Possible considerations to teleport fermionic particles via studying on teleportation of two-particle state with a four-particle pure entangled state

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Abstract:

In this research we have firstly studied some evolutionary theorems in new conceptual quantum information matters such as *EPR* gedanken experiment, *Bell* states and *CHSH* inequality. Then we have introduced those kinds of experiments which are dealing with spin-correlation of photons and protons by which the mentioned inequalities are violated. Proton as a famous fermionic-hadronic particle in low and high energy ***QCD*** interactions has been the center of our attentions to focus. This study has been followed by an idea we have, on possible considerations on proton teleportation by discussing about probabilistic teleportation of an unknown two-particle state with a four-particle pure entangled state and positive operator valued measure (POVM).

Introduction:

The paradox of *Einstein, Podolsky* and *Rosen* [1] was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables, by which the causality and locality theories are restored.

It is well-known that *Einstein, Podolsky* and *Rosen* (***EPR***) [1] asserted quantum mechanics is incomplete in terms of local realism, which has been a basic view on nature of classical theories. According to the local realistic theories, objects should have definite properties whether they are measured or not (reality), and there is no action-at a-distance in nature (locality). Some attempts were made to explain quantum mechanical phenomena from a view of the local realistic theories. *Bell*, however, showed quantum mechanical correlations between entangled systems, can be stronger than those by the local realistic theories [2]. Since Bell's proof was given by an inequality which could be tested experimentally, many experiments have been devoted to test the inequality by measuring polarization correlations between entangled two photons and have yielded results which are consistent with quantum mechanics. A brief history of these experiments is summarized in Refs. [3] and [4]. The mentioned movement to pursue and check experimentally the bell inequality in ***QCD*** interactions started with the key paper of *Clauser, Horne, Shimony and Holt* (***CHSH***) [5] who generalized Bell's theorem such that it applies to realizable experimental tests with pair detection of all local hidden variable theories.

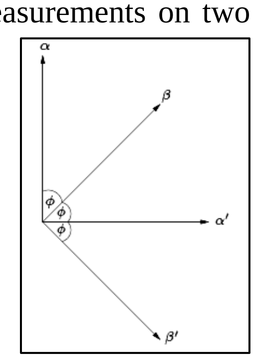
A proof of the *CHSH* inequality by *Bell* can also be found in [6] here, we rather give the final result. For any local hidden variable theory the expectation values E(i; j) of spin measurements on two particles have to fulfill the following inequality: $i=\alpha,\alpha' \; and \quad j=\beta,\beta'$

$$-2 \le S_{cl}(\alpha,\alpha',\beta,\beta') =$$

$$E(\alpha,\beta)-E(\alpha,\beta')+E(\alpha',\beta)+E(\alpha',\beta') \le 2$$

$$S(\alpha,\alpha',\beta,\beta')_{QM} = 2\sqrt{2} > |S_{cl}|$$

Orientations of angels for maximal violation of CHSH inequality



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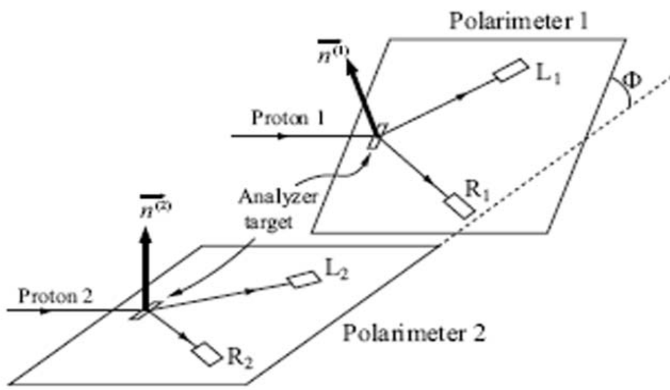
Quantum mechanically thinking, the mentioned inequality can not be totally true though. To check this non reliability for photons and most importantly for ***QCD*** like scatterings such as *Proton-Proton* (called ***PP***) scattering in low and rarely for high energies to actually observe such a violation of Bell's inequality in a laboratory some experiments was done. It was first shown for a set of local measurements on the two subsystems of an *EPR* singlet state by *Fredman* and *Clauser* and later most prominently by *A. Aspect* et al. [7] with photons and by *M. Lamehi-Rachti* and *W. Mittig* [14] with protons in entangled systems. In this experiment (figure below), we see a spin-correlation measurement of two protons produced by the $^{1}H(d,^{2}He)n$ reaction.



