Spectator model in D Meson Decays

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In this research we describe effective Hamiltonian theory and apply this theory to the calculation of current-current Q_{12} and QCD penguin $Q_{3\ldots6}$ decay rates We calculate the decay rates of semileptonic and hadronic of charm quark in effective Hamiltonian theory. We investigate the decay rates of **D meson decays** according to **Spectator Quark Model(SQM)** for the calculation of **D** meson decays. We obtain the total decay rates of semileptonic and hadronic of charm quark in effective Hamiltonian according to colour Favoured (C-F) and colour Suppressed (C-S), and then to added amplitude of processes colour Favoured and colour Suppressed (F-S) and obtain the decay rates of them.

1. Effective Hamiltonian

The Effective Hamiltonian with QCD effects $(c \to q_i q_k \overline{q_j})$ is given by

$$H_{eff} = 2\sqrt{2}G_F \sum_{i=1}^{6} d_i(\mu)Q_i(\mu)$$

The operators $Q_i(\mu)$ can be grouped into two categories [1],[2]:i=1,2-current-current operators;$d_i(\mu) = V_{CKM}C_i(\mu)$ denotes the relevant CKM factors that are:

$$d_{1,2} = V_{ic}V_{tk}^{*}C_{1,2} \quad , \quad d_{3,\ldots,6} = -V_{tc}V_{tk}^{*}C_{3,\ldots,6} \quad (1)$$

We should take the variable$P_i$andP_k,or x and y as,

$$P_i = x\frac{M_c}{2} \quad , \quad P_k = y\frac{M_c}{2} \quad (2)$$

The partial decay rate in the c rest frame is,

$$d^2\Gamma_{Q_1,\ldots,Q_2}/dxdy = \Gamma_{0c}(\alpha_1 I_{ps}^1 + \alpha_2 I_{ps}^2 + \alpha_3 I_{ps}^3) \quad (3)$$

Here

$$\alpha_1 = |d_1 + d_2 + d_3 + d_4|^2 + 2|d_1 + d_4|^2 + 2|d_2 + d_3|^2$$

$$\alpha_2 = |d_5 + d_6|^2 + 2|d_5|^2 + 2|d_6|^2)$$

$$\alpha_3 = Re(3d_1 + d_2 + d_3 + 3d_4)d_5^{*}$$

$$I_{ps}^1 = 6xy.f_{ab}.(1 - h_{abc}), I_{ps}^2 = 6xy.f_{bc}.(1 + h_{bca}),$$

$$I_{ps}^3 = 6xy.f_{ac}.h_{xa}.h_{yc}$$

$$h_{xa} = [1-(x^2/(x^2+a^2))]^{1/2}, h_{yc} = [1-(y^2/(y^2+c^2))]^{1/2},$$

$$h_{xb} = [1-(x^2/(x^2+b^2))]^{1/2}, h_{ya} = [1-(y^2/(y^2+a^2))]^{1/2},$$

$$\Gamma_{0c} = G_f^2 M_c^5/192\pi^3, f_{ab} = 2 - \sqrt{x^2+a^2} - \sqrt{y^2+b^2},$$

$$f_{bc} = 2 - \sqrt{x^2+b^2} - \sqrt{y^2+c^2},$$

$$f_{ac} = 2 - \sqrt{x^2+a^2} - \sqrt{y^2+c^2}$$

$$h_{abc} = \frac{(f_{ab})^2 - (c^2+x^2+y^2)}{2\sqrt{x^2+a^2}\sqrt{y^2+b^2}},$$

$$h_{bca} = \frac{(f_{bc})^2 - (a^2+x^2+y^2)}{2\sqrt{x^2+b^2}\sqrt{y^2+c^2}}$$

$$h_{bca} = \frac{(f_{bc})^2 - (a^2+x^2+y^2)}{2\sqrt{x^2+b^2}\sqrt{y^2+c^2}}$$

2. Spectator Model

In the spectator model [3] the spectator quark is given a non-zero momentum having in this work a Gaussian distribution,represented by a free (but adjustable) parameter,Λ :

$$P(|P_s|^2) = (1/\pi^{3/2}\Lambda^3)e^{-(p_s^2/\Lambda^2)} \quad (4)$$

The total meson decay rate through a particular mode is then assumed to be

$$\Gamma_{total} = \int \frac{d^2\Gamma}{dp_i dp_k} P(|P_s|^2) d^3p_s dp_i dp_k \quad (5)$$

equal to the initiating decay rate. We have

$$\frac{d^2\Gamma}{dM_{is}dM_{k\bar{j}}} = \frac{2\pi M_{is}M_{k\bar{j}}}{m_c} \int \frac{E_i P_s}{P_i^2} \frac{d^2\Gamma}{dp_i dp_k} P(|P_s|^2) dP_s dP_k \quad (6)$$

