Classical field theory approach to neutrino flavor oscillations

Maxim Dvornikov

Institute of Terrestrial Magnetism,

Ionosphere and Radiowave Propagation

RUSSIA

Development of neutrino flavor oscillations theory

Neutrino flavor oscillations in vacuum

- C.Giunti et al., Phys.Rev.**D 48** (1993) 4310
- M.Blasone and G.Vitiello, Ann.Phys. **244** (1995) 283
- W.Grimus and P.Stockinger, Phys.Rev.**D 54** (1996) 3414
- M.Beuthe, Phys.Rep. **375** (2003) 105
- M.Dvornikov, Phys.Lett.**B 610** (2005) 262

Neutrino flavor oscillations in matter

- L.Wolfenstein, Phys.Rev.**D 17** (1978) 2369
- S.Mikheev and A.Smirnov, Sov.J.Nucl.Phys. **42** (1985) 913
- P.Mannhein, Phys.Rev.**D 37** (1988) 1935
- C.Cardall and D.Chung, Phys.Rev.**D 60** (1999) 073012

Pontecorvo formula

$$P(t) = \sin^2(2\theta)\sin^2\left(\frac{\pi}{L}t\right)$$

- Neutrinos were supposed to be scalar particles.
- Pontecorvo formula is valid only for high energy particles.
- Flavor or mass eigenstates?
- Energies or momentums or velocities are equal?

Mixed classical flavor neutrinos

- Let us study two flavor neutrinos (ν_1,ν_2) interacting with an external axial-vector field f^μ

$$\mathcal{L}(\nu_1,\nu_2)=\sum_{k=1,2}\mathcal{L}_0(\nu_k)$$

$$+g\bar{\nu}_1\nu_2+g^*\bar{\nu}_2\nu_1-\bar{\nu}_2\gamma_\mu^L\nu_2 f^\mu,$$

$$\mathcal{L}_0(\nu_k)=\bar{\nu}_k(i\gamma^\mu\partial_\mu-\mathrm{m}_k)\nu_k$$

Classical (first quantized) fermions

$$\frac{1}{c}\frac{\partial \psi}{\partial t} + \boldsymbol{\alpha}\nabla\psi + \frac{imc}{\hbar}\beta\psi = 0$$

- Dirac equation contains Plank constant $\hbar$. However the neutrino wave function ψ is a c***-number*** function rather than an ***operator***
- First quantized approach to neutrino flavor oscillations was also discussed in C.Nishi, Phys.Rev.**D 73** (2006) 053013

Neutrino interaction with moving and polarized matter

- In this case $\nu_1 = \nu_\mu$ or ν_τ and $\nu_2 = \nu_e$. Axial-vector field f^μ has the form (M.Dvornikov and A.Studenikin, JHEP **09** (2002) 016)

$$f^{\mu} = \sqrt{2} G_F \sum_{f=e,p,n} \left(j_f^{\mu} q_f^{(1)} + \lambda_f^{\mu} q_f^{(2)} \right),$$

$$j_f^{\mu} = \left(n_f, n_f \mathbf{v}_f \right),$$

$$\lambda_f^{\mu} = \left(n_f \left(\zeta_f \mathbf{v}_f \right), n_f \sqrt{1 - v_f^2} + \frac{n_f \mathbf{v}_f \left(\zeta_f \mathbf{v}_f \right)}{1 + \sqrt{1 - v_f^2}} \right)$$

Initial conditions problem

- We suppose that initial fields distributions of flavor neutrinos are known functions.
- Let us search for the fields distributions at subsequent moments of time

$$\nu_\mu(\mathbf{r},0)=0$$

$$\nu_e(\mathbf{r},0)=\xi(\mathbf{r})$$

$$\nu_\mu(\mathbf{r},t)=?$$

$$\nu_e(\mathbf{r},t)=?$$

Mass eigenstates

- "Mass eigenstates" are introduced by means of the matrix transformation
- Instead of the vacuum case the Lagrangian is not diagonal

$$\nu_i(\mathbf{r},t) = \sum_{k=1}^{2} M_{ik}\psi_k(\mathbf{r},t)$$

$$\mathcal{L}(\psi_1,\psi_2) = \sum_{k=1,2}\mathcal{L}_0(\psi_k)$$

$$-[\sin^2\theta\bar{\psi}_1\gamma_\mu^L\psi_1 + \cos^2\theta\bar{\psi}_2\gamma_\mu^L\psi_2$$

