Fermion Electric Dipole Moments in R-parity violating Supersymmetry.

- Dipole Moments of fermions.
- R-parity violating Supersymmetry.
- A general method of analysis of the EDM's.
- Application of the method in R-parity violating Supersymmetry.
- Constraints on the R-parity violating couplings from EDM's of e, n and Hg.
- EDM's in R-parity violating SUSY with bilinear R-parity violation.

1. Dipole Moments, whether magnetic/electric, diagonal or transition, probe loop physics.

2. Examples:
   - Starting from the QED tests to the possible glimpse of Supersymmetry provided by $(g-2)_\mu$
   - Let lepton and flavour number violating transition moments: $\mu \rightarrow e\gamma, b \rightarrow s\gamma$ etc. for BSM contributions to FCNC,
   - Small Majorana mass for the neutrinos.

3. The CP odd electric dipole moment $d_n$ for neutrons only at three loops in the SM, and for an electron at order $G_F^2$ induced by $d_n$.

4. As a result these dipole moments are a good place to hunt for new physics. In many cases, such as lepton flavour violation, CP violating phases in SUSY, these dipole moments provide strong constraints on the corresponding new physics parameters.
• EDM's of fermions (electron, neutron) expected in the SM very small.
  
  – The only source of CPV in the SM is CKM phases or QCD $\theta$ term. With CKM phases, $d_n \sim 10^{-32\pm 1}$ e cm.
  
  – In the SM no leptonic CP violation. Hence $d_e \sim (G_F M_n^2)^2 d_n \sim 10^{-42}$ e cm. In literature estimated values in the SM, with more refined calculations $\sim 10^{-41}$ e cm.
  
  – These expectations are way below the current limits: for the neutron $|d_n| < 6.3 \times 10^{-26}$ e cm.
  
  – $d_n$ induces in turn $d_D, d_{Hg}$ etc. The experimental limits on them are $\sim 10^{-27}$ – $\sim 10^{-28}$ e cm and are indeed competitive with the expectations in some models and hence can constrain the new physics.

• Any additional source of CP-violation (CPV) in the physics beyond the SM will induce EDM's. If they are at the one-loop or two-loop level, then inspite of the typical mass suppressions expected due to physics beyond the SM, expectations are normally higher than in the SM and thus can constrain the new physics.

• MSSM where $R$ parity ($R_p$) is conserved, leptoquark models, $R_p$ violating supersymmetric models, extended Higgs sector: all have additional sources of $CP$ phases in both the flavour violating and flavour conserving sector.
1. Dipole moment operators flip chirality, hence proportional to fermion mass.

2. In leptoquark models, eg., EDM’s of the $\tau$ receives an enhancement $m_t/m_\tau$ and thus the predictions are large enough to be tested in collider experiments.

3. Our work has been to set up a method of analysis which would allow us to obtain the leading mass dependence of the coefficients of the induced dipole moments in a given class of models of new physics. The case of $R_p$ violating Supersymmetry is taken as an illustration.

\[
\begin{align*}
\mathcal{L} &= \bar{f}_1 (p-q) \sigma_{\mu \nu} q^\nu (F_T^\gamma + \gamma_5 F_T^\gamma) u_2 (p), \quad (\gamma = \gamma, Z)
\end{align*}
\]
$R_p$ violating Supersymmetric models

Phenomenology of the MSSM and beyond

TeV Scale SUSY

Theory and Phenomenology of Sparticles

Manual Drees
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Probir Roy

Theory and Phenomenology of Sparticles

A four-dimensional $N=1$ super symmetry in High Energy Physics

World Scientific

May 15–20, 2006
$R_p = (-1)^{3B+L+2S}$: Most studies consider case where $R_p$ is conserved.

- $R_p$ or $R_p$ conservation neither is mandatory from theoretical point of view.

- Actually $B, L$ symmetries of the SM but NOT of the MSSM

- Supersymmetry and Gauge Invariance allow $R_p$ terms in the Superpotential

\[
W_{R_p} = \frac{1}{2} \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \frac{1}{2} \chi''_{ijk} U_i^c D_j^c D_k^c + \kappa_i L_i H_2,
\]

$L_i, Q_i$: doublet Lepton, Quark superfields, $E_i, U_i, D_i$: the singlet Lepton and Quark superfields.
● No DM candidate, as the $\tilde{\chi}_1^0$ not stable.

● Proton will decay **very rapidly** for TeV scale SUSY breaking.

● Can be cured by adopting $B$ conservation $\lambda'' = 0$. This choice preferred if we don’t want $R_p$ terms to wash out baryogenesis.

● Unified string theories actually prefer models with $B$ conservation and $R_p$ violation.


These models treat the Lepton and the Quark fields differently and have two discrete symmetries. $B$ conservation and $R_p$ eliminates not just the dimension 4 operators for proton decay BUT also dimension 5.

