The k=2 string tension for SU(4) gauge group

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Confinement

- The mechanism of confinement is still unknown and is an attractive open subject to the particle physicists.

- The confinement implies that the potential between static sources has the following form:

$$V \propto \sigma r$$

String tension (σ)

The distance between two sources (r)

- For the SU(N) gauge groups with N>3, there are more than one asymptotic string tensions which are called k-strings.

- Consider sources which transform under a global gauge transformation in the center of the group.

N-ality of the representation

The center of SU(N) gauge group

$$\Psi(x) \rightarrow z^k \Psi(x) \qquad z \in Z_N$$

- Since glouns transform trivially under the center, they can not change the N-ality of the representation
- The asymptotic string tensions depend on the N-lity of the representation not on the dimension of the representation.
- The k-string is the flux tube with the smallest N-ality and so it is a stable string which all other strings will decay to it for a long life.
- For SU(N), we have stable strings, int(N/2).

- For the asymptotic regime, There are different theories about the ratio of k-string tensions to the string tension of the fundamental quarks.
  - k independent tubes:

$$\sigma_k = \tilde{k}\sigma_{N-k} \qquad ; \qquad \tilde{k} = \min\{k, N-k\}$$

  - Casimir scaling:

$$\frac{\sigma_k}{\sigma_f} = \frac{k(N-k)}{N-1}$$

  - Sine-law scaling:

$$\frac{\sigma_k}{\sigma_f} = \frac{\sin\frac{k\pi}{N}}{\sin\frac{\pi}{N}}$$

- For intermediate regime, there are two different theories about the string tension.
  - The string tension is representation dependent and roughly proportional to the eigenvalue of quadratic Casimir operator of the representation. The proportionality of the potential to the Casimir operator is called Casimir scaling.

$$\frac{\sigma_r}{\sigma_{fund}} = \frac{C_r}{C_{fund}}$$

  - Flux tube counting is another idea which claims that the string tension is proportional to the number of fundamental flux tubes embedded into the representation

Center vortex model

- Based on vortex theory, QCD vacuum is filled with closed magnetic vortices that have the topology of lines or surfaces of finite thickness.
- A center vortex carries magnetic flux quantized in the element of the center of the gauge group.
- The inter-quark potential induces by the vortices is:

$$V = \sum_{x} \ln\{1 - \sum_{n=1}^{N-1} f_n (1 - \mathrm{Re}\, g_r[\vec{\alpha}_c^n(x)])$$

f_n: the probability that any given unit area is pierced by a vortex

$\vec{\alpha}_c^n(x)$: the location of the center of the vortex

where

$$g_r[\vec{\alpha}] = \frac{1}{d_r} Tr \exp[i\vec{\alpha}.\vec{H}]$$

d_r: dimension of the representation

$\vec{H}$: generators of Cartan sub algebra

$\vec{\alpha}$: Vortex profile

- For SU(N) gauge group, there are N-1 possible vortices .
- Vortices of type n and N-n are the same, except that the magnetic fluxes are in the opposite direction:

$$f_n = f_{N-n} \qquad g_r[\vec{\alpha}_c^{\,n}(x)] = g_r^*[\vec{\alpha}_c^{\,N-n}(x)\ ]$$

- The induced potential of the model for SU(4) may be written as:

$$V(R) = \sum_{m=-\infty}^{m=+\infty} \ln\{1 - 2f_1(1 - \mathrm{Re}[\vec{\alpha}_C^{\,1}(x)]) - f_2(1 - \mathrm{Re}[\vec{\alpha}_C^{\,2}(x)])\}$$

- One of the pre-tested profiles which seems to be physical, have been used

$$\alpha_j^i(x) = N_j^i[1 - \tanh(ay(x) + \frac{b}{R})]$$

normalization factor

$$y(x) = \begin{cases} x - R & \text{for} \quad |R - x| \le |x| \\ -x & \text{for} \quad |R - x| > |x| \end{cases}$$