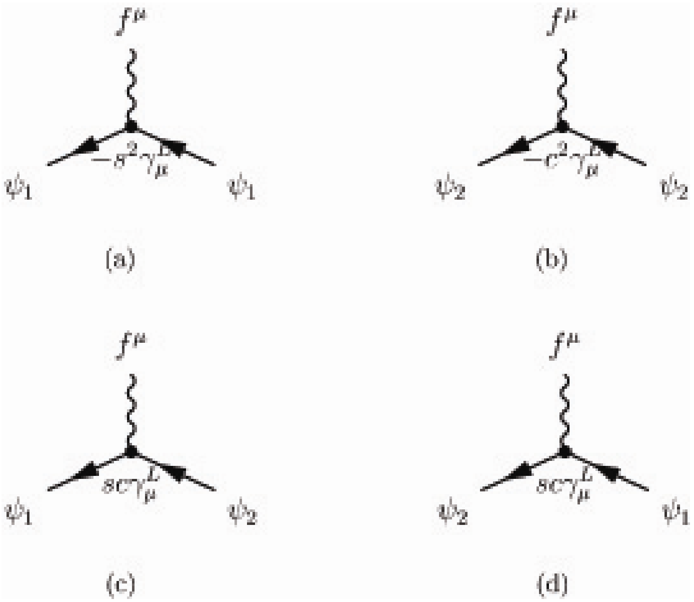
$$-\sin\theta\cos\theta(\bar{\psi}_1\gamma_\mu^L\psi_2 + \bar{\psi}_2\gamma_\mu^L\psi_1)]f^\mu,$$

$$\mathcal{L}_0(\psi_k) = \bar{\psi}_k(i\gamma^\mu\partial_\mu - m_k)\psi_k$$

Perturbation theory

- Let us describe the evolution of the mass eigenstates with help of the perturbation theory
- If we had solved this problem using the quantum field theory, we would have taken into account four Feynman diagrams

$$\psi_k(\mathbf{r},t) = \psi_k^{(0)}(\mathbf{r},t) + \psi_k^{(1)}(\mathbf{r},t) + \cdots$$



Evolution of flavor eigenstates in vacuum

- General expression for the final fields distributions (M.Dvornikov, Phys.Lett.**B 610** (2005) 262)

$$\nu_j^{(0)}(\mathbf{r},t) = \sum_{ik=1}^{2} M_{jk}(M^{-1})_{ke}$$

$$\times \int d^3\mathbf{r}' S_k(\mathbf{r}'-\mathbf{r},t)(-i\gamma^0)\xi(\mathbf{r}')$$

- Pauli-Jordan function for a spinor field

$$S_k(\mathbf{r},t) = (i\gamma^\mu \partial_\mu + m_\kappa) D_k(\mathbf{r},t),$$

$$D_k(\mathbf{r},t) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} e^{i\mathbf{p}\mathbf{r}} \frac{\sin \mathcal{E}_k t}{\mathcal{E}_k}$$

Neutrino evolution in vacuum

- Initial field distribution of $\boldsymbol{\nu}_{\mathbf{e}}$ is the plane wave
- Field distribution of $\boldsymbol{\nu}_{\boldsymbol{\mu}}$ in the ultrarelativistic limit *(ω⑦$m_{1,2}$)*

$$\xi(\mathbf{r}) = e^{i\boldsymbol{\omega}\mathbf{r}}\xi_0$$

$$\nu_\mu^{(0)}(\mathbf{r},t) \approx \sin 2\theta \sin\left[\Delta(\omega)t\right] e^{i\boldsymbol{\omega}\mathbf{r}} \times$$

$$\left\{\sin\left[\sigma(\omega)t\right] + i\left(\boldsymbol{\alpha}\mathbf{n}\right)\cos\left[\sigma(\omega)t\right]\right\}\xi_0,$$

$$\sigma(\omega) \to \omega + \frac{m_1^2 + m_2^2}{4\omega}, \quad \Delta(\omega) \to \frac{\Delta m^2}{4\omega}$$

