Dynamics of $O(N)$ model in a strong magnetic field

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Based on

- *A. Jafari Salim and N.S.; Phys. Rev. D 73, 065023 (2006)*

Also on

- *Phys. Rev. D72, 065014 (2005)*
- *Phys. Lett. B611, 207 (2005)*
- *Phys. Rev. D70, 105007 (2004)*

by V.A. Miransky et al.

I. Dynamics of NJL model in a strong magnetic field as a modified noncommutative field theory

$\hookrightarrow$ *V.A. Miransky et al.; Phys. Rev. D70, 105007 (2004)*

I. Modified noncommutative field theory

- The NJL model consists of two composites σ and π
- In a strong magnetic background field, *i.e.* in the regime of LLL dominance , the low energy effective action of σ and π is, apart from a certain modification, comparable with the action of the ordinary NCFT
- Modification: In contrast to the ordinary NCFT a natural cutoff appears here ▸▸ **NO** UV / IR mixing occurs here
- Same phenomena observed in QED

  ↪ *Phys. Rev. D72, 065014 (2005)*

  ↪ *Phys. Lett. B611, 207 (2005)*

II. Noncommutative (NC) field theory

II. Noncommutative (NC) field theory

- Noncommutativity of space-time ▸ $[x_i, x_j] = i\Theta_{ij}$

  ▸ Noncommutative Moyal $\star$-product

$$f(x) \star g(x) \equiv f(x+\xi) \exp\left(i\Theta^{\mu\nu}\partial_\mu^\xi \partial_\nu^\zeta\right) g(x+\zeta)\Big|_{\xi=\zeta=0}$$

- Action of NC-U(1)

$$\mathcal{S}[A_\mu, \bar{\psi}, \psi] = -\frac{1}{4}\int d^4x\, F_{\mu\nu} \star F^{\mu\nu} + \int d^4x\, \bar{\psi}(x) \star (i\not{D} - m)\, \psi(x)$$

  ▸ Field strength tensor

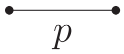
$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu + ig\left[A_\mu, A_\nu\right]_\star$$

  ▸ Covariant derivative

$$D_\mu \psi(x) \equiv \partial_\mu \psi(x) + igA_\mu(x) \star \psi(x)$$

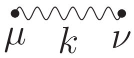
Perturbative aspects

- Fermion propagator (not modified)



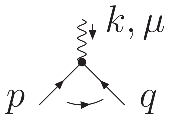
$$S(p) = \frac{i}{\not{p} - m}$$

- Photon propagator ($\xi = 1$) (not modified)



$$D_{\mu\nu}(k) = \frac{-ig_{\mu\nu}}{k^2}$$

- Vertex (modified by a momentum dependent phase factor)



$$V_\mu = (2\pi)^4\, \delta^4(p+q+k)\, ig\gamma_\mu \exp\left(i\Theta_{\eta\sigma} p^\eta q^\sigma\right)$$

  - Consequence: Theory consists of planar and nonplanar Feynman integrals
  - The nonplanar integral exhibits a certain UV / IR mixing

UV / IR mixing

- Ordinary NCFT $\ni$ New scale $|\Theta p|$
- For finite $|\Theta p|$ ▸▸ Familiar UV divergent integrals are regulated
- BUT, $|\Theta p| \to 0$ limit ▸▸ New IR poles appears

Equivalently: The limit $|\Theta p| \to 0$ is singular

- UV limit $\Lambda \to \infty$ and IR limit $|\Theta p| \to 0$ do not commute ▸▸ UV / IR mixing

Consequence

- Noncommutative gauge theories are non-renormalizable

Miransky's Setup: In contrast to the ordinary NCFT a natural cutoff appears
▸▸ **NO** UV / IR mixing occurs

III. Dynamics of $O(N)$-model in a strong magnetic field

$\hookrightarrow$ A. Jafari Salim and N.S.; Phys. Rev. D 73, 065023 (2006)

- In the presence of a magnetic background field this model consists of a condensate
- Look for:
  - Effective action of this condensate in the LLL approximation
  - Effective action of the composite field from the world-line formalism (new method)
- Comparison with the ordinary NCFT