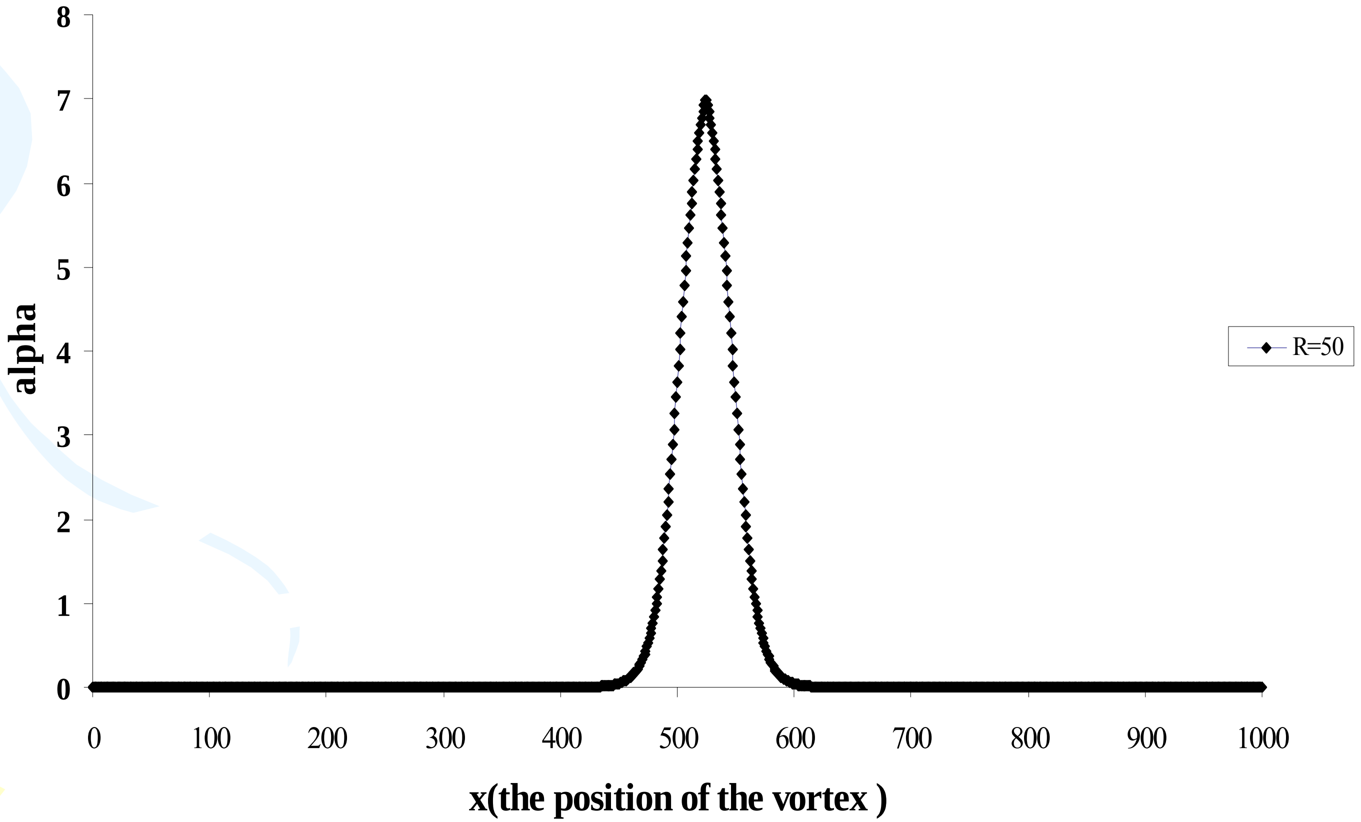
- The normalization factors must be calculated from the assumptions of the model:

  - A center vortex linked to a Wilson loop, in the fundamental representation of SU(N), has the effect of multiplying the Wilson loop by an element of gauge group:

$$W(C) \rightarrow \exp\left(\frac{2\pi i n}{N}\right) \qquad n = 1, 2, \dots, N-1$$

  - For higher representations, each time that the minimal surface is pierced by a center vortex, a center element should be inserted somewhere along the loop.