Here the decay of particle in the spin-singlet state[$^{1}S_{0}$] into two spin-half particles, for two protons;

. Making use of this reaction, our experiment has a distinguished feature that the directions of the spin-measuring axes are not pre-fixed by the experimental setup, which enabled us to solve the problem in this experiment.

They used strong interaction to test the Bell's inequality. Since the strong interaction is a short range interaction, entangled particles are produced with extremely short coherence length. It is of considerable interest to investigate whether an entanglement between two particles is robustly maintained even if the two particles are spatially separated from each other by a distance extremely beyond their coherence length.
Measured spin-correlations between two protons in the spin-singlet ($^{1}S_{0}$) state which was produced by the proton-proton S-wave elastic scattering. This is because proton-proton scattering at large angle and low energy, say a few MeV, goes mainly in S wave But the antisymmetry of the final wave function then requires the antisymmetries singlet spin state. In this state, when one spin is found 'up' the other is found 'down'. This follows formally from the quantum expectation value, $\langle \sin glet | \sigma_z(1)\sigma_z(2) | \sin glet \rangle = -1$.
In these experiments, protons of ***14 MeV*** lab energy are scattered at a lab angle of ***45°***, and spin correlation of scattered and recoil protons are measured.
They obtained results which showed violation of Bell's inequality, we consider there is a crucial weak point in this experiment. In their setup, one of the spin-measuring axes of the polarimeters was fixed along the normal of the scattering plane of the pp-scattering. $^{2}He[^{1}S_{0}] \rightarrow P_1 + P_2$

$[^{1}S_{0}]: |\Psi_{12}\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2\}$ *entangled state* (EPR pair) $\{C_{QM}(\theta) = -\cos\theta\} \neq \{C_{BI}(\theta) = -1 + \frac{2}{\pi}\theta\}$.

This experiment anyhow, shows agrees with quantum mechanics and disagrees (in the sense of a certain extrapolation) with the locality inequality, and is the first serious test of Bell inequality by spin $= \frac{1}{2}$ fermion system with mass. Violating the *Bell* inequality for the sake of *EPR* states, opens up the world of teleportation for us, that has recently been the source of many researches in this domain [9].

Quantum Teleportation

Quantum teleportation is a technique for moving quantum states around, even in the absence of a quantum communications channel linking the sender of quantum state to the recipient. Now it seems necessary to explain some crucial concepts, namely, entanglement, qubit and Bell state, which have key roles to perceive teleportation.

What is the entanglement?

In order to understand the concept of quantum communication the phenomena called entanglement, described for the first time at 1935 by *Einstein*, *Podolsky* and *Rosen*. They explained the theoretical possibility, from the point of view of quantum physics, to create a system composed by two particles with a strong interaction between them and which is known as *EPR* system in honour of its discoverers. The interaction which was described in these particles is so deep to the limit that if one of them was modified, the other one was instantly affected on a determined and unique way. Until this point, nothing escapes to the classical physics being this kind of interactions present, for example, at a planet–satellite system. But the EPR interactions go further, and have three main characteristics which make them unique and extremely interesting:

• Any modification applied to one of the particles, from now on particle 1, would imply a variation on its couple (particle 2). The alteration on particle 2 would depend on the modification done on particle1. Different modifications would imply different variations.

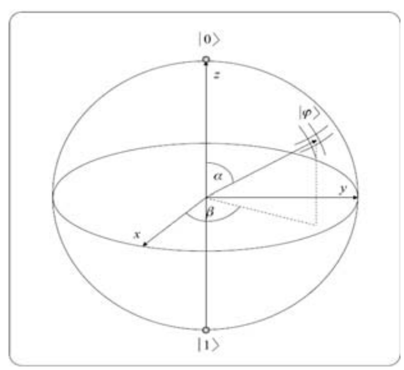
• The alteration of particle 2, due to the modification of particle 1 does not depend on the distance between Them.

