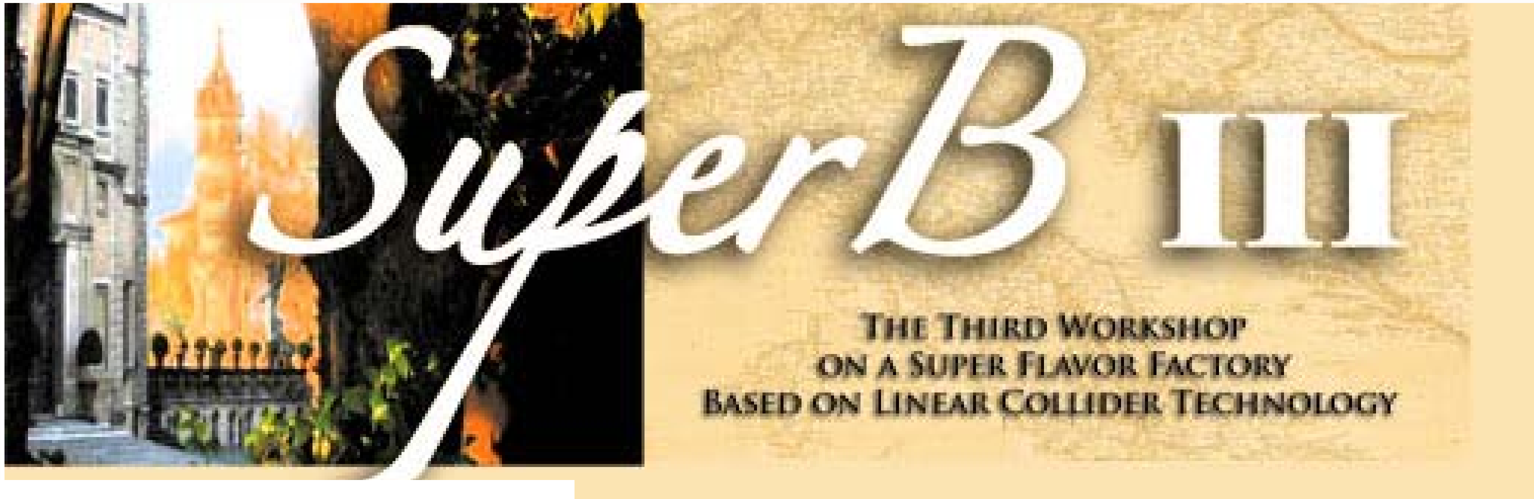
# Novel Tests of QCD at Super B




Stanford Linear Accelerator Center
14 - 16 June, 2006

Stan Brodsky
SLAC

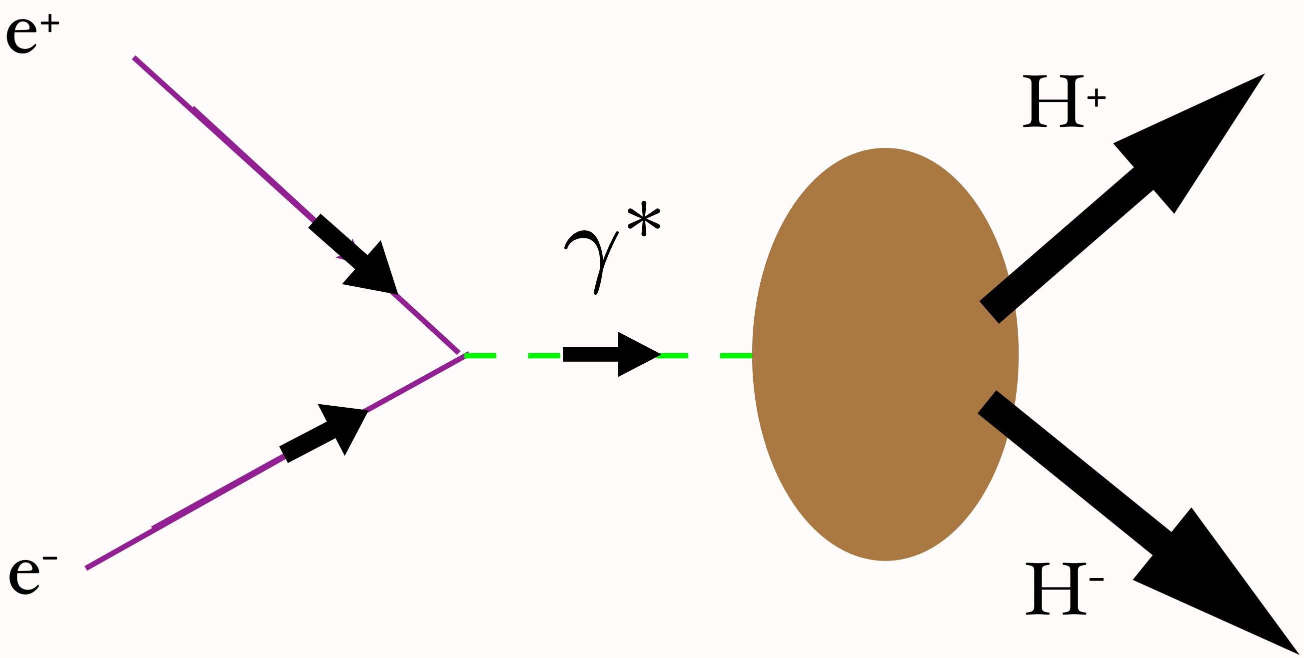
# Super B: Precision QCD Machine

- Hadronization
- Exotic Spectroscopy
- Subtle Spin Effects: Single spin asymmetries
- Measure Fundamental QCD Coupling
- Exclusive Channels: QCD at Amplitude Level
- Compton Processes
- Hidden Color

# *Hadron Dynamics at the Amplitude Level*

- DIS studies have primarily focussed on probability distributions: integrated and unintegrated.
- Test QCD at the amplitude level: Phases, multi-parton correlations, spin, angular momentum, exclusive amplitudes
- Impact of ISI and FSI: Single Spin Asymmetries, Diffractive Deep Inelastic Scattering, Shadowing, Antishadowing
- Hadron wavefunctions: Fundamental QCD Dynamics,
- Hadron wavefunctions: crucial for exclusive B decays
- Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

# Complete measurement of hadron time-like form factors angular distributions 3 independent form factors for spin-one pairs




Leading power in QCD

$$F_H(s) \propto [\frac{1}{s}]^{n_H - 1}$$

**Test QCD Counting Rules**

**Conformal Symmetry: AdS/CFT**

**Hadron Helicity Conservation**

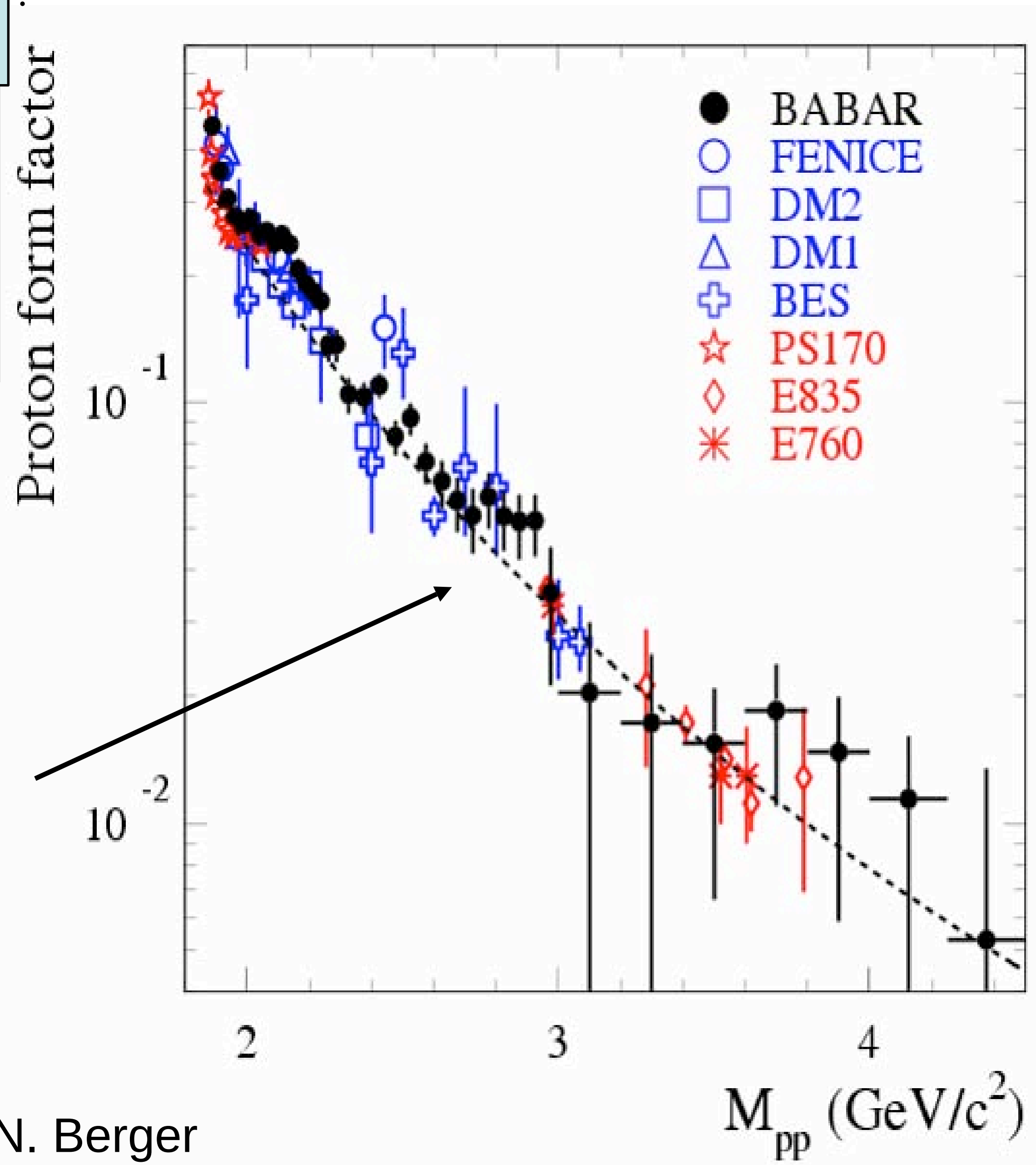
$$\sum_{\text{initial}} \lambda_H - \sum_{\text{total}} \lambda_H = 0\,,$$

- Define "Effective" form factor by

$$\sigma = \frac{4\pi\alpha^2\beta C}{3m_{p\bar{p}}^2}|F|^2, \quad |F| = \sqrt{|G_M|^2 + \frac{2m_p^2}{m_{p\bar{p}}^2}|G_E|^2}.$$

- Peak at threshold, sharp dips at 2.25 GeV, 3.0 GeV.
- Good fit to pQCD prediction for high $m_{pp}$.

New ISR measurement from BaBar




Uses radiative return

$$F(s) \propto \frac{\log^{-2}\frac{s}{\Lambda^2}}{s^2}$$

# Breakdown of Rosenbluth Formula for $G_E$, $G_M$ separation

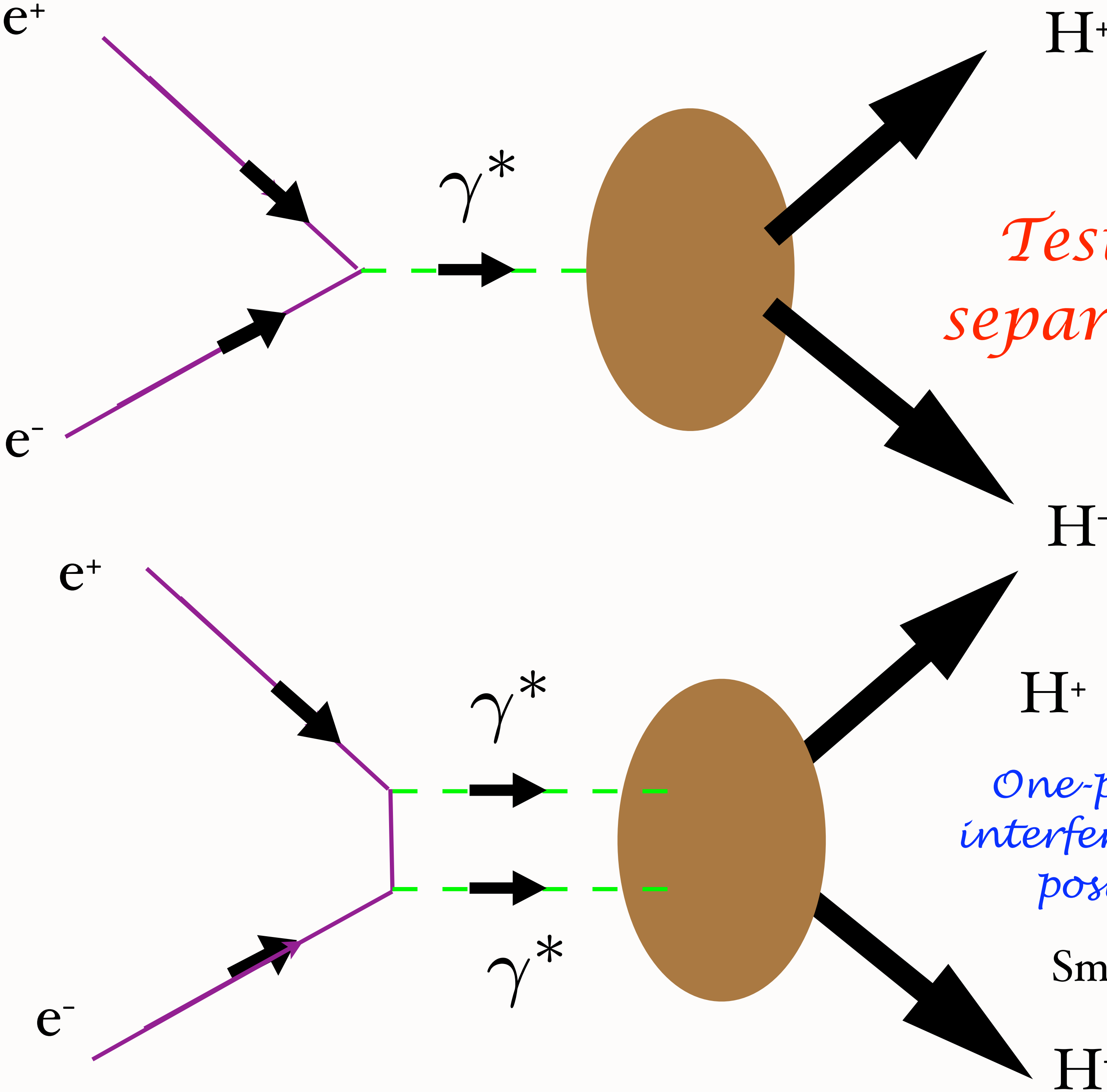
- Two-photon exchange correction, elastic and inelastic nucleon channels, give significant; interference with one-photon exchange, destroys Rosenbluth method

Blunden, Melnitchouk; Afanasev, Chen,Carlson, Vanderhaegen, sjb

- Use J-Lab polarization transfer method

- Timelike form factors from radiative return; angular separation

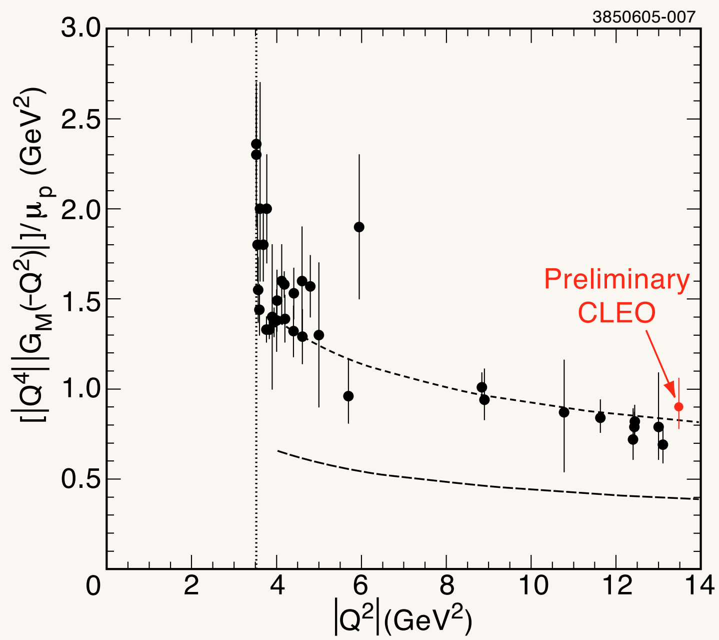
- $e^+ e^-$ charge asymmetry from interference of 1 and 2 photon amplitudes



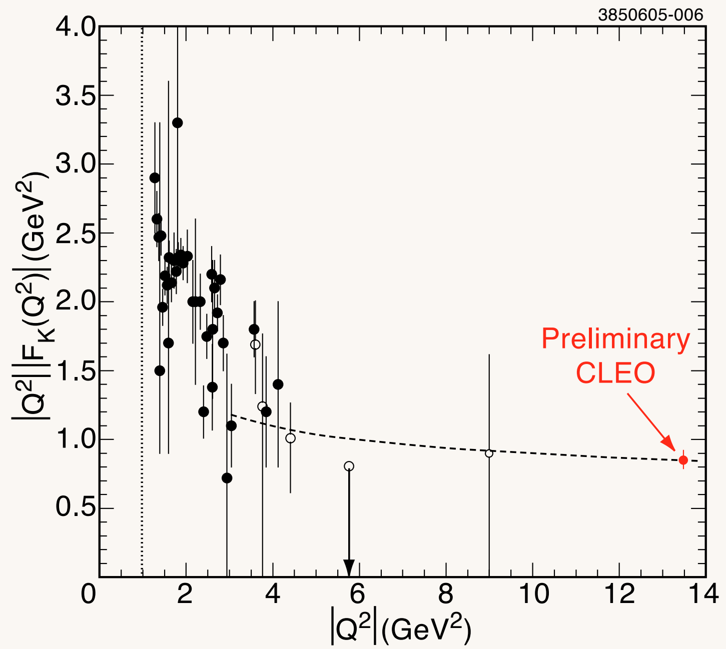

*Test Rosenbluth separation problem*

*One-photon/two-photon interference gives electron-positron asymmetry*

Small Effect from Z°




Proton timelike form factor.




Kaon timelike form factor.

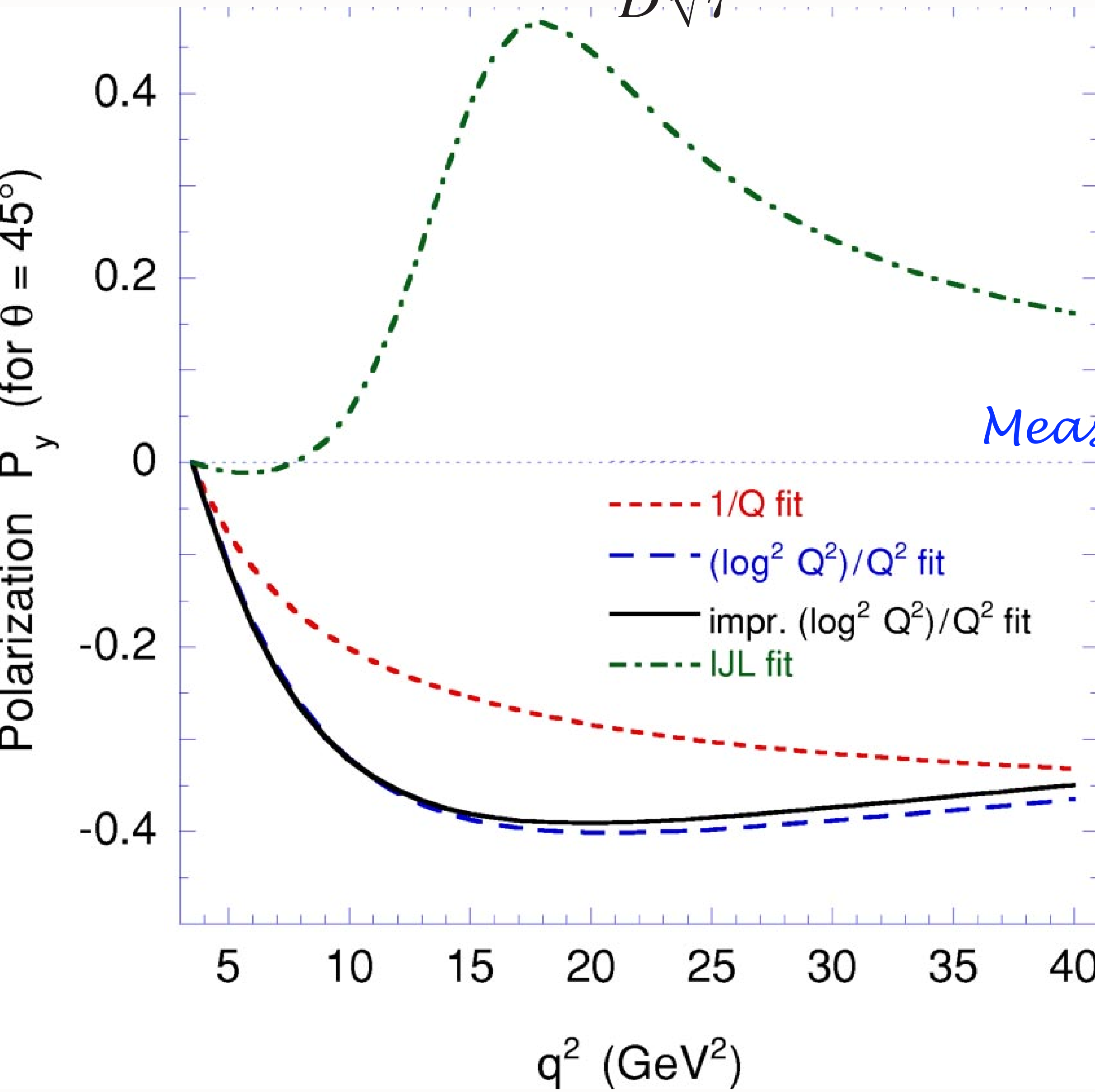
## New results from CLEO

$$
\begin{aligned}
Q^2|F_K(13.48\ \mathrm{GeV}^2)| &= 0.85 \pm 0.05(\mathrm{stat}) \pm 0.02(\mathrm{syst})\ \mathrm{GeV}^2 \\
Q^4|G_M^p(13.48\ \mathrm{GeV}^2)| &= 2.54 \pm 0.36(\mathrm{stat}) \pm 0.16(\mathrm{syst})\ \mathrm{GeV}^4
\end{aligned}
$$

The proton magnetic form factor result agrees with that measured in the reverse reaction $p\bar{p} \to e^+e^-$ at Fermilab. **The kaon form factor measurement is the first ever direct measurement at $|Q^2| > 4$ GeV$^2$.**

$$\mathcal{P}_y = \frac{\sin 2\theta \,\mathrm{Im}\, G_E^* G_M}{D\sqrt{\tau}} = \frac{(\tau - 1)\sin 2\theta \,\mathrm{Im}\, F_2^* F_1}{D\sqrt{\tau}}$$