$O(N)$-Model

- Lagrangian density

$$\mathcal{L} = -|D_\mu \Phi|^2 - m^2 \Phi^* \Phi - \frac{1}{2}\lambda(\Phi^*\Phi)^2,$$

with $\Phi = (\varphi_1, \cdots, \varphi_N)$ and $D_\mu \Phi = \partial_\mu \Phi + igA_\mu \Phi$

- Introducing
  - Composite field $\sigma \equiv \lambda \Phi^* \Phi$
  - 't Hooft Coupling $g \equiv \lambda N$
- Lagrangian density in terms of σ

$$\mathcal{L} = -|D_\mu \Phi|^2 - m^2 \Phi^* \Phi - \sigma \Phi^* \Phi + \frac{N}{2g}\sigma^2$$

- In a strong magnetic field a condensate is built even in $N = 1$

$$\langle \varphi^*(x) \varphi(x) \rangle \sim |eB|$$

- Look for the effective action for the composite σ

Effective action for the composite σ

- Take the Lagrangian density in terms of the composite field σ

$$\mathcal{L} = -|D_\mu \Phi|^2 - m^2 \Phi^* \Phi - \sigma \Phi^* \Phi + \frac{N}{2g}\sigma^2$$

- Integrating out Φ^* and Φ ▸ Effective action for σ

$$\begin{aligned} e^{\widetilde{\Gamma}[\sigma]} &= \int \mathcal{D}\Phi^* \, \mathcal{D}\Phi \, \exp\left(-\int d^4x \left\{\Phi^*[-D_\mu D^\mu + m^2 + \sigma]\Phi + \frac{N}{2g}\,\sigma^2\right\}\right) \\ &= \exp\left(-\text{tr}\ln[-D_\mu D^\mu + m^2 + \sigma] + \frac{N}{2g}\int d^4x\,\sigma^2\right), \end{aligned}$$

- The full effective action

$$\widetilde{\Gamma}[\sigma] = \Gamma[\sigma] + \frac{N}{2g}\int d^4x\,\sigma^2$$

$$\Gamma[\sigma] = -\text{tr}\ln\left(-D_\mu D^\mu + m^2 + \sigma\right)$$

$$D_\mu = \partial_\mu + igA_\mu$$

III.A. Effective action for n composites σ in the LLL approximation

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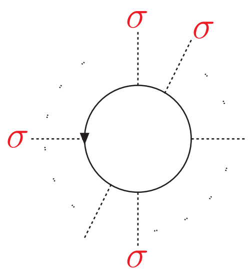
- Full effective action for σ

$$\Gamma[\sigma] \equiv \sum_{n=0}^{\infty} \frac{1}{n!}\Gamma_n[\sigma];$$



$$= \Gamma_n[\sigma]$$

- Effective action for n composite fields σ

$$\Gamma_n[\sigma] = \int d^4x_1 \cdots d^4x_n \left[\sigma(x_1)G(x_1,x_2)\sigma(x_2)G(x_2,x_3)\cdots\sigma(x_n)G(x_n,x_1)\right]$$

Here: $G(x_i, x_j)$ is the full propagator between the insertion points x_i and x_j

► In the presence of electromagnetic field A_μ

Aim

- Effective action for n composite fields σ in the LLL approximation

$$\Gamma_n[\sigma] = \int d^4x_1 \cdots d^4x_n \left[\sigma(x_1)G(x_1,x_2)\sigma(x_2)G(x_2,x_3)\cdots\sigma(x_n)G(x_n,x_1)\right]$$

- Compare $\Gamma_n[\sigma]$ with the action of NCFT for σ

Strategy

- ▸ Use Schwinger proper time formalism ▸ Full propagator $G(x_i, x_j)$
  - ▸ Constant magnetic field
- ▸ Determine $G(x_i, x_j)$ in the LLL approximation

