The Charm Quark Contribution to the Proton Structure Function

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The charm quark structure function F_2^c and the longitudinal structure function F_l^p is directly sensitive to the gluon content of proton and is therefore crucial in undersatanding of proton structure function, in particular at low momentum transfer Q^2 and low Bjorken x. In the framework pf perturbative QCD the charm structure function is calculated in the leading order (LO) and the proton strcture function is investigated in the next leading order (NLO) at small x region. The valance quark distribution is obtained from the relativistic quark-exchange model.The Calculated $F_2^c(x,Q^2)$, $F_2^p(x,Q^2)$ and $F_L^p(x,Q^2)$, are compared with theb persent avilable experimental data.

1 Valence Quark Distribution

1.1 Quark-Exchange Formalism

Now, let us very briefly discuss the evaluation of valence quark distribution. We assume the nucleon is composed of three valence-quarks in the following way [1]:

$$|\alpha\rangle = \mathcal{N}^{\alpha^\dagger}|0\rangle = \frac{1}{\sqrt{3!}}\mathcal{N}^{\alpha}_{\mu_1\mu_2\mu_3} q^\dagger_{\mu_1} q^\dagger_{\mu_2} q^\dagger_{\mu_3}|0\rangle, \tag{1}$$

where α designate the nucleon states $\{\vec{P}, M_S, M_T\}$ and μ stand for the quark states $\{\vec{k}, m_s, m_t, c\}$. With the convention that there is a summation on the repeated indices as well as integration over $\vec{k}$. $q^\dagger$ ($\mathcal{N}^{\alpha^\dagger}$) are the creation operators for quarks (nucleons) and $\mathcal{N}^{\alpha}_{\mu_1\mu_2\mu_3}$ are the totally antisymmetric nucleon wave functions, i.e.

$$\mathcal{N}^{\alpha}_{\mu_1\mu_2\mu_3} = D(\mu_1,\mu_2,\mu_3;\alpha_i) \times \delta(\vec{k}_1+\vec{k}_2+\vec{k}_3-\vec{P})\phi(\vec{k}_1,\vec{k}_2,\vec{k}_3,\vec{P}). \tag{2}$$

The $D(\mu_1,\mu_2,\mu_3;\alpha_i)$ depend on the Clebsch-Gordon coefficients $C^{j_1 j_2 j}_{m_1 m_2 m}$ and the color factor $\epsilon_{c_1c_2c_3}$,

$$D(\mu_1,\mu_2,\mu_3;\alpha_i) = \frac{1}{\sqrt{3!}}\epsilon_{c_1c_2c_3}\frac{1}{\sqrt{2}}\sum_{s,t=0,1} C^{\frac{1}{2}s\frac{1}{2}}_{m_{s_\sigma}m_s M_{S_{\alpha_i}}} C^{\frac{1}{2}\frac{1}{2}s}_{m_{s_\mu}m_{s_\nu}m_s} C^{\frac{1}{2}t\frac{1}{2}}_{m_{t_\sigma}m_t M_{T_{\alpha_i}}} C^{\frac{1}{2}\frac{1}{2}t}_{m_{t_\mu}m_{t_\nu}m_t} \tag{3}$$

The $\phi(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{P})$ are the nucleon wave functions in terms of quarks and we write it in a Gaussian form (b $\simeq$ nucleon radius) :

$$\phi(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{P}) = \left(\frac{3b^4}{\pi^2}\right)^{\frac{3}{4}} \exp[-b^2(\frac{(k_1^2 + k_2^2 + k_3^2)}{2} + \frac{b^2 P^2}{6}]. \quad (4)$$

We can define the nucleus state based on nucleon creation operators, i.e.

$$|\mathcal{A}_i = 3\rangle = (3!)^{-\frac{1}{2}} \chi^{\alpha_1\alpha_2\alpha_3} \mathcal{N}^{\alpha_1 \dagger} \mathcal{N}^{\alpha_2 \dagger} \mathcal{N}^{\alpha_3 \dagger} |0\rangle, \quad (5)$$

where $\chi^{\alpha_1\alpha_2\alpha_3}$ are the complete antisymmetric nuclear wave functions (they are taken from Faddeev calculations with Reid soft core potential.
The quark momentum distributions with fixed flavour in a three nucleon system are defined as,

$$\rho_{\bar{\mu}}(\vec{k}; \mathcal{A}_i) = \frac{\langle \mathcal{A}_i = 3 | q_{\bar{\mu}}^{\dagger} q_{\bar{\mu}} | \mathcal{A}_i = 3\rangle}{\langle \mathcal{A}_i = 3 | \mathcal{A}_i = 3\rangle}, \quad (6)$$

where the sign bar means no summation on m_t and integration over $\vec{k}$ in the μ indices. By using the above definition, we can calculate the quark momentum distribution for each flavour. In above equation we use, $\chi(x, y, cos\theta)$, the Fourier transform of the nucleus wave function .