• Particle 2 is instantly altered once particle 1 is modified.

What is qubit?

Qubit is a fundamental element for quantum computation and quantum information. In the early days of quantum mechanics the qubit structure was not at all obvious, and people struggle with phenomena that we may now understand in terms of qubits, that is, in the terms of two level quantum systems.

A decisive early experiment indicating the qubit structure we conceived by *Stern* in 1921 and performed with *Gerlach* in 1922 in Frankfurt.



Qubit can be in a state other than $|0\rangle$ or $|1\rangle$. It is also possible to form $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$ where the numbers α and β are complex numbers. In general a qubit's state is a unit vector in a two-dimensional complex vector space. Because $|\alpha^2| + |\beta^2| = 1$ we write, $|\Psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$ where the angels θ and φ define a point on the unit three-dimensional sphere. This sphere is often called the *Bloch* sphere, which provides a useful means of visualizing the state of single qubit and often serves as an excellent testbed for ideas about quantum computation and quantum information.

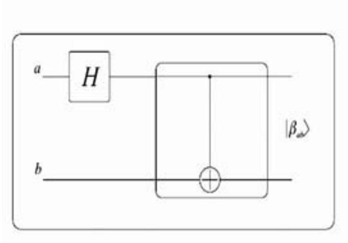
Bloch sphere representation of a qubit

What is Bell state?

Suppose we have two qubit system whose four computational basis states are denoted by $|00\rangle, |01\rangle, |10\rangle, |11\rangle$. Obviously an infinite number of different entangled pairs can be defined by only varying probability amplitudes of $|\Phi\rangle = a|00\rangle + b|11\rangle$. Moreover computational basis states $|01\rangle$ and $|10\rangle$ also form a suitable subset of all possible two-qubit basis vectors.
There are four distinguished entangled pairs called *Bell states* or *EPR pairs*

$$|\beta_{00}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}} \qquad |\beta_{01}\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

$$|\beta_{10}\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}} \qquad |\beta_{11}\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$



Bell state generator quantum circuit

Bell states have an often exploited important property, namely they form an orthonormal vector set which is equivalent to the fact – as we will see later – that they can be distinguished unambiguously.
Now here’s how quantum teleportation works. **Alice** and **Bob** met long ago but now live far apart. While together they generated an EPR pair, each taking one qubit of the EPR pair when they separated. Many years later, Bob is in hiding, and Alice's mission, should she choose to accept it, is to deliver a qubit $|\Psi\rangle$ to Bob. She does not know the state of the qubit, and moreover can only send *classical* information to Bob.
In outline, the steps of the solution are as follows: Alice interacts the qubit $|\Psi\rangle$ with half of EPR pair (singlet state), and then measures the two qubits in her possession, obtaining one of four possible classical results, 00, 01, 10 and 11. She sends this information to Bob. Depending on Alice's classical message, Bob performs one of four operations on his half of the EPR pair. Amazingly, by doing this he recovers the original state $|\Psi\rangle$!.
The state to be teleported is $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$, where α and β are unknown amplitudes. EPR pair of Alice and Bob as seen below is the package B,C. Qubit A which belongs to Alice, now should be teleported to Bob.

Using a special quantum circuit figured in [9, pp. 27] the state input into the quantum circuit $|\Psi_0\rangle$ is $|\Psi_0\rangle = \frac{1}{\sqrt{2}}\left[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|00\rangle + |11\rangle)\right]$.
The first two qubits (on the left) belong to Alice, and the third qubit to Bob. Alice sends her qubits through a CNOT gate, and then sends the first qubit through a Hadamard gate, obtaining,

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}\left[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|10\rangle + |01\rangle)\right]$$

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}\left[\alpha(|0\rangle + |1\rangle)(|00\rangle + |11\rangle) + \beta(|0\rangle - |1\rangle)(|10\rangle + |01\rangle)\right]$$

The latter state may be re-written in the following way, simply by regrouping terms:

