Gravi-Leptogenesis

Leptogenesis from Gravity Waves in Models of Inflation

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Plan of the Talk

■ Need for matter anti-matter asymmetry

■ Sakharov Conditions

■ Leptogenesis vs. Baryogenesis

■ Our model, An overview
- Ingredients
- Realization of Sakharov conditions in our model

■ Inflation in a nutshell

■ Analysis of Gravity Waves
- Classical e.o.m for GWs and their solutions
- Quantization of GWs (tensor perturbations) and their two point function

■ Computation of Lepton number

■ Numeric values and discussions

■ Summary & works in progress

▶ Matter anti-matter asymmetry

• by now is established as an experimental fact, e.g. the CMBR observations lead to

$\frac{n_B}{n_\gamma} = (6.5 \pm 0.4) \times 10^{-10}$ astro-ph/0302209.

$n_B = n_b - n_{\bar{b}}$ and $n_\gamma = \#$density of photons.

• This number, while non-zero, is very small.

▶ About 40 years ago Sakharov stated that: to explain matter anti-matter asymmetry from a symmetric soup of everything one should have

• Baryon number violation,

• C & CP violating interaction vertices,

• the above interactions should happen in a phase where the system is out of thermal equilibrium.

▶ Leptogenesis

In the usual SM B & L are both classically conserved. At quantum level

$B+L$ is anomalous, mediated by sphaelerons. (Sphaelerons are electroweak instantons thermally activated at temperatures not less than $1-10\ TeV$ and interconvert Baryons to Leptons.)

$B-L$ can only have gravitational anomaly.

In the Beyond standard models:

e.g. in MSSM $B+L$ is anomalous perturbatively if R parity is violated, and
in GUTs we have vertices (at classical level) which interchange quarks and leptons..........

Hence to create non-zero n_B one can come up with a model which creates n_L, leptogenesis.

C & CP violation in the usual models of baryo or lepto genesis come from (Dirac) phases of the mixing matrices in quark or neutrino sectors. In MSSM large CP violating phases are highly constrained [hep-ph/0208043].

One might also use phases of Yukawa couplings in MSSM [Fukugita-Yanagida model] as sources for CP violation...

Out-of-equilibrium condition in the usual models is the hardest part:

It only happens during (first order) phase transitions or spontaneous symmetry breakings where we cannot generically perform perturbative calculations...........

In our model, however, all Sakharov conditions are met in a common thread:

Gravity waves in a pseudo-scalar (axionic) driven inflation model.

Our Model

- Inflation is now becoming a part of observationally tested/testable physics.

- Out-of-Equil. condition is naturally satisfied if we cook up our model during inflation.

- During inflation we produce gravity waves (as quantum perturbations) in abundance. If the situation is prepared, $B-L$ can become anomalous via gravitational (triangle) chiral anomaly:

$$\partial_\mu J_l^\mu = \frac{\mathcal{A}}{16\pi^2} R\tilde{R}$$

$$J_l^\mu = \sum_i \bar{l}_i \gamma^\mu l_i + \bar{\nu}_i \gamma^\mu \nu_i, \quad R\tilde{R} = \frac{1}{2}\epsilon^{\alpha\beta\rho\sigma} R_{\alpha\beta\mu\nu} R_{\rho\sigma}{}^{\mu\nu}$$

$\mathcal{A}$ is the anomaly coefficient counting difference between # left and right handed fermions.

Here I would only consider SM, and its field content. Our model can work with any beyond SM with a non-zero $\mathcal{A}$.

• A pseudo-scalar inflaton can break parity and hence CP. Such axionic inflaton would naturally couple to $R\tilde{R}$ as

$$\Delta\mathcal{L} = F(\phi) R\tilde{R} \qquad\qquad (\star)$$

where under P and CP:

$$\phi \to -\phi \;\&\; \; F(\phi) \to -F(\phi).$$

Such terms would generically appear once we integrate out heavy fermions axially coupled to ϕ. Effects of such terms on the evolution of CMBR have been analyzed by

Lue, Wang and Kamionkowski (LWK) [astro-ph/9812088] and by
S. Alexander, J. Martin [hep-th/0410230].

• Terms like $(\star)$ appear naturally in string theory via Green-Schwarz mechanism with

$$F(\phi) = \frac{\mathcal{N}}{16\pi^2}\frac{\phi}{M_{Pl}}$$

To have an idea what $\mathcal{N}$ is let us take a de tour to string theory....