1.2 The Q^2 Dependence of Valence Quarks

By considering the relativistic correction, the valence parton distribution at each Q^2 can be related to momentum distribution for each flavour according to the following equation,

$$q^v(x, Q^2; \mathcal{A}_i) = \frac{1}{(1-x)^2} \int \rho_q(\vec{k}; \mathcal{A}_i) \delta(\frac{x}{(1-x)} - \frac{k_+}{M})\, d\vec{k}. \quad (7)$$

After doing the angular integration, we get,

$$q^v(x, Q^2; \mathcal{A}_i) = \frac{2\pi M}{(1-x)^2} \int_{k_{min}}^{\infty} \rho_q(\vec{k}; \mathcal{A}_i) k \; dk \quad (8)$$

with

$$k_{min}(x) = \frac{(\frac{xM}{1-x} + \epsilon_0)^2 - m^2}{2(\frac{xM}{1-x} + \epsilon_0)}, \quad (9)$$

where m (M) is the quark (nucleon) mass , k_+ is the light-cone momentum of initial quark and ϵ_0 is the quark binding energy. The valence quark distribution of a bound nucleon can be derived from the free nucleon valence quark distribution function by using the convolution approximation,

$$q^v(x,Q^2;\mathcal{A}_i) = \sum_N \int q^v(\frac{x}{y_{\mathcal{A}_i}},Q^2;N) f_{N/\mathcal{A}_i}(y_{\mathcal{A}_i})\ dy_{\mathcal{A}_i}, \tag{10}$$

where $f_{N/\mathcal{A}_i}(y_{\mathcal{A}_i})$ are the nucleon momentum distributions in the nucleus. By taking into account the fact that $f_{N/\mathcal{A}_i}(y_{\mathcal{A}_i})$ are large only around $\frac{x}{\langle y_{\mathcal{A}_i}\rangle}$ we can write [23]

$$\Delta q^v(\frac{x}{\langle y_{\mathcal{A}_i}\rangle},Q^2;N) = \Delta q^v(x,Q^2;\mathcal{A}_i) \tag{11}$$

with $\langle y_{\mathcal{A}_i}\rangle = 1+\frac{\bar{\epsilon}}{M}$ and $\bar{\epsilon}$ being the average removal energy of the nucleon. A typical ansatz for the parton distribution is usually parameterized as [15,18,19],

$$x\ \mathcal{P}(x,Q^2) = A_{\mathcal{P}}\eta_{\mathcal{P}}x^{a_{\mathcal{P}}}(1-x)^{b_{\mathcal{P}}}(1+\gamma_{\mathcal{P}}x+\varrho_{\mathcal{P}}x^{\frac{1}{2}}) \tag{12}$$

where $A_{\mathcal{P}}$ the normalization factor is given by,

$$A_{\mathcal{P}}^{-1} = (1+\gamma_{\mathcal{P}}\frac{a_{\mathcal{P}}}{a_{\mathcal{P}}+b_{\mathcal{P}}+1}Beta(a_{\mathcal{P}}+b_{\mathcal{P}}-1,b_{\mathcal{P}}+1))+\varrho_{\mathcal{P}}Beta(a_{\mathcal{P}}+b_{\mathcal{P}}-\frac{1}{2},b_{\mathcal{P}}+1) \tag{13}$$

2 NLO Evolution Procedure

In the NLO, $F_2^p(x,Q^2)$ is related to the quark, antiquark and gluon distributions, as follows:

$$F_2^p(x;Q^2) = x\sum_{q=u,d,s} e_q^2\{q(x,Q^2)+\bar{q}(x,Q^2)\}$$

$$+\frac{\alpha_s(Q^2)}{2\pi}[C_q\otimes(q(x,Q^2)+\bar{q}^N(x,Q^2)+2C_g\otimes G(x,Q^2)]+F_2^c(x,Q^2,m_c^2), \tag{14}$$

where $\otimes$ means the convolution and it is defined as,

$$C_{\mathcal{P}}\otimes\mathcal{P} = \int_x^1 \frac{dy}{y}C_{\mathcal{P}}(\frac{x}{y})\mathcal{P}(y,Q^2). \tag{15}$$