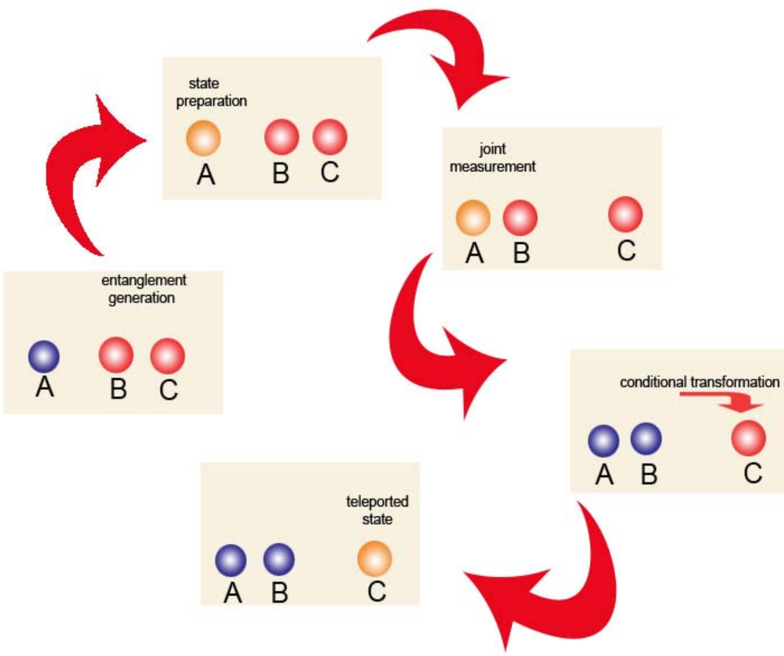
$$|\Psi_2\rangle = \frac{1}{2}[|00\rangle(\alpha|0\rangle + \beta|1\rangle) + |01\rangle(\alpha|1\rangle + \beta|0\rangle) + |10\rangle(\alpha|0\rangle - \beta|1\rangle) + |11\rangle(\alpha|1\rangle - \beta|0\rangle)].$$

Obviously Alice obtains one of the four possible two-bit results among 00, 01, 10 or 11 as measurement outcome. Each of them is in close connection with the state of Bob's qubit hence Alice sends these two classical bits to Bob. After a short hesitation Bob compares $|\Psi\rangle$ to the potential states of his half Bell pair. It is easy to realize the

$$00 \quad \rightarrow \quad \frac{\alpha|0\rangle + \beta|1\rangle}{2} = \mathrm{I}|\Psi\rangle \qquad 01 \quad \rightarrow \quad \frac{\alpha|1\rangle + \beta|0\rangle}{2} = X|\Psi\rangle$$

$$10 \quad \rightarrow \quad \frac{\alpha|0\rangle - \beta|1\rangle}{2} = Z|\Psi\rangle \qquad 11 \quad \rightarrow \quad \frac{\alpha|1\rangle - \beta|0\rangle}{2} = ZX|\Psi\rangle$$

Therefore Bob has only to apply the inverse of the appropriate transform(s) in compliance with the received classical bits.



So as it can be seen, quantum teleportation is a process of transmission of an unknown quantum state via a previously shared EPR pair with the help of only two classical bits transmitted through a classical channel [8]. It was regarded as one of the most striking progress of quantum information theory [9]. It may have a number of useful applications in quantum computer [10, 11], quantum dense coding [12], quantum cryptography [13] and quantum secure direct communication. The first scheme for teleporting an unknown quantum state from a sender to a receiver was proposed by Bennett et al.

About three years ago, YAN Feng-Li , DING He-Wei designed a scheme for probabilistic teleporting an unknown two particle state with a four-particle pure entangled state and a projective measurement on an auxiliary particle. It is shown that by performing two Bell state measurements, a proper POVM and a unitary transformation, the unknown two-particle state can be teleported from the sender Alice to the receiver Bob with certain probability.
Suppose Alice has two particles 1and 2 in an unknown state,

$$\left|\Psi\right\rangle_{12} = (a\left|00\right\rangle + b\left|01\right\rangle + c\left|10\right\rangle + d\left|11\right\rangle) \qquad |a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$$