(A de tour to string theory

To see how this term appears in string theory, consider e.g. the Heterotic SUGRA action:

$$S = M_{10}^8 \int d^{10}x \sqrt{\det g_{10}} \Big(\mathcal{R} + \frac{1}{2}(\partial\Phi)^2 + \frac{1}{12} e^{-\Phi} H_{ABC}^2 + \frac{1}{4} e^{-\Phi/2} (F_{AB})^2 \Big)$$

where

$$H_3 = dB_2 - \frac{1}{4}\Big(\Omega_3(A) + \alpha' \Omega_3(\omega)\Big)$$

($\Omega_3(A)$ and $\Omega_3(\omega)$ are the gauge and gravitational Chern-Simons three-forms, respectively. Explicitly,

$$\Omega_3(\omega) = Tr(\omega \wedge d\omega + \frac{2}{3}\omega \wedge \omega \wedge \omega)$$

In particular note that

$$^*(d\Omega_3(\omega)) = R\tilde{R}$$

Upon compactification to four dimensions, the H^2 term leads to

$$S \sim M_{10}^8 \int d^6y \int d^4x \; e^{-\Phi}(dB + \alpha' \Omega_3(\omega))^2$$
$$= M_{10}^8 \frac{V_6}{g_s} \int d^4x [(dB)^2 + 2\alpha'({}^*dB) \wedge \Omega_3(\omega) + \cdots]$$
$$= \frac{M_{Pl}^2}{g_s} \int d^4x ((\partial\phi)^2 - 2\alpha' \phi R\tilde{R})$$

where the pseudoscalar ϕ (our inflaton field) is dual to the two form B_2 and the 4d Planck length, M_{Pl} is defined as

$$M_{Pl}^2 = M_{10}^8 V_6.$$

And hence, finally:

$$\mathcal{N} = 8\pi^2 \frac{M_{Pl}^2 \alpha'}{g_s} = 8\pi^2 \left(\frac{M_{Pl}}{M_{10}}\right)^2 \frac{1}{\sqrt{g_s}}$$

end of the de tour)

• P & CP violation in our model is coming from the fact that the (classical) expectation value for the axionic inflaton is non-vanishing during inflation.

• ϕ is not necessarily P-Q. or QCD axion.

■ Inflation in a nutshell

• *Equations of Motion:*

$$\ddot{\phi} + 3H\dot{\phi} - V'(\phi) = 0,$$

$$M_{pl}^2 H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right),$$

where $M_{pl}^{-2} = 8\pi G_N$ is the reduced Planck mass, $M_{pl} = 2.44 \times 10^{18}$ GeV.

- *Number of e-folds:*

$$N_e(t) \equiv \ln \frac{a(t_{end})}{a(t)} = \frac{1}{M_{pl}^2} \int_{\phi_e}^{\phi_i} \, d\phi \frac{V}{V'},$$

- *Slow-roll Conditions:*

$$\ddot{\phi} \ll H\dot{\phi} \ , \quad \dot{\phi}^2 \ll V(\phi).$$

And hence in slow-roll inflationary models

$$\dot{\phi} = \frac{1}{3H} V'(\phi), \quad M_{pl}^2 H^2 = \frac{1}{3} V(\phi).$$

- *Slow-roll Parameters:*

$$\epsilon = \frac{M_{pl}^2}{2} \left(\frac{V'}{V} \right)^2 \ , \quad \eta = M_{pl}^2 \frac{V''}{V}.$$

Velocity of the field $\dot{\phi}$ in terms of slow-roll parameters:

$$\frac{\dot{\phi}}{M_{Pl}^2} = \sqrt{2\epsilon} \ \frac{H}{M_{Pl}}.$$

- *Density of scalar perturbations:*

$$\left(\frac{\delta\rho}{\rho}\right)_{scalar} = \frac{2}{5}\mathcal{P}_R^{1/2}$$
$$= \frac{1}{5\pi\sqrt{2}}\frac{H}{M_{Pl}}\frac{1}{\sqrt{\epsilon}}$$
$$= 1.91 \times 10^{-5} \qquad COBE\ results.$$

$\mathcal{P}_R$ is the *power spectrum* of scalars,

$$\mathcal{P}_R = \left(\frac{H}{M_{Pl}}\right)^2 \frac{1}{\epsilon}.$$

- Power spectrum of gravity waves (tensor perturbations):

$$\mathcal{P}_{gr} = \left(\frac{H}{M_{Pl}}\right)^2$$

- Based on the WMAP data [astro-ph/0302225]

$$r \equiv \mathcal{P}_{gr}/\mathcal{P}_R < 0.71, \quad \frac{H}{M_{Pl}} < 1 \times 10^{-4}, \quad \epsilon \sim 0.01$$

Analysis of Gravity Waves

$$ds^2 = -(1+2\varphi)dt^2 + w_i dt dx^i + a^2(t)\left[\left((1+2\psi)\delta_{ij} + h_{ij}\right) dx^i dx^j\right]$$

φ, ψ are scalar, w_i vector and h_{ij} are tensor perturbations.

