New approach in the extracting of parton distributions, based on parameterized inverse Mellin technique

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IPM-LHP06 school and conference,
15-20 May 2006, Tehran

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Introduction

- Quark parton model is one the feature of strong interaction field which is investigated by many phenomenological model.

- Constituent quark model seems give us a better insight as to how to construct the hadron structure from the parton distributions.

- According to this model, a constituent parton is defined as a cluster of valence quarks accompanied by a cloud of sea quarks and gluons.

- To improve and increase the ability of calculation, we try to use the Complete Renormalization Group Improvement (CORGI) approach.

- In this approach the standard perturbative series of QCD observable reconstructed in terms of scheme-invariant quantities. So it is expected to get more reliable results with respect to what we obtain in standard QCD approach.

- Finally, in order to report directly all exist parameters, just by using the available experimental data, we use from the Inverse Mellin Technique, not in a numerical form but in a parameterized form.

Phenomenological constituent quark model

- The idea of quark cluster is not new. In this model, the hadron is envisaged as a bound state of valence quark clusters. For example the bound state of π^- consists of a "anti-up" and "down" constituent quarks

- To facilitated the phenomenological analysis the following simple form for the exclusive constituent quark inside the proton are assumed

$$G_{UUD/p}(y_1, y_2, y_3) = g\,(y_1 y_2)^{\alpha}\, y_3^{\beta}\,\delta(y_1 + y_2 + y_3 - 1),$$

where α and β are two free parameters and y_i is the momentum fraction of the i'th constituent quark. The U and D type inclusive constituent quark distributions can be obtained by double integration over the specified variables,

- Here we particularly concentrate on calculation of sea quark distribution, based on the constituent quark model.

• This model suggests that the structure function of a hadron involves a convolution of two distributions. Constituent quark distributions in the proton and structure function for each constituent quark so as

$$F_2^p(x, Q^2) = \sum_v \int_x^1 dy\, G_{v/p}(y)\, F_2^v(z = \frac{x}{y}, Q^2)\,, \quad (1)$$

• Since the calculation in moment n-space is easier than the calculation in x-space, we now go to the moment of distribution , defining

$$M_{2,3}(n, Q^2) = \int_0^1 dx\, x^{n-2} \left\{ \begin{array}{c} F_2 \\ xF_3 \end{array} \right\} (x, Q^2),$$

It then follows from Eq. (1) that

$$M^N(n, Q^2) = \sum_v M_{v/N}(n)\, M^v(n, Q^2). \qquad (2)$$

Using the Eq. 2, we obtain

$$M_{sea}(n, s) = \frac{1}{2f}\left[2U(n) + D(n)\right]\left[M^S(n, s) - M^{NS}(n, s)\right],$$

where M^S and M^{NS} are singlet and non-singlet evolution function given in leading order by

$$M^{NS}(n,Q^2) = \exp\left(-d_{NS}\, s\right),$$

$$M^S(n,Q^2) = \frac{1}{2}(1+\rho)\, \exp\left(-d_+\, s\right) + \frac{1}{2}(1-\rho)\, \exp\left(-d_-\, s\right).$$

and in the NLO approximation, we have

$$M^{NS}(n,Q^2) = \quad \left[1 + \frac{\alpha_s(Q^2)-\alpha_s(Q_0^2)}{4\pi}\left(\frac{\gamma_1^{NS}}{2\beta_0} - \frac{\beta_1\gamma_0^{qq}}{2\beta_0^2}\right)\right]$$

$$\left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{\gamma_0^{qq}/2\beta_0},$$

and

$$\begin{pmatrix} M^S(n,Q^2) \\ M^g(n,Q^2) \end{pmatrix} = \left[P_- - \frac{1}{2\beta_0}\frac{\alpha_s(Q_0^2)-\alpha_s(Q^2)}{4\pi} P_-.\gamma.P_-\right.$$

$$-\left(\frac{\alpha_s(Q_0^2)}{4\pi} - \frac{\alpha_s(Q^2)}{4\pi}\left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{(\lambda_+-\lambda_-)/2\beta_0}\right)$$

$$\left. .\frac{P_-.\gamma.P_+}{2\beta_0+\lambda_+-\lambda_-}\right]\left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{\lambda_-/2\beta_0} + (+ \longleftrightarrow -).$$