The charm quark contribution to the proton structure function, $F_2^c(x,Q^2,m_c^2)$ has the following form in the LO limit, if $\frac{1}{x}\geq 1+(\frac{2m_c}{Q})^2$, (note that for small x, with this condition Q^2 can become less than m_c^2, also see Roberts [1]),

$$F_2^c(x, Q^2, m_c^2) = 2xe_c^2 \frac{\alpha_s(\mu'^2)}{2\pi} \int_{ax}^{1} \frac{dy}{y} C_g^c(\frac{x}{y}, (\frac{m_c}{Q})^2) g(y, \mu'^2), \tag{16}$$

where $a = 1 + 4\frac{m_c^2}{Q^2}$ and (n_f is the number of active flavours)

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda^2)} - \frac{4\pi\beta_1}{\beta_0^3} \frac{\ln\ln(Q^2/\Lambda^2)}{\ln(Q^2/\Lambda^2)} \tag{17}$$

with ($\Lambda_{\overline{MS}} = 200MeV$)

$$\beta_0 = 11 - \frac{2}{3}n_f, \quad \beta_1 = 102 - \frac{38}{3}n_f. \tag{18}$$

Similar to $F_2^p(x; Q^2)$ we can calculate the longitudinal SF, $F_L^p(x; Q^2)$:

$$F_L^p(x; Q^2) = x\frac{\alpha_s(Q^2)}{2\pi} \sum_{q=u,d,s} [C_{q,L} \otimes (q(x, Q^2) + \bar{q}^N(x, Q^2) + 2C_{g,L} \otimes G(x, Q^2)]$$

$$+F_L^c(x, Q^2, m_c^2) \tag{19}$$

with

$$C_{q,L} = \frac{8}{3}z, \quad C_{g,L} = 2z(z-1). \tag{20}$$

For $F_L^c(x, Q^2, m_c^2)$ we use the LO expression, which is identical to equation (3) i.e.

$$F_L^c(x, Q^2, m_c^2) = 2xe_c^2 \frac{\alpha_s(\mu'^2)}{2\pi} \int_{ax}^{1} \frac{dy}{y} C_{g,L}^c(\frac{x}{y}, (\frac{m_c}{Q})^2) g(y, \mu'^2), \tag{21}$$

Here we assume the SU(3) flavour-symmetric sea quark distributions $\bar{q} = \bar{u} = \bar{d} = \bar{s} = s$. In addition we consider the sea quark and gluon contributions to vanish in the static point $\mu_0^2 \ll Q^2$, (we set $\mu_0^2 = 0.32GeV^2$),i.e,

$$G(x, \mu_0^2) = 0 \qquad \bar{q}(x, \mu_0^2) = 0. \tag{22}$$

Finally we use the Mellin and inverse Mellin transformation to calculate the NLO parton distributions in the (x, Q^2)-plane.

3 Results and discussions

In figure 1, we present the charm quark contribution to the SF of proton at (a) $Q^2 = 6GeV^2$ and (b) $Q^2 = 10GeV^2$ with different charm mass values i.e.

$m_c = 1.1 GeV$ (small-dash curve), $m_c = 1.2 GeV$ (dash-small-dash curve), $m_c = 1.3 GeV$(full curve) and $m_c = 1.4 GeV$ (dash curve). The data are those of H1 [2] and ZEUS [3] collaborations , i.e. the squares (ZEUS,1997), circles (H1,2000), diamonds (ZEUS,2000) and triangles (ZEUS,2004). Our results are in very good agreement with the present available data. By reducing the charm mass, the charm structure function of proton increases but it still passes though the data. It also becomes zero for $x > 0.1$. Obviously the structure function becomes larger as we increase Q^2 (by comparing figures 2(a) and 2(b) in agreement with the data). The calculated $F_2^c(x, Q^2)$ by using the gluon distribution of GRV (heavy-full curves) with $m_c = 1.3$ show less charm quark contribution to the SF of proton.
Figures 2(a) and 2(b) show the comparison of Q^2 dependence of charm contribution to the SF of proton for various x values with the available ZEUS data (the triangles (1997), circles (2000) and squares (2004)). We get a reasonable result with respect to the data [2].
The Q^2 dependence of longitudinal SF of proton are given in figures 3(a) and 3(b). The data are from H1 collaboration experiments [3]:the circles (H1,2001) and triangles (H1,1996). Again there are good agreement between our calculated results and the experimental predictions. These show that our gluon distribution can reasonably well treat the PGF process.