- Intensity (transition probability)

$$I^{(0)}(t) = \left|\nu_\mu^{(0)}\right|^2 = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2}{4\omega}t\right)$$

$$+\mathcal{O}\left(\frac{m_k}{\omega}\right)$$

Neutrino evolution in matter

- Field distribution of ν_μ in the ultrarelativistic limit *(ω⑦$m_{1,2}$)*

$$\nu_\mu^{(1)}(\mathbf{r},t) \approx \frac{i}{4}\sin 2\theta\left[f^0-(\mathbf{fn})(\mathbf{\Sigma n})\right]\left(1+\gamma^5\right)e^{i\omega\mathbf{r}}\times$$
$$\left\{\frac{\sin\Delta(\omega)t}{\Delta(\omega)}\cos 2\theta\left(\cos[\sigma(\omega)t]-i(\boldsymbol{\alpha}\mathbf{n})\cos[\sigma(\omega)t]\right)\right.$$
$$+t\left(\sin^2\theta\cos[\mathcal{E}_1(\omega)t]-\cos^2\theta\cos[\mathcal{E}_2(\omega)t]\right.$$
$$\left.\left.-i(\boldsymbol{\alpha}\mathbf{n})\left\{\sin^2\theta\sin[\mathcal{E}_1(\omega)t]-\cos^2\theta\sin[\mathcal{E}_2(\omega)t]\right\}\right)\right\}\xi_0$$

- Intensity (transition probability)

$$I(t)=I^{(0)}(t)+I^{(1)}(t)=\sin^2(2\theta)\Big\{\sin^2[\Delta(\omega)t]+$$
$$\cos 2\theta\sin[\Delta(\omega)t]\left(\frac{\sin[\Delta(\omega)t]}{\Delta(\omega)}-t\cos[\Delta(\omega)t]\right)\left[f^0-(\mathbf{fn})\right]\Big\}$$
$$+\mathcal{O}\left(\frac{m_k}{\omega}\right)$$

Comparison with previous results

- Neutrino transition probability in moving and polarized matter (H.Nunokava, V.Semikoz, A.Smirnov, J.Valle, Nucl.Phys.**B 501** (1997) 17; A.Grigoriev, A.Lobanov and A.Studenikin, Phys.Lett.**B 535** (2002) 187)
- Effective mixing angle and oscillation length

$$P_{\nu_e \to \nu_\mu}(t) = \sin^2(2\theta_{\text{eff}})\sin^2\left(\frac{\pi t}{L_{\text{eff}}}\right)$$

$$\sin^2(2\theta_{\text{eff}}) = \frac{\Delta^2(\omega)\sin^2(2\theta)}{\left[\Delta(\omega)\cos 2\theta - A/2\right]^2 + \Delta^2(\omega)\sin^2(2\theta)},$$

$$\frac{\pi}{L_{\text{eff}}} = \sqrt{\left[\Delta(\omega)\cos 2\theta - A/2\right]^2 + \Delta^2(\omega)\sin^2(2\theta)},$$

$$A = f^0 - (\mathbf{fn})$$

Limit of the “weak” external field

- Expanding effective mixing angle and oscillations length over the small parameter $A=\Delta(\omega)$ (and $At\cos 2\theta=1$) we get for the transition probability