▸▸ Effective action for n-composites in the LLL approximation

Full propagator from Schwinger proper time formalism

- Schwinger propagator

$$G(x', x'') = P(x', x'')D(x' - x'')$$

  - Schwinger line integral

$$P(x', x'') \equiv \exp\left(ie\int_{x''}^{x'} d\xi^{\mu} A_{\mu}(\xi)\right)$$

  - Translational invariant part

$$D(x' - x'') \equiv \frac{1}{(4\pi)^2}\int_0^{\infty}\frac{ds}{s^2}e^{-ism^2}\exp\left(\frac{-1}{2}\mathrm{tr}\ln\left[\frac{\sinh eFs}{eFs}\right]\right)\exp\left(-\frac{i}{4}(x' - x'')eF\coth(eFs)(x' - x'')\right)$$

- Fixing the gauge (symmetric gauge for $\vec{B} = B\hat{e}_3$)

$$A_{\mu} = \frac{B}{2}(0, x_2, -x_1, 0)$$

►►►

- Schwinger line integral

$$P(x',x'') = e^{\frac{ieB}{2}\epsilon^{ab}x'_a x''_b}, \qquad a,b=1,2$$

- Translational invariant part in momentum space

$$\tilde{D}(k) = -\frac{1}{eB}\int_0^\infty \frac{ds}{\cosh s}\, e^{-s\rho-\alpha\tanh(s)} \qquad \left\{\begin{array}{ccc} \mathbf{k}_\parallel^2 & \equiv & k_0^2 - k_3^2 \\ \mathbf{k}_\perp^2 & \equiv & k_1^2 + k_2^2 \\ \alpha & \equiv & \mathbf{k}_\perp^2/|eB| \\ \rho & \equiv & \left(m^2 - \mathbf{k}_\parallel^2\right)/|eB| \end{array}\right.$$

- Full propagator

$$G(x',x'') = e^{\frac{ieB}{2}\epsilon^{ab}x'_a x''_b} \int \frac{d^4k}{(2\pi)^4}\widetilde{D}(k)\, e^{-ik(x'-x'')}$$

- To determine Schwinger propagator in LLL approximation ▸▸▸

Schwinger propagator in LLL approximation

- $$\tilde{D}(k) = \frac{e^{-\alpha}}{2|eB|}\sum_{n'=0,m'=1}^{\infty}(-1)^{n'}L_{n'}^{(-1)}(2\alpha)\left(\frac{\rho+2n'-3}{m'(4m'+\rho+2n'-3)}-\frac{\rho+2n'-1}{m'(4m'+\rho+2n'-1)}\right)$$

  $L_{n'}^{\beta}$ generalized Laguerre polynomial, m' is related to the expansion of a digamma function $\psi(z)$

- To determine (m', n') related to LLL form of $\widetilde{D}(k)$:
  - Energy spectrum of a KG particle in an external magnetic field

    $$E_{\ell'} = \sqrt{m^2 + |eB|(2\ell'+1) + k_3^2}, \qquad \ell' = 0, 1, 2, \cdots, \infty$$

  - E_0 from this energy spectrum

    $\equiv$

    Poles of the propagator

  - ▸▸ LLL energy $\ell' = 0$ ▸ $m' = 1$ and $n' = 0$ in $\widetilde{D}(k)$

- LLL propagator

- LLL propagator

$$G(x', x'') = G_{\perp}(\mathbf{x}'_{\perp}, \mathbf{x}''_{\perp}) G_{\parallel}(\mathbf{x}'_{\parallel} - \mathbf{x}''_{\parallel}),$$

  - The transverse part ▸ $\mathbf{x}_{\perp} = (x_1, x_2)$

$$G_{\perp}(\mathbf{x}'_{\perp}, \mathbf{x}''_{\perp}) = \frac{|eB|}{2\pi} e^{\frac{ieB}{2}\epsilon^{ab} x'_a x''_b} \exp\left(-\frac{|eB|}{4}(\mathbf{x}'_{\perp} - \mathbf{x}''_{\perp})^2\right)$$