- h_{ij} is symmetric, traceless and divergence free, and represents the two degrees of freedom of graviton, the gravity waves.

- Only h_{ij} gives contribution to $R\tilde{R}$ so we only consider tensor perturbations here. For gravity waves moving along the z direction:

$$ds^2 = -dt^2 + a^2(t)\Big[(1-h_+)dx^2 + (1+h_+)dy^2 + 2h_\times dx dy + dz^2\Big]$$

In our case: $a(t) = e^{Ht}$ with a constant H and $h_+,\ h_\times$ are functions of t, z.

- Helicity basis:

$$h_L = \frac{1}{\sqrt{2}}(h_+ - ih_\times)\ , \qquad h_R = \frac{1}{\sqrt{2}}(h_+ + ih_\times)$$

Note: $h_R = h_L^{c.c}$.

Evolution of Gravity Waves:

$$\mathcal{L} = \frac{1}{2} M_{pl}^2 \sqrt{-\det\ g}\ R + F(\phi) R\tilde{R}$$

where

$$\sqrt{-\det\ g}\ R = -(h_L \Box h_R + h_R \Box h_L) + \mathcal{O}(h^3),$$

$$\Box = \frac{\partial^2}{\partial t^2} + 3H\frac{\partial}{\partial t} - \frac{1}{a^2}\frac{\partial^2}{\partial z^2}$$

and

$$R\tilde{R} = \frac{4i}{a^3}\Bigg[\left(\frac{\partial^2}{\partial z^2} h_R \frac{\partial^2}{\partial t \partial z} h_L - \frac{\partial^2}{\partial z^2} h_L \frac{\partial^2}{\partial t \partial z} h_R\right) + a^2\left(\frac{\partial^2}{\partial t^2} h_R \frac{\partial^2}{\partial t \partial z} h_L - \frac{\partial^2}{\partial t^2} h_L \frac{\partial^2}{\partial t \partial z} h_R\right) + Ha^2\left(\frac{\partial}{\partial t} h_R \frac{\partial^2}{\partial t \partial z} h_L - \frac{\partial}{\partial t} h_L \frac{\partial^2}{\partial t \partial z} h_R\right)\Bigg] + \mathcal{O}(h^4)$$

- Note: $R\tilde{R}$, similarly to $\sqrt{-g}R$, starts at $\mathcal{O}(h^2)$.

- In order to have non-vanishing $R\tilde{R}$ we need cosmological birefringence.

Cosmological Birefringence

Ignoring the third order derivative terms in e.o.m. we have

$$\Box h_L = -2i\frac{\Theta}{a}\dot{h}'_L \ , \qquad \Box h_R = +2i\frac{\Theta}{a}\dot{h}'_R$$

with

$$\Theta = \frac{4}{M_{pl}^2}(F''\ddot{\phi} + F''\dot{\phi}^2 + 2HF'\dot{\phi})$$
$$\simeq \frac{8}{M_{pl}^2}HF'\dot{\phi}$$

For the choice of linear $F(\phi)$

$$\Theta = \sqrt{2\epsilon}\mathcal{N}(H/M_{pl})^2/2\pi^2$$

Taking $h_L(t,z) = h_L(t)e^{ikz}$ and introducing conformal time η instead of the comoving time t:

$$\eta = \frac{1}{Ha} = \frac{1}{H}e^{-Ht} \ ,$$

(note: conformal time η runs in the opposite direction from t), e.o.m. becomes:

$$(\frac{d^2}{d\eta^2} - 2\frac{1}{\eta}\frac{d}{d\eta} + k^2)h_L = 2k\Theta\frac{d}{d\eta}h_L$$

Note: using the conformal time all factors of $a(t)$ disappear from the e.o.m.

Solution to e.o.m

If $\Theta = 0$ solutions are spherical Bessel functions (with +ve or -ve frequencies):

$$h_L^+(k,\eta) = e^{+ik(\eta+z)}(1 - ik\eta)$$

For $\Theta \neq 0$:

$$h_L = (-ik\eta)e^{k\Theta\eta}g(k\eta)$$

where

$$\frac{d^2}{d\eta^2}g + \left[k^2(1-\Theta^2) - \frac{2}{\eta^2} - \frac{2k\Theta}{\eta}\right] g = 0.$$

Note: This is a Coulomb wave equation, *i.e.* equation of a Schrödinger particle with $l = 1$ in a Coulomb potential proportional to $k\Theta$.