References

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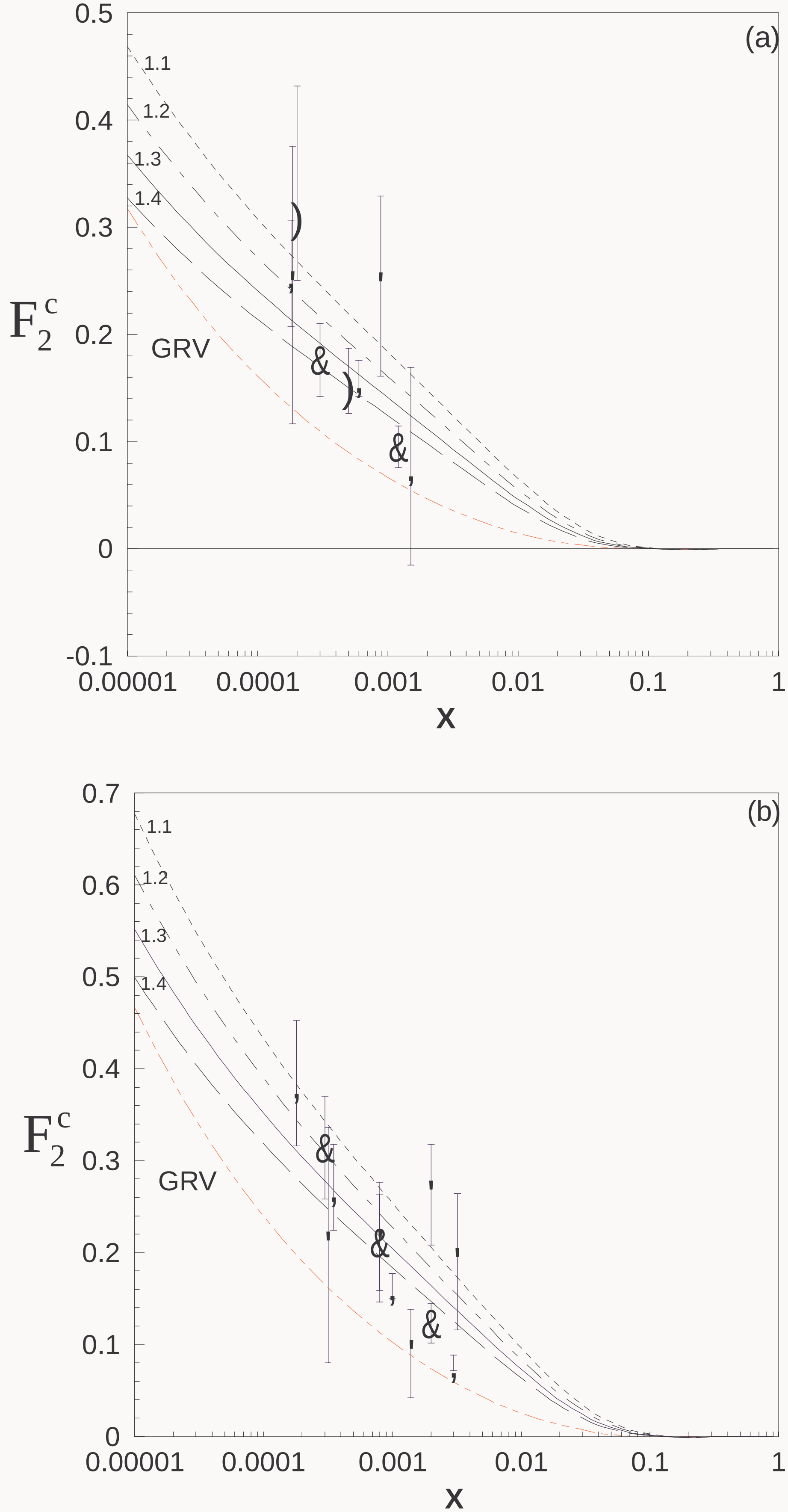


(a)
1.1
1.2
1.3
1.4
GRV
F_2^c
x
0.5
0.4
0.3
0.2
0.1
0
-0.1
0.00001
0.0001
0.001
0.01
0.1
1
(b)
1.1
1.2
1.3
1.4
GRV
F_2^c
x
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.00001
0.0001
0.001
0.01
0.1
1