Carlson, Hiller, Hwang, sjb



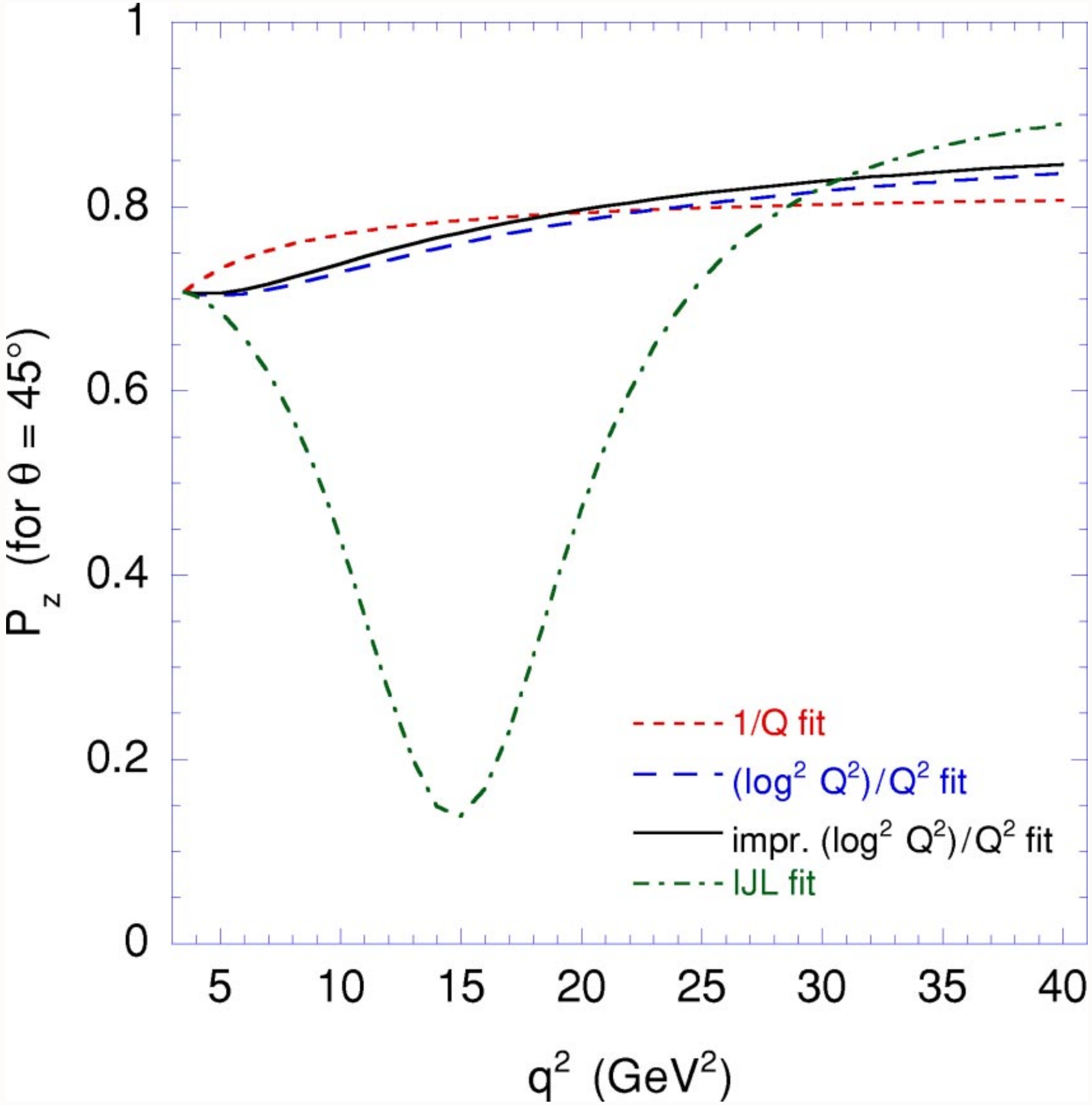

$$D = |G_M|^2(1+\cos^2\theta) + \frac{1}{\tau}|G_E|^2\sin^2\theta;$$

$$\tau \equiv q^2/4m_B^2$$

*Measure interference of form factors*

Test with hyperon pairs

## Single-spin polarization effects and the determination of timelike proton form factors

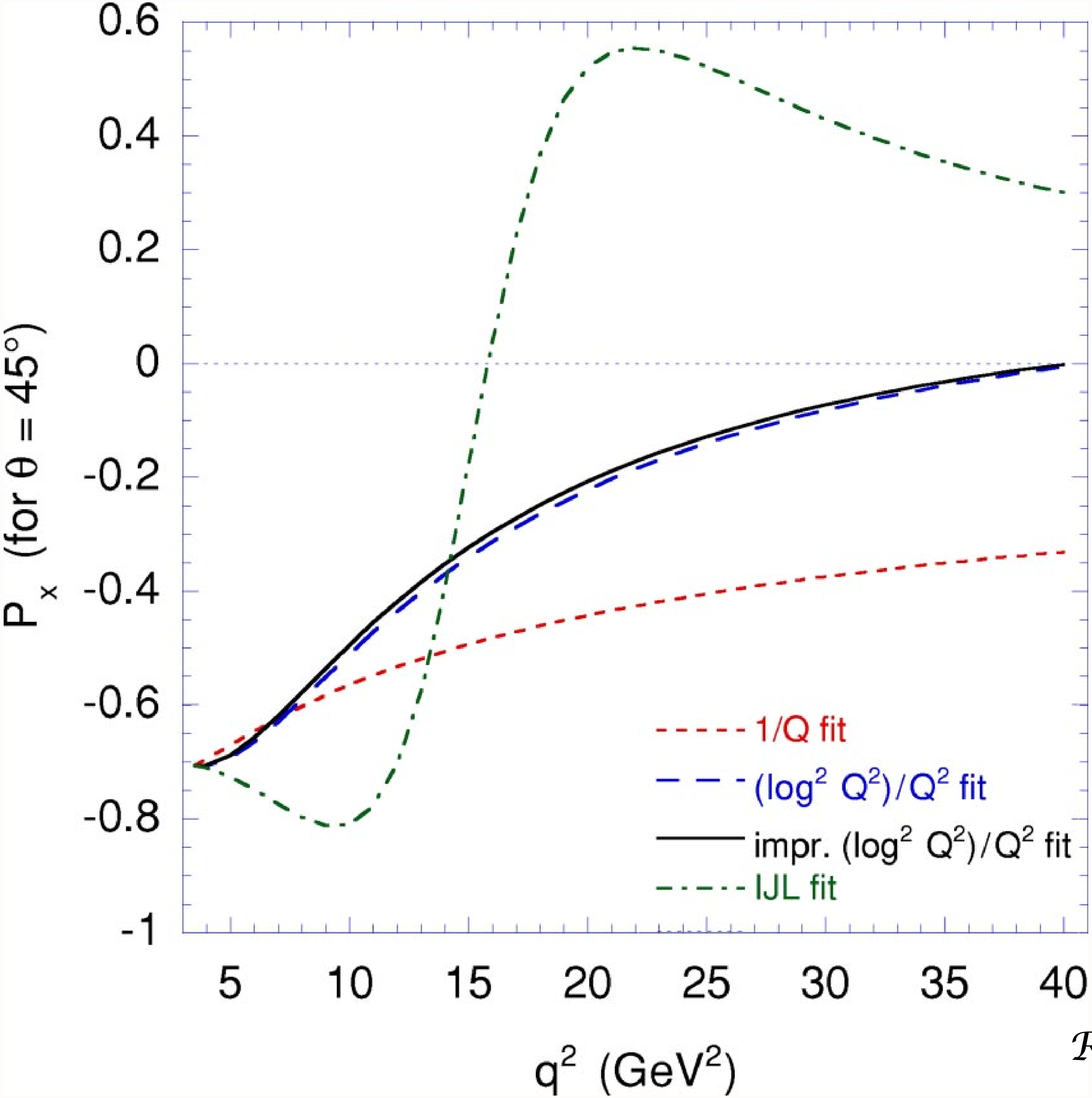



Carlson, Hiller, Hwang, sjb

$$\mathcal{P}_z = P_e \frac{2\cos\theta |G_M|^2}{D}$$

$$D = |G_M|^2(1+\cos^2\theta) + \frac{1}{\tau}|G_E|^2 \sin^2\theta;$$

*Requires beam polarization*

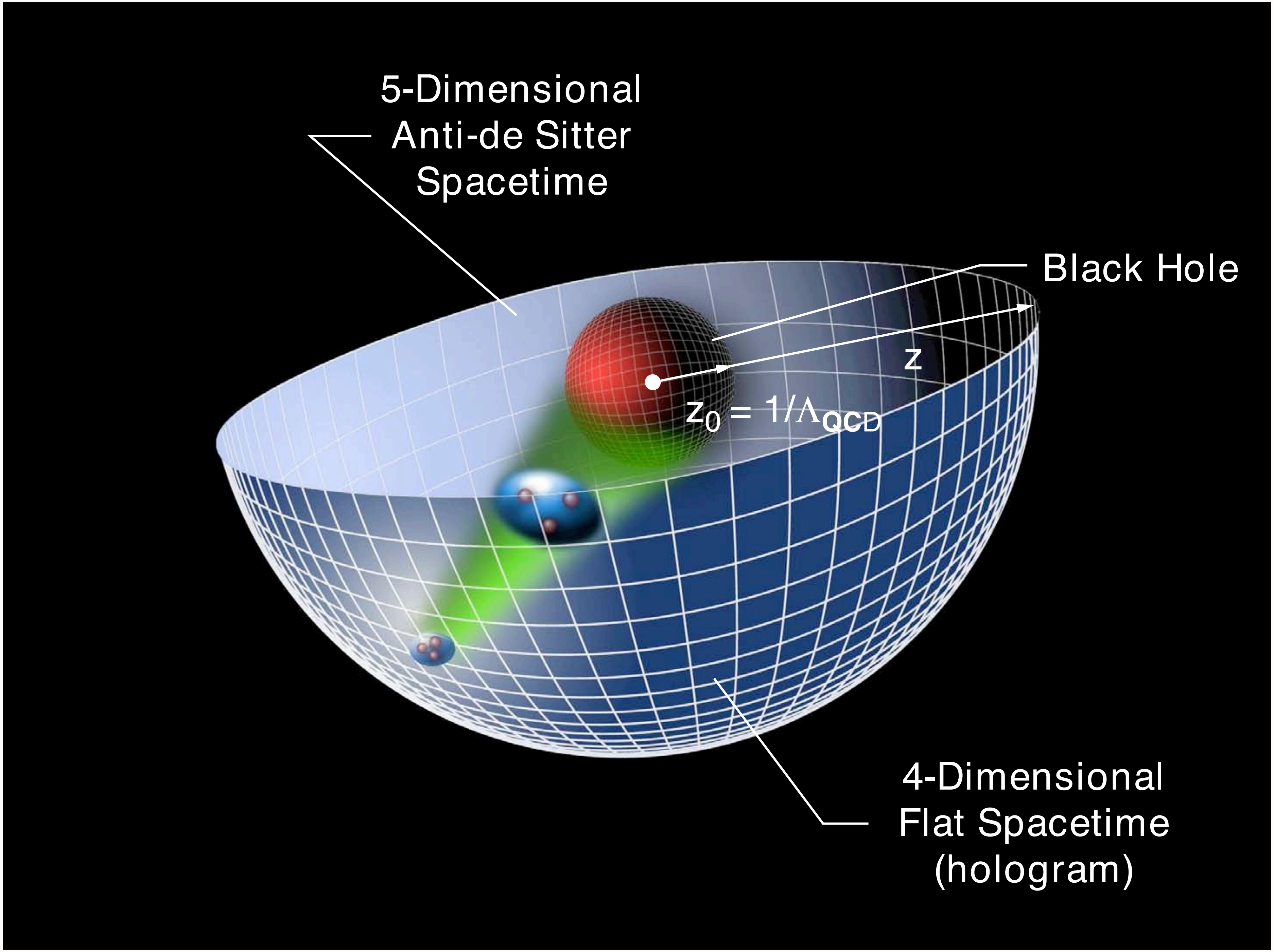



Carlson, Hiller, Hwang, sjb

$$\mathcal{P}_x = -P_e \frac{2 \sin\theta \,\mathrm{Re}\, G_E^* G_M}{D\sqrt{\tau}}$$

$$D = |G_M|^2(1+\cos^2\theta) + \frac{1}{\tau}|G_E|^2 \sin^2\theta;$$

*Requires beam polarization*


5-Dimensional
Anti-de Sitter
Spacetime
Black Hole
z
$z_0 = 1/\Lambda_{QCD}$
4-Dimensional
Flat Spacetime
(hologram)

Predictions of AdS/CFT

Only one parameter!

Entire light quark baryon spectrum

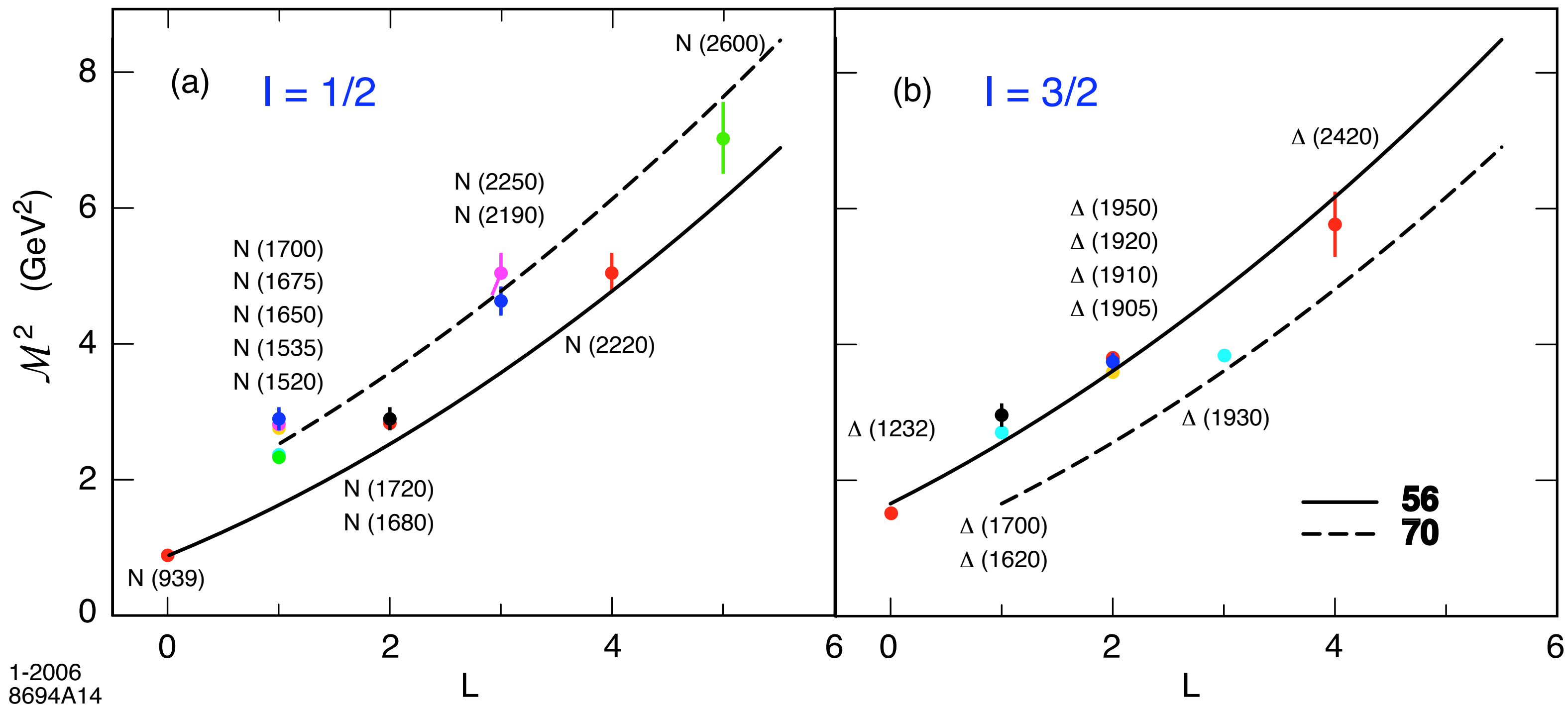



Fig: Predictions for the light baryon orbital spectrum for $\Lambda_{QCD}$ = 0.25 GeV. The $\mathbf{56}$ trajectory corresponds to $L$ even $P = +$ states, and the $\mathbf{70}$ to $L$ odd $P = -$ states.

Guy de Teramond
SJB

- AdS/CFT builds in conformal symmetry at short distances, quark counting rules for form factors and hard exclusive processes

- Non-perturbative derivation Polchinski, Strassler

- Goal: Use AdS/CFT to provide models of hadron structure: confinement at large distances, near conformal behavior at short distances

- Holographic Model: Initial “classical” approximation to QCD: Remarkable agreement with light hadron spectroscopy

- Use AdS/CFT wavefunctions as expansion basis for diagonalizing $H^{LF}{}_{QCD}$ ; variational methods

# *Light-Front Wavefunctions*

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_\perp)$$

$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of $P^\mu$

$$\mathrm{H}_{LF}^{QCD}|\psi> = M^2|\psi>$$

*Bethe-Salpeter Wavefunctions at fixed LF time. Minkowski space*

New insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

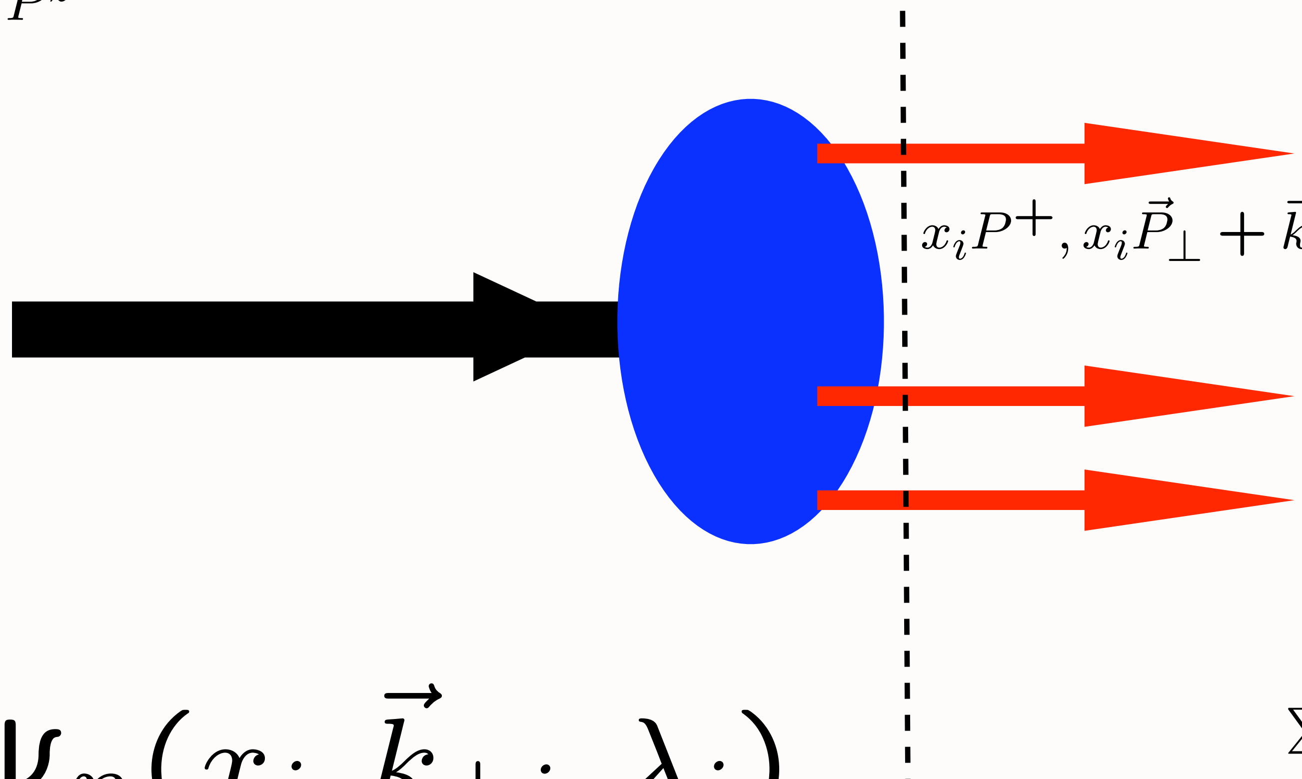
# Light-Front Wavefunctions

Fixed $\tau = t + z/c$

$P^+ = P^0 + P^z$

$x_i = \frac{k_i^+}{P^+}$

$P^+, \vec{P}_\perp$




$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

*Invariant under boosts! Independent of $P^\mu$*

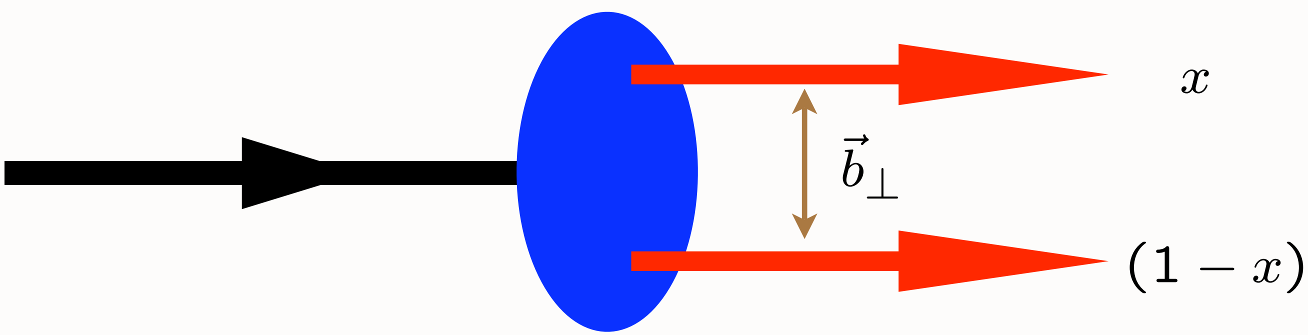
# *Holography: Map AdS/CFT to 3+1 LF Theory*

Relativistic radial equation: *Frame Independent*

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

G. de Teramond, sjb

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$

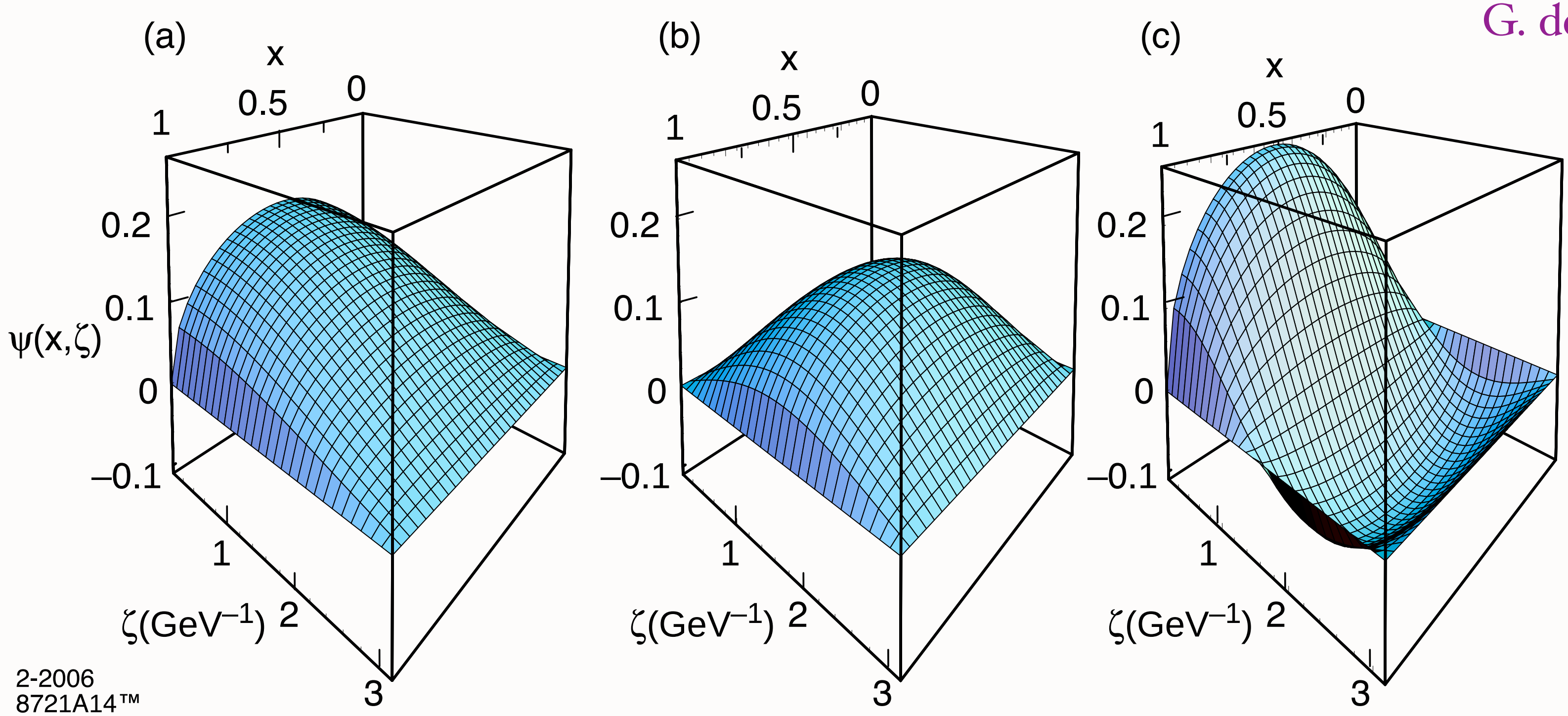



Effective conformal potential:

$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

# AdS/CFT Prediction for Meson LFWF

G. de Teramond
SJB



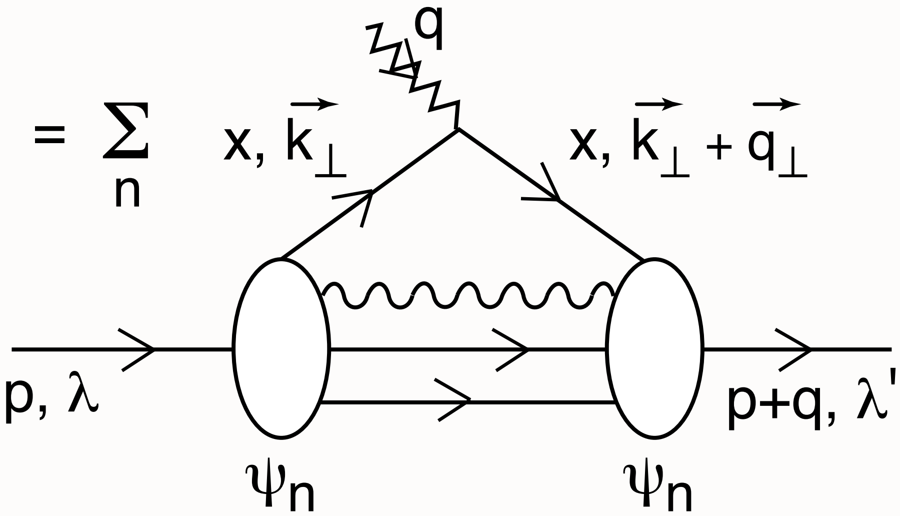

Two-parton holographic LFWF in impact space $\widetilde{\psi}(x,\zeta)$ for $\Lambda_{QCD} = 0.32$ GeV: (a) ground state $L = 0,\ k = 1$; (b) first orbital exited state $L = 1,\ k = 1$; (c) first radial exited state $L = 0,\ k = 2$. The variable $\zeta$ is the holographic variable $z = \zeta = |b_\perp|\sqrt{x(1-x)}$.

$$\boxed{\widetilde{\psi}(x,\zeta) = \frac{\Lambda_{\rm QCD}}{\sqrt{\pi}J_1(\beta_{0,1})}\sqrt{x(1-x)}J_0\left(\zeta\beta_{0,1}\Lambda_{QCD}\right)\theta\left(z \le \Lambda^{-1}_{\rm QCD}\right)}$$

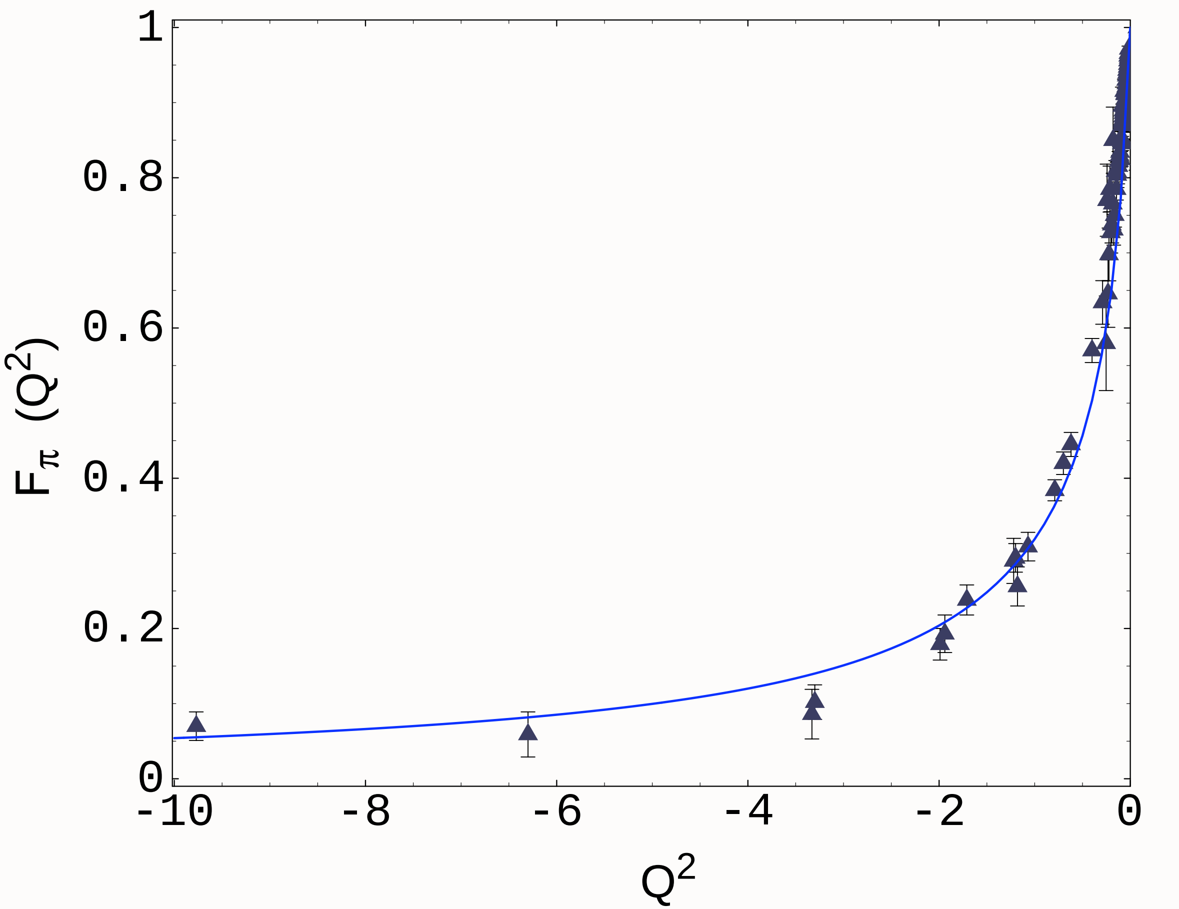
# Light-Front Calculation of Form Factors

Form Factors $\ell p \to \ell' p'$ $\langle p' \lambda' | J^+(0) | p \lambda \rangle$

$F_{\lambda\lambda'}(Q^2) = \sum_n$



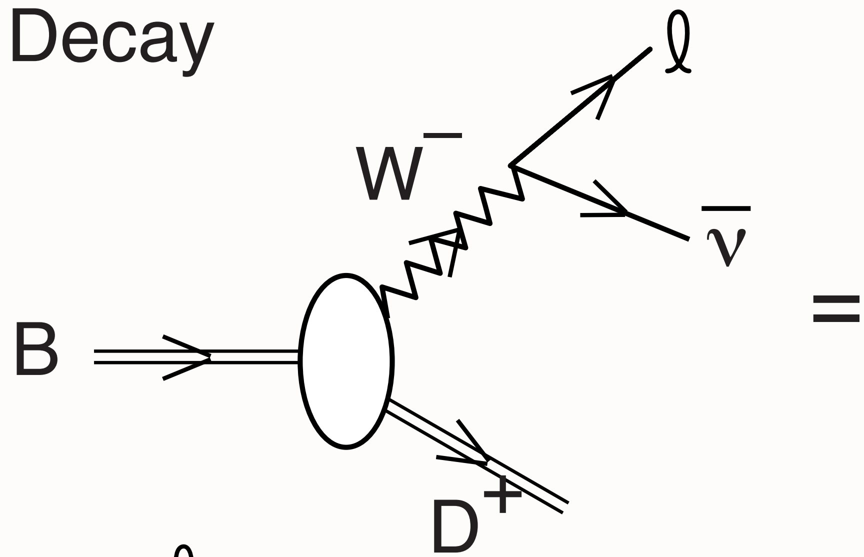

Convolute $\psi(x, \vec{k}_\perp)$ $\psi(x, \vec{k}_\perp + (1-x)\vec{q}_\perp)$




Space-like pion form factor in holographic model for $\Lambda_{QCD} = 0.2$ GeV.

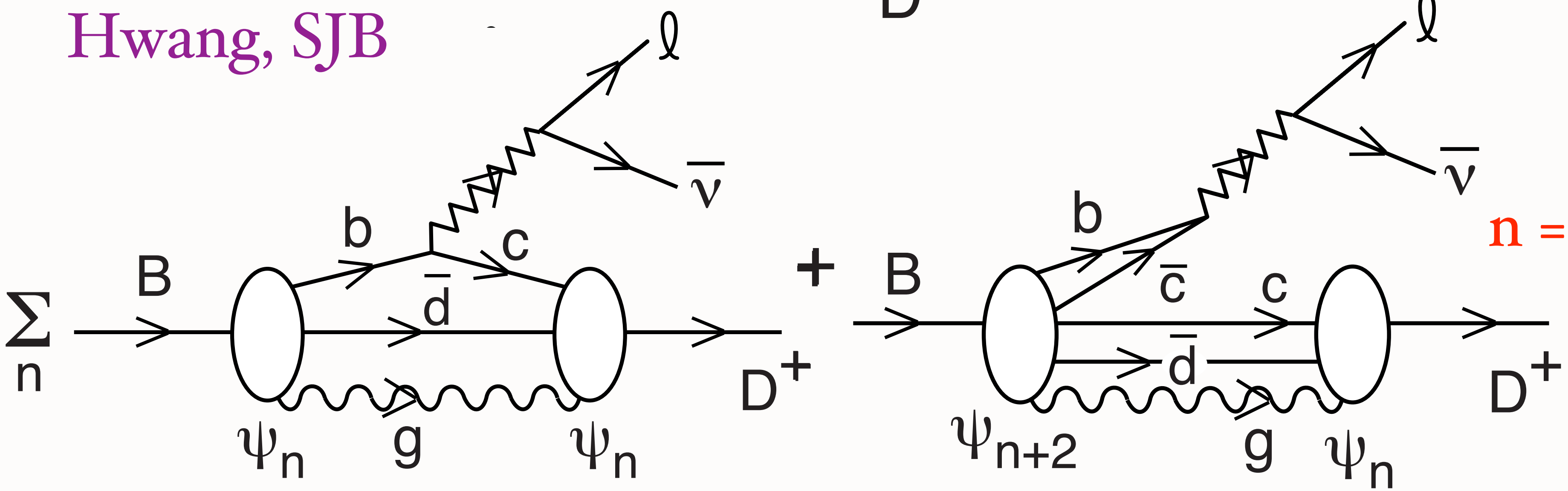
# Weak Exclusive Decay

$\langle D \,|\, J^{+}(0) \,|\, B \rangle$




Exact Formula!