Note: The Coulomb potential for h_L and h_R have a different sign.

For $k\eta \gg 1$ (sub-horizon modes):

$$g(k\eta) = \exp[i(1-\Theta^2)^{1/2}k\eta(1+\alpha(k\eta))]$$

where $\alpha(\eta) \sim \log \eta/\eta$.
Note: For sub-horizon modes waves are effectively propagating in a flat background (instead of de Sitter).

Quantization of gravity waves

To compute $\langle R\tilde{R}\rangle$ we need to know the two point functions:

$$G(x,t;x',t') \equiv \langle h_L(x,t)h_R(x',t')\rangle$$
$$= \int \frac{d^3k}{(2\pi)^3} e^{ik\cdot(x-x')} G_k(\eta,\eta')$$

where $G_k(\eta,\eta')$ satisfies

$$\left[\frac{d^2}{d\eta^2} - 2(\frac{1}{\eta} + k\Theta)\frac{d}{d\eta} + k^2\right] G_k = i\frac{(H\eta)^2}{M_{pl}^2}\delta(\eta-\eta')$$

For $\Theta = 0$ solution is

$$G_k^{(0)}(\eta,\eta') = \begin{cases} \frac{H^2}{2k^3 M_{pl}^2} h_L^+(k,\eta)h_R^-(-k,\eta'), & \eta < \eta' \\ \\ \frac{H^2}{2k^3 M_{pl}^2} h_L^-(k,\eta)h_R^+(-k,\eta') & \eta' < \eta \end{cases}$$

For $\Theta \neq 0$, up to leading order in Θ,

$$G_k(\eta,\eta') = e^{-k\Theta\eta}\; G_k^{(0)} e^{+k\Theta\eta'}$$

In fact for the sub-horizon modes the Θ corrections to Green's function G are subleading and not important.

Using this Green's function we have

$$\langle R\tilde{R}\rangle = \frac{16}{a}\int \frac{d^3k}{(2\pi)^3}\frac{H^2}{2k^3M_{pl}^2}(k\eta)^2\cdot k^4\Theta + \mathcal{O}(\Theta^3)$$

Note: The main contribution to the integral is coming from sub-horizon modes with $k\eta \gg 1$.

Note: $\langle R\tilde{R}\rangle$ is non-zero and is proportional to Θ. Recall that Θ is the parameter measuring the amount of CP violation.

The CP violation which is a result of the axionic (CP-odd) inflaton has been communicated to the gravity waves. In other words, we have a classical background which is CP-asymmetric.

Noting the anomaly equation, and integrating both sides over the space, we have:

$$\frac{d}{dt}n = \frac{1}{a}\frac{d}{d\eta}n = \frac{1}{16\pi^2}\langle R\tilde{R}\rangle$$

Computation of Lepton Number

$$n = \int_0^{H^{-1}} d\eta \int^{\mu} \frac{d^3k}{(2\pi)^3} \frac{8H^2k^3\eta^2\Theta}{16\pi^2 M_{pl}^2}$$

The "cut-off" μ is the scale at which our effective Lagrangian description breaks down.

As a good estimate, μ should be less than the fundamental Planck scale, M_{10}. It should, in principle, also be compared with the scale of our beyond SM particle physics model where the new physics sets in, e.g. the right-handed neutrino mass or SUSY GUT scale.

The dominant contributions come from large k, and for $\mu > H$, that is the sub-horizon modes.

Note that the tensor perturbations usually considered and discussed in the inflationary context are the super-horizon modes which leave the horizon during inflation and re-enter horizon as classical modes after inflation.

$$n = \frac{1}{72\pi^4}\left(\frac{H}{M_{pl}}\right)^2 \Theta H^3 \left(\frac{\mu}{H}\right)^6$$

- n is # density of leptons right after inflation.
- $\left(H/M_{pl}\right)^2$ is power spectrum of gravity waves.
- Θ is the CP-violation parameter.
- H^{-3} is volume of space at the end of inflation.
- $(\mu/H)^6$ is the enhancement factor coming from the fact that we are dealing the short distance (sub-horizon) quantum fluctuations.

Calculation of n_B/n_γ

- n_B red-shifts by the Hubble expansion as $n_B \sim 1/a^3$. n_γ also red-shifts like the volume. Therefore, n_B/n_γ is not red-shifting and is a constant during the history of the Universe.