Where a, b, c and d are arbitrary complex numbers. Alice and Bob share quantum entanglement in the form of the following four-particle pure entangled state, which is used as the quantum channel below,

$$\left|\Psi\right\rangle_{3456} = (\alpha\left|0000\right\rangle + \beta\left|1001\right\rangle + \gamma\left|0110\right\rangle + \delta\left|1111\right\rangle)_{3456}$$

The particles 3 and 4, and particle pair (1, 2) are in Alice's possession, and particles 5 and 6 are in Bob's possession.The overall state of six particles is

$$\left|\Psi\right\rangle_{w} = \left|\Psi\right\rangle_{12} \otimes \left|\Psi\right\rangle_{3456}$$

If the outcomes of the Alice's two Bell state measurements are $\left|\Phi^+\right\rangle_{23} and \left|\Phi^+\right\rangle_{14}$, then the particles 5 and 6 are in the state,

$$\left|\Psi_0\right\rangle_{56} = \frac{1}{\sqrt{|a\alpha|^2 + |b\beta|^2 + |c\gamma|^2 + |d\delta|^2}} \cdot (a\alpha\left|00\right\rangle + b\beta\left|01\right\rangle + c\gamma\left|10\right\rangle + d\delta\left|11\right\rangle)$$

Now, we will not write out the states of the particles 5 and 6 corresponding to the other outcomes of Alice's two Bell state measurements [16]. Then Alice informs Bob her two Bell state measurements on particles 2, 3 and 1, 4 Without loss of generality, we give the case for $\left|\Psi_0\right\rangle_{56}$, all other cases can be deduced similarly. In order to realize the teleportation, Bob introduces two auxiliary qubits a and b in the state $\left|00\right\rangle_{ab}$. Thus the state of particles 5, 6, a, and b becomes,

$$\left|\Psi_0\right\rangle_{56}\left|00\right\rangle_{ab} = \frac{1}{\sqrt{|a\alpha|^2 + |b\beta|^2 + |c\gamma|^2 + |d\delta|^2}} \cdot (a\alpha\left|00\right\rangle + b\beta\left|01\right\rangle + c\gamma\left|10\right\rangle + d\delta\left|11\right\rangle)_{56}\left|00\right\rangle_{ab}$$

Then Bob performs two controlled-not operations(C NOT gate) with particles 5 and 6 as the control qubits and the auxiliary particles a and b as the target qubits respectively. After completing this operation the particles 5, 6, a, and b are in the following state,

$$\left|\Psi_0^{'}\right\rangle_{56ab} = \frac{1}{\sqrt{|a\alpha|^2 + |b\beta|^2 + |c\gamma|^2 + |d\delta|^2}} \cdot (a\alpha\left|0000\right\rangle + b\beta\left|0101\right\rangle + c\gamma\left|1010\right\rangle + d\delta\left|1111\right\rangle)_{56ab}$$

After some rearrangement one obtains,

$$\left|\Psi'_0\right\rangle_{56ab} = \frac{1}{\sqrt{|a\alpha|^2+|b\beta|^2+|c\gamma|^2+|d\delta|^2}} \cdot [(a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle)_{56} \otimes (\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle)_{ab}$$

$$+ (a|00\rangle + b|01\rangle - c|10\rangle - d|11\rangle)_{56} \otimes (\alpha|00\rangle + \beta|01\rangle - \gamma|10\rangle - \delta|11\rangle)_{ab}$$

$$+ (a|00\rangle - b|01\rangle + c|10\rangle - d|11\rangle)_{56} \otimes (\alpha|00\rangle - \beta|01\rangle + \gamma|10\rangle - \delta|11\rangle)_{ab}$$

$$+ (a|00\rangle - b|01\rangle - c|10\rangle + d|11\rangle)_{56} \otimes (\alpha|00\rangle - \beta|01\rangle - \gamma|10\rangle + \delta|11\rangle)_{ab}\,]$$

Now it should be noted that, there is a mathematical tool known as the POVM formalism which is especially well adapted to the analysis of the measurements. [17]

Now Bob makes an optimal POVM [15,18] on the ancillary [17, pp.282] particles a and b to conclusively distinguish the above states.