● All this makes $R_p$ worth investigating.
- $\nu$ masses can be generated in an economical way without introducing any new fields, only the $R_p$ couplings are 'new'.

- Tree level via the Bilinear $\kappa_i$, and Quantum one or two loop level via the Trilinears $\lambda, \lambda'$,

- Kamioka, SNO $\rightarrow$ Unambiguous proof of $\nu$ masses.

- Enough freedom to generate the mass patterns required by all the data. Testable predictions at the Colliders.

- A large no. (48) Yukawa type couplings. No theoretical indications about their sizes.
Many of the unknown couplings constrained by low energy processes, e.g. Proton Decay, $\mu$ decay, $\mu \to e\gamma$, dipole moments, cosmological arguments, e.g. the baryogenesis.


A host of references!!

Majority of the constraints come from virtual effects caused by sparticle exchanges via $R_p$ interactions or from loops; Both CP-conserving majorana mass of the neutrinos, $\mu \to e\gamma$ and $\not\!p$ such as EDM's, if the couplings have nonzero phases.

Many discussions of direct collider probes of $R_p$ which can further help in the model building (not discussed further here).

We wanted to look at the EDM's induced by $R_p$ couplings. In leptoquark models large enhancements of the EDM's compared to the SM expectations were found. Since, the sfermions behave like leptoquarks for $R_p$ case, we wanted to see whether same is possible for this case.

Consider only the trlinears to begin with.
Define five global charges $Q_{l_L}, Q_{e_R}, Q_{q_L}, Q_{d_R}$ and $Q_{u_R}$ corresponding to five different $U(1)$ transformations.

$Q_{l_L} = 1$ for $e_{iL}, \nu_i, \tilde{e}_{iL}, \tilde{\nu}_i$ ($i = 1 - 3$)

$Q_{l_L} = 0$ for all the other particles/sparticles.

The charge is independent of the generation. The value of all the charges for all the Gauge bosons and Higgs bosons (plus their SUSY partners) are zero.

the superpotential Yukawa interactions, $A$ terms and $R_p$ terms do not conserve these charges where as the gauge (and gaugino) interactions do.

These charges are some kind of 'superchirality' in that they are nonzero even for spin zero sfermions.
Sources of chirality flip required for giving rise to dipole operators:

1) Yukawa interactions (in the SM this is the only source).

2) A-terms and Higgsinos (once we include the MSSM).

3) $R_p$ interactions (once one includes the possibilit of $R_p$).

The charge assignment provides a systematic way to analyze when a dipole moment can be induced.

For example, for a leptonic moment to be induced $Q_{l_L}$ and $Q_{e_R}$ to change by one unit in equal and opposite directions, with no change in the other charges.

Similarly for $Q_{q_L}$ and $Q_{u_R}$ ($Q_{d_R}$ and $Q_{q_L}$) for $u(d)$ dipole moments.
Yukawa interactions (say for leptons): stands for the lepton mass term, lepton-Higgs coupling, Higgs/slepton coupling from superpotential and also from A term, slepton mixing terms.

Two of the trilinear Higgs sfermion interaction from the D-term and F-term, cause the charges to change in a manner difft. from the Yukawa.
The net changes in various charges:

Let 

\( P, S \) and \( R \) : number of down-quark, up-quark and lepton Yukawa interactions

\( P^*, S^*, R^* \) : the number of insertions corresponding to the \((h.c.)\).

\( N, M, L \) : number of vertices corresponding to interactions proportional to \( \lambda, \lambda', \lambda'' \) etc.

\( T, T^* \) : number of the trilinear or quartic vertex

\[ \Delta Q_{lL} = -2\Delta N - \Delta M - \Delta R \]

\[ \Delta Q_{eR} = \Delta N + \Delta R \]

\[ \Delta Q_{qL} = -\Delta M - \Delta P - \Delta S \]

\[ \Delta Q_{dR} = 2\Delta L + \Delta P + \Delta M + \Delta T \]

\[ \Delta Q_{uR} = \Delta L + \Delta S - \Delta T \]
For nonzero leptonic moment one requires:
\[ \Delta Q_{lL} = -1, \Delta Q_{eR} = 1 \]
This means,
\[ \Delta N = 1 - \Delta R \]
\[ \Delta M = \Delta R - 1 \]
\[ \Delta P = 1 - \Delta R - \Delta T \]
\[ \Delta L = 0, \Delta S = \Delta T. \]
For such a diagram, the dipole moment, with appropriate factors of \( M_W, M_{SUSY} \) is given by,
\[
\mathcal{D}_l \propto m_{l_i}^{R+R^*} m_{d_j}^{P+P^*} m_{u_k}^{S+S^*} (m_{u_l} m_{d_i})^{T+T^*}
\]
1) SM/MSSM: \( \Delta L = \Delta M = \Delta N = 0 \rightarrow \Delta R = 1 \). \( R \) or \( R^* \) must be non-zero \( \rightarrow \) Moment \( \propto m_l \).