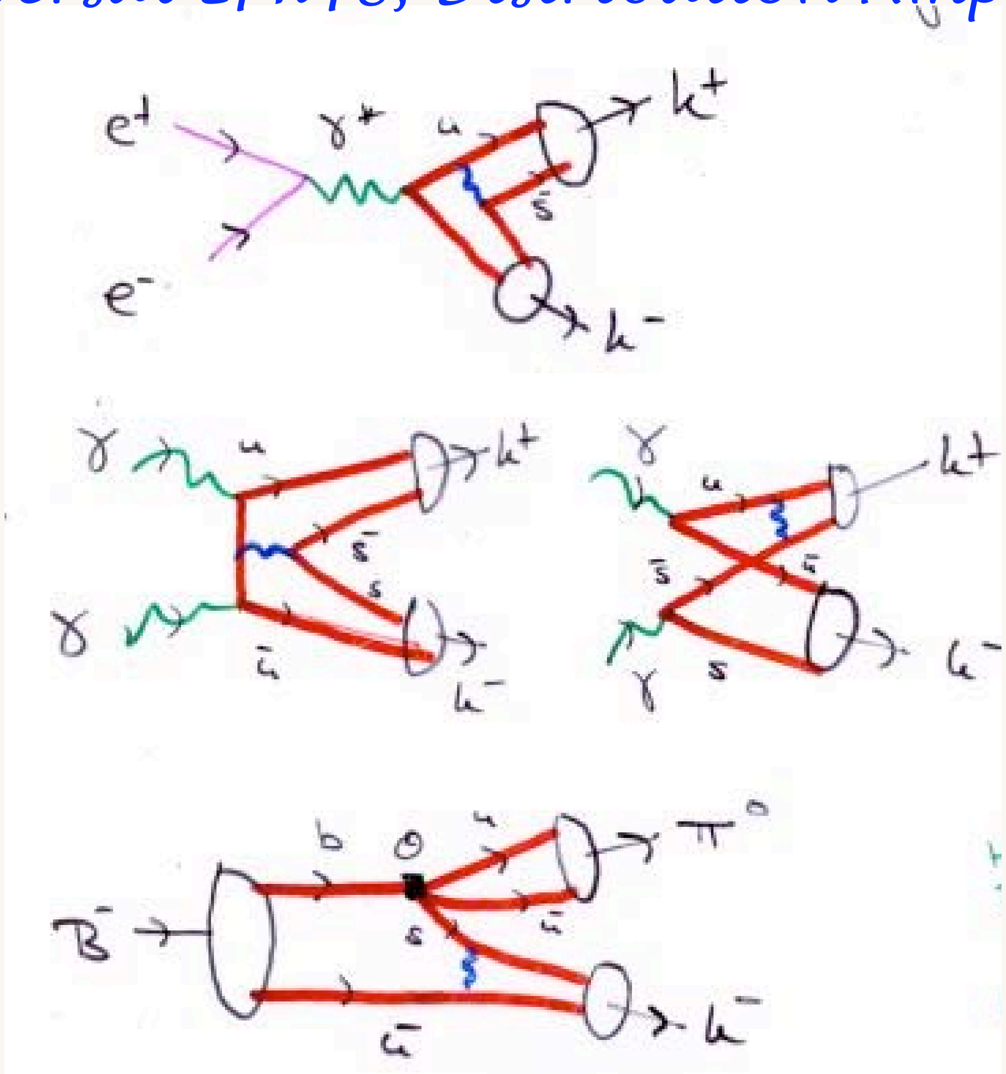
Hwang, SJB




n = n' + 2

Annihilation amplitude needed for Lorentz Invariance

# Common Ingredients:
# Universal LFWFS, Distribution Amplitudes

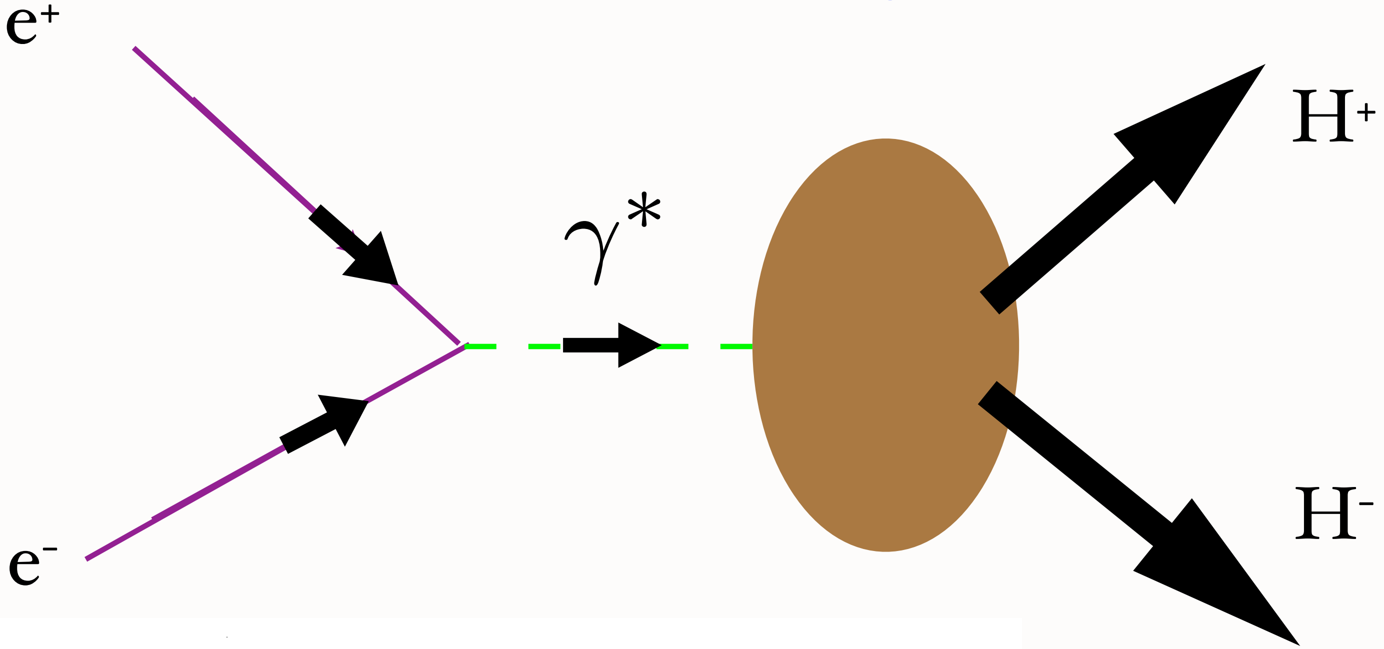


**Light-front wavefunctions underly exclusive B decays**

Exclusive Photon-Induced Reactions

Lepage, sjb

# Perturbative QCD: Hadron Helicity Conservation at leading twist




$$\frac{d\sigma}{d\cos\theta}(e^+e^- \to B\bar{B}) \propto 1+\cos^2\theta \quad \text{(baryons)}\,, \qquad |\lambda_A| = |\lambda_B| = \tfrac{1}{2}$$

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \to M\bar{M}) \propto \sin^2\theta \quad \text{(mesons)}\,. \qquad |\lambda_A| = |\lambda_B| = 0$$

**Exclusive Production Of Higher Generation Hadrons And Form-Factor Zeros In Quantum Chromodynamics.**
Stanley J. Brodsky, **Chueng-Ryong Ji** (**SLAC**) **. SLAC-PUB-3756, Aug 1985. 16pp.**
**Published in Phys.Rev.Lett.55:2257,1985**

# *Cross section for spin-one pairs*

$$4\pi \frac{d\sigma}{d\Omega}(e^+e^- \to M_\lambda \overline{M}_{\bar{\lambda}})$$

$$= \frac{3}{4}\beta\sigma_{e^+e^-\to\mu^+\mu^-}\Bigg\{\frac{1}{2}\beta^2 \sin^2\theta\Bigg[|F_{0,0}(q^2)|^2 + \frac{1}{(1-\beta^2)^2}\Big((3-2\beta^2+3\beta^4)|F_{1,1}(q^2)|^2$$

$$-4(1+\beta^2)\mathrm{Re}[F_{1,1}(q^2)F^*_{0,1}(q^2)] + 4|F_{0,1}(q^2)|^2\Big)\Bigg]$$

$$+ \frac{3\beta^2}{2(1-\beta^2)}(1+\cos^2\theta)|F_{0,1}(q^2)|^2\Bigg\}$$

Dominates in PQCD

$$q^2 = s = 4M_H^2\overline{q}^2$$

$$\beta = (1-4M_H^2/q^2)$$

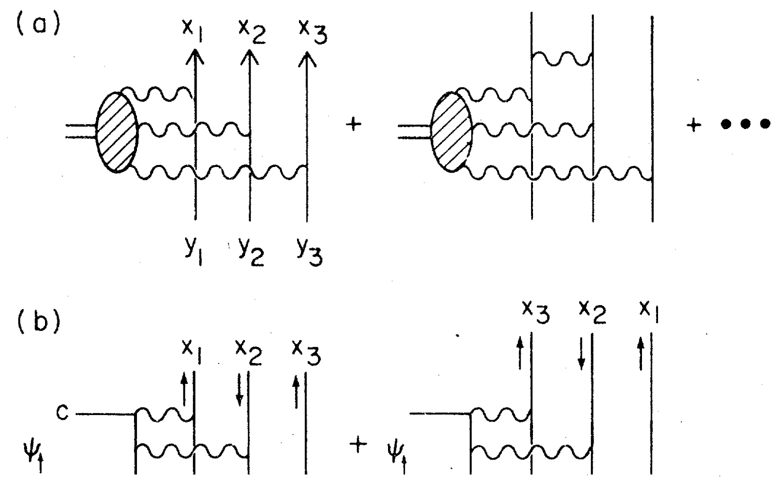
| | $e^+e^- \to h_A(\lambda_A)\bar{h}_B(\lambda_B)$ | Angular distribution | $\dfrac{\sigma(e^+e^- \to h_A\bar{h}_B)}{\sigma(e^+e^- \to \mu^+\mu^-)}$ |
|---|---|---|---|
| Allowed in QCD | $e^+e^- \to \pi^+\pi^-, K^+K^-$ | $\sin^2\theta$ | $\frac{1}{4}\lvert F(s)\rvert^2 \sim c/s^2$ |
| | $\rho^+(0)\rho^-(0), K^{*+}K^{*-}$ | $\sin^2\theta$ | $\frac{1}{4}\lvert F(s)\rvert^2 \sim c/s^2$ |
| | $\pi^0\gamma(\pm 1), \eta\gamma, \eta'\gamma$ | $1+\cos^2\theta$ | $(\pi\alpha/2)s\lvert F_{M\gamma}(s)\rvert^2 \sim c/s$ |
| | $e^+e^- \to p(\pm\frac{1}{2})\bar{p}(\mp\frac{1}{2}), n\bar{n}, \ldots$ | $1+\cos^2\theta$ | $\lvert G(s)\rvert^2 \sim c/s^4$ |
| | $p(\pm\frac{1}{2})\bar{\Delta}(\mp\frac{1}{2}), \bar{n}\Delta, \ldots$ | $1+\cos^2\theta$ | $\lvert G(s)\rvert^2 \sim c/s^4$ |
| | $\Delta(\pm\frac{1}{2})\bar{\Delta}(\mp\frac{1}{2}), y^*\bar{y}^*, \ldots$ | $1+\cos^2\theta$ | $\lvert G(s)\rvert^2 \sim c/s^4$ |
| Suppressed in QCD | $e^+e^- \to \rho^+(0)\rho^-(\pm 1), \pi^+\rho^-, K^+K^{*-}, \ldots$ | $1+\cos^2\theta$ | $< c/s^3$ |
| | $\rho^+(\pm 1)\rho^-(\pm 1), \ldots$ | $\sin^2\theta$ | $< c/s^3$ |
| | $e^+e^- \to p(\pm\frac{1}{2})\bar{p}(\pm\frac{1}{2}), p\bar{\Delta}, \Delta\bar{\Delta}, \ldots$ | $\sin^2\theta$ | $< c/s^5$ |
| | $p(\pm\frac{1}{2})\bar{\Delta}(\pm\frac{3}{2}), \Delta\bar{\Delta}, \ldots$ | $1+\cos^2\theta$ | $< c/s^5$ |
| | $\Delta(\pm\frac{3}{2})\bar{\Delta}(\pm\frac{3}{2}), \ldots$ | $\sin^2\theta$ | $< c/s^5$ |

**Helicity** Selection **Rules And Tests Of Gluon Spin In Exclusive Qcd Processes.**
Stanley J. Brodsky (SLAC) , G.Peter Lepage (Cornell U., LNS) . **SLAC-PUB-2746, May 1981. 29pp.**
**Published in Phys.Rev.D24:2848,1981**

# Exclusive Hadronic Decays of Quarkonia in PQCD

Lepage, sjb

$$T(s,\theta=0)=\int_0^1 [dx][dy]\,\phi^*(y_i,s)T_H(x_i,y_i,s)\times\phi(x_i,s)\,.$$




$$\frac{\Gamma(\psi\to p\bar{p})}{\Gamma(\psi\to \text{hadrons})}=(3.2\times10^6)\alpha_s{}^3(s)\frac{|\vec{p}_{\text{c.m.}}|}{\sqrt{s}}\frac{\langle T\rangle^2}{s^4}\,,$$

where $|\vec{p}_{\text{c.m.}}|/\sqrt{s}\simeq 0.4$, $s=9.6\ \text{GeV}^2$, and

$$\langle T\rangle\equiv\int_0^1[dx][dy]\frac{\phi^*(y_i,s)}{y_1y_2y_3}\frac{x_1y_3+x_3y_1}{[x_1(1-y_1)+y_1(1-x_1)][x_3(1-y_3)+y_3(1-x_3)]}\frac{\phi(x_i,s)}{x_1x_2x_3}\,.$$

## Super B: Measure Exclusive Upsilon decays

# QCD Puzzle

**Infamous $J/\psi \to \rho\pi$ decay:** BR = 1.27 +/- 0.09 %

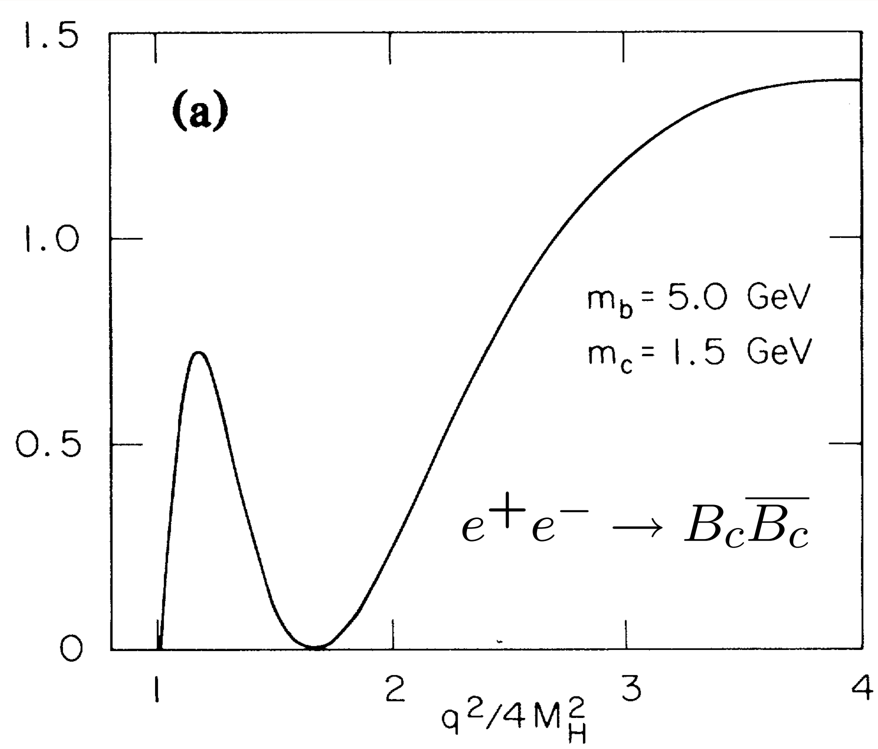
**Violates hadron helicity conservation**

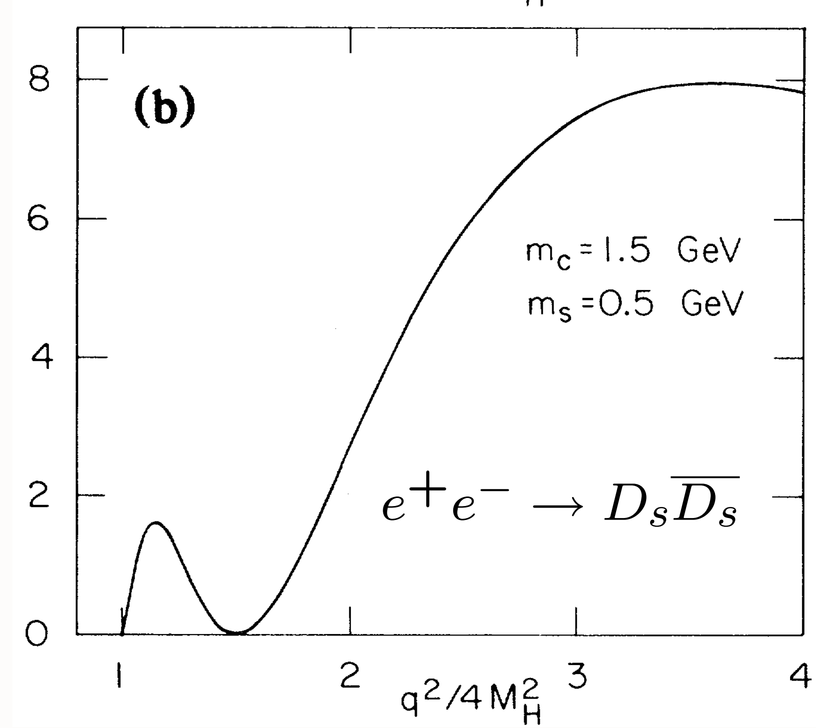
$$\sum_{\text{initial}} \lambda_H - \sum_{\text{total}} \lambda_H = 0\,,$$

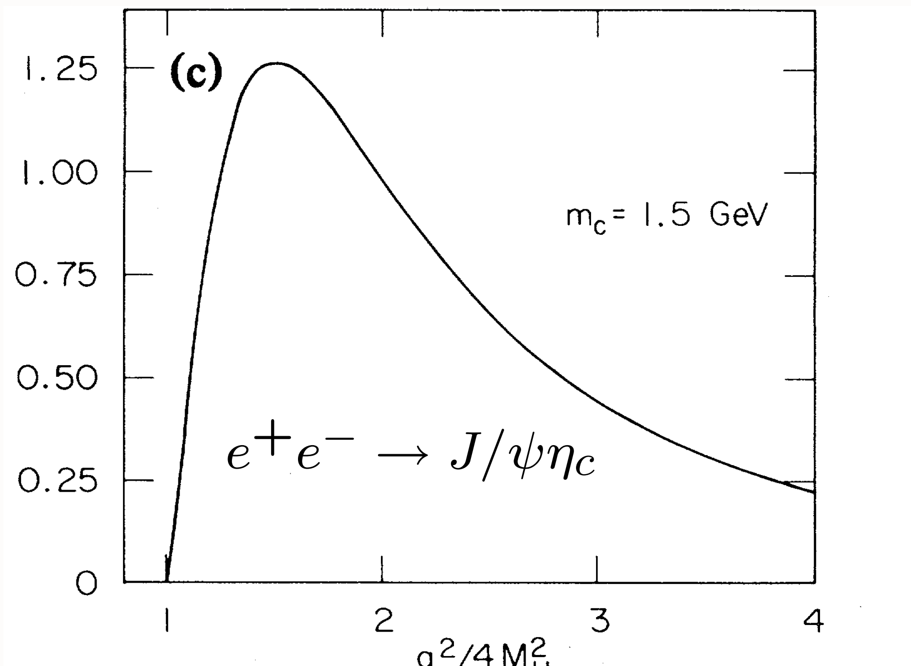
$\psi' \to \rho\pi$ **and** $\psi'' \to \rho\pi$ **suppressed**

$< 8.3 \quad \times 10^{-5} \quad$ CL=90%

**Is there an $\Upsilon \to \rho\pi$ puzzle?**




# *Novel Form-Factor Zeroes for Heavy Hadron Pairs*

$S_z = 0$

C. Ji, sjb

**Phys.Rev.Lett.55:2257,1985**

# Scaling of deuteron FFs

**CCR in elastic scattering**

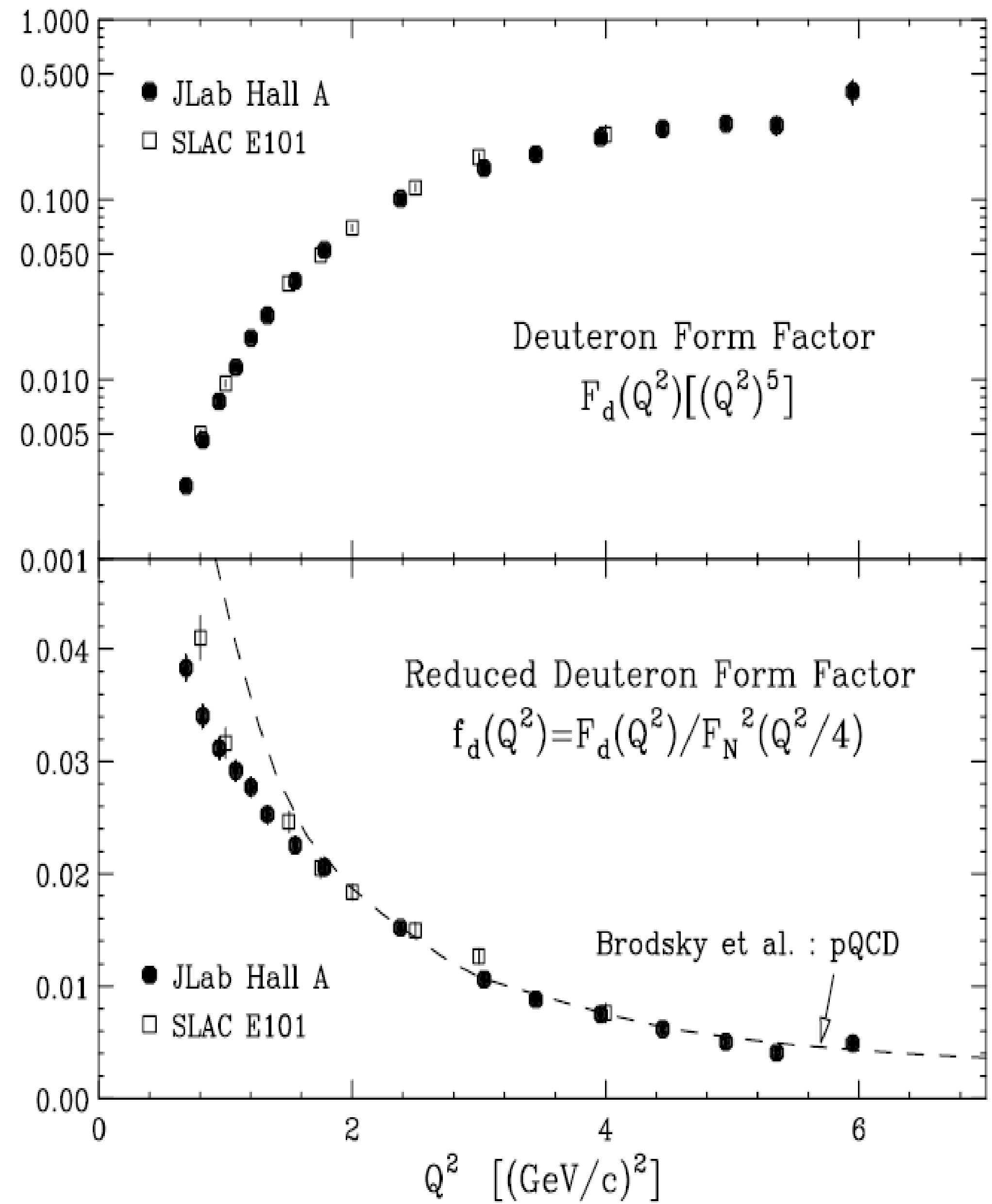
leading term: $FF \propto \left(Q^2\right)^{1-n_d}$

the "deuteron FF": $F_d \equiv \sqrt{A} \propto Q^{-10}$

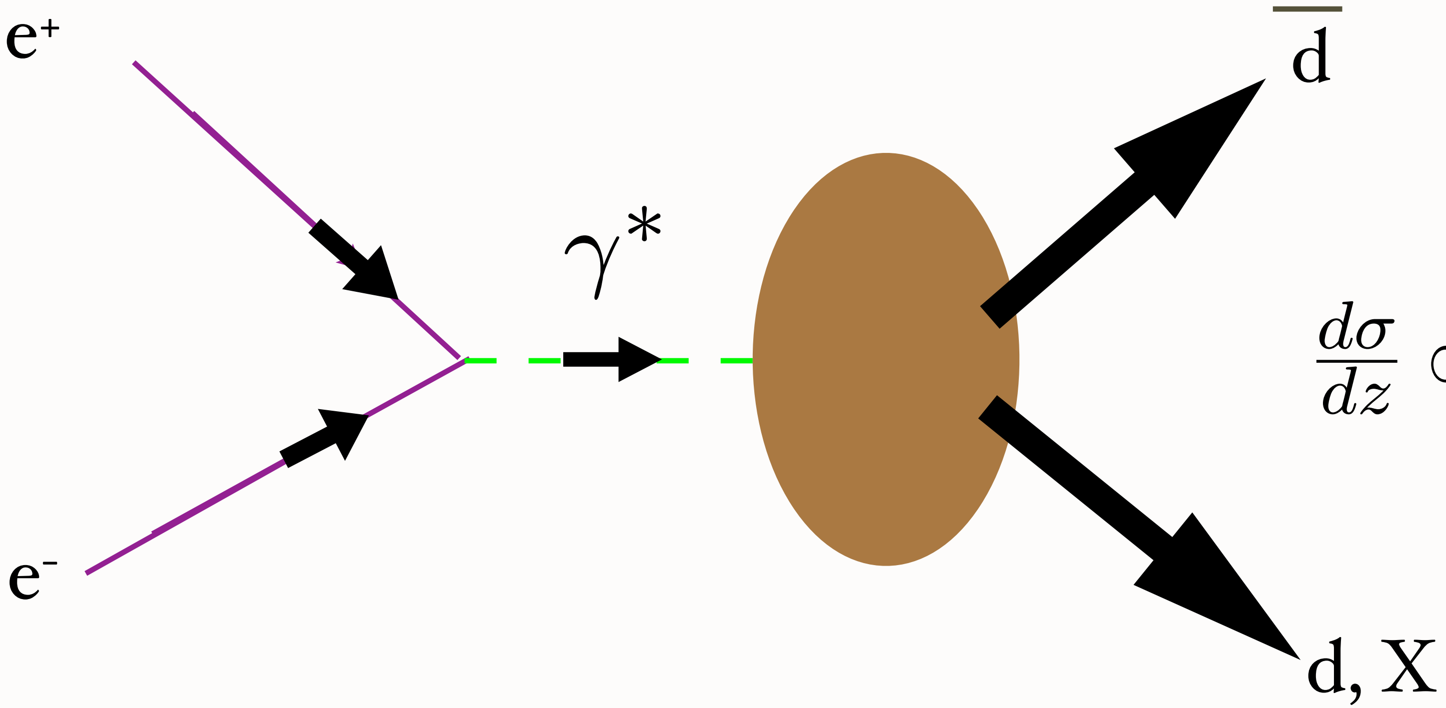
For $Q^2$ above ≈4 GeV$^2$ data are consistent with CCR

Super B: Measure deuteron pairs

Alexa et al., PRL 82,1374 (1999)



# *Challenge:*
# *Exclusive or Inclusive Deuteron Production*




$$\frac{d\sigma}{dz} \propto (1-z)^9$$

*Set scale for pentaquark and other multiparticle state production*

# QCD Prediction for Deuteron Form Factor

$$F_d(Q^2) = \left[\frac{\alpha_s(Q^2)}{Q^2}\right]^5 \sum_{m,n} d_{mn} \left(\ln \frac{Q^2}{\Lambda^2}\right)^{-\gamma_n^d - \gamma_m^d} \left[1 + O\left(\alpha_s(Q^2), \frac{m}{Q}\right)\right]$$

Chertok, Lepage, Ji, sjb

Define “Reduced” Form Factor

$$f_d(Q^2) \equiv \frac{F_d(Q^2)}{F_N{}^2(Q^2/4)}\,.$$

Same large momentum transfer behavior as pion form factor

$$f_d(Q^2) \sim \frac{\alpha_s(Q^2)}{Q^2} \left(\ln \frac{Q^2}{\Lambda^2}\right)^{-(2/5)\, C_F/\beta}$$