- In the current phase of the Universe, the ratio of present baryon number to the lepton number generated in the leptogensis [Kuzmin-Rubakov-Shaposhnikov PLB**155** (1985) 36]

$$n_B/n = 4/11\ , \quad (T_\nu/T_\gamma)^3 = 4/11.$$

- The number density of photons and their entropy density are related as

$$s = 1.8 g_*\ n_\gamma\ ,$$

and today (as only photons are relativistic now) $g_* = 2$.

- The above implies that

$\frac{n}{s} = 2.4 \times 10^{-10}$ the observed value.

▶ Computation of s

Assuming instant reheating

$$\rho = 3H^2 M_{Pl}^2 = \pi^2 g_* T^4/30$$

and

$$s = 2\pi^2 g_* T^3/45.$$

Hence $s = 2.3 g_*^{1/4} (H M_{Pl})^{3/2}$.

Taking $g_* \sim 100$

$$\frac{n}{s} = 2 \times 10^{-4} \left(\frac{H}{M_{pl}}\right)^{7/2} \ominus \left(\frac{\mu}{H}\right)^6$$

$$\sim 1 \times 10^{-6} \cdot \mathcal{N}\sqrt{\epsilon} \left(\frac{H}{M_{Pl}}\right)^{11/2} \left(\frac{\mu}{H}\right)^6$$

$$\sim 1 \times 10^{-4} \cdot \sqrt{\epsilon} \left(\frac{M_{Pl}}{M_{10}}\right)^2 \left(\frac{\mu}{M_{Pl}}\right)^6 \left(\frac{H}{M_{Pl}}\right)^{-1/2}$$

- Note: $n \propto H$ and $n/s \propto H^{-1/2}$.

- In the 2nd and 3rd lines of the above, the explicit expression of $\ominus$

$$\ominus = \sqrt{2\epsilon}\mathcal{N}(H/M_{Pl})^2/2\pi^2$$

has been used.

• In the last line the expression for $\mathcal{N}$, assuming $g_s \sim 1$, has been used.

$$\frac{n}{s} \sim 1 \times 10^{-4} \cdot \sqrt{\epsilon} \left(\frac{M_{Pl}}{M_{10}}\right)^2 \left(\frac{\mu}{M_{pl}}\right)^6 \left(\frac{H}{M_{Pl}}\right)^{-1/2}$$

Numeric values

Acceptable range for H:

An upper bound from WMAP $H < 10^{-4} M_{Pl}$, and $\epsilon \sim 0.01$;

a lower bound from $T_{reh} \gtrsim 1$ TeV, leads to $H \gtrsim 10^{-2}$ eV.

As for μ

$$H \lesssim \mu \lesssim M_{10}$$

For $H \sim 10^{-4} M_{Pl}$ and $\epsilon \sim 0.01$,

$$\frac{n}{s} \sim 10^{-3} \left(\frac{M_{Pl}}{M_{10}}\right)^2 \left(\frac{\mu}{M_{Pl}}\right)^6$$

Some estimates:

- $M_{pl} \sim M_{10}$, $\mu \sim 10^{-1} M_{pl}$, $H \sim 10^{-4} M_{Pl}$

$$\frac{n}{s} \sim 10^{-10}.$$

(For this case, the CP violating parameter, $\Theta = 4\sqrt{2\epsilon}\left(\frac{H}{M_{10}}\right)^2 \sqrt{g_s} \sim 10^{-9}$.)

- $\mu \sim M_{10} \sim 10^{-1} - 10^{-2} M_{Pl}$, $H \sim 10^{-4} M_{Pl}$,

$$\frac{n}{s} \sim 10^{-10}.$$

(For this case, the CP violating parameter, $\Theta \sim 10^{-5}$.)

■ Summary & works in progress

- Using inflation as a period when things are out of thermal equilibrium.

- Our model is novel in the sense that all the Sakharov conditions are satisfied from one common thread, gravity waves.

- P and CP violation is a result of pseudo-scalar driven inflation.

- In our model, there are imprints of parity violation in the spectrum of tensor perturbations, in principle could be observable in CMBR experiments [astro-ph/9812088].

This could also be related to the so-called "axis of evil" discrepancy, i.e. the recently reported parity dependence in the CMBR at low l.

- Gravitational chiral anomaly is responsible for lepton number violation.

Things to be studied in further detail:

- Here we assumed the SM for our particle physics model and in particular we assumed

$$\mathcal{A} = n_{f_L} - n_{f_R} \neq 0$$

one can imagine particle physics models in which $\mathcal{A}$ is a function of time and changes by cosmic evolution.

Moreover, the particle physics models will give us a better idea how large we can push μ.....

- Here we assumed instant reheating, which is not so realistic/appealing. Working on details of a more realistic reheating model would generically reduce T_{reheat} and the amount of entropy produced, is yet to be done......