Complete RG Improvement approach

- In this approach, all perturbative terms are scheme independent, so there is no dependency to renormalization scale (μ) or factorization scale (M) in these terms.
- If an observable $R(Q)$ in a standard approach has a perturbative series like

$$R(Q) = a + r_1 a^2 + r_2 a^3 + \cdots + r_n a^{n+1} + \cdots , \quad (3)$$

then in the new approach, this observable has a perturbative series as

$$R(Q) = a_0 + X_2 a_0^3 + X_3 a_0^4 + \cdots + X_n a_0^{n+1} + \cdots . \quad (4)$$

- In Eq. (3) all terms depend on renormalization scale (μ), while in Eq. (4), $a_0 = a_0(Q)$ and $X_2, X_3, \cdots$ are constants and scheme invariants.

- Here a_0 is defined in terms of Lambert-W function as

$$a_0 = -\frac{1}{c[1 + W_{-1}(z(Q))]} \ , \ z(Q) = -\frac{1}{e}\left(\frac{Q}{\Lambda_{\mathcal{R}}}\right)^{-b/c},$$

where b and c are the first two universal terms of QCD β-function.

- For the case of two μ and M scale dependency, we have

$$\Lambda_{\mathcal{R}} = \Lambda_{\overline{MS}} \left(\frac{2c}{b}\right)^{-c/b} \exp\left(\frac{d_1(n)}{bd(n)} + \frac{r_1(n)}{d(n)}\right),$$

where $d_1(n)$ is the $\overline{MS}$ NLO anomalous dimension coefficient, and $r_1(n)$ is computed in the $\overline{MS}$ scheme with $\mu = Q$.

- Each term in Eq. (4) involves a resummation of infinite terms at specified order. For instance, the first term in Eq. (4), a_0, is a representation of resummation over NLO contribution of all terms in Eq.(3) and the second term $X_2 a_0^3$ as a representation of resummation for the NNLO contribution of the terms.

- if we intend to employ the CORGI approach to extract sea quark distribution, in the LO approximation we need just to change the RG-coupling constant a to a_0 which exists in definition of evolution parameter s that is defined by $s = \log \frac{a(Q_0^2)}{a(Q^2)}$.

Inverse Mellin Technique - Parameterized Solution

- In order to report directly all exist parameters, just by using the available experimental data, we can use from the Inverse Mellin Technique, not in a numerical form but in a parameterized form.

- For this propose, we take the Inverse Mellin of moment of distribution in complex space:

$$F(x,s) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \frac{dn}{x^{n-1}}\, M(n,s). \tag{5}$$

- Since the variable c should be placed on the right hand side of all singularities and considering this point that all singularities of moments will occur for n less than 2, so we choose c equal to 3.

- The integrated interval was first $[3-10i, 3+10i]$. If we choose the interval $[3-20i, 3+20i]$, there will be a difference only of order about 10^{-5} with respect to last interval.

- Since the function $M(n,s)$ is not a simple function, our machine will not be able to compute Eq. (5).

- We use from this point that this integral with respect to real axis, is symmetric. So first we do integral for interval $[c = c + 0i, c + mi]$ where the final result is twice this result.

- The technique which we used to integrate the $M(n,Q^2)$ in (5) is that we first choose a small interval, say $[c+0i, c+2\epsilon i]$. Then we expand $M(n,Q^2)$ about $c+\epsilon i$. If ϵ is small enough, we can keep just the first term of this expansion.

- In next step, we repeat the calculation for interval $[c+2\epsilon i, c+4\epsilon i]$ and expand $M(n,Q^2)$ about $c+3\epsilon i$. Repeating this procedure and adding all results, we are able to calculate Eq. (5) completely in parameterized form.