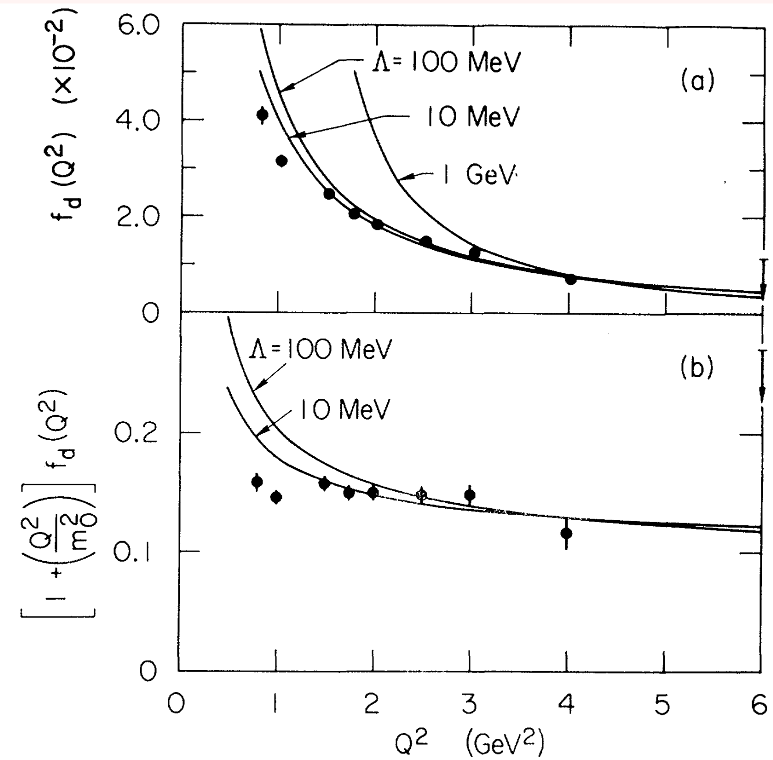
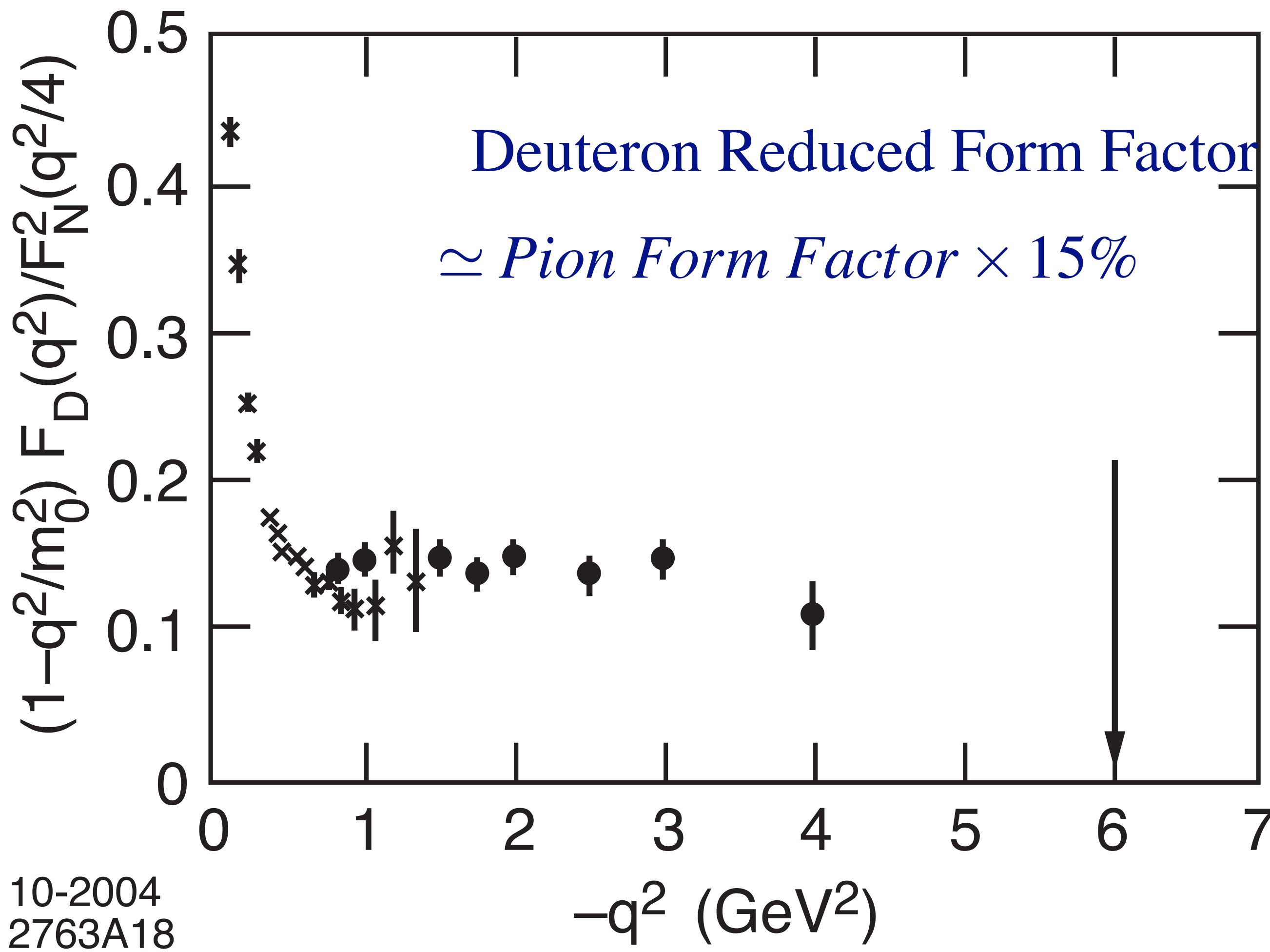


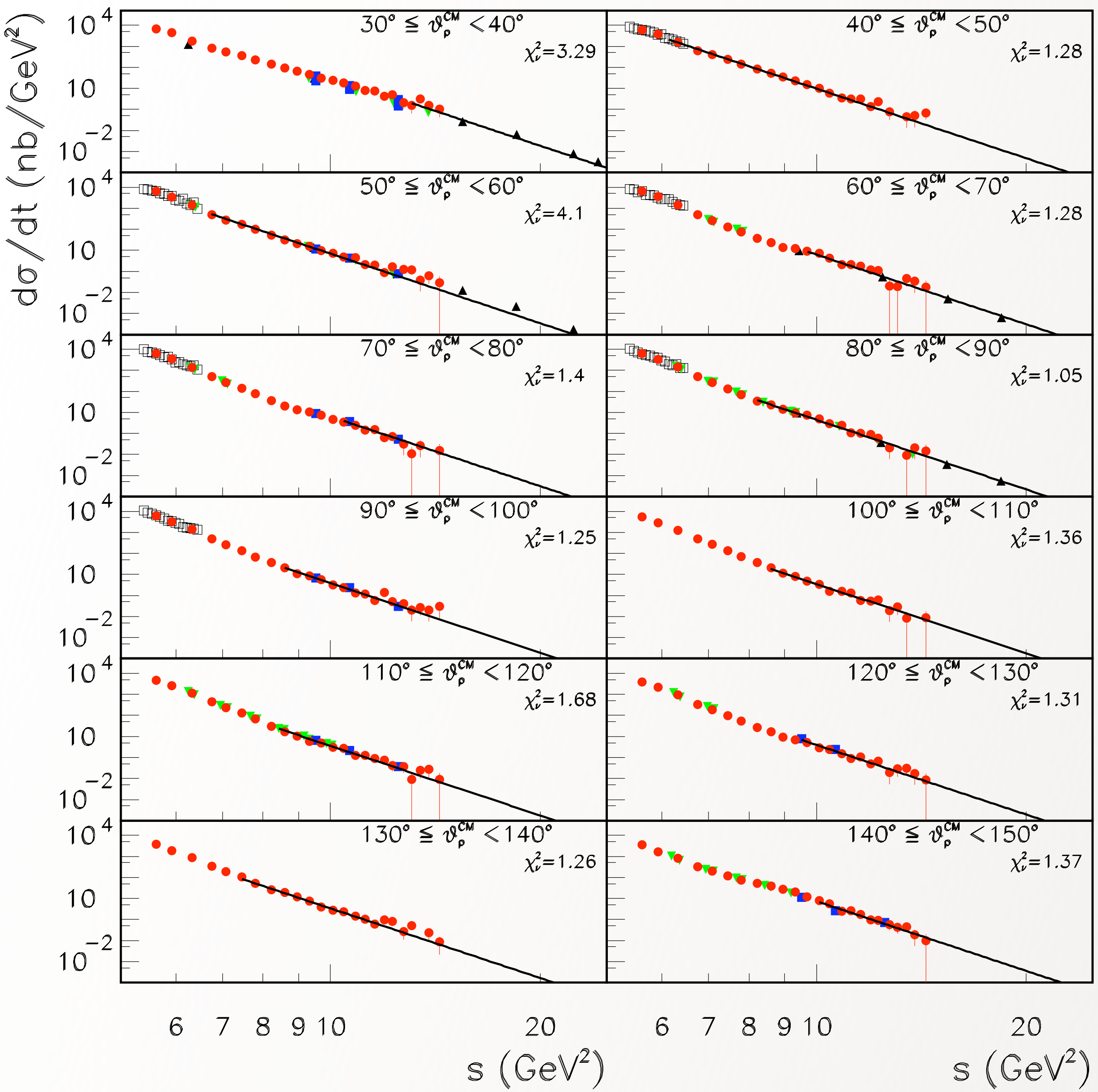

FIG. 2. (a) Comparison of the asymptotic QCD prediction $f_d(Q^2) \propto (1/Q^2)[\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/\beta}$ with final data of Ref. 10 for the reduced deuteron form factor, where $F_N(Q^2) = [1 + Q^2/(0.71\ \mathrm{GeV}^2)]^{-2}$. The normalization is fixed at the $Q^2 = 4\ \mathrm{GeV}^2$ data point. (b) Comparison of the prediction $[1 + (Q^2/m_0{}^2)]\, f_d(Q^2) \propto [\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/\beta}$ with the above data. The value $m_0{}^2 = 0.28\ \mathrm{GeV}^2$ is used (Ref. 8).




Resembles pion form factor times 15%

- Indicates: ~ 15% Hidden Color in the Deuteron

# Deuteron Photodisintegration & Dimensional Counting Rules




PQCD and AdS/CFT:

$$s^{n_{tot}-2}\frac{d\sigma}{dt}(A+B\to C+D)= F_{A+B\to C+D}(\theta_{CM})$$

$$s^{11}\frac{d\sigma}{dt}(\gamma d\to np)=F(\theta_{CM})$$

$n_{tot}-2=$
(1 + 6 + 3+ 3 ) - 2 = 11

Exclusive Photon-Induced Reactions

- Remarkable Test of Quark Counting Rules
- Deuteron Photo-Disintegration $\gamma d \to np$
- $$\frac{d\sigma}{dt} = \frac{F(t/s)}{s^{n_{tot}-2}}$$
- $$n_{tot} = 1 + 6 + 3 + 3 = 13$$

Scaling characteristic of
scale-invariant theory at short distances

Conformal symmetry

Hidden color: $$\frac{d\sigma}{dt}(\gamma d \to \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \to pn)$$

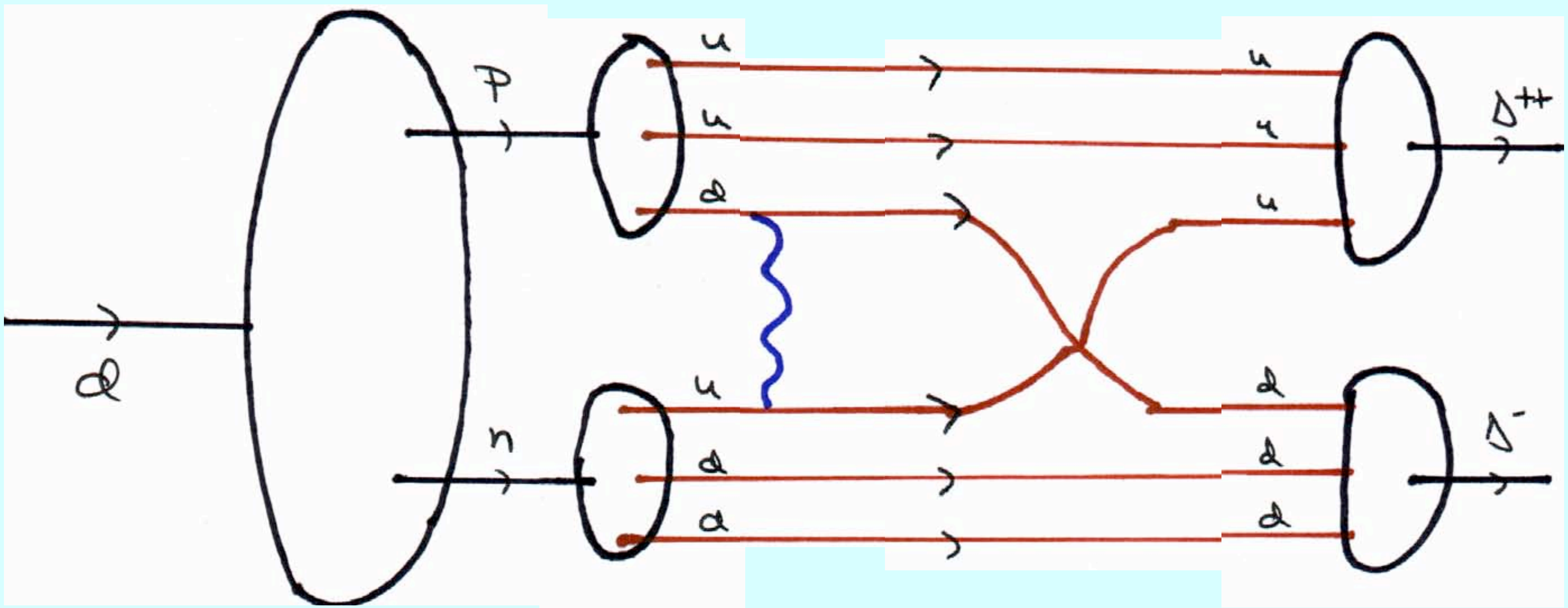
at high $p_T$

Exclusive Photon-Induced Reactions

# Hidden Color in QCD

- Deuteron six quark wavefunction: Lepage, Ji, sjb
- 5 color-singlet combinations of 6 color-triplets -- one state is |n p>
- Components evolve towards equality at short distances
- Hidden color states dominate deuteron form factor and photodisintegration at high momentum transfer
- Predict $\frac{d\sigma}{dt}(\gamma d \to \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \to pn)$ at high $Q^2$

# Structure of Deuteron in QCD




Hidden Color Fock State

Delta-Delta Fock State

The evolution equation for six-quark systems in which the constituents have the light-cone longitudinal momentum fractions $x_i$ ($i = 1, 2, \ldots, 6$) can be obtained from a generalization of the proton (three-quark) case.[2] A nontrivial extension is the calculation of the color factor, $C_d$, of six-quark systems[5] (see below). Since in leading order only pairwise interactions, with transverse momentum $Q$, occur between quarks, the evolution equation for the six-quark system becomes $\{[dy] = \delta(1 - \sum_{i=1}^{6} y_i)\prod_{i=1}^{6} dy_i$, $C_F = (n_c{}^2 - 1)/2n_c = \frac{4}{3}$, $\beta = 11 - \frac{2}{3}n_f$, and $n_f$ is the effective number of flavors$\}$

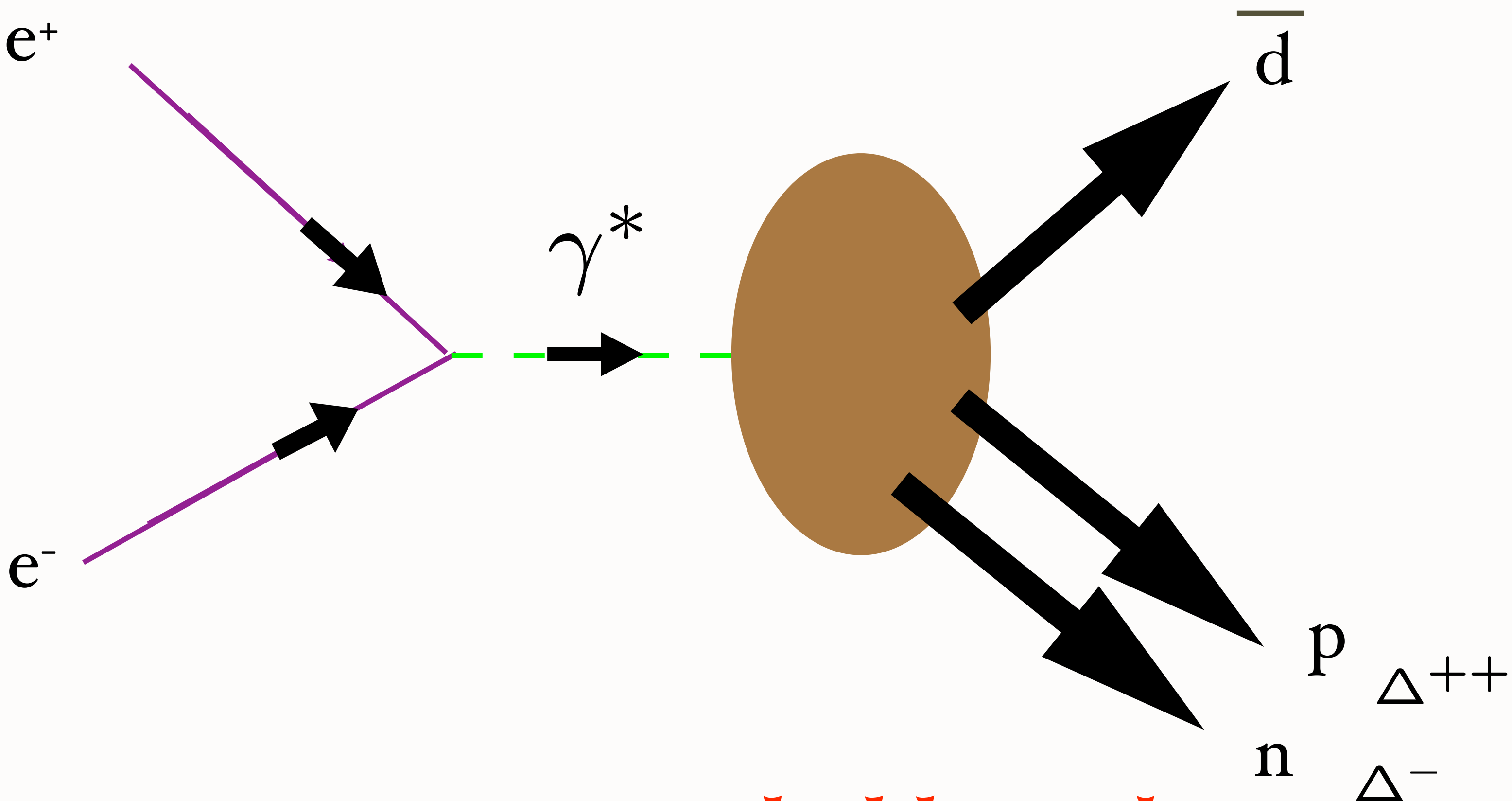
$$\prod_{k=1}^{6} x_k \left[\frac{\partial}{\partial\xi} + \frac{3C_F}{\beta}\right] \tilde{\Phi}(x_i, Q) = -\frac{C_d}{\beta}\int_0^1 [dy]\, V(x_i, y_i)\, \tilde{\Phi}(y_i, Q),$$

$$\xi(Q^2) = \frac{\beta}{4\pi}\int_{Q_0^2}^{Q^2} \frac{dk^2}{k^2}\,\alpha_s(k^2) \sim \ln\left(\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}\right).$$

$$V(x_i, y_i) = 2\prod_{k=1}^{6} x_k \sum_{i \neq j}^{6} \theta(y_i - x_i) \prod_{l \neq i,j}^{6} \delta(x_l - y_l)\frac{y_j}{x_j}\left(\frac{\delta_{h_i \bar{h}_j}}{x_i + x_j} + \frac{\Delta}{y_i - x_i}\right)$$

where $\delta_{h_i \bar{h}_j} = 1$ (0) when the helicities of the constituents $\{i, j\}$ are antiparallel (parallel). The infrared singularity at $x_i = y_i$ is cancelled by the factor $\Delta\tilde{\Phi}(y_i, Q) = \tilde{\Phi}(y_i, Q) - \tilde{\Phi}(x_i, Q)$ since the deuteron is a color singlet.

# Deuteron Production -- Test for Hidden Color

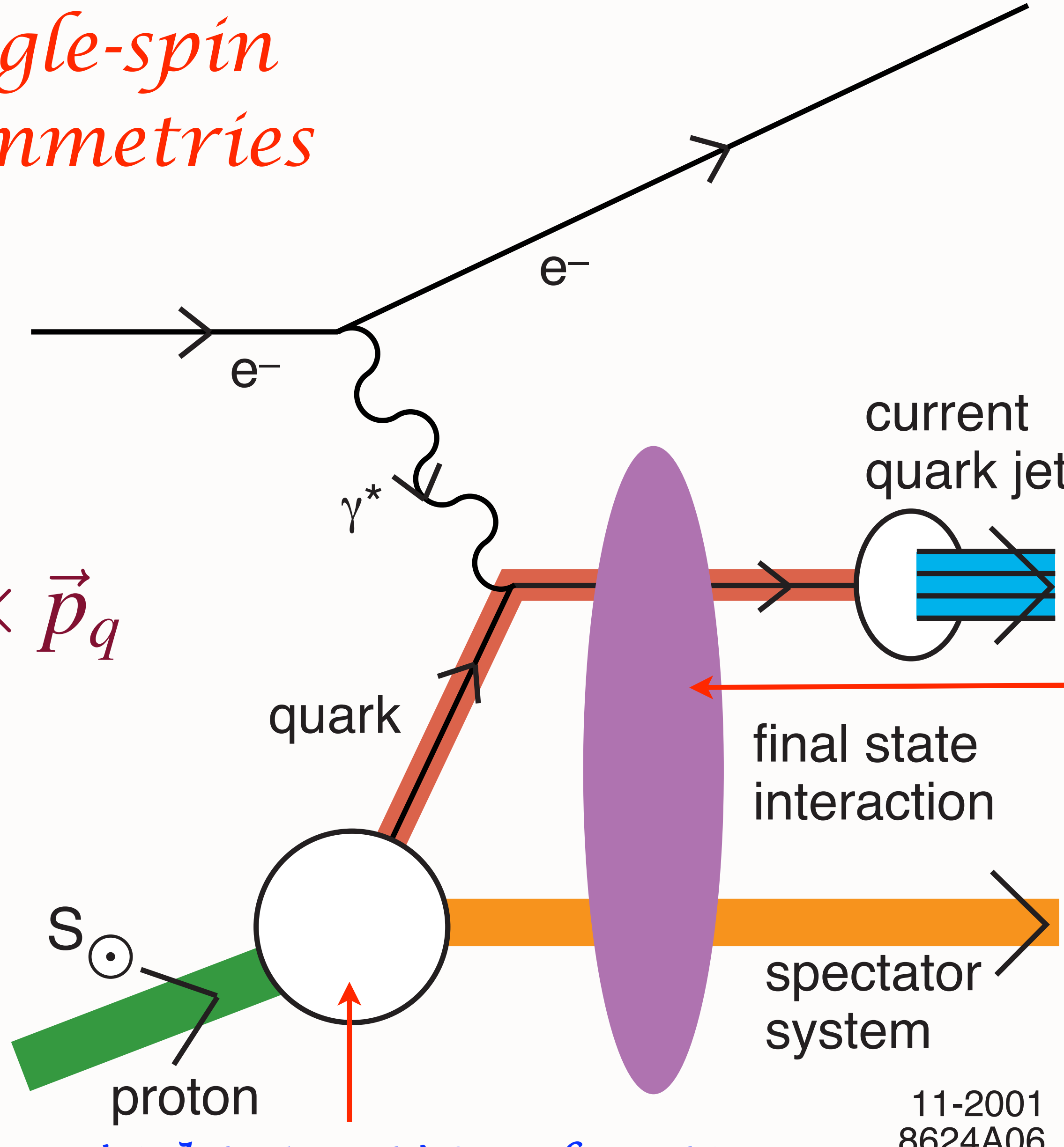



Ratio sensitive to hidden color Fock state probability

# *Single-spin asymmetries*

## Sivers Effect

$$\vec{S}_p \cdot \vec{q} \times \vec{p}_q$$




*QCD S- and P-Coulomb Phases*

*Light-Front Wavefunction S and P- Waves*

Hwang, Schmidt. sjb

# Final-State Interactions Produce T-Odd (Sivers Effect)

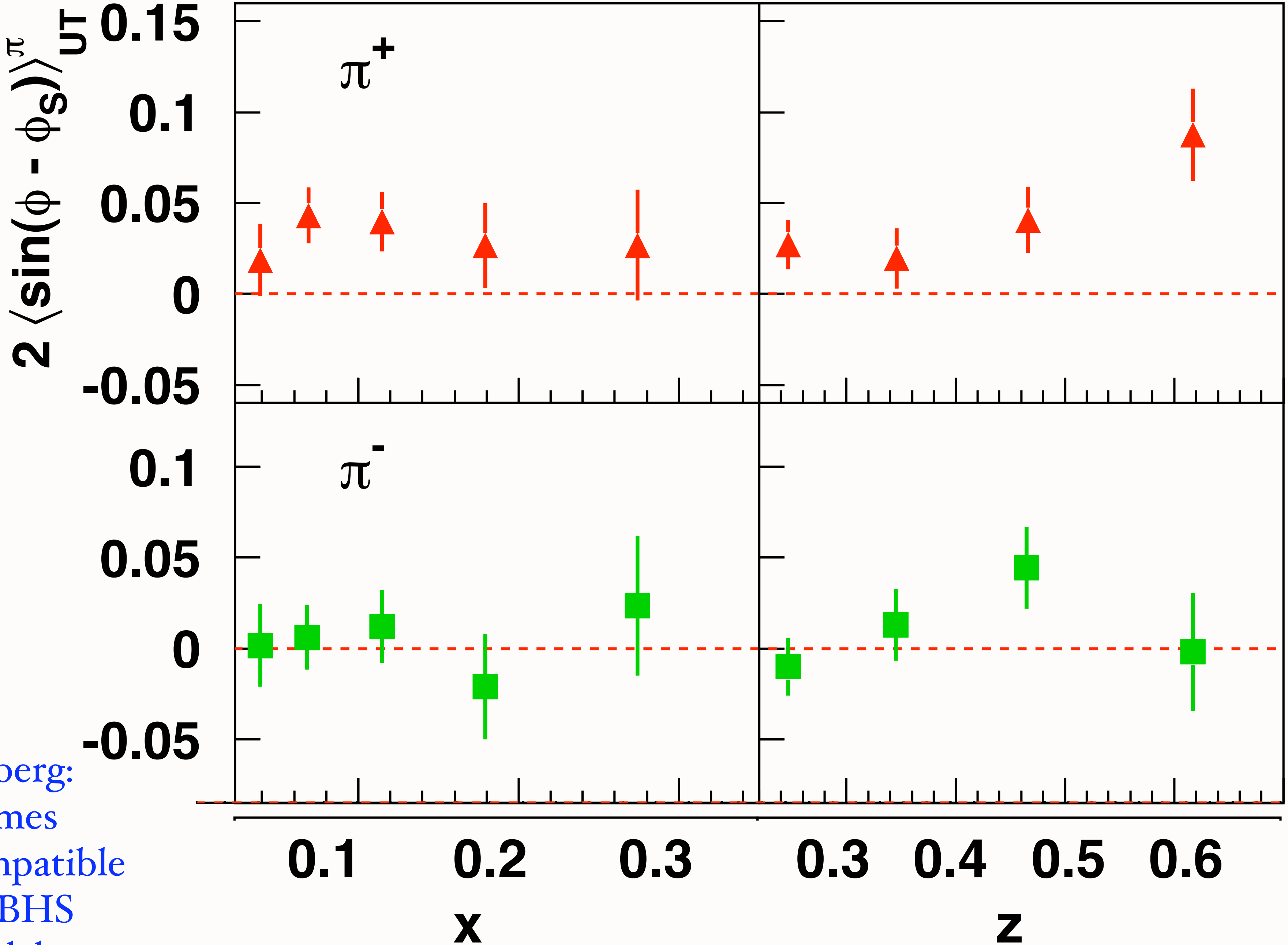
$$i \, \vec{S} \cdot \vec{p}_{jet} \times \vec{q}$$

- Bjorken Scaling!
- Arises from Interference of Final-State Coulomb Phases in S and P waves
- Relate to the quark contribution to the target proton anomalous magnetic moment

Hwang, Schmidt. sjb;
Burkardt

HERMES coll., A. Airapetian et al., Phys. Rev. Lett. 94 (2005) 012002.

