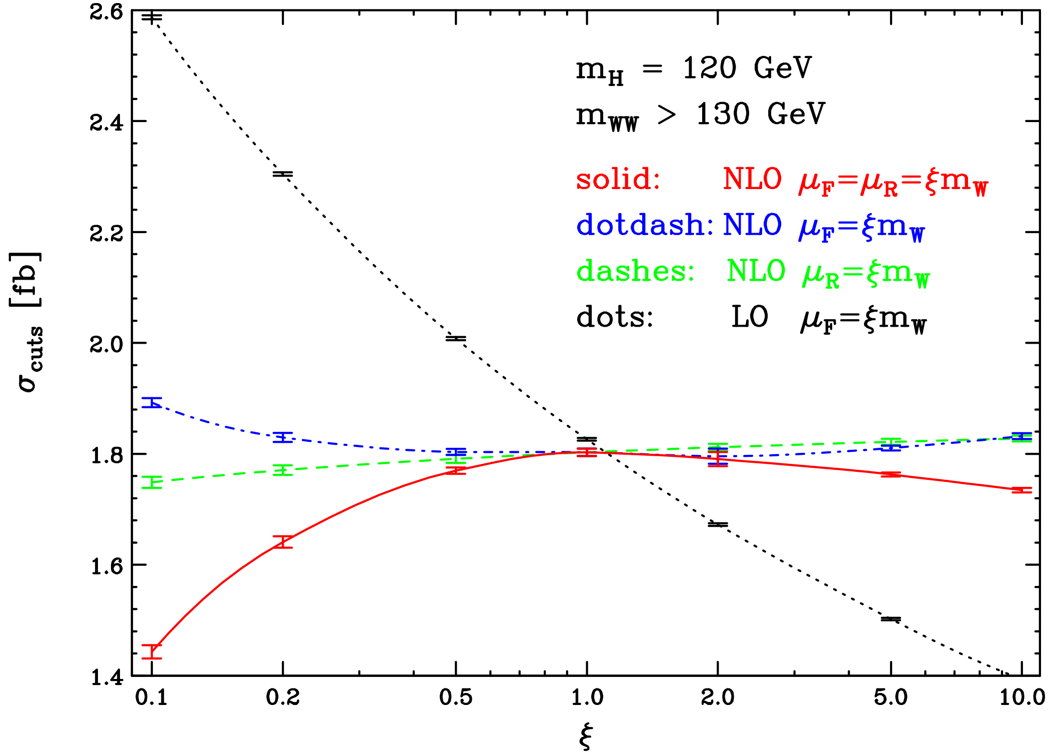
$K \approx 0.98$ at $\mu_\mathrm{fact} = \mu_\mathrm{ren} = M_\mathrm{W}$

scale dependence $\lesssim 10\%$ in NLO

Note:

larger corrections to distributions,

distortion of jet shapes



5 Conclusions

Goals scored in recent years:

- NNLO splitting and DIS coefficient functions
- NNLO (and beyond) calculations for static quantities, vertices, $2 \to 2$ amplitudes ($\Delta\rho$, μ decay, $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$, $gg \to \mathrm{H}$, Drell–Yan, Bhabha, etc.)
- first $2 \to 4$ processes at NLO (ee$\to 4f$, ee$\to \nu\nu\mathrm{HH}$, $6g$ amplitudes)
- progress in many-particle production (matrix elements, showers, etc.)
- great technical and conceptual progress in perturbative QFT
- etc.

Goals for the (near?) future:

- full NNLO calculations to $2 \to 2$ processes
- further important NLO predictions for $2 \to 3, 4, \ldots$ processes
- proper matching of NLO predictions with parton showers
- phenomenologically useful results from twistor-inspired methods
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"It feels good, I am excited about what is ahead of us." (Paul Gascoigne)