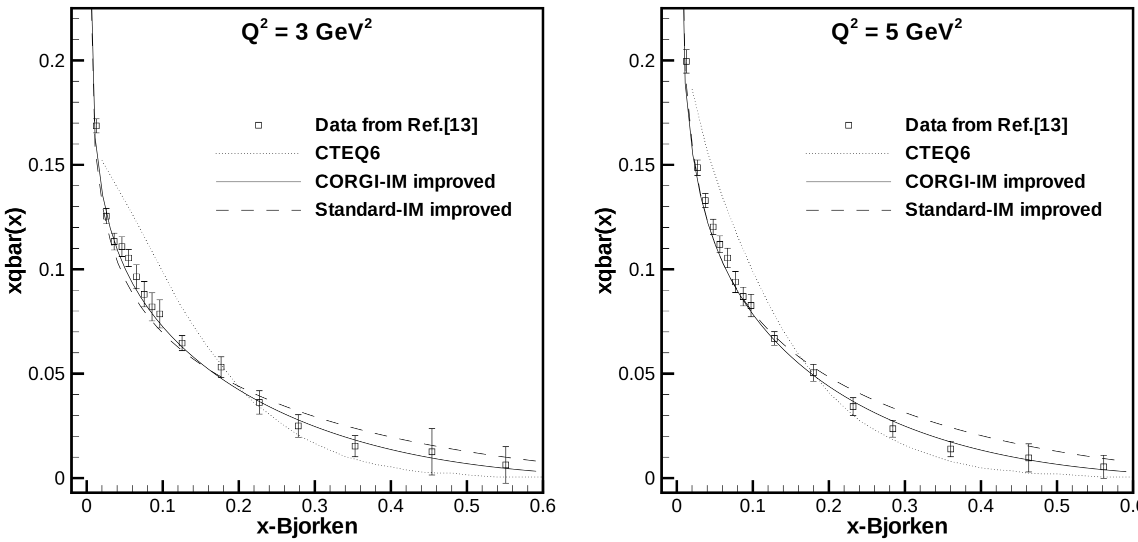

Figure 1: The results of Inverse Mellin (IM) technique for the standard and CORGI approaches in the LO approximation

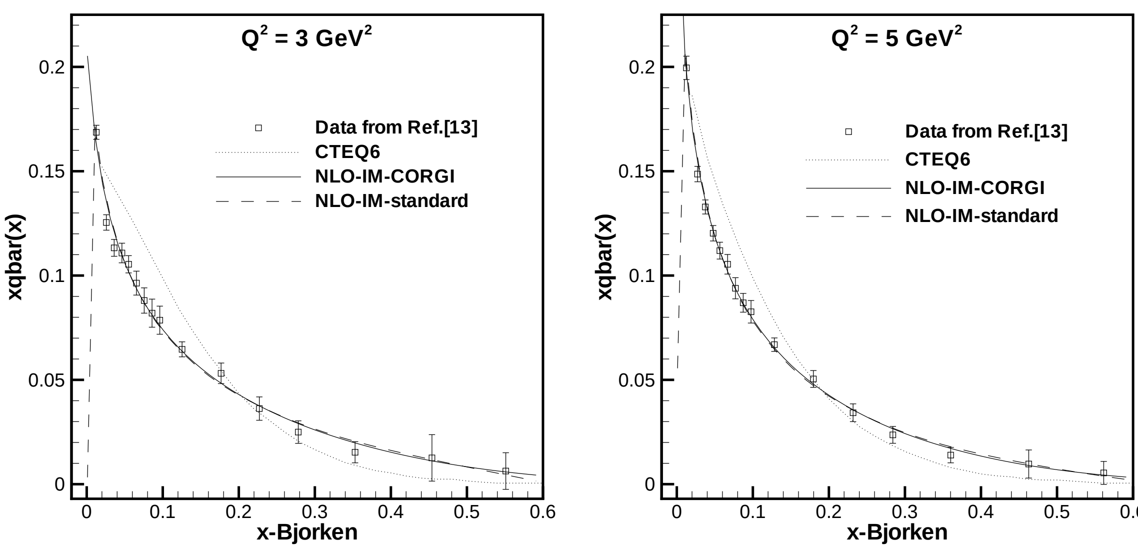

Figure 2: The results of Inverse Mellin (IM) technique for NLO approximation in both of Standard and CORGI approach.

Conclusions

- Constituent quark model as a good candidate to extract sea quark distribution inside the nucleon is used.

- It is possible to use from this technique in the LO and NLO approximation, using two different standard and CORGI.

- In order to get the unknown parameters of the model from the fitting over the available experimental data, parameterized Inverse Mellin technique is used. approaches.

- The compared results will show that the CORGI approach indicates better consistency with available experimental data.

- These calculations can be extended to the higher order of standard and CORGI approaches, using the recent analytical calculations for Wilson coefficients and anomalous dimensions.