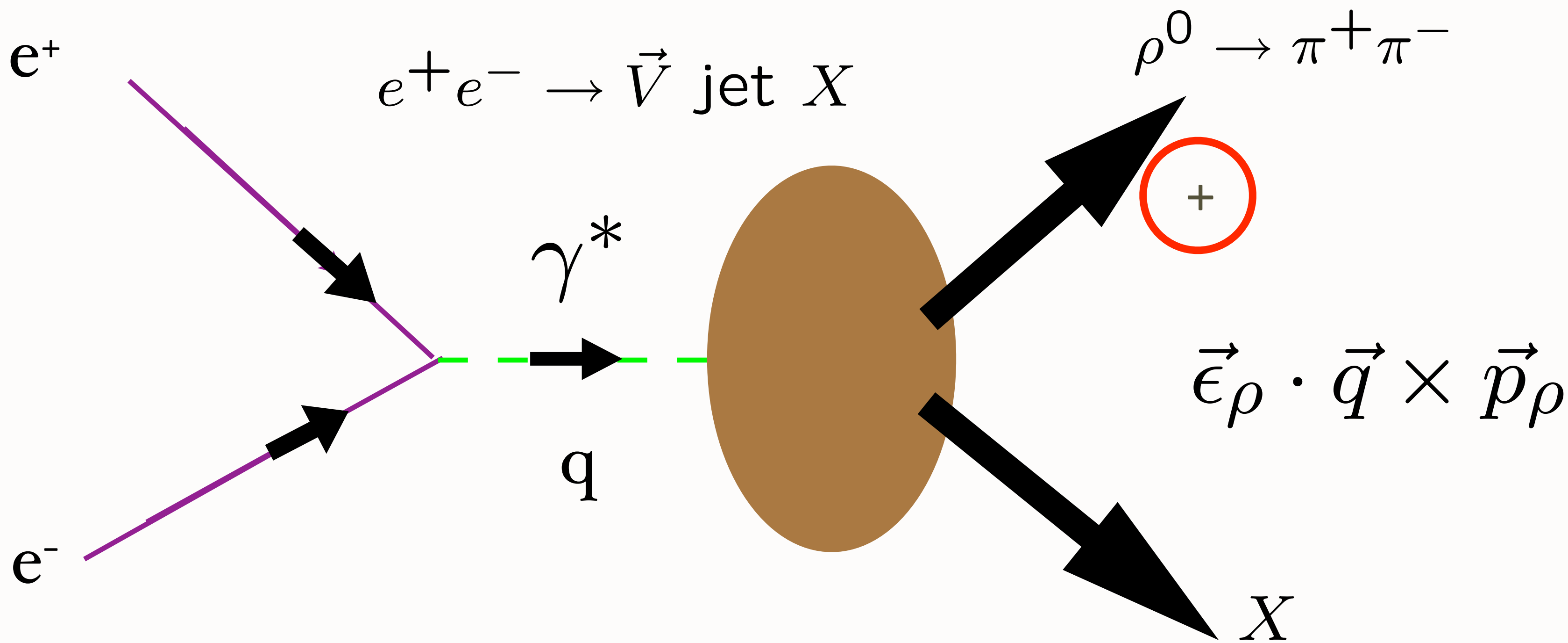
# Sivers asymmetry from HERMES




Gamberg: Hermes data compatible with BHS model

# Measure Time-like T-odd SSA

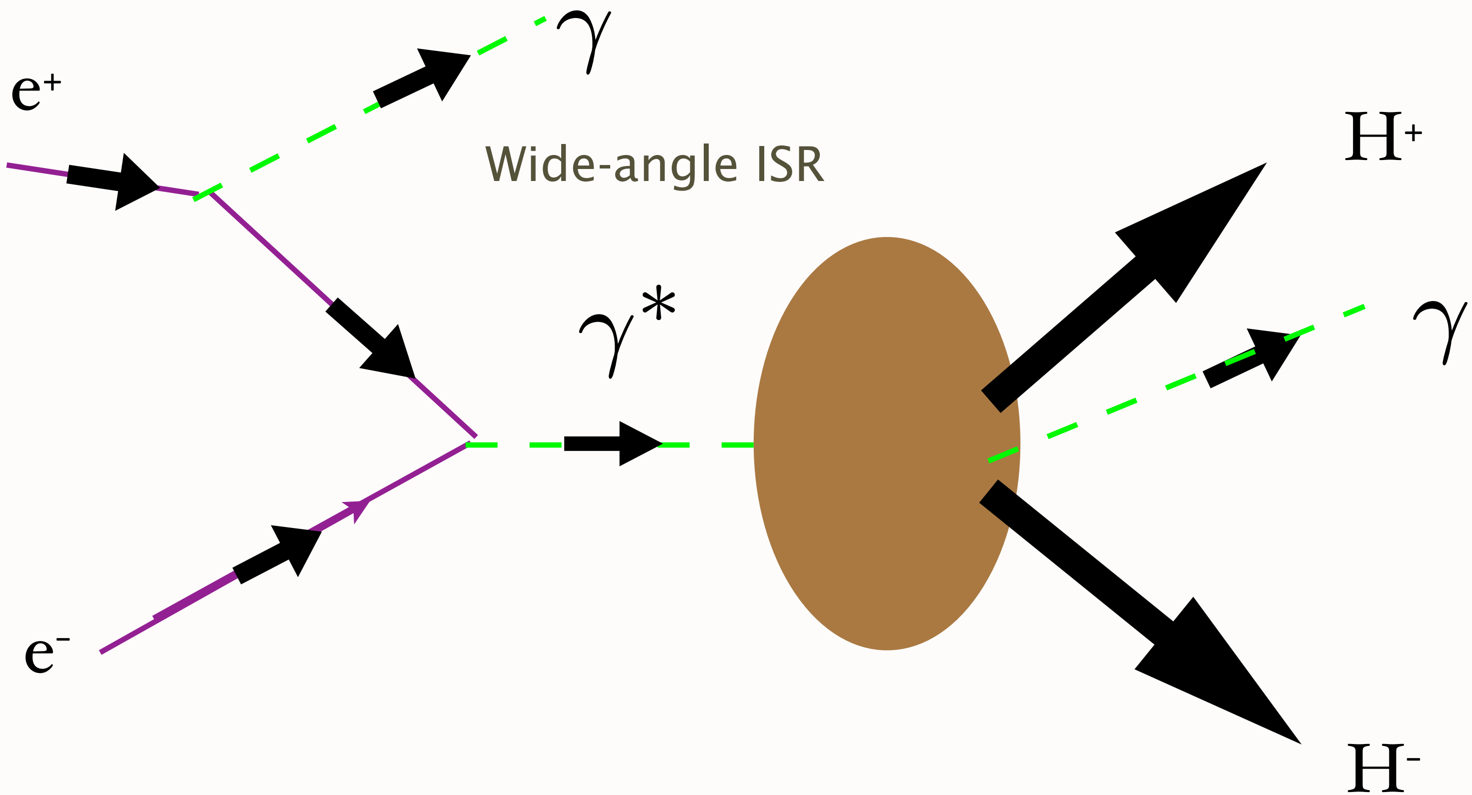
Test both Sivers and Collins Effect in Quark Fragmentation




$$\epsilon_{\mu\nu\sigma\tau}\ \epsilon_V^\mu\ p_V^\nu\ p_{\text{jet}}^\sigma\ q^\tau$$

Measure spin projection of detected hadron normal to production plane; use asymmetric B-factory

# Time-like Deeply Virtual Compton Scattering
# Time-like Generalized Parton Distributions



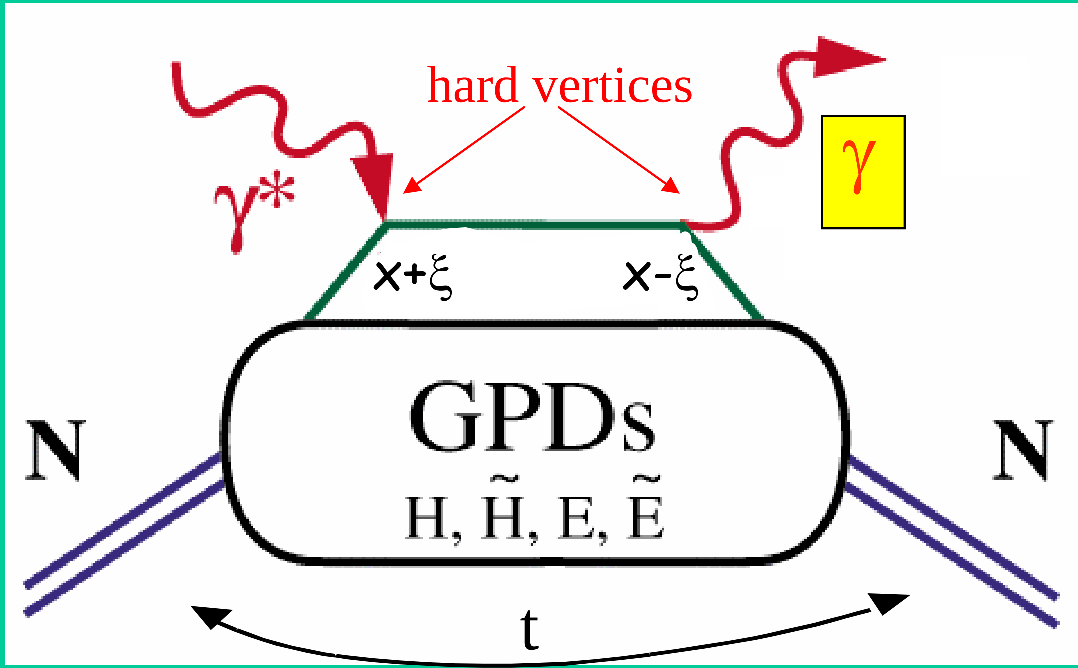

Interference of timelike DVCS amplitude $T(\gamma^* \to H^+ H^- \gamma)$
with timelike form factor produces charge asymmetry

$$e^+ e^- \to H^+ H^- \gamma$$

# GPDs & Deeply Virtual Exclusive Processes

## "handbag" mechanism

### Deeply Virtual Compton Scattering (DVCS)




x – longitudinal quark momentum fraction

$2\xi$ – longitudinal momentum transfer

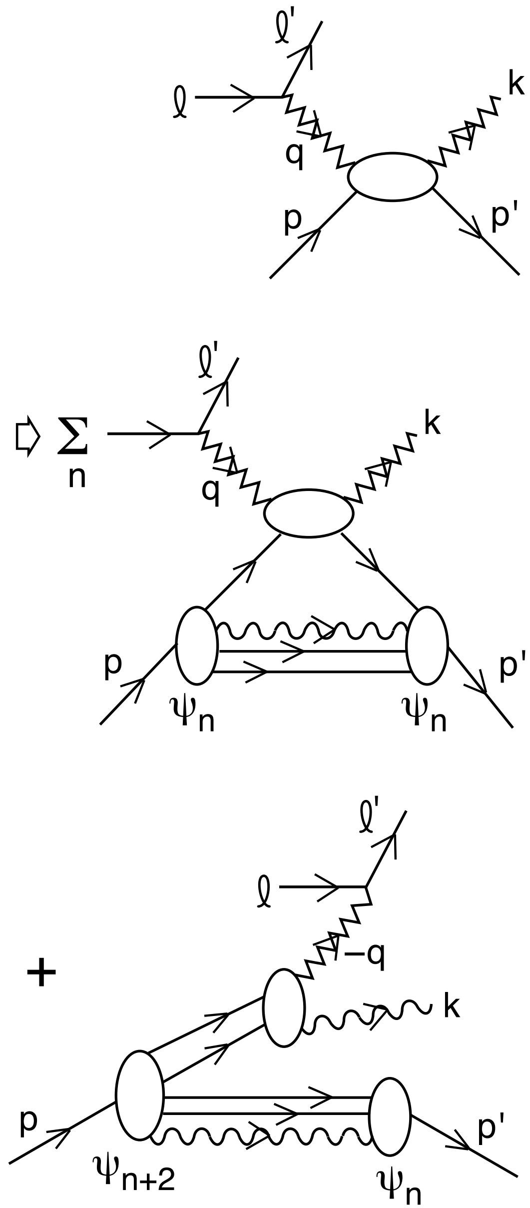
$\sqrt{-t}$ – Fourier conjugate to transverse impact parameter

$H(x,\xi,t), E(x,\xi,t), \ldots$

$$\xi = \frac{x_B}{2-x_B}$$

$\langle p'\,\lambda' |\, J^{\mu}(z)\, J^{\nu}(0)\, | p\,\lambda \rangle$

Large $-q^2 = Q^2$




$\gamma^* p \to \gamma p'$

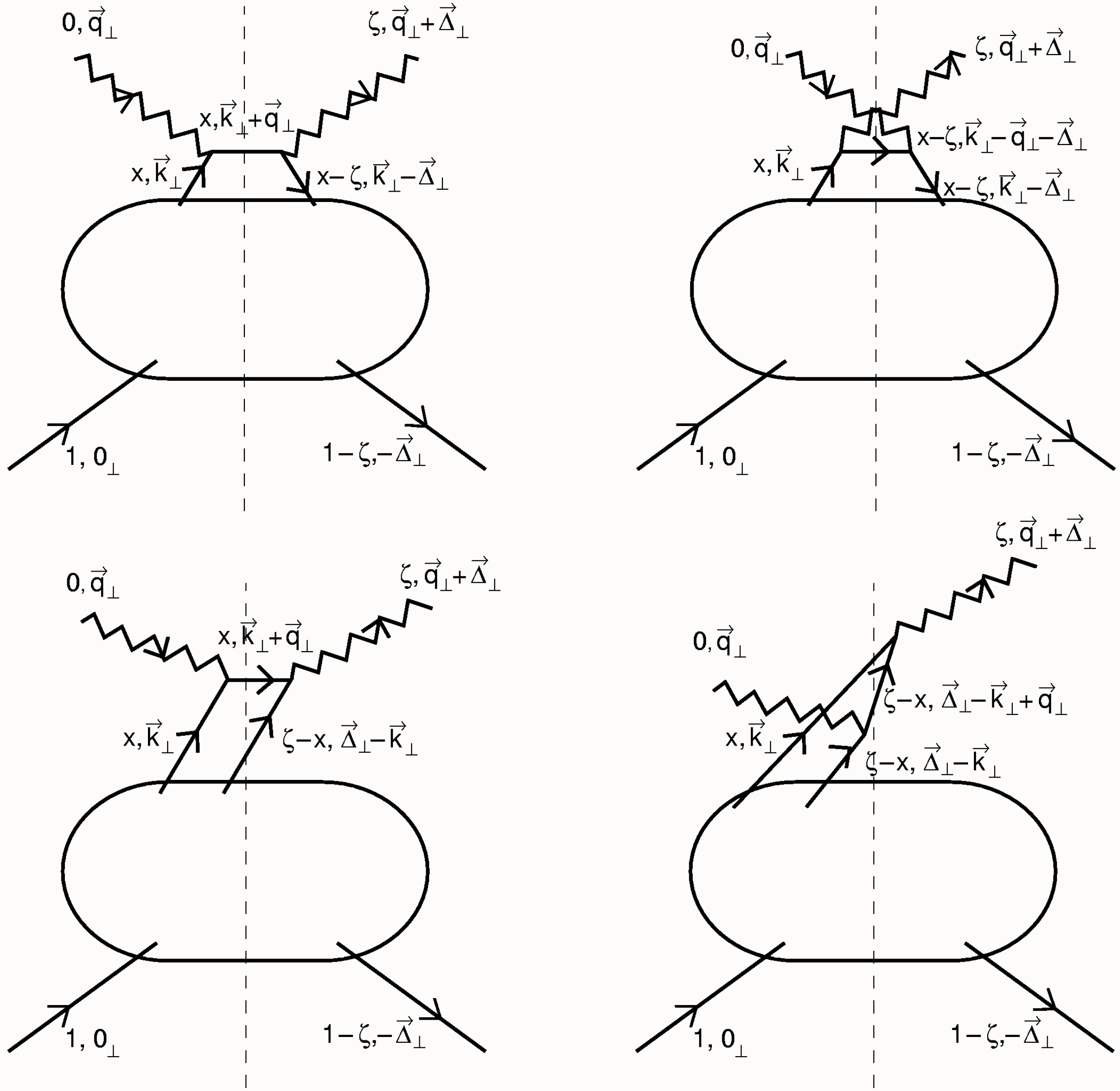
Deeply Virtual Compton Scattering

*Given LFWFs, compute all GPDs !*

ERBL Evolution

**n = n' + 2**

Required for Lorentz Invariance




# Light-cone wavefunction representation of deeply virtual Compton scattering ☆


Stanley J. Brodsky [a], Markus Diehl [a,1], Dae Sung Hwang [b]

# Link to DIS and Elastic Form Factors

DIS at $\xi = t = 0$

$$H^q(x,0,0) = q(x), \quad -\bar{q}(-x)$$
$$\tilde{H}^q(x,0,0) = \Delta q(x), \quad \Delta\bar{q}(-x)$$

Form factors (sum rules)

$$\int_{-1}^{1} dx \sum_q \left[H^q(x,\xi,t)\right] = F_1(t) \text{ Dirac f.f.}$$
$$\int_{-1}^{1} dx \sum_q \left[E^q(x,\xi,t)\right] = F_2(t) \text{ Pauli f.f.}$$
$$\int_{-1}^{1} dx\, \tilde{H}^q(x,\xi,t) = G_{A,q}(t), \quad \int_{-1}^{1} dx\, \tilde{E}^q(x,\xi,t) = G_{P,q}(t)$$

$$H^q, E^q, \tilde{H}^q, \tilde{E}^q(x,\xi,t)$$

Verified using LFWFs

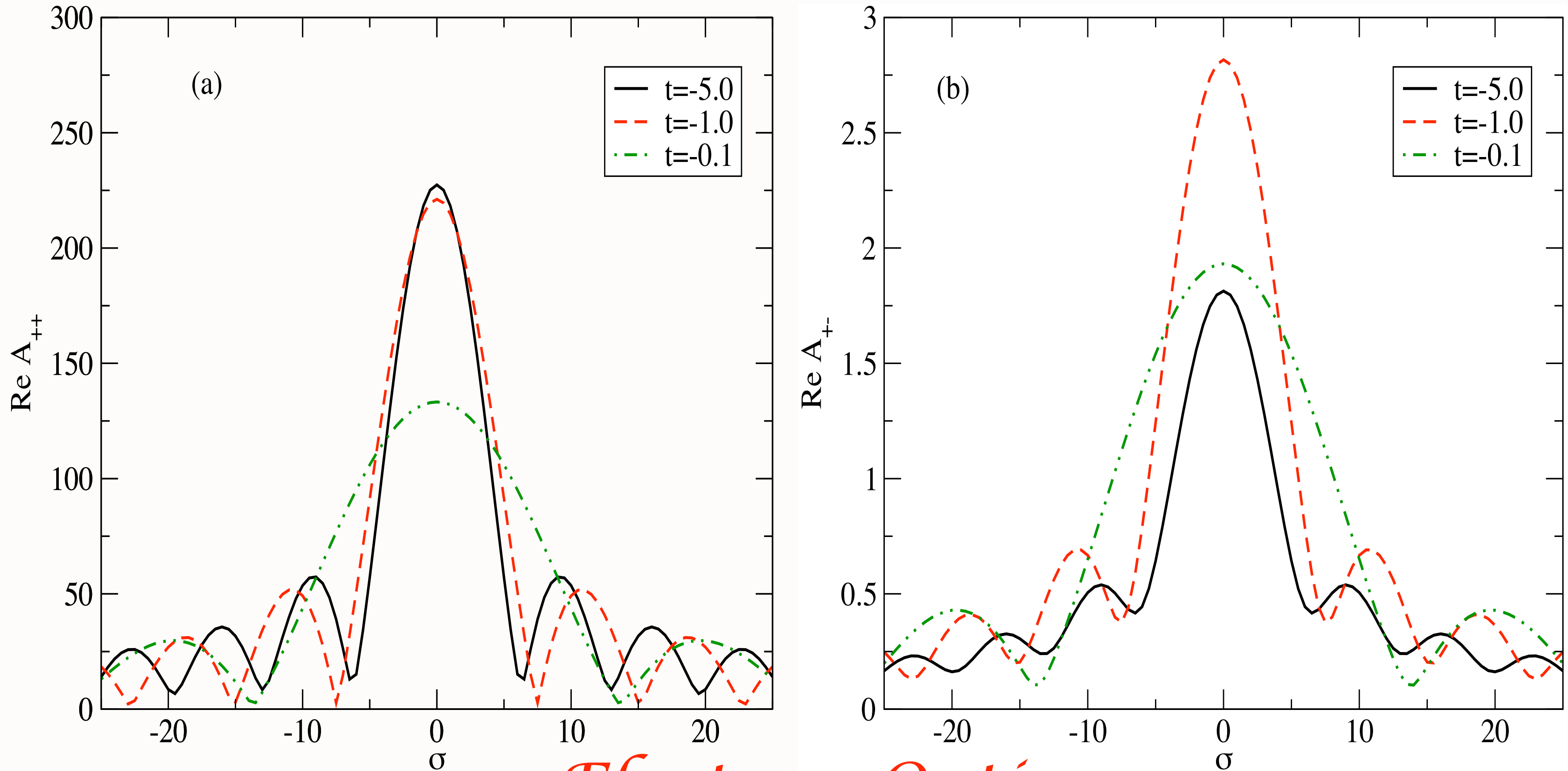
Diehl,Hwang, sjb

Quark angular momentum (Ji's sum rule)

$$J^q = \frac{1}{2} - J^G = \frac{1}{2}\int_{-1}^{1} x\,dx \left[H^q(x,\xi,0) + E^q(x,\xi,0)\right]$$

X. Ji, Phy.Rev.Lett.78,610(1997)

S. J. Brodsky[a], D. Chakrabarti[b], A. Harindranath[c], A. Mukherjee[d], J. P. Vary[e,a,f]




# Electron Optics

Fourier spectrum of the real part of the DVCS amplitude of an electron vs. $\sigma$ for $M = 0.51$ MeV, $m = 0.5$ MeV, $\lambda = 0.02$ MeV, (a) when the electron helicity is not flipped; (b) when the helicity is flipped. The parameter $t$ is in MeV$^2$.

$$A(\sigma, \Delta_\perp) = \frac{1}{2\pi} \int d\zeta e^{\frac{i}{2}\sigma\zeta} M(\zeta, \Delta_\perp) \qquad \zeta = \frac{Q^2}{2p\cdot q}$$

# Example of LFWF representation of GPDs (n => n)

Diehl,Hwang, sjb

$$\frac{1}{\sqrt{1-\zeta}}\frac{\Delta^1 - i\,\Delta^2}{2M} E_{(n\to n)}(x,\zeta,t)$$

$$= \left(\sqrt{1-\zeta}\right)^{2-n} \sum_{n,\lambda_i} \int \prod_{i=1}^{n} \frac{\mathrm{d}x_i\,\mathrm{d}^2\vec{k}_{\perp i}}{16\pi^3}\, 16\pi^3 \delta\left(1-\sum_{j=1}^{n} x_j\right) \delta^{(2)}\left(\sum_{j=1}^{n}\vec{k}_{\perp j}\right)$$

$$\times\,\delta(x-x_1)\,\psi^{\uparrow *}_{(n)}(x'_i,\vec{k}'_{\perp i},\lambda_i)\,\psi^{\downarrow}_{(n)}(x_i,\vec{k}_{\perp i},\lambda_i),$$

where the arguments of the final-state wavefunction are given by

$$x'_1 = \frac{x_1-\zeta}{1-\zeta}, \qquad \vec{k}'_{\perp 1} = \vec{k}_{\perp 1} - \frac{1-x_1}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the struck quark,}$$

$$x'_i = \frac{x_i}{1-\zeta}, \qquad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + \frac{x_i}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the spectators } i=2,\ldots,n.$$

# Example of LFWF representation of GPDs (n+1 => n-1)

Diehl, Hwang, sjb

$$\frac{1}{\sqrt{1-\zeta}} \frac{\Delta^1 - i\Delta^2}{2M} E_{(n+1\to n-1)}(x,\zeta,t)$$

$$= \left(\sqrt{1-\zeta}\right)^{3-n} \sum_{n,\lambda_i} \int \prod_{i=1}^{n+1} \frac{\mathrm{d}x_i\, \mathrm{d}^2\vec{k}_{\perp i}}{16\pi^3} 16\pi^3 \delta\left(1 - \sum_{j=1}^{n+1} x_j\right) \delta^{(2)}\left(\sum_{j=1}^{n+1} \vec{k}_{\perp j}\right)$$

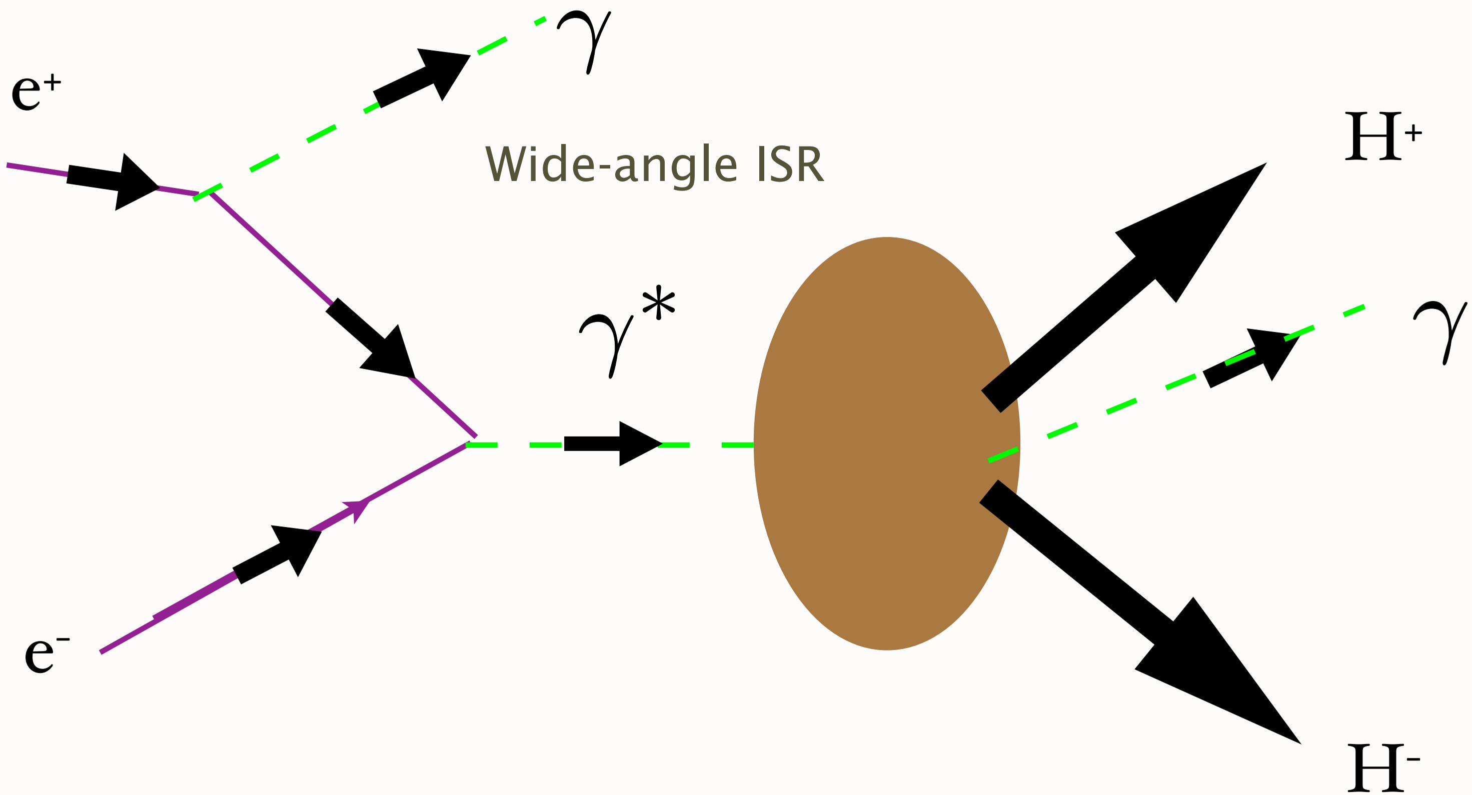
$$\times 16\pi^3 \delta(x_{n+1} + x_1 - \zeta) \delta^{(2)}(\vec{k}_{\perp n+1} + \vec{k}_{\perp 1} - \vec{\Delta}_\perp)$$

$$\times \delta(x - x_1) \psi^{\uparrow *}_{(n-1)}(x'_i, \vec{k}'_{\perp i}, \lambda_i) \psi^{\downarrow}_{(n+1)}(x_i, \vec{k}_{\perp i}, \lambda_i) \delta_{\lambda_1 - \lambda_{n+1}},$$

where $i = 2, \ldots, n$ label the $n-1$ spectator partons which appear in the final-state hadron wavefunction with

$$x'_i = \frac{x_i}{1-\zeta}, \qquad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + \frac{x_i}{1-\zeta} \vec{\Delta}_\perp.$$