FIG. 1

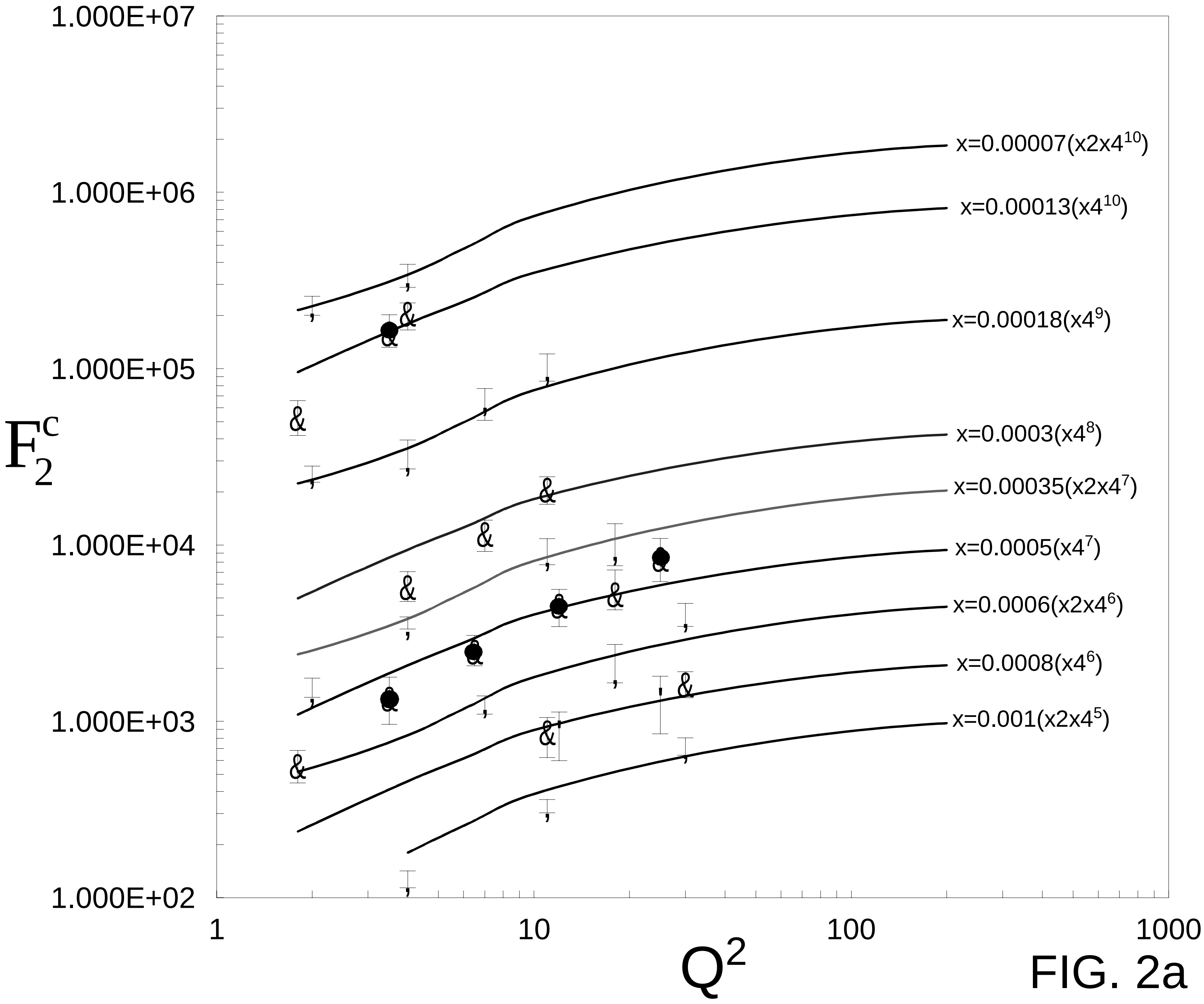