We choose the optimal POVM in this subspace as follows:

$$\mathrm{P}_i = \frac{1}{x}|\Psi_i\rangle\langle\Psi_i|, \qquad i=1,2,3,4;$$

$$P_5 = \mathrm{I} - \frac{1}{x}\sum_{i=1}^{4}|\Psi_i\rangle\langle\Psi_i| = \mathrm{I} - P_1 - P_2 - P_3 - P_4$$

$$|\Psi_1\rangle = F(\frac{1}{\alpha}|00\rangle + \frac{1}{\beta}|01\rangle + \frac{1}{\gamma}|10\rangle + \frac{1}{\delta}|11\rangle)_{ab}$$

$$|\Psi_2\rangle = F(\frac{1}{\alpha}|00\rangle + \frac{1}{\beta}|01\rangle - \frac{1}{\gamma}|10\rangle - \frac{1}{\delta}|11\rangle)_{ab}$$

$$|\Psi_3\rangle = F(\frac{1}{\alpha}|00\rangle - \frac{1}{\beta}|01\rangle + \frac{1}{\gamma}|10\rangle - \frac{1}{\delta}|11\rangle)_{ab}$$

$$|\Psi_4\rangle = F(\frac{1}{\alpha}|00\rangle - \frac{1}{\beta}|01\rangle - \frac{1}{\gamma}|10\rangle + \frac{1}{\delta}|11\rangle)_{ab}$$

Where, $F = \frac{1}{\sqrt{\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} + \frac{1}{\delta^2}}}$

the matrix I in matrix P_5 above, is an identity operator, x is a coefficient relating to $\alpha, \beta, \gamma, \delta$, $1 \le x \le 4$ and makes, P_5 to be a positive operator. For exactly determining x, we would like to write the five operators P_1, P_2, P_3, P_4 and P_5 in the matrix form,

$$P_1 = \frac{F^2}{x}\begin{pmatrix} \frac{1}{\alpha^2} & \frac{1}{\alpha\beta} & \frac{1}{\alpha\gamma} & \frac{1}{\alpha\delta} \\ \frac{1}{\alpha\beta} & \frac{1}{\beta^2} & \frac{1}{\beta\gamma} & \frac{1}{\beta\delta} \\ \frac{1}{\alpha\gamma} & \frac{1}{\beta\gamma} & \frac{1}{\gamma^2} & \frac{1}{\gamma\delta} \\ \frac{1}{\alpha\delta} & \frac{1}{\beta\delta} & \frac{1}{\gamma\delta} & \frac{1}{\delta^2} \end{pmatrix} \qquad P_2 = \frac{F^2}{x}\begin{pmatrix} \frac{1}{\alpha^2} & \frac{1}{\alpha\beta} & -\frac{1}{\alpha\gamma} & -\frac{1}{\alpha\delta} \\ \frac{1}{\alpha\beta} & \frac{1}{\beta^2} & -\frac{1}{\beta\gamma} & -\frac{1}{\beta\delta} \\ -\frac{1}{\alpha\gamma} & -\frac{1}{\beta\gamma} & \frac{1}{\gamma^2} & \frac{1}{\gamma\delta} \\ -\frac{1}{\alpha\delta} & -\frac{1}{\beta\delta} & \frac{1}{\gamma\delta} & \frac{1}{\delta^2} \end{pmatrix}$$

$$P_3 = \frac{F^2}{x}\begin{pmatrix} \frac{1}{\alpha^2} & -\frac{1}{\alpha\beta} & \frac{1}{\alpha\gamma} & -\frac{1}{\alpha\delta} \\ -\frac{1}{\alpha\beta} & \frac{1}{\beta^2} & -\frac{1}{\beta\gamma} & \frac{1}{\beta\delta} \\ \frac{1}{\alpha\gamma} & -\frac{1}{\beta\gamma} & \frac{1}{\gamma^2} & -\frac{1}{\gamma\delta} \\ -\frac{1}{\alpha\delta} & \frac{1}{\beta\delta} & -\frac{1}{\gamma\delta} & \frac{1}{\delta^2} \end{pmatrix} \qquad P_4 = \frac{F^2}{x}\begin{pmatrix} \frac{1}{\alpha^2} & -\frac{1}{\alpha\beta} & -\frac{1}{\alpha\gamma} & \frac{1}{\alpha\delta} \\ -\frac{1}{\alpha\beta} & \frac{1}{\beta^2} & \frac{1}{\beta\gamma} & -\frac{1}{\beta\delta} \\ -\frac{1}{\alpha\gamma} & \frac{1}{\beta\gamma} & \frac{1}{\gamma^2} & -\frac{1}{\gamma\delta} \\ \frac{1}{\alpha\delta} & -\frac{1}{\beta\delta} & -\frac{1}{\gamma\delta} & \frac{1}{\delta^2} \end{pmatrix}$$