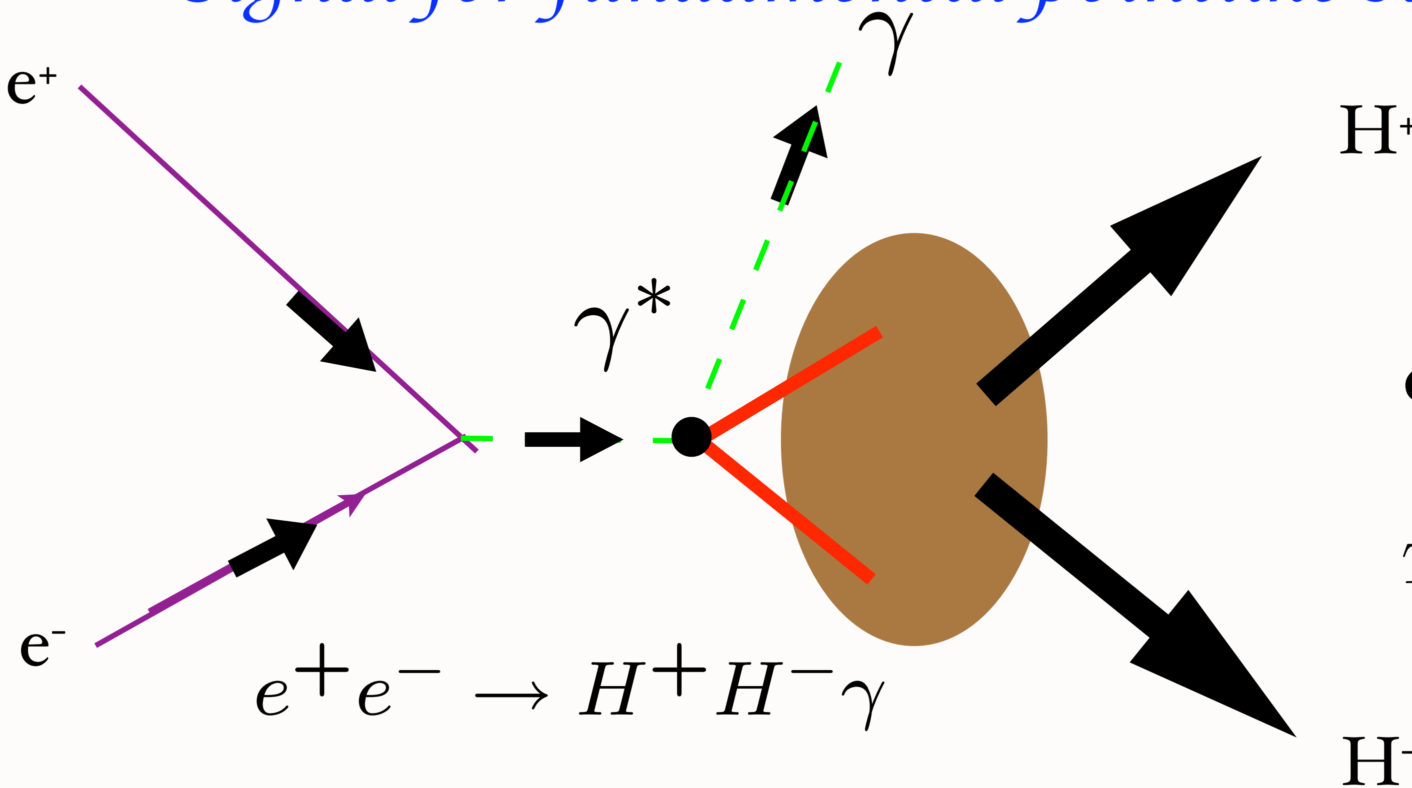
# Time-like Deeply Virtual Compton Scattering
# Time-like Generalized Parton Distributions




*Interference of timelike DVCS amplitude* $T(\gamma^* \to H^+ H^- \gamma)$
*with timelike form factor produces charge asymmetry*

$$e^+e^- \to H^+H^-\gamma$$

# *Time-like Deeply Virtual Compton Scattering*
# *J=0 Fixed Pole*
# *Signal for fundamental pointlike structure*



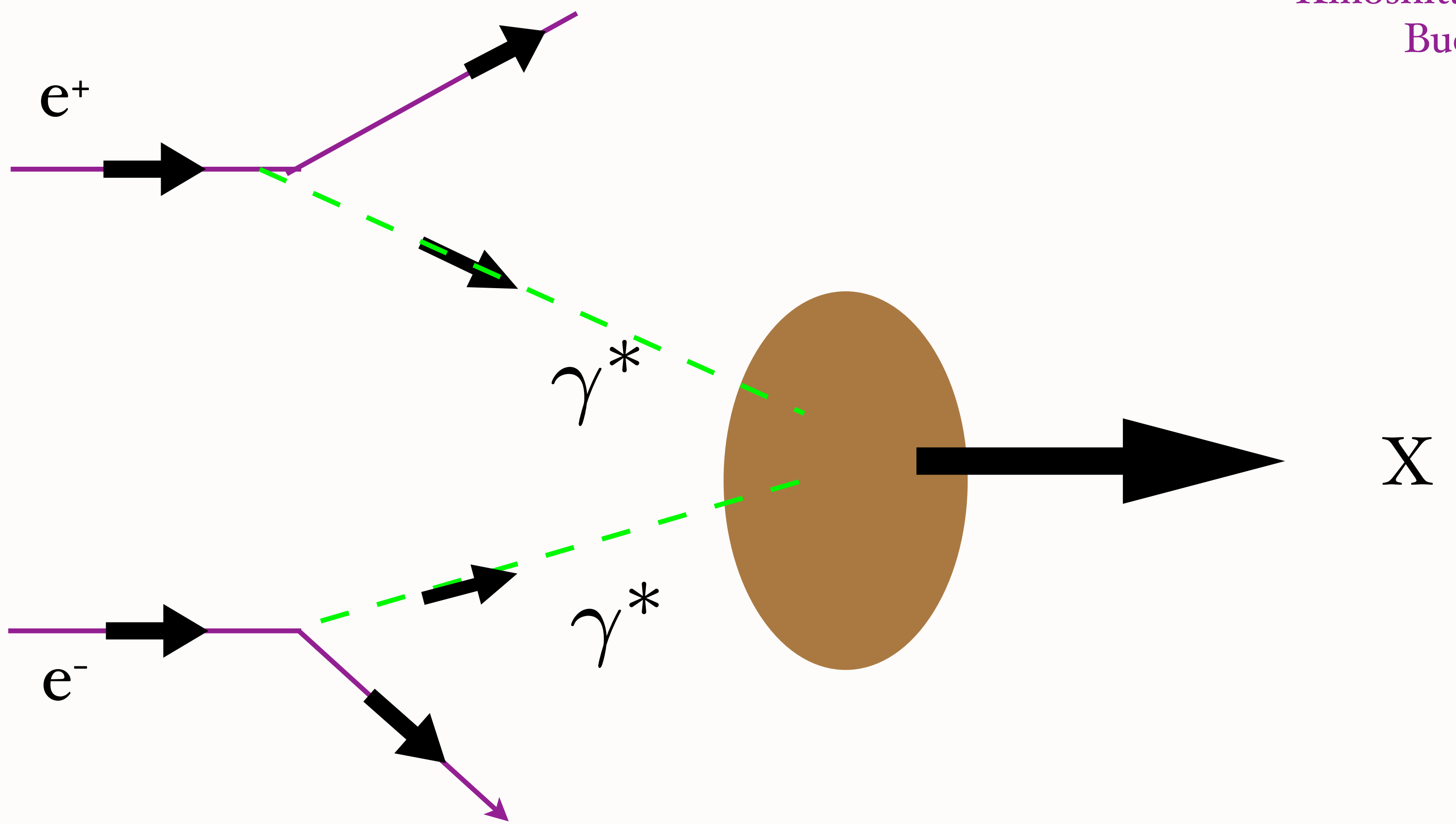

Instantaneous quark exchange

$$T_{\mathrm{seagull}} = F(M^2_{H\bar{H}})$$

Local "seagull" interaction of two photons at same point produces isotropic real amplitude, independent of photon virtuality at fixed pair mass

# Two-Photon Processes

Kinoshita, Terazawa, sjb;
Budnev et al




Production of all C=+ Hadronic States

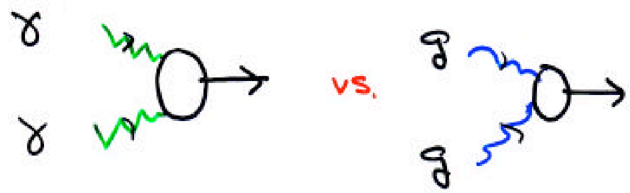
virtual photons $q_a^2, q_b^2$

# Photon-Photon Fusion: Remarkable laboratory for testing QCD

- $C = +$ Resonances
- Heavy Quarkonium
- Photon-to-Meson Transition Form Factors
- Exclusive Two-Photon Reactions
- Timelike Compton Reactions
- Hard QCD Jets
- Photon Structure Function
- Nature of Pomeron and Odderon

$\gamma\gamma \to$ Resonances

Discovery tool for Gluonium




$$S = \frac{\Gamma_{gg}}{\Gamma_{\gamma\gamma}}$$ "stickiness"

Chanowitz

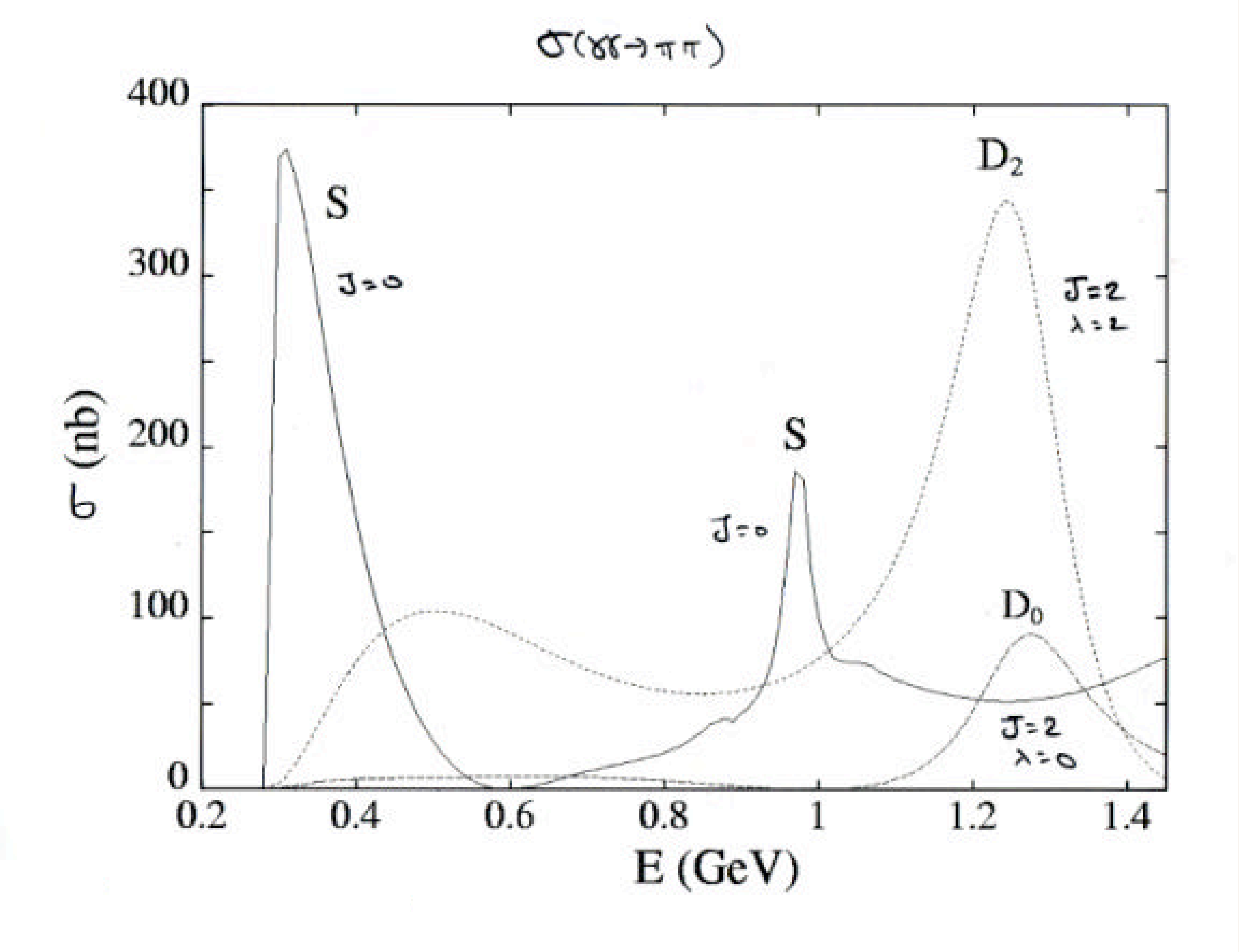
CLEO: upper limit for $\Gamma$

Sivertz
Poor

$\gamma\gamma \to f_0(2220)$
$\hookrightarrow K_s K_s$

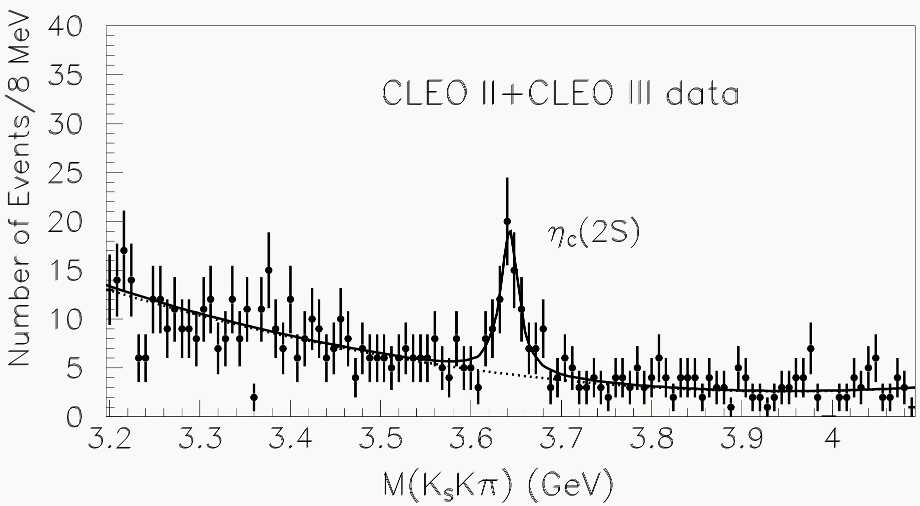
$S > 82$ (95% c.l.) !

also: $J/\psi\ \gamma$
MARK III
BES

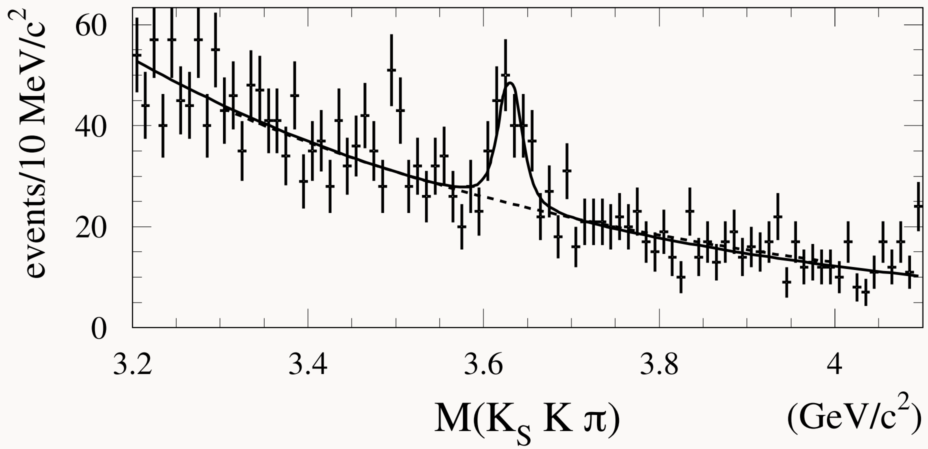



Roglioni, Pennington

# The Discovery of $\eta_c'(2^1S_0)$




CLEO II+III: 27 fb$^{-1}$ $(\gamma\gamma \to K_S K \pi)$




BaBar: 86 fb$^{-1}$ $(\gamma\gamma \to K_S K \pi)$

# Z(3931) observed in two photon fusion

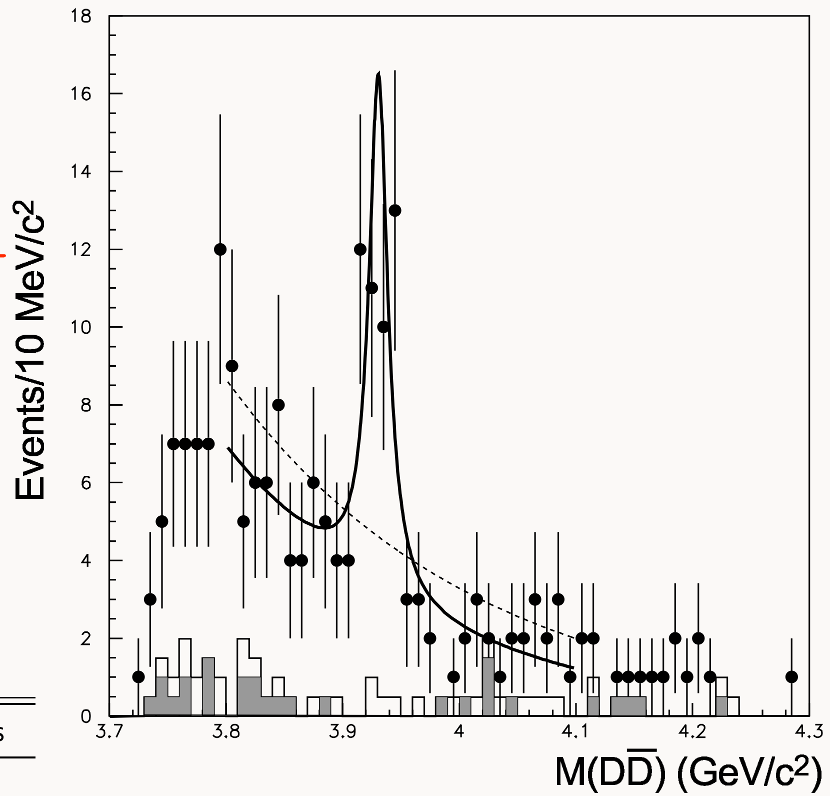
$$e^+e^- \to e^+e^-(\gamma\gamma),\ \gamma\gamma \to D\bar{D}$$

M(Z)$=3931 \pm 4 \pm 2$ MeV, significance$=5.5\sigma$

$\Gamma$(Z)$=20 \pm 8 \pm 3$ MeV

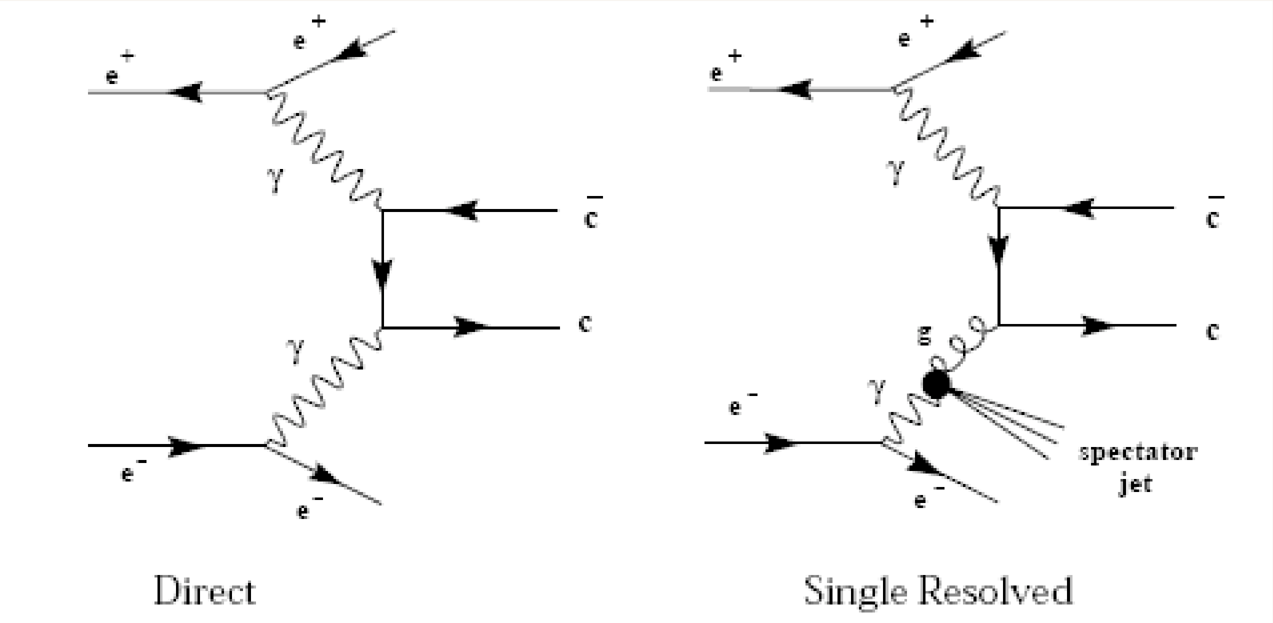
$\Gamma_{\gamma\gamma} \times \mathcal{B}(\to D\bar{D}) = 0.23 \pm 0.06 \pm 0.04$ keV

Candidate for charmonium state $\chi_2'(2^3P_2)$.

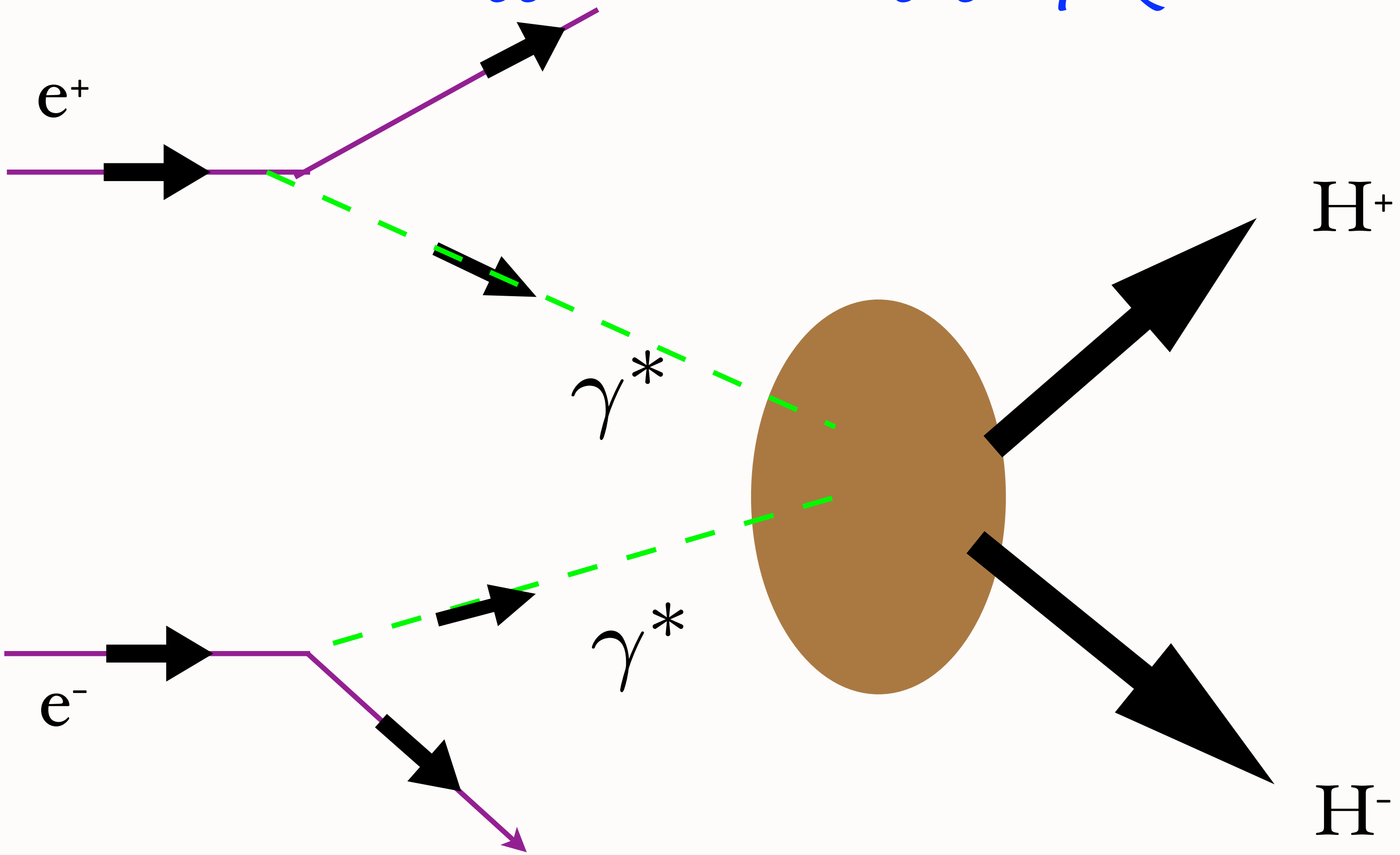



| | M(MeV) | $\Gamma$(MeV) | Formed in | Decays in | not in | suggests |
|---|---|---|---|---|---|---|
| X | $3943 \pm 6 \pm 6$ | $15 \pm 10$ | $e^+e^- \to J/\psi(c\bar{c})$ | $D^*\overline{D}$ | $D\overline{D}$, $\omega J/\psi$ | ? |
| Y | $3943 \pm 11 \pm 13$ | $87 \pm 22$ | $B \to K(\omega J/\psi)$ | $\omega J/\psi$ | $D^*\overline{D}$(?) | $c\bar{c}$ hybrid? |
| Z | $3931 \pm 4 \pm 2$ | $20 \pm 8 \pm 3$ | $\gamma\gamma$ fusion | $D\overline{D}$ | | $\chi'_{c2}(2^3P_2)$ |

# Inclusive Charm Production



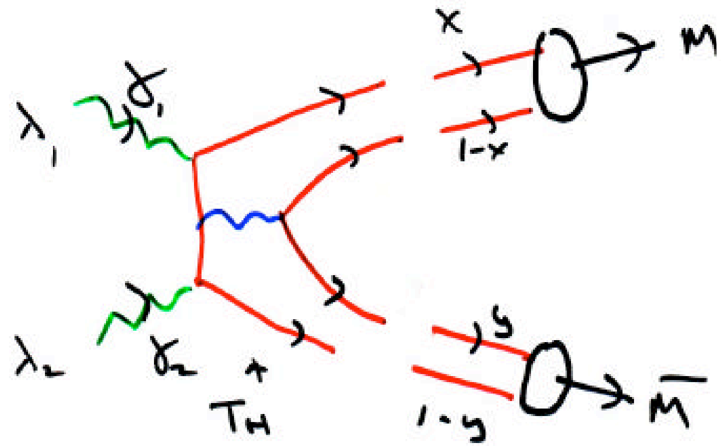
# Two-Photon Processes : Essential tests of QCD




Production of all Hadron Pairs
via real or virtual photons
timelike Compton Scattering

# Two-Photon Exclusive Reactions

PQCD Factorization
at large s, t




no FSI!

$$\boxed{\mathcal{M}(\gamma_1,\gamma_2 \to M\bar{M}) = \int_0^1 dx \int_0^1 dy \cdot T_H(x,y;s,\theta_{cm})\ \phi_M(x,\tilde{Q})\ \phi_{\bar{M}}(y,\tilde{Q})}$$

$$\phi_M(x,\tilde{Q}) = \int^{\tilde{Q}} d^2k_\perp\ \psi_{q\bar{q}/M}(x,\vec{k_\perp})$$

# Two-Photon Exclusive Amplitudes

$$F_M(s)=\frac{16\pi\alpha_s}{3s}\int_0^1 dx\,dy\,\frac{\phi_M^*(x,\tilde{Q}_x)\phi_M^*(y,\tilde{Q}_y)}{x(1-x)y(1-y)}$$

when $\phi_M(x,Q)=\phi_M(1-x,Q)$ is assumed.[7] Thus much of the dependence on $\phi(x,Q)$ can be removed from $\mathcal{M}_{\lambda\lambda'}$ by expressing it in terms of the meson form factor—i.e.,

$$\left.\begin{matrix}\mathcal{M}_{++}\\ \mathcal{M}_{--}\end{matrix}\right\}=16\pi\alpha F_M(s)\left[\frac{\langle(e_1-e_2)^2\rangle}{1-\cos^2\theta_{\rm c.m.}}\right],$$

Lepage, SJB

$$\left.\begin{matrix}\mathcal{M}_{+-}\\ \mathcal{M}_{-+}\end{matrix}\right\}=16\pi\alpha F_M(s)\left[\frac{\langle(e_1-e_2)^2\rangle}{1-\cos^2\theta_{\rm c.m.}}+2\langle e_1e_2\rangle g[\theta_{\rm c.m.};\phi_M]\right],$$

up to corrections of order $\alpha_s$ and $m^2/s$. Now the only dependence on $\phi_M$, and indeed the only unknown quantity, is in the $\theta$-dependent factor

$$g[\theta_{\rm c.m.};\phi_M]=\frac{\displaystyle\int_0^1 dx\,dy\,\frac{\phi_M^*(x,\tilde{Q})\phi_M^*(y,\tilde{Q})}{x(1-x)y(1-y)}\,\frac{a[y(1-y)+x(1-x)]}{a^2-b^2\cos^2\theta_{\rm c.m.}}}{\displaystyle\int_0^1 dx\,dy\,\frac{\phi_M^*(x,\tilde{Q})\phi_M^*(y,\tilde{Q})}{x(1-x)y(1-y)}}.$$