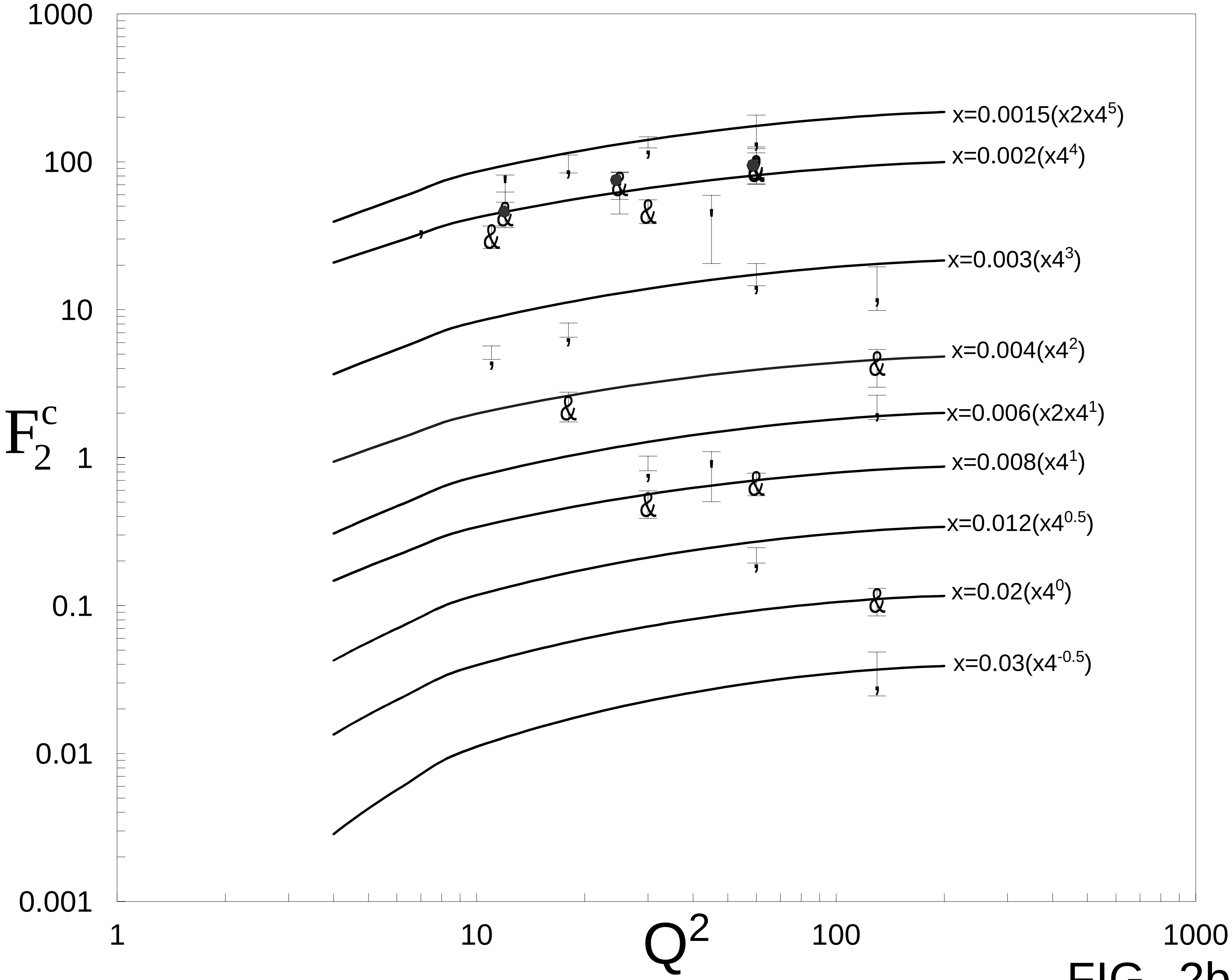