2) in \( R_p \) third condition \( \rightarrow \) moment possible ONLY with an insertion of down-type OR lepton Yukawa insertion. With \( R_p \) there is also Lepton Flavour Violation \( \rightarrow \) dipole moment enhanced relative to SM/MSSM by \( m_b/m_l \).

3) Same conditions need be satisfied for Majorana mass/dipole moment of the \( \nu, \mu \rightarrow e\gamma \) etc.
For non-zero down-wuark moments we need:

\[ \Delta Q_{qL} = -1, \Delta Q_{dR} = 1, \]

with all the other charges remaining unchanged.

\[ \Delta M = 1 - \Delta P - \Delta T \]
\[ \Delta N = \Delta P - 1 + \Delta T \]
\[ \Delta R = 1 - \Delta P - \Delta T \]
\[ \Delta L = 0, \Delta S = \Delta T. \]

1) **SM/MSSM**: dipole moments \( \propto \) down type quark mass.

2) With \( R \), require \( \Delta R \neq 0 \) or \( \Delta P \neq 0 \). \( \propto \) lepton/down-quark mass.

3) No big enhancement involving top quark possible.
Similar analysis can be done for the $t$ quark case as well. Our mass dependencies agree with those obtained in literature whenever available.

Unlike the leptoquark case no enhancements, relative to SM/MSSM, possible for the $R_p$ eventhough the $\bar{q}/\bar{l}$ behave like a leptoquark for a $R_p$ case.

Reason can be traced to the fact that due to SUSY the couplings remain chiral. This forbids large enhancements of the dipole moments relative to SM/MSSM.

These are the conditions for a dipole moment in general. For an EDM the digram should be complex.

The lowest order $R_p$ contribution needs $N = N^* = 1$ or $M = M^* = 1$. Our framework then tells us that the diagram $\propto |\lambda|^2$ or $|\lambda'|^2$. No one loop contribution to EDM is possible from the trilinear $R_p$ couplings.
These diagrams are possible if $\nu$ have Majorana type masses (and lepton number violating senutrino masses). We expect that these contributions may be rather small due to smallness of the $\nu$ masses.
One numerical estimate

\[ d_e \sim \frac{(e^2, g_Z^2)}{4\pi^2} \frac{1}{4\pi^2} \sum_{i,j} m_l \lambda_{i11} \lambda_{i11} \frac{1}{m_{\tilde{\nu}_i}^2}. \]

Of course only a qualitative estimate. Note an enhancement by \( m_\tau/m_e \) as expected from general analysis. Enhancement w/out paying any price for mixing angles. Using the bound on \( d_e \) one gets,

\[ \Im (\lambda_{211} \lambda_{233}^*) < 5 \times 10^{-4} \left( \frac{\tilde{m}_{111}}{1\text{TeV}} \right)^2. \]

For some of the Higgs exchange diagrams explicit calculations for the two loop contributions exist and our crude estimates are in reasonable agreement with them. Using the bound on \( d_e \) then one can constrain the \( R_p \) couplings:

\[ \Im \sum_{i \neq 1} \lambda_{i11} \lambda_{i33}^* < 0.6 \times 10^{-4} \left( \frac{\tilde{m}_{111}}{1\text{TeV}} \right)^2. \]
A. Faessler, Kovalenko et al: hep-ph/0604026

They use a one pion-exchange model with CP-odd pion-nucleon interactions. The former are generated through CP violating four quark interactions by $\lambda'$ interactions.

Limits improve using the predictions for $d_{Hg}$ as opposed to $d_n$, by an order of magnitude.

Of course this limit has model dependence.
In the presence of Bilinear $R_p$ violation, there are also soft SUSY breaking scalar bilinears.

Our analysis needs modification when the bilinear mass term and the sneutrino VEV's are of the same order.

Indeed Keum and collaborators precisely found that one loop contributions are possible in this case: Phys.Rev.Lett.86:393-396,2001.

However the induced EDM's are not numerically suppressed due to strong constraints coming from $\nu$ masses.

In principle a more general analysis including both the bilinear and the trilinears may be possible.
Conclusions:

We presented a general analysis for obtaining the fermion mass dependence of the induced dipole moment operator.

We illustrated it with the example for the case of trilinear $R_p$ interactions.

The estimates we obtain agree with the ones present in literature when they are available. Our results show that the unlike the leptoquark model big enhancements by factors of $m_t/m_f$ do not occur in this case.

The analysis needs to be modified for including the bilinear $R_p$ terms.