The spin-averaged cross section follows immediately from these expressions

$$\frac{d\sigma}{dt}=\frac{2}{s}\frac{d\sigma}{d\cos\theta_{\rm c.m.}}=\frac{1}{16\pi s^2}\frac{1}{4}\sum_{\lambda\lambda'}|\mathcal{M}_{\lambda\lambda'}|^2$$

$$=16\pi\alpha^2\left|\frac{F_M(s)}{s}\right|^2\left\{\frac{\langle(e_1-e_2)^2\rangle^2}{(1-\cos^2\theta_{\rm c.m.})^2}+\frac{2\langle e_1e_2\rangle\langle(e_1-e_2)^2\rangle}{1-\cos^2\theta_{\rm c.m.}}g[\theta_{\rm c.m.};\phi_M]\right.$$

$$\left.+2\langle e_1e_2\rangle^2g^2[\theta_{\rm c.m.};\phi_M]\right\}.$$

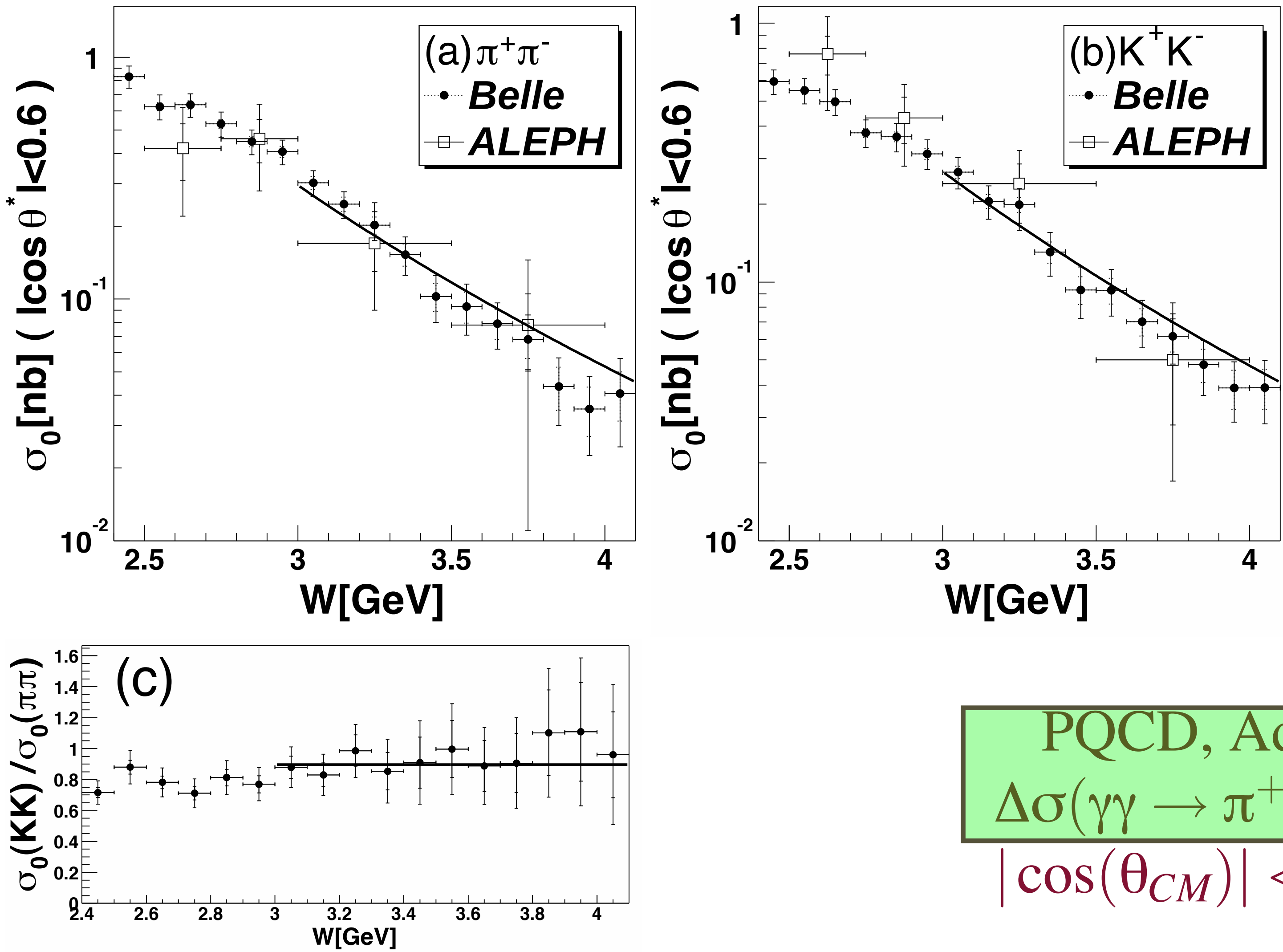



Fig. 5. Cross section for (a) $\gamma\gamma \to \pi^+\pi^-$, (b) $\gamma\gamma \to K^+K^-$ in the c.m. angular region $|\cos\theta^*| < 0.6$ together with a $W^{-6}$ dependence line derived from the fit of $s|R_M|$. (c) shows the cross section ratio. The solid line is the result of the fit for the data above 3 GeV. The errors indicated by short ticks are statistical only.

# Two Photon Reactions

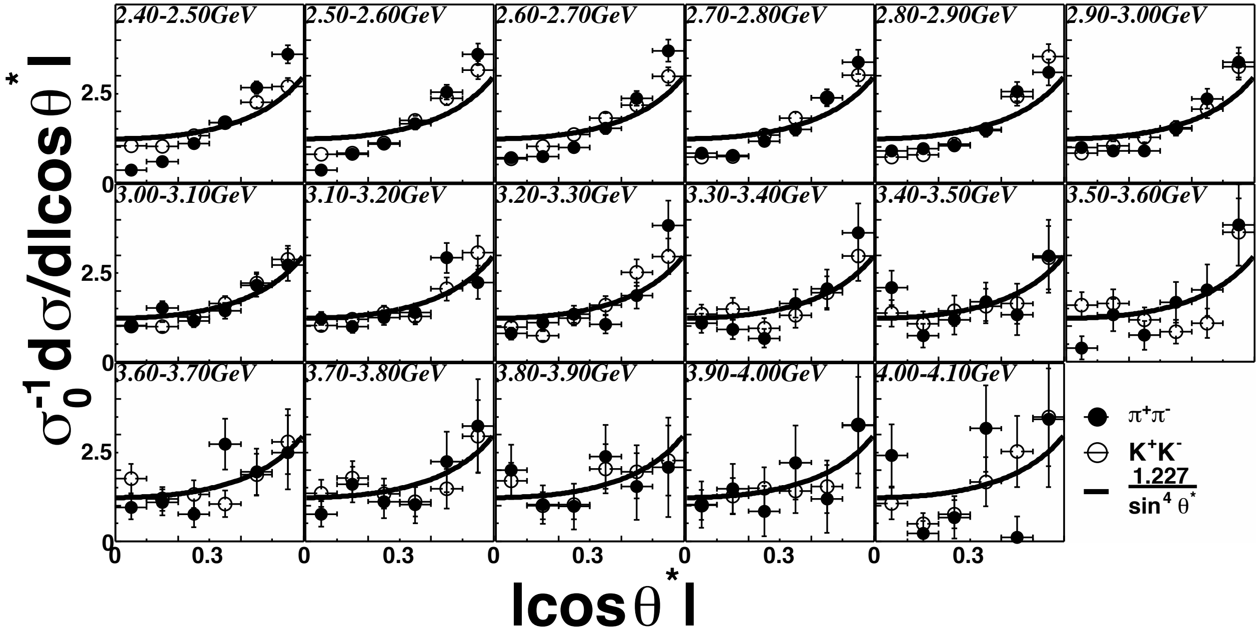
Hard Exclusive Processes: Fixed angle

PQCD, AdS/CFT:

$$\Delta\sigma(\gamma\gamma \to \pi^+\pi^-, K^+, K^-) \sim 1/W^6$$

$$|\cos(\theta_{CM})| < 0.6$$

$$\text{PQCD:}\quad \frac{d\sigma}{d|\cos\theta^*|}(\gamma\gamma \to M^+M^-) \approx \frac{16\pi\alpha^2}{s}\frac{|F_M(s)|^2}{\sin^4\theta^*},$$




Angular dependence of the cross section, $\sigma_0^{-1} d\sigma/d|\cos\theta^*|$, for the $\pi^+\pi^-$(closed circles) and $K^+K^-$(open circles) processes. The curves are $1.227 \times \sin^{-4}\theta^*$. The errors are statistical only.

**Measurement of the $\gamma\gamma \to \pi^+\pi^-$ and $\gamma\gamma \to K^+K^-$ processes at energies of 2.4–4.1 GeV**

Belle Data: Consistent with QCD predictions - energy and angular dependence

# Hadron Distribution Amplitudes

$$\phi(x_i, Q) \equiv \Pi_{i=1}^{n-1} \int^Q d^2\vec{k}_\perp \;\psi_n(x_i, \vec{k}_{\perp i})$$

- Fundamental measure of valence wavefunction
- Gauge Invariant (includes Wilson line)
- Evolution Equations, OPE
- Conformal Expansion
- Hadronic Input in Factorization Theorems

Lepage; SJB
Efremov, Radyuskin

# Critical Test of PQCD vs. "Handbag"

sjb, gpl

Crucial Ratio $\dfrac{\Delta\sigma(\gamma\gamma \to \pi^0\pi^0)}{\Delta\sigma(\gamma\gamma \to \pi^+\pi^-)}$

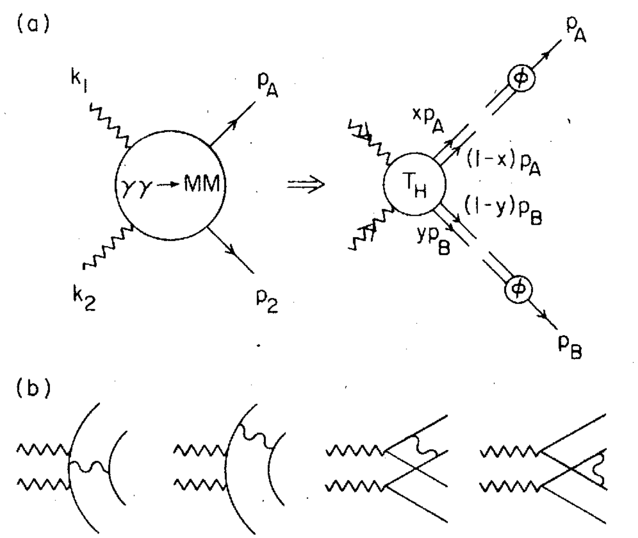



FIG. 1. (a) Factorized structure of the $\gamma\gamma \to M\bar{M}$ amplitude in QCD at large momentum transfer. The $T_H$ amplitude is computed with quarks collinear with the outgoing mesons. (b) Diagram contributing to $T_H$ $(\gamma\gamma \to M\bar{M})$ to lowest order in $\alpha_s$.

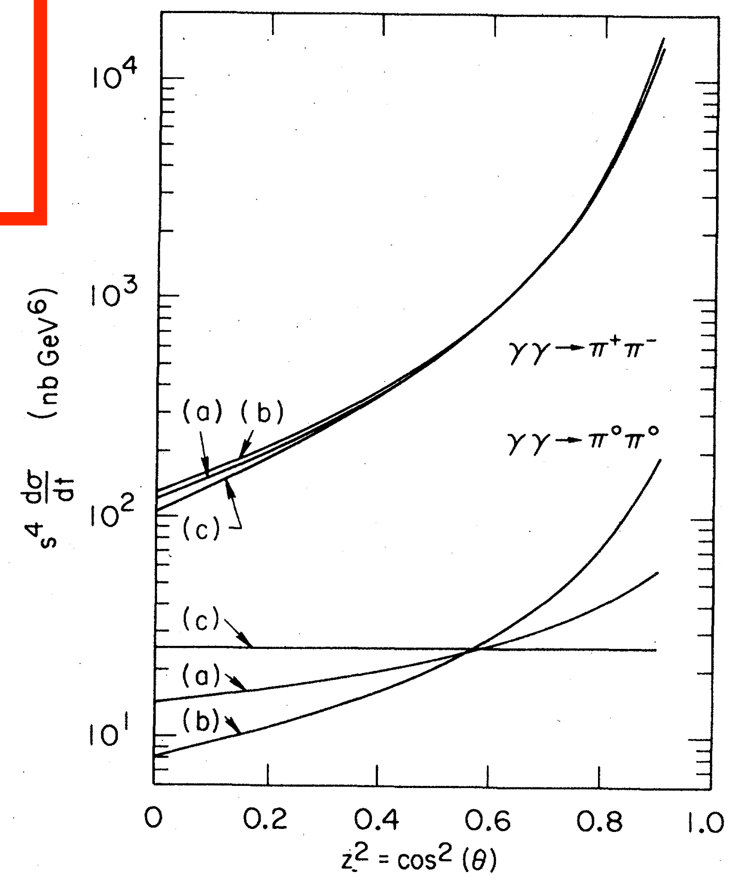



FIG. 3. QCD predictions for $\gamma\gamma \to \pi\pi$ to leading order in QCD. The results assume the pion-form-factor parametrization $F_\pi(s) \sim 0.4$ GeV$^2/s$. Curves (a), (b), and (c) correspond to the distribution amplitudes $\phi_M = x(1-x)$, $[x(1-x)]^{1/4}$, and $\delta(x-\frac{1}{2})$, respectively. Predictions for other helicity-zero mesons are obtained by multiplying with the scale constants given in Table I.

Handbag model (Diehl, Kroll et al ) neglects $e_1 \times e_2$ cross terms

$\gamma\gamma \to \pi^+\pi^-$

$\gamma\gamma \to \pi^0\pi^0$

Critical discriminant: Handbag vs.PQCD

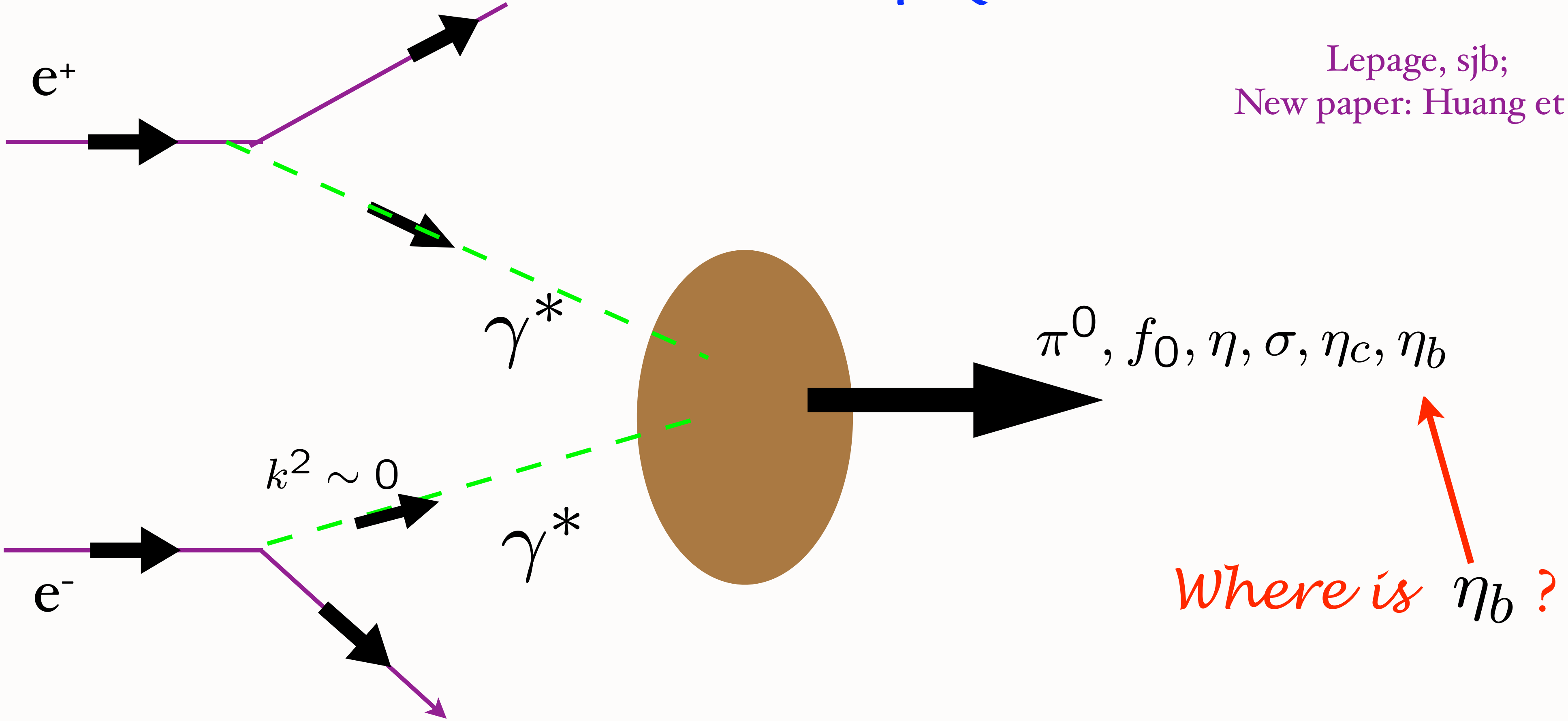
$\gamma\gamma \to K^+K^-$

$\gamma\gamma \to p\bar{p}$

$\gamma^*\gamma \to H\overline{H}$ Timelike DVCS!

# Two-Photon Processes : Essential tests of QCD

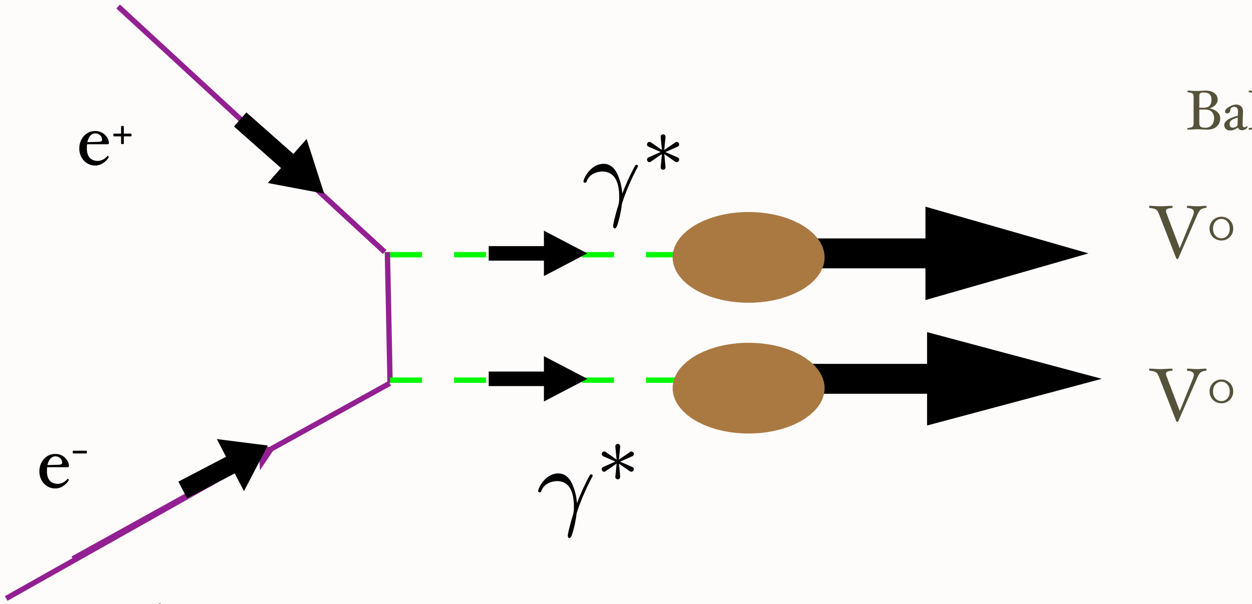
Lepage, sjb;
New paper: Huang et al



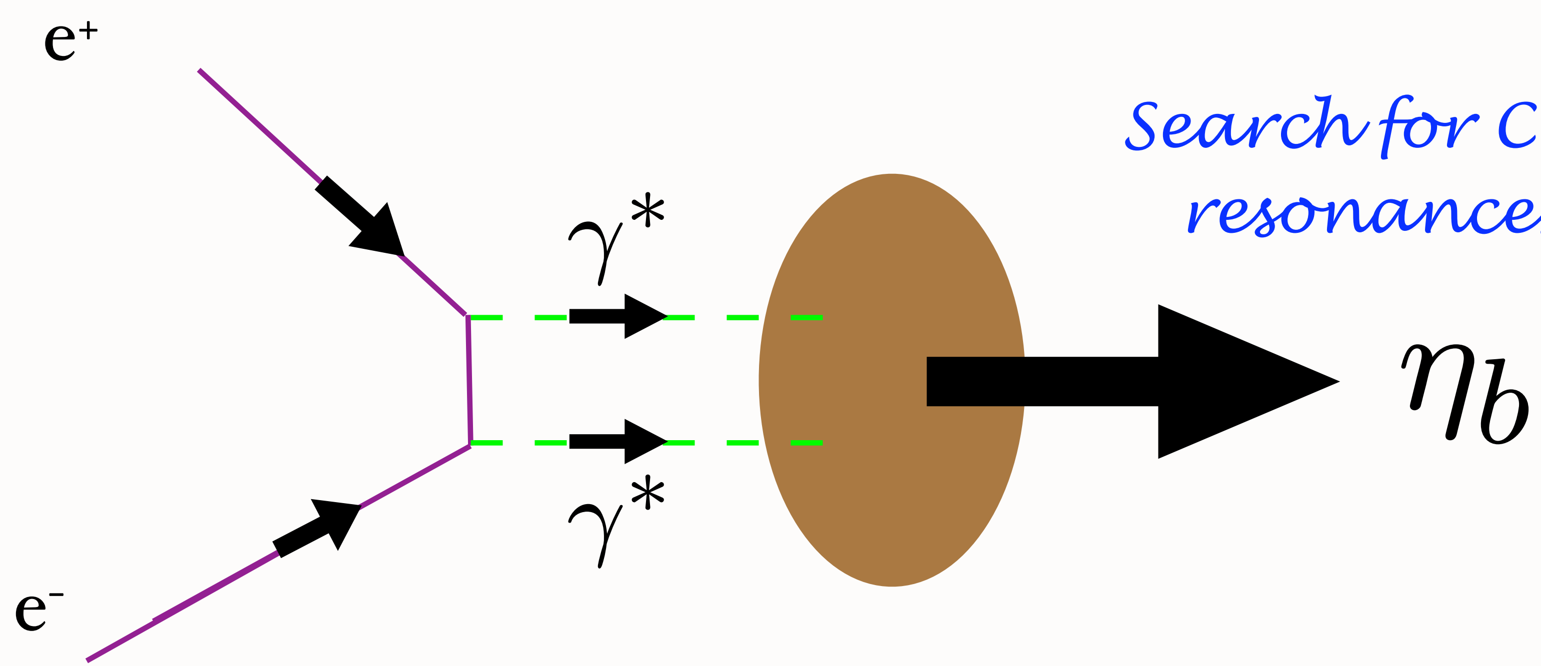

## Transition Form Factors
## Window to hadron distribution amplitudes

# Production of C=+ States

BaBar: Yi Snyder Davier

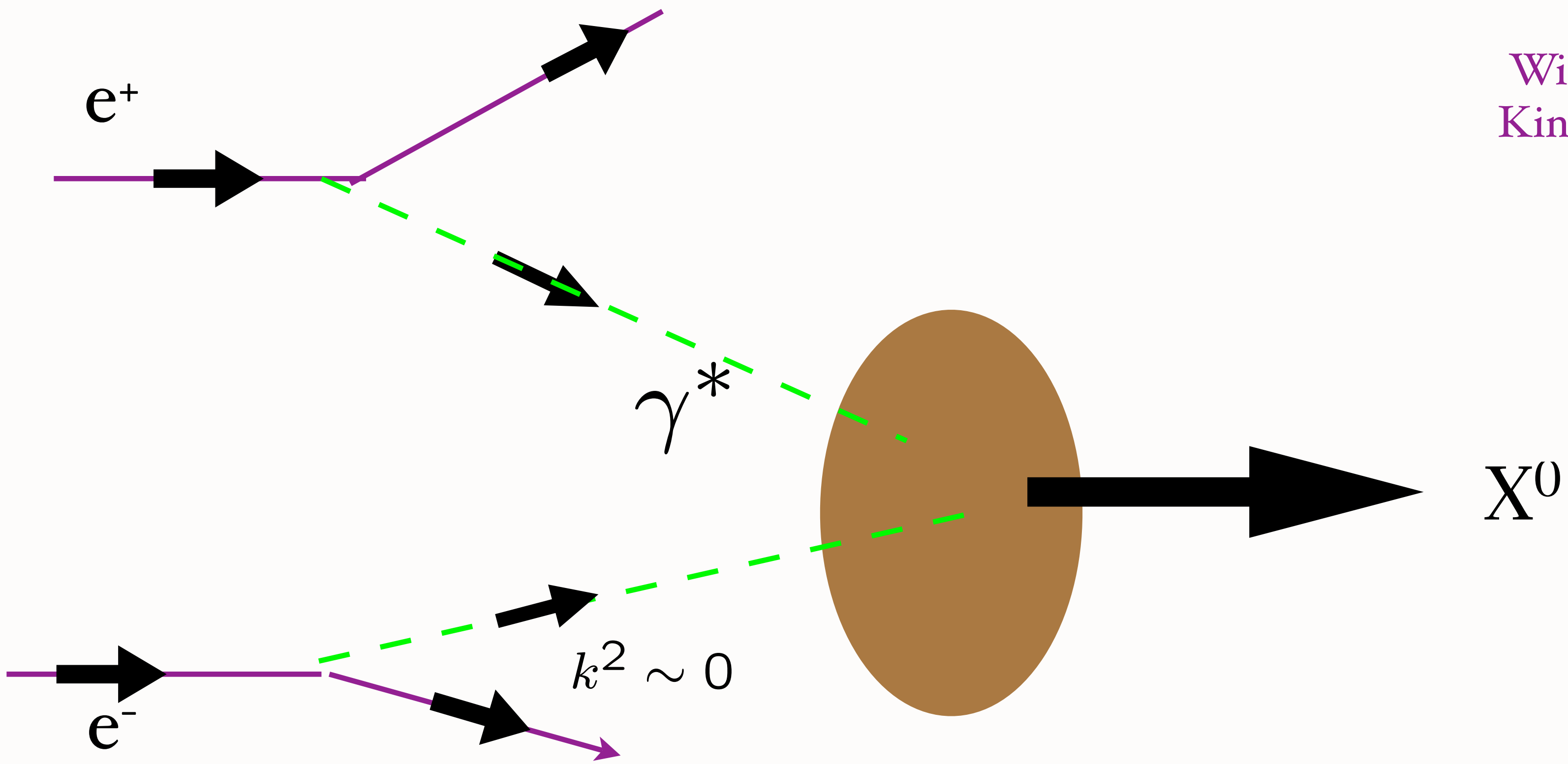



Search for C=+ resonances



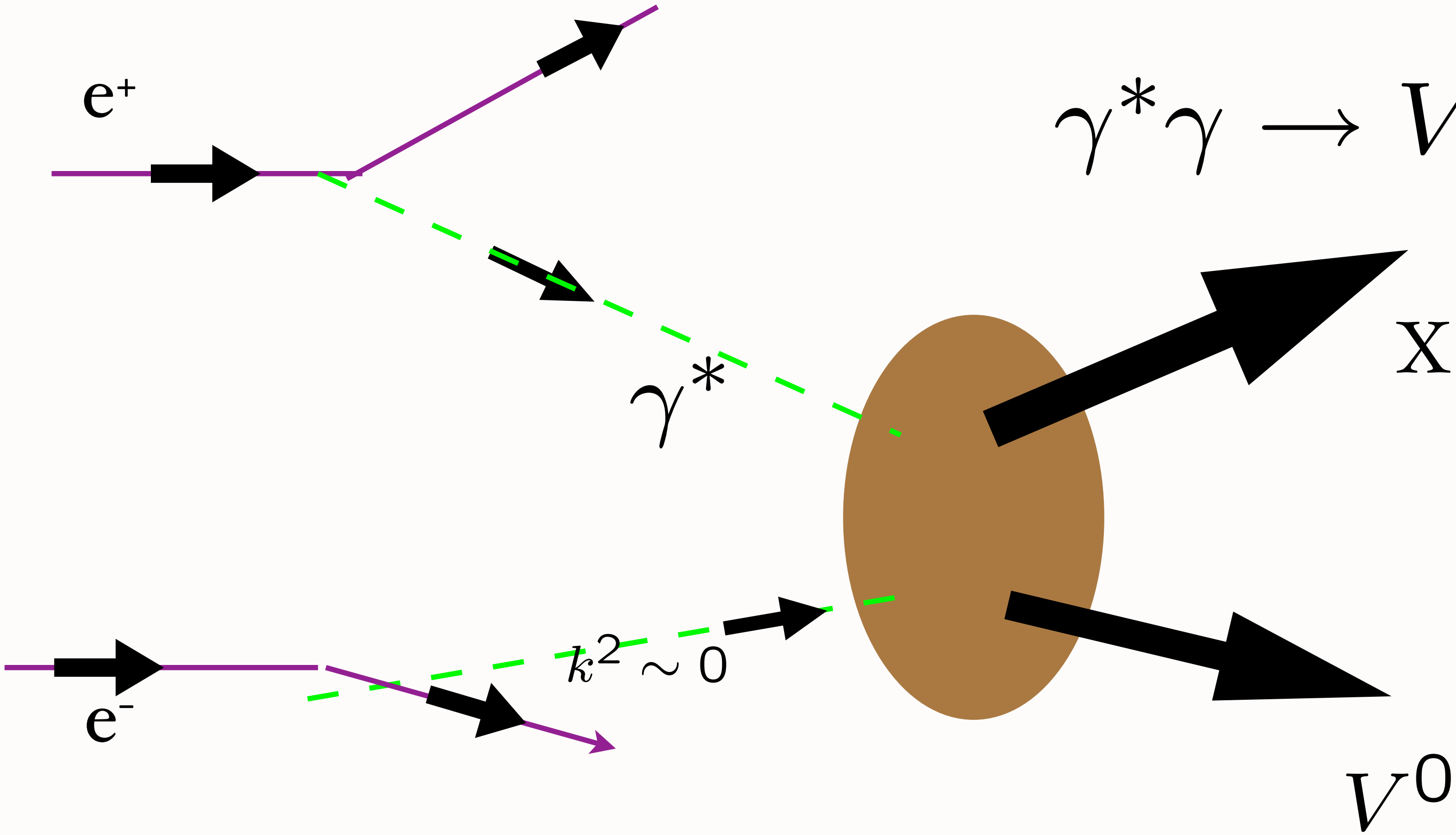
# Photon Structure Function

Witten: Walsh, Zerwas;
Kinoshita, Terazawa, sjb;