  - The longitudinal part ▸ $\mathbf{x}_{\parallel} = (x_0, x_3)$

$$G_{\parallel}(\mathbf{x}'_{\parallel} - \mathbf{x}''_{\parallel}) = \int \frac{d^2 k_{\parallel}}{(2\pi)^2} \frac{1}{\mathbf{k}_{\parallel}^2 - (m^2 + |eB|)} e^{i\mathbf{k}_{\parallel} \cdot (\mathbf{x}' - \mathbf{x}'')_{\parallel}}$$

- n-point vertex function for n-composites σ in LLL approximation

Result

- n-point contribution to the effective action in the LLL approximation

$$\Gamma_n[\sigma] = 2\pi N|eB| \int d^2\mathbf{x}_{1\parallel}\cdots d^2\mathbf{x}_{n\parallel} \frac{d^4p_1}{(2\pi)^4}\cdots\frac{d^4p_n}{(2\pi)^4}$$
$$\times\delta^2\left(\sum_{i=1}^{n}\mathbf{p}_{i\perp}\right)\exp\left(-\frac{1}{4|eB|}\sum_{i=1}^{n}\mathbf{p}_{i\perp}^2\right)\exp\left(-\frac{i}{2}\sum_{i<j=1}^{n}\mathbf{p}_i\times\mathbf{p}_j\right)$$
$$\times\exp\left(i\sum_{i=1}^{n}\mathbf{p}_{i\parallel}\cdot\mathbf{x}_{i\parallel}\right)\left[\sigma(p_1)G_\parallel(\mathbf{x}_{1\parallel},\mathbf{x}_{2\parallel})\sigma(p_2)G_\parallel(\mathbf{x}_{2\parallel},\mathbf{x}_{3\parallel})\cdots\sigma(p_n)G_\parallel(\mathbf{x}_{n\parallel},\mathbf{x}_{1\parallel})\right]$$

Here $\mathbf{p}_i\times\mathbf{p}_j \equiv \epsilon_{ab}p_i^a p_j^b$ and $a,b=1,2$

- Full effective action in the LLL approximation

$$\Gamma[\sigma] \equiv \sum_{n=0}^{\infty}\frac{1}{n!}\Gamma_n[\sigma]$$

III.B. Effective action for σ from worldline formalism

$\hookrightarrow$ *Phys. Rev. D73, 065023 (2006) for more details*

Comparing two results

- 1PI n-point function for n composites σ from worldline formalism

$$\Gamma_{1\mathrm{PI}}[p_1,\cdots,p_n] = N|eB|(-1)^n(2\pi^2)\,\delta\left(\sum_{i=1}^{n} p_i\right) \times \exp\left(-\frac{1}{4|eB|}\sum_{i=1}^{n}\mathbf{p}_{i\perp}^2\right)\exp\left(-\frac{i}{2}\sum_{i<j=1}^{n}\mathbf{p}_i\times\mathbf{p}_j\right) \times \int_0^\infty \frac{dT}{T^{2-n}} e^{-(m^2+|eB|)T}\prod_{i=1}^{n}\int_0^1 du_i \exp\left(\sum_{i<j=1}^{n}[G_{Bij}\,g_\|]\,p_i\cdot p_j\right)$$

- Effective action for n composites σ in the LLL approximation

$$\Gamma_n[\sigma] = 2\pi N|eB|\int d^2\mathbf{x}_{1\|}\cdots d^2\mathbf{x}_{n\|}\frac{d^4p_1}{(2\pi)^4}\cdots\frac{d^4p_n}{(2\pi)^4}\delta^2\left(\sum_{i=1}^{n}\mathbf{p}_{i\perp}\right) \times \exp\left(-\frac{1}{4|eB|}\sum_{i=1}^{n}\mathbf{p}_{i\perp}^2\right)\exp\left(-\frac{i}{2}\sum_{i<j=1}^{n}\mathbf{p}_i\times\mathbf{p}_j\right) \times \exp\left(i\sum_{i=1}^{n}\mathbf{p}_{i\|}\cdot\mathbf{x}_{i\|}\right)\left[\sigma(p_1)G_\|(\mathbf{x}_{1\|},\mathbf{x}_{2\|})\sigma(p_2)G_\|(\mathbf{x}_{2\|},\mathbf{x}_{3\|})\cdots\sigma(p_n)G_\|(\mathbf{x}_{n\|},\mathbf{x}_{1\|})\right]$$