- Flux distributions must satisfy the following conditions:
  - For fixed R, as $x \rightarrow \infty$, $\alpha \rightarrow 0$.
  - If the vortex core is entirely contained within the Wilson loop, then for example for SU(2), $\alpha = 2\pi$.
  - As $R \rightarrow 0$ then $\alpha \rightarrow 0$.



Asymptotic string tensions from the vortex model

- The asymptotic string tensions for different 4-ality classes can be predicted by the model:

  - Fundamental representation as the representative of 4-ality=1 class:

$$\sigma_f \cong 2f_1 + 2f_2$$

  - 6 dimensional representation as the representative of 4-ality=2 class:

$$\sigma_6 \cong 4 f_1$$

  - adjoint representation as the representative of 4-ality=0 class:

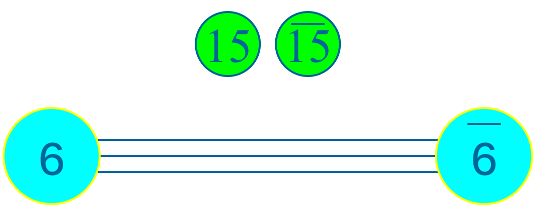
$$\sigma_{adj} = 0$$

- So, the ratio of diquark string tension to that of the fundamental one depends on f_1 and f_2.

$$\frac{\sigma_6}{\sigma_f} = \frac{4 f_1}{2 f_1 + 2 f_2}$$

- $f_1 = f_2 \quad \Rightarrow \quad \frac{\sigma_6}{\sigma_{fund}} = 1$

a universal string tension will be obtained.



$$15 \otimes 6 = 64 \oplus 10 \oplus \overline{10} \oplus 6$$

$$15 \otimes 10 = 70 \oplus 64 \oplus \overline{10} \oplus 6$$

- $$f_2 = 0 \quad \Rightarrow \quad \frac{\sigma_6}{\sigma_{fund}} = 2$$

The k= 2 string is nothing more than two separate fundamental strings joining two charges

- We use lattice results of the ratio of diquark string tension to that of the fundamental one:

B.luicini, *et. al*.: $$\frac{\sigma_6}{\sigma_{fund.}} \cong 1.377 \quad \rightarrow \quad f_2 = 0.045$$