$$F_2^{\gamma}(x, q^2)$$

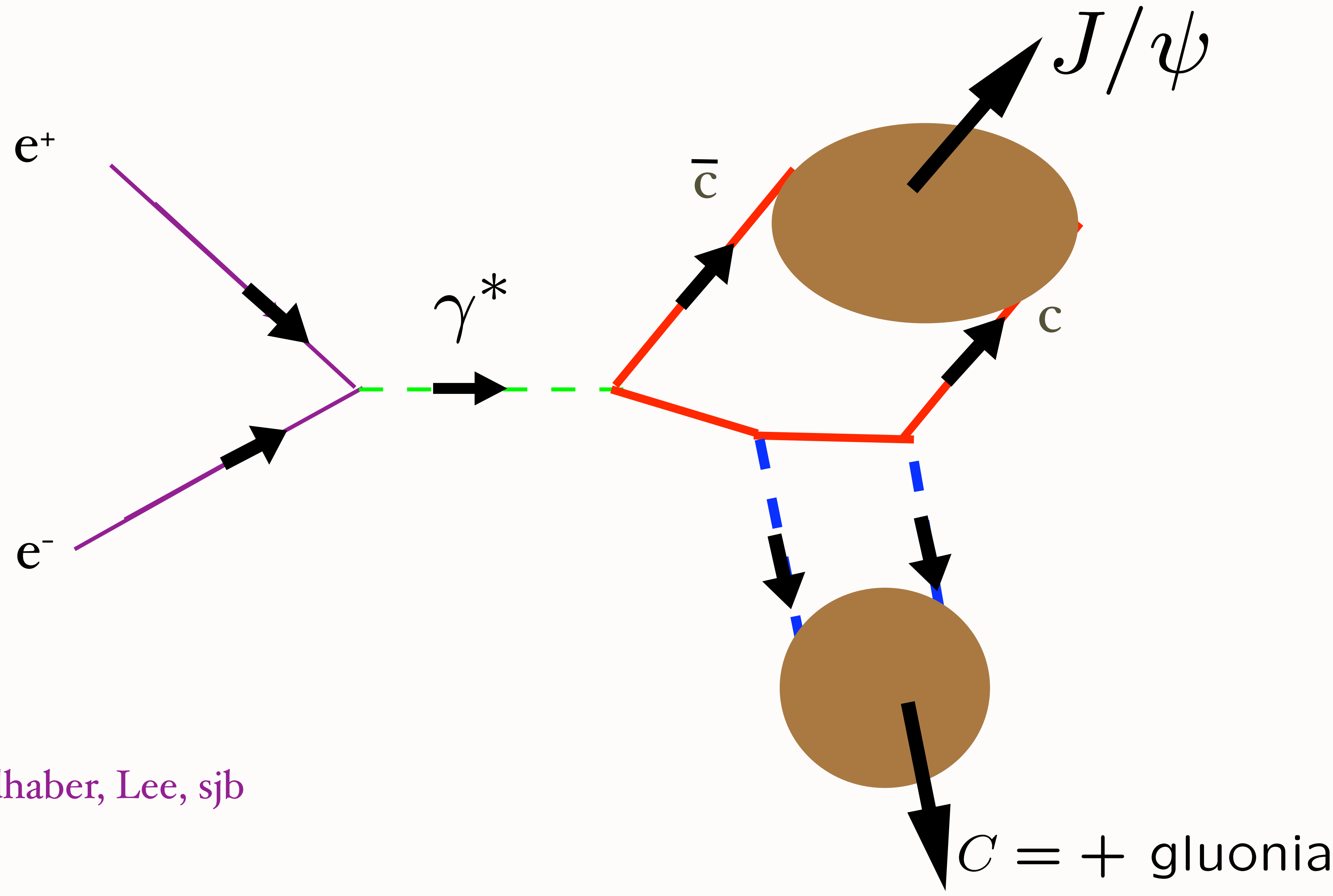
# Photon Diffractive Structure Function




$$\gamma^*\gamma \to V^0 X$$

Diffractive deep inelastic scattering
on a photon target

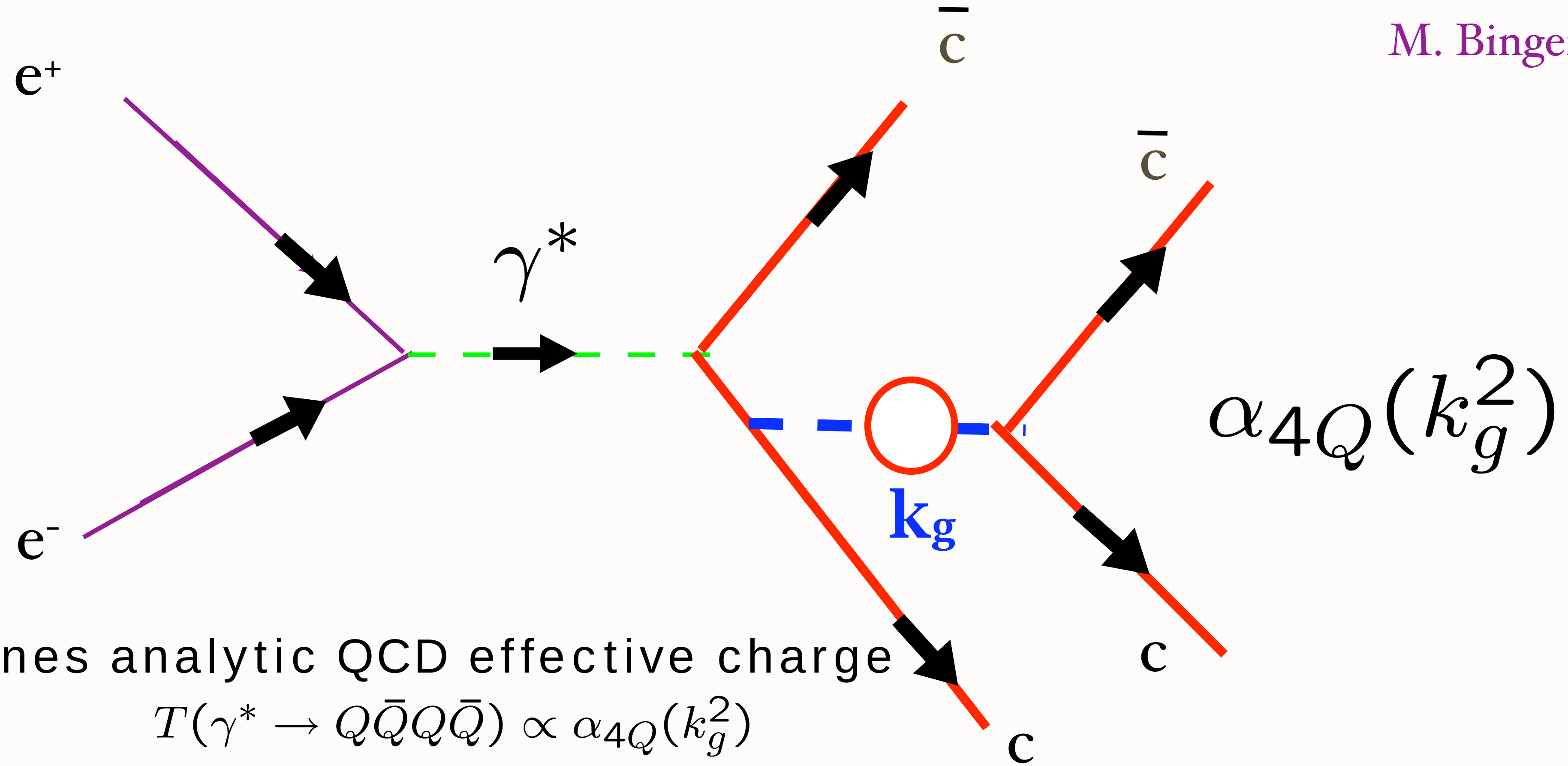
# *Glueball Factory*




Goldhaber, Lee, sjb

# Production of four heavy-quark jets

M. Binger, sjb




Defines analytic QCD effective charge

$$T(\gamma^* \to Q\bar{Q}Q\bar{Q}) \propto \alpha_{4Q}(k_g^2)$$

time-like values not same as space-like

coupling similar to "pinch" scheme

complex for time-like argument

# Define QCD Coupling from Observable

Grunberg

$$R_{e^+e^-\to X}(s) \equiv 3\Sigma_q e_q^2 \ [1 + \frac{\alpha_R(s)}{\pi}]$$

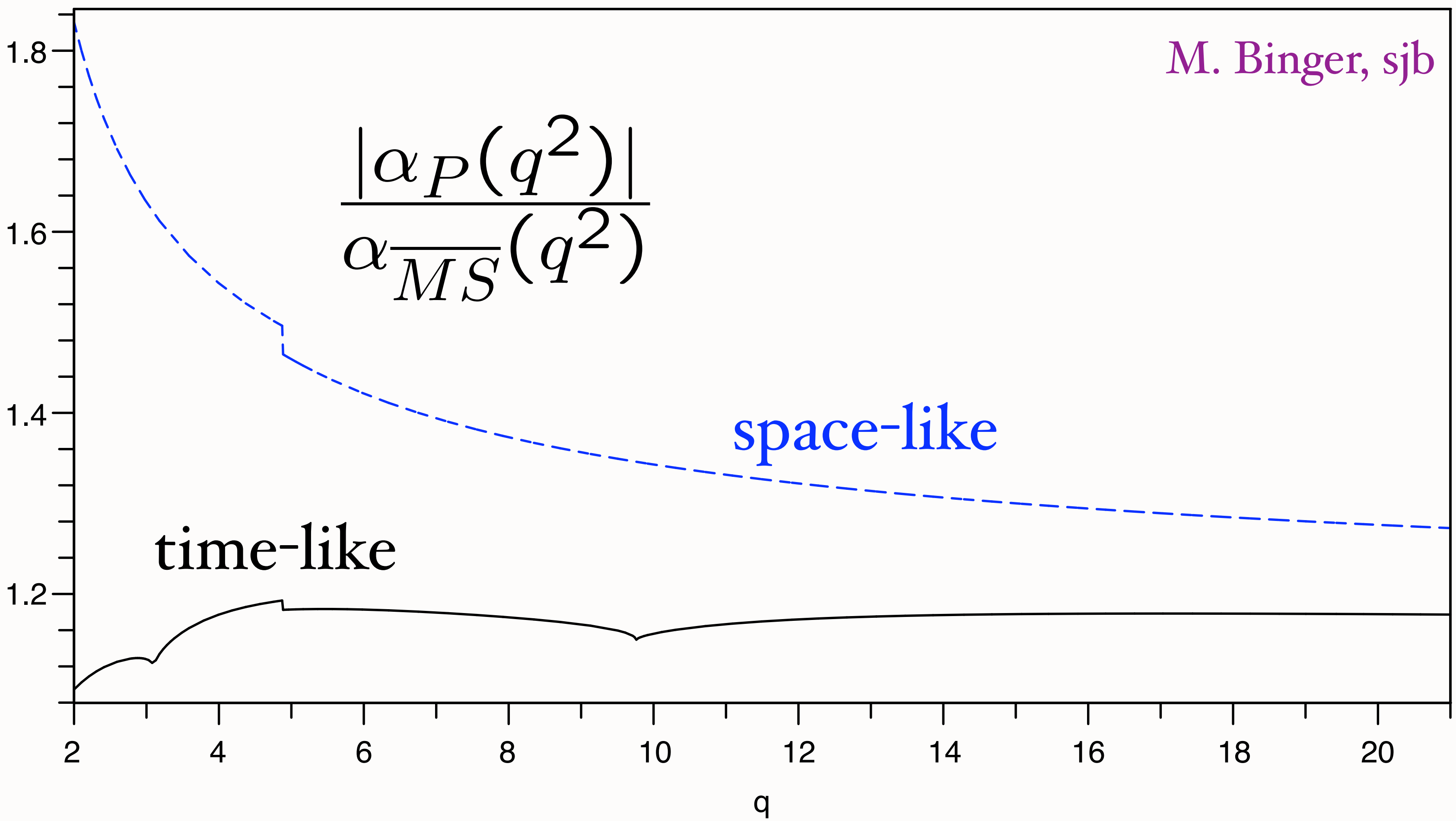
$$\Gamma(\tau \to X e\nu)(m_\tau^2) \equiv \Gamma_0(\tau \to u\bar{d}e\nu)\times[1+\frac{\alpha_\tau(m_\tau^2)}{\pi}]$$

Relate observable to observable at commensurate scales

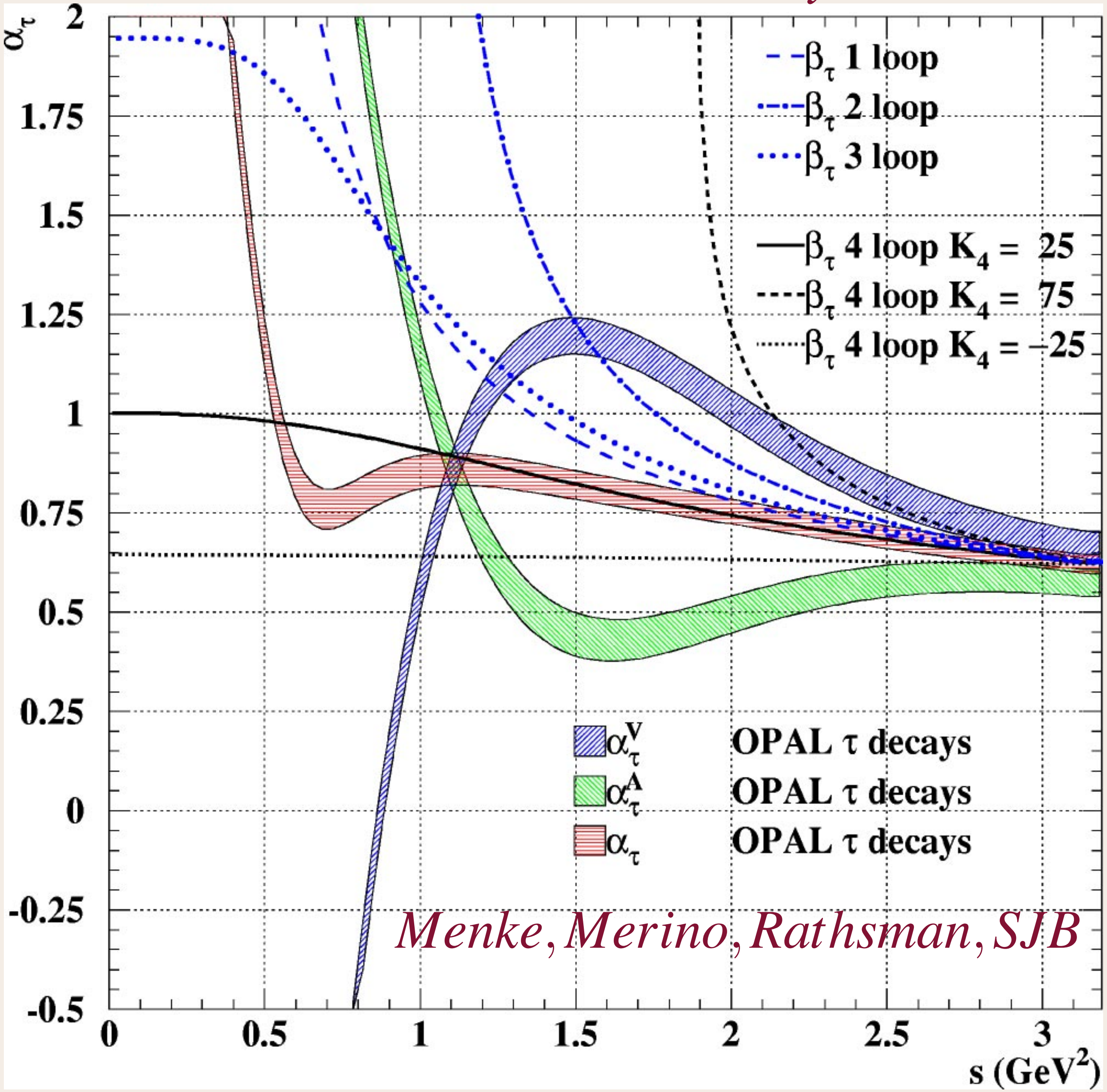
H.Lu, sjb

$\alpha_{\overline{MS}}(q^2)$ is nonanalytic; has same value for spacelike and timelike arguments

$\alpha_P(q^2)$ is analytic; $\alpha_P(q^2)$ is complex-valued for timelike arguments similar to $\alpha_{QED}$



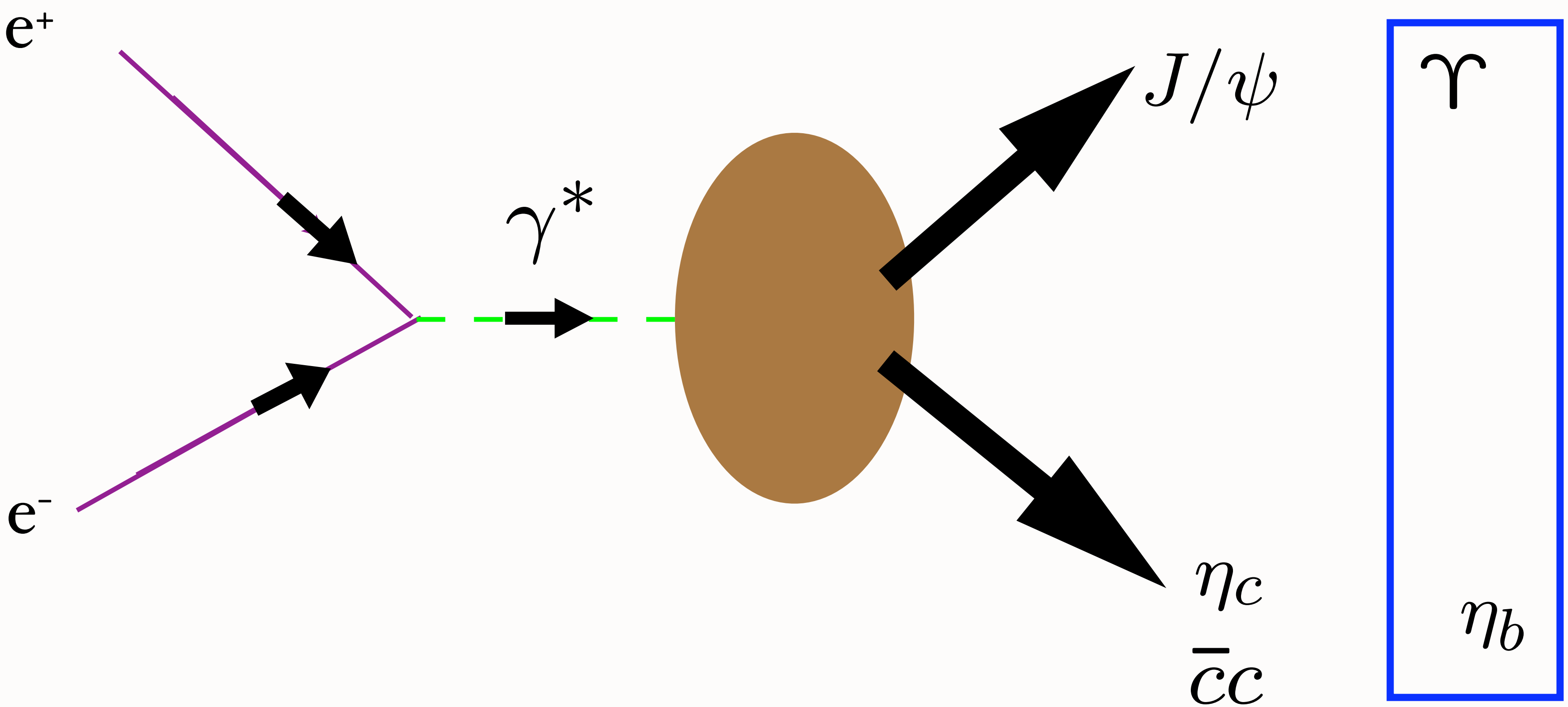
# QCD Effective Coupling from *hadronic τ decay*




*Menke, Merino, Rathsman, SJB*

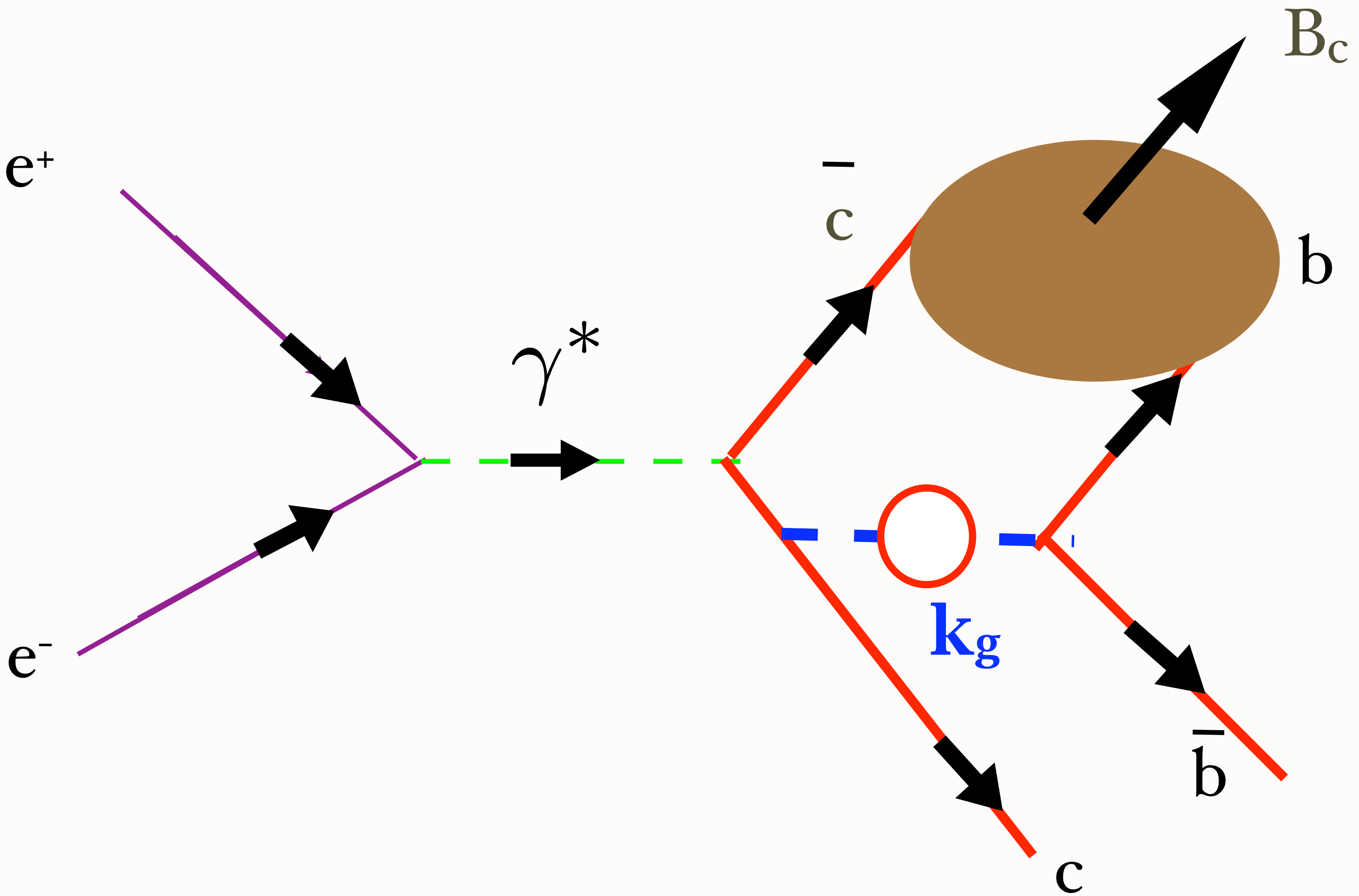
# *Quarkonium Production*

**Test at high energy B factory**




*Cross section appears anomalously high :*
*Challenge to NRQCD, Color-Octet, PQCD*

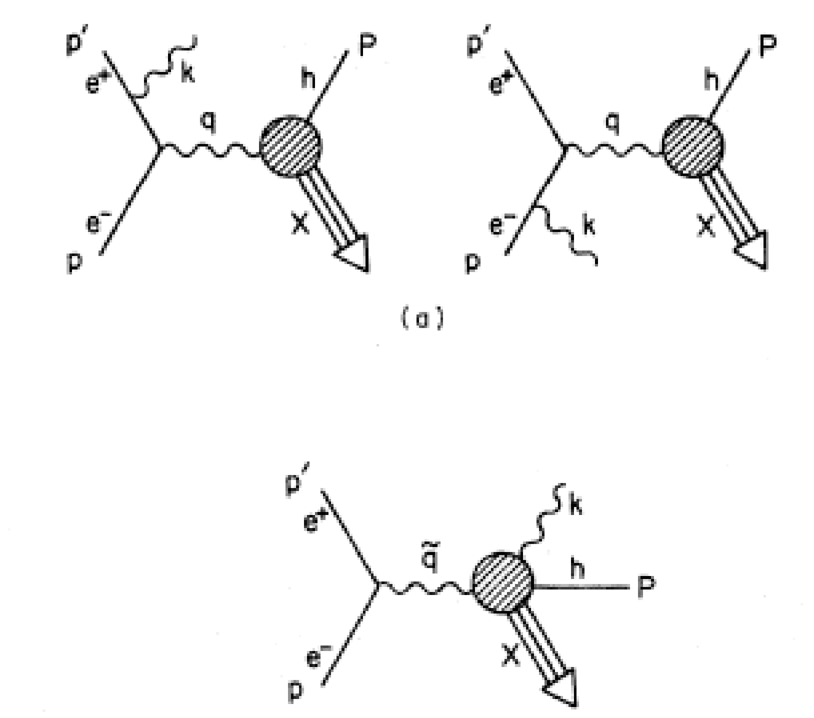
# *Semi-Exclusive Quarkonium*




Related to anomalous magnitude of $\gamma^* \to J/\psi + \eta_c$

**Charge asymmetry in $e^+e^- \to \gamma$+hadrons: New tests of the quark-parton model and fractional charge**

**Phys.Rev.D14:2264-2272,1976** Carlson, Suaya, sjb




*Charge asymmetry measures quark charge cubed*

S. J. Brodsky, C. E. Carlson and R. Suaya,
"Charge Asymmetry In $e^+e^- \to \gamma +$ Hadrons:
New Tests Of The Quark - Parton Model And Fractional Charge,"
Phys. Rev. D **14**, 2264 (1976).

$$R_h^{(3)}(x) \equiv \frac{\Delta_h}{k_0 d\sigma/d^3k\, d\Omega_\mu + k_0 d\sigma/d^3k\, d\Omega_{\bar{\mu}}} = \sum_q \frac{e_q^{\,3}}{e^3}\left[D_q^h(x) - D_{\bar{q}}^h(x)\right].$$

$$\Delta_h \equiv \frac{d\sigma(e^+e^- \to \gamma h X)}{(d^3k/k_0)\, d\Omega_h\, dx} - \frac{d\sigma(e^+e^- \to \gamma \bar{h} X)}{(d^3k/k_0)\, d\Omega_{\bar{h}}\, dx}$$

Charged-Cubed Sum Rule!

*Quantum-number sum rules.* We can define the effective multiplicity of hadron $h$ from fragmentation of quark $q$ as

$$n_q^h = \int_0^1 dx\, D_q^h(x). \tag{10}$$

Then

$$\int_0^1 R_h^{(2)}(x)\, dx = \sum_q \frac{e_q^{\,2}}{e^2}(n_q^h + n_{\bar{q}}^h) = (n_h + n_{\bar{h}}) \sum_q \frac{e_q^{\,2}}{e^2}\,, \tag{11}$$

$$n_h + n_{\bar{h}} = \frac{1}{\sigma}\int_0^1 dx\, \frac{d\sigma}{dx}(e^+e^- \to hX)$$

is the hadron multiplicity in $e^+e^-$ annihilation. The integral of the hadron asymmetry is

$$\int_0^1 dx\, R_h^{(3)}(x) = \sum_q \frac{e_q^{\,3}}{e^3}(n_q^h - n_q^{\bar{h}}). \tag{12}$$

Note that Eq. (12) is convergent because of the absence of the Pomeron contribution.

# Super B: Precision QCD Machine

- Hadronization
- Exotic Spectroscopy
- Subtle Spin Effects: Single spin asymmetries
- Measure Fundamental QCD Coupling
- Exclusive Channels: QCD at Amplitude Level
- Compton Processes
- Hidden Color