1.000E+07
1.000E+06
1.000E+05
1.000E+04
1.000E+03
1.000E+02
F_2^c
1
10
100
1000
Q^2
x=0.00007(x2x4^10)
x=0.00013(x4^10)
x=0.00018(x4^9)
x=0.0003(x4^8)
x=0.00035(x2x4^7)
x=0.0005(x4^7)
x=0.0006(x2x4^6)
x=0.0008(x4^6)
x=0.001(x2x4^5)

FIG. 2a



1000
100
10
1
0.1
0.01
0.001
1
10
100
1000
F_2^c
Q^2
x=0.0015(x2x4^5)
x=0.002(x4^4)
x=0.003(x4^3)
x=0.004(x4^2)
x=0.006(x2x4^1)
x=0.008(x4^1)
x=0.012(x4^0.5)
x=0.02(x4^0)
x=0.03(x4^-0.5)

FIG. 2b

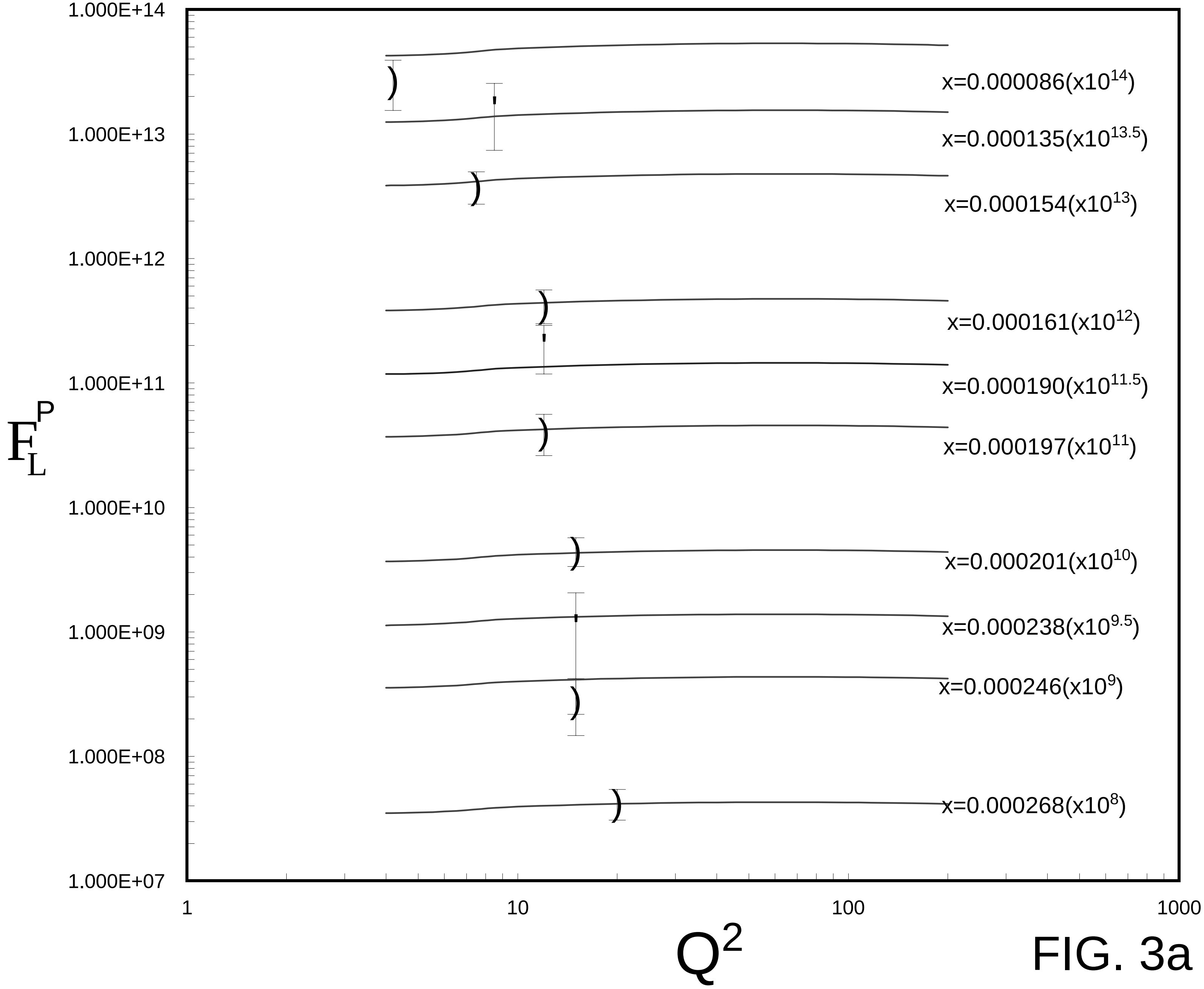

1.000E+14
1.000E+13
1.000E+12
1.000E+11
1.000E+10
1.000E+09
1.000E+08
1.000E+07
F_L^P
x=0.000086(x10^{14})
x=0.000135(x10$^{13.5}$)
x=0.000154(x10^{13})
x=0.000161(x10^{12})
x=0.000190(x10$^{11.5}$)
x=0.000197(x10^{11})
x=0.000201(x10^{10})
x=0.000238(x10$^{9.5}$)
x=0.000246(x10^{9})
x=0.000268(x10^{8})
1
10
100
1000
Q^2