B. Lucini, M. Teper, Phys.Rev. D**64** 105019 (2001)

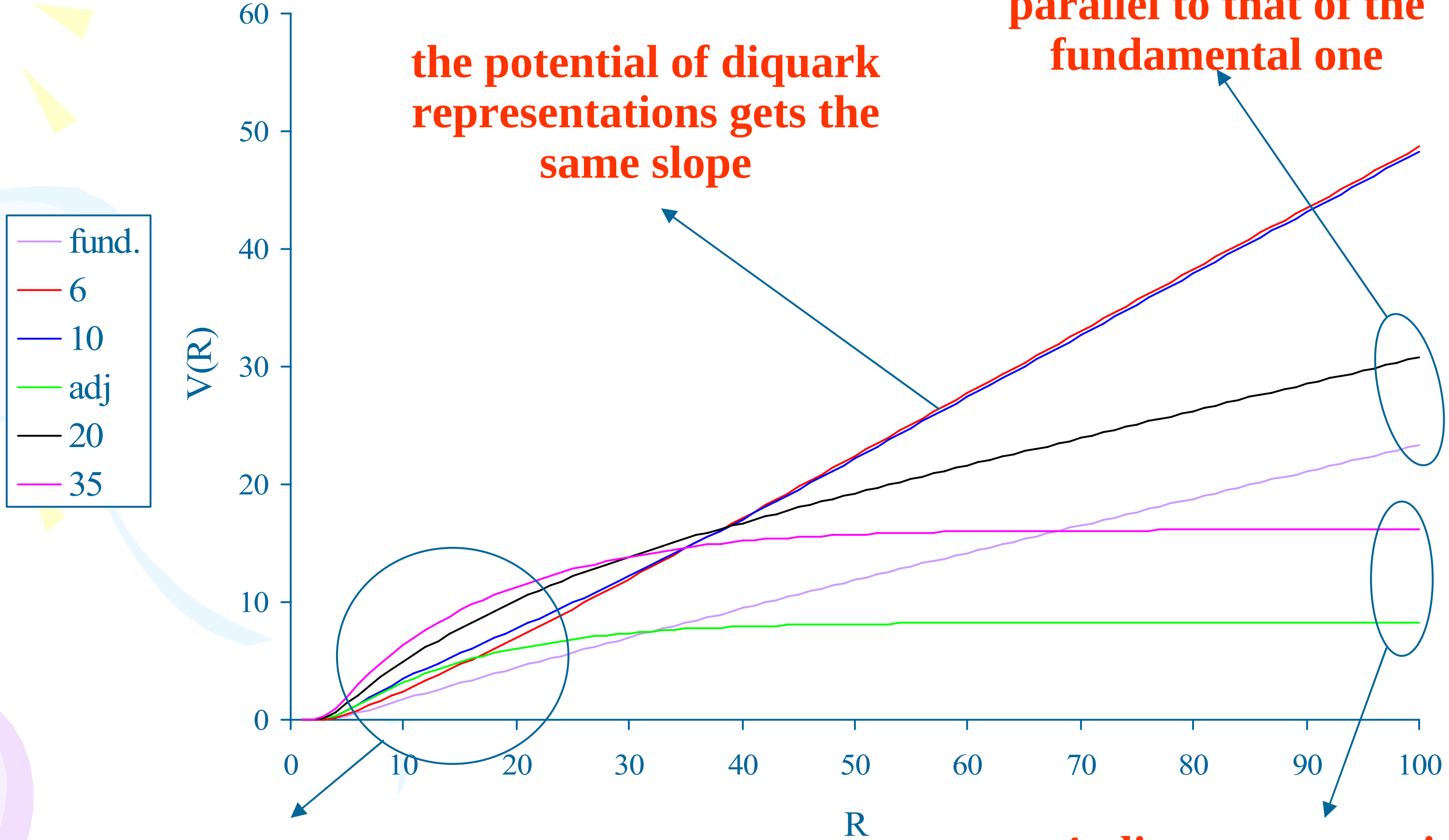
S. Ohta, *et. al*: $$\frac{\sigma_6}{\sigma_{fund,}} \cong 1.3 \quad \rightarrow \quad f_2 = 0.053$$

S.Ohta, M.Wingate, Nucl. Phys. **73** (proc suppl.), 435 (1999)

Potentials of various representations versus R
f_1=0.1 , f_2=0

representation 20 becomes parallel to that of the fundamental one

the potential of diquark representations gets the same slope



linear region

zero 4-ality representations are screened

Intermediate string tensions

probabilities	$\sigma_6/\sigma_{fund.}$	$\sigma_{15}/\sigma_{fund}$	$\sigma_{10}/\sigma_{fund}$	$\sigma_{20}/\sigma_{fund}$	$\sigma_{35}/\sigma_{fund}$
f_2**=0.043,**f_1**=0.1**	1.6	1.57	1.86	2.52	3.21
f_2**=0.03, ,**f_1**=0.1**	1.49	1.60	1.88	2.58	3.31
f_2**=0, ,**f_1**=0.1**	1.51	1.56	1.76	2.31	2.66
Casimir ratio	1.6	1.57	1.86	2.52	3.21
no. of fund. fluxes	2	2	2	3	4

conclusion

- Trying a couple of different probabilities, it is observed that intermediate string tensions are not sensitive to the amount of f_1 and f_2.
- For all representations at intermediate distances, there is a linear region in agreement with both Casimir scaling and flux counting prediction.
- At large distances:
  - zero 4-ality representations **(15,35)** are screened.
  - representation **20** has become parallel to the **fundamental** representation.
  - **diquark** representations **(6,10)** get the same slope.


Thank you