IV. Modified noncommutative $O(N)$-model

Ordinary noncommutative field theory

- General structure of an n-point vertex

$$\int d^D x\, \overbrace{\phi(x) \star \cdots \star \phi(x)}^{n\text{-times}} =$$

$$= \int \frac{d^D p_1}{(2\pi)^D} \cdots \frac{d^D p_n}{(2\pi)^D}\, \delta^D \left(\sum_{i=1}^{n} p_i \right) \exp\left(-\frac{i}{2} \sum_{i<j=1}^{n} \mathbf{p}_i \times \mathbf{p}_j \right) \phi(p_1) \cdots \phi(p_n)$$

Here $\mathbf{p}_i \times \mathbf{p}_j \equiv \epsilon_{ab} p_i^a p_j^b$ and $a, b = 1, 2$

Modified noncommutative field theory

- The new vertex (as we have seen)

$$\exp\left(-\frac{1}{4|eB|} \sum_{i=1}^{n} \mathbf{p}_{i\perp}^2 - \frac{i}{2} \sum_{i<j=1}^{n} \mathbf{p}_i \times \mathbf{p}_j \right) \equiv \exp\left(-\frac{\mathbf{i}}{\mathbf{2}} \sum_{i<j=1}^{n} \mathbf{p}_i \widehat{\times} \mathbf{p}_j \right),$$

Ordinary noncommutative field theory

- The ordinary vertex

$$\exp\left(-\frac{i}{2}\sum_{i<j=1}^{n}\mathbf{p}_i\times\mathbf{p}_j\right)\qquad \text{arises from}$$

$$f(x)\star g(x)\equiv f(x)\,\exp\left(i\overleftarrow{\nabla}\cdot\Theta\cdot\overrightarrow{\nabla}\right)g(x)\qquad (\Theta)=\Theta\begin{pmatrix}0 & +1\\ -1 & 0\end{pmatrix}$$

Modified noncommutative field theory

- The new vertex

$$\exp\left(-\frac{1}{4|eB|}\sum_{i=1}^{n}\mathbf{p}_{i\perp}^2-\frac{i}{2}\sum_{i<j=1}^{n}\mathbf{p}_i\times\mathbf{p}_j\right)\equiv\exp\left(-\frac{i}{2}\sum_{i<j=1}^{n}\mathbf{p}_i\widehat{\times}\mathbf{p}_j\right)\qquad \text{arises from}$$

$$\star\longrightarrow\widehat{\star}\qquad \text{or equivly}\qquad (\Theta)\longrightarrow\widehat{\Theta}=\frac{1}{|eB|}\begin{pmatrix}i & \mathrm{sign}(eB)\\ -\mathrm{sign}(eB) & i\end{pmatrix}$$

Modified noncommutative $O(N)$ model

- The effective action of n composites

$$\Gamma_{n\sigma} \sim \int d^2x_{\parallel}\, d^2x_{\perp}\, \underbrace{\sigma(\mathbf{x}_{\perp}) \,\widehat{\star}\, \cdots \,\widehat{\star}\, \sigma(\mathbf{x}_{\perp})}_{n\text{-times}},$$

- Alternatively

  Use the ordinary noncommutativity

$$[x_i, x_j] = i\Theta_{ij} \qquad \text{with} \qquad \Theta_{ij} = \Theta\epsilon_{ij}$$

BUT modify the fields with a Gaussian damping factor

$$\Sigma(x) \equiv e^{\frac{\vec{\nabla}_{\perp}^2}{4|eB|}}\sigma(x)$$

to write

$$\Gamma_{n\Sigma} \sim \int d^2x_{\parallel}\, d^2x_{\perp}\, \underbrace{\Sigma(\mathbf{x}_{\perp}) \star \cdots \star \Sigma(\mathbf{x}_{\perp})}_{n\text{-times}},$$

Outlook

- Varying magnetic field
- Constant electric field (vacuum polarization effect)