$$P_5 = F^2\begin{pmatrix} A & 0 & 0 & 0 \\ 0 & B & 0 & 0 \\ 0 & 0 & C & 0 \\ 0 & 0 & 0 & D \end{pmatrix}$$

$$A = (1-\frac{4}{x})\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} + \frac{1}{\delta^2}$$

$$B = (1-\frac{4}{x})\frac{1}{\beta^2} + \frac{1}{\alpha^2} + \frac{1}{\gamma^2} + \frac{1}{\delta^2}$$

$$C = (1-\frac{4}{x})\frac{1}{\gamma^2} + \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\delta^2}$$

$$D = (1-\frac{4}{x})\frac{1}{\delta^2} + \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$$

Obviously, we should carefully choose x such that all the diagonal elements of P_5 be nonnegative. If the result of Bob's POVM is P_1, then Bob can safely conclude that the state of the particles 5 and 6 is then,

$$|\Psi_{56}\rangle = (a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle)_{56}$$

If Bob's POVM outcome is P_2, Bob can obtain $|\Psi\rangle_{56}$ by applying the unitary transformation $\sigma_z \otimes \mathrm{I}$ on the particles 5 and 6. Two other possible results can be achieved similarly. Therefore, in these four cases stated above, the teleportation is realized successfully. However if Bob's POVM outcome is P_5, Bob can infer nothing about the identity of the state of the particles 5 and 6. In this case the teleportation fails. As a matter of fact, the key point is that Bob never makes a mistake identifying the state of the particles 5 and 6. This infallibility comes at the price that sometime Bob obtains no information about the identity of the state of the particles 5 and 6. By the similar method we can make the teleportation successful in the other outcomes of Alice's two Bell state measurements.

Evidently, when the Bell states $\left|\Phi^+\right\rangle_{23}, \left|\Phi^+\right\rangle_{14}$ are acquired in Alice's two Bell state measurements, the probability of successful teleportation is $\left|{}_{14}\left\langle\Phi^+\right|{}_{23}\left\langle\Phi^+\right\|\Phi\right\rangle_w\Big|^2 (\mathrm{I} - {}_{56ab}\left\langle\Psi_0'\right|P_5 \otimes \mathrm{I}\left|\Psi_0'\right\rangle_{56ab}) = \frac{F^2}{x}$

Synthesizing all Alice's Bell state measurement cases (sixteen kinds in all), the probability of successful teleportation in this scheme is $\frac{16F^2}{x}$. Hence the smallest x corresponds to the highest probability of successful teleportation. Because P_5 is a positive operator, i.e., A,B,C, and D are nonnegative, it is easy to obtain that the smallest x must be $\frac{4F^2}{\mu^2}$. Here μ^2 is the smallest one in the set of $\{\alpha^2, \beta^2, \gamma^2, \delta^2\}$. Hence if $x = \frac{4F^2}{\mu^2}$ is chosen, the probability of successful teleportation in this scheme is $4\mu^2$.

Conclusion:

It was shown that the possibility of teleportation of protons as fermions (in low energy scales) can be focused using some available experimental techniques. However in the last, we have used a two-particle state in one hand, with a four-particle pure entangled state in other hand to teleport a fermionic characteristic (spin), but it should be noted that we have no idea on how to prepare a successful setup to examine our suggestion. It seems that the mathematics behind it works well. But it is honestly a long way between a mythic and gedanken Idea and a viewable-experimental idea to have reasonable data in laboratory.

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