FIG. 3a

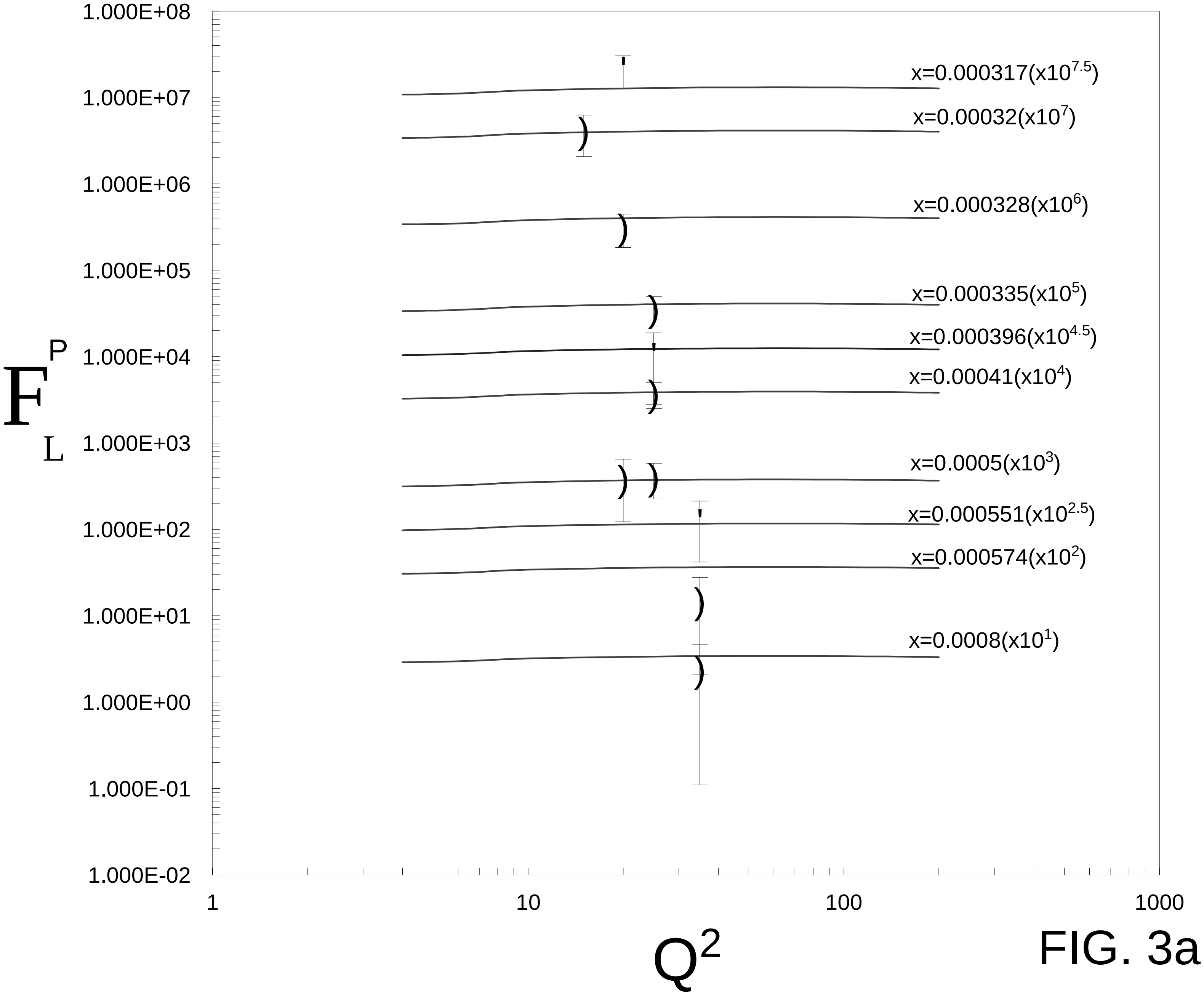

1.000E+08
1.000E+07
1.000E+06
1.000E+05
1.000E+04
1.000E+03
1.000E+02
1.000E+01
1.000E+00
1.000E-01
1.000E-02
1
10
100
1000
x=0.000317(x10^7.5)
x=0.00032(x10^7)
x=0.000328(x10^6)
x=0.000335(x10^5)
x=0.000396(x10^4.5)
x=0.00041(x10^4)
x=0.0005(x10^3)
x=0.000551(x10^2.5)
x=0.000574(x10^2)
x=0.0008(x10^1)
F_L^P
Q^2

FIG. 3a