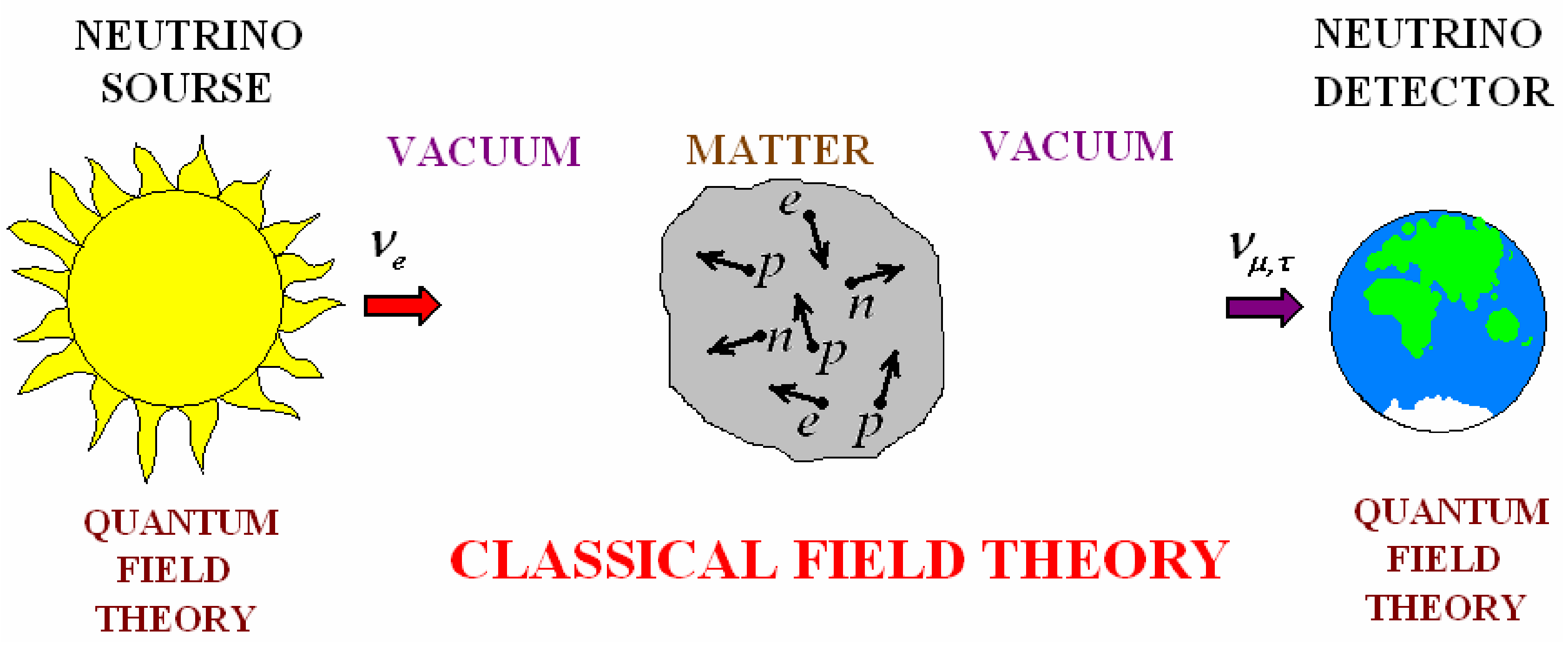
$$P_{\nu_e\to\nu_\mu}(t)\approx\sin^2(2\theta)\Big\{\sin^2[\Delta(\omega)t]+$$

$$\cos 2\theta\sin[\Delta(\omega)t]\left(\frac{\sin[\Delta(\omega)t]}{\Delta(\omega)}-t\cos[\Delta(\omega)t]\right)\left[f^0-(\mathbf{fn})\right]\Big\}$$

New terms in transition probability formula

- We receive that new rapidly oscillating terms on frequency $\omega_{rapid}=\omega+(m_1^2+m_2^2)/\omega$ (suppressed by the small factor $m_k/\omega=1$) appear in the transition probability formula.
- Recently it was claimed (M.Blasone and G.Vitiello, Ann.Phys. **244** (1995) 283) that analogous terms arise in quantum field theory treatment of neutrino flavor oscillations.
- We demonstrate that these terms appear even in the classical field theory approach.

Neutrino oscillations process



Conclusion

- The dynamics of two mixed spinor fields was examined in frames of the classical field theory. We solved the Cauchy problem for this system.
- The evolution of two fermions in vacuum as well as in an external axial-vector field was discussed.
- We applied the results of our studies to neutrino flavor oscillations in vacuum and in moving and polarized matter.
- The Pontecorvo formula for the neutrino transition probability was re-derived and the corrections to this expression were discussed.
- It was shown that the process of neutrino flavor oscillations could be treated within the classical approach.

Acknowledgements

- The organizers of the IPM School and Conference on Lepton and Hadron Physics
- Russian Science Support Foundation