Here

$$M_{is}^2 = (P_i+P_s).(P_i+P_s) = m_i^2+m_s^2+2(E_iE_s-P_iP_s\cos\theta_{is})$$

$$M^2_{k\bar{j}} = (P_k + P_{\bar{j}}).(P_k + P_{\bar{j}}) = m^2_K + m^2_{\bar{j}} +$$

$$2(E_K E_{\bar{j}} - P_K P_{\bar{j}} \cos\theta_{k\bar{j}})$$

The integration range is restricted by $|\cos\theta_{kj}| \leq 1$.We call this mode of quark and antiquark combination (colour favoured). It is also possible that the spectator antiquark combines with the quarkq_k,for which we get

$$\frac{d^2\Gamma}{dM_{ks}dM_{k\bar{j}}} = \frac{2\pi M_{ks}M_{i\bar{j}}}{M_c}\int \frac{E_k P_s}{P_k^2}\frac{d^2\Gamma}{dP_i dP_k}$$

$$P(|P_s|^2)dP_s dP_i \qquad (7)$$

We call this **process(C-S)** (colour suppressed). Summing, the decay rates of B mesons for **process(C-F)** and **process(C-S)** are:

$$\Gamma_{(C-F)} = \int_{m_{minis}}^{m_{cutis}}\int_{m_{mink\bar{j}}}^{m_{cutk\bar{j}}} \frac{d^2\Gamma}{dM_{is}dM_{k\bar{j}}} dM_{is}dM_{k\bar{j}}$$

$$\Gamma_{(C-F)} = \int_{m_{minks}}^{m_{cutks}}\int_{m_{mini\bar{j}}}^{m_{cuti\bar{j}}} \frac{d^2\Gamma}{dM_{ks}dM_{i\bar{j}}} dM_{ks}dM_{i\bar{j}} \qquad (8)$$

where $m_{minis} = (m_{q_i} + m_{q_s}), m_{cutis} = M_{q_i q_s}$ and so on.

3. Effective Hamiltonian Spectator Model

The differential decay rates for two boson system in the spectator quark model for current-current plus penguin operators in the Effective Hamiltonian is given by,

$$\frac{d^2\Gamma_{Q_1,...,Q_6}}{d(q_{si}/M_c)d(q_{k\bar{j}}/M_c)} = \Gamma_{0c}\frac{8q_{si}q_{k\bar{j}}}{\sqrt{\pi}M_c}\frac{\beta^2}{\Lambda}$$

$$\frac{\sqrt{(2m_i/M_c)^2} + x^2}{x^2}\int_0^1 dy \int_0^1 dz \zeta^{eff}_{ps(q,z)} z e^{-\beta^2 z^2} \qquad (9)$$

where $\zeta^{eff}_{ps(q,z)} = \alpha_1\zeta^{eff}_1 + \alpha_2\zeta^{eff}_2 + \alpha_3\zeta^{eff}_3$
The integration region is restricted by the condition $\cos\theta_{is} \leq 1$,thus

$$\zeta^{eff}_1, \zeta^{eff}_2, \zeta^{eff}_3 = \begin{cases} \zeta^{eff}_{1ps}, \zeta^{eff}_{2ps}, \zeta^{eff}_{3ps} & if \quad (f_{si(z)})^2) \leq 1 \\ o & otherwise \end{cases} \qquad (10)$$

where

$$f_{si(z)} = \Big([(m_i + m_s)/M_c]^2 - (q_{si}/M_c)^2 +$$

$$(1/M_c)\sqrt{m_s^2 + (\beta\Lambda z)^2} \times \sqrt{(2m_i/M_c)^2 + x^2}\Big)/(\beta\Lambda xz/M_c)$$

$$\zeta^{eff}_1, \zeta^{eff}_2, \zeta^{eff}_3 = \begin{cases} \zeta^{eff}_{1ps}, \zeta^{eff}_{2ps}, \zeta^{eff}_{3ps} & if \quad (f_{si(z)})^2 \leq 1 \\ o & otherwise \end{cases} \qquad (11)$$

where

$$f_{si(z)} = \Big([(m_i + m_s)/M_c]^2 - (q_{si}/M_c)^2 +$$

$$(1/M_c)\sqrt{m_s^2 + (\beta\Lambda z)^2} \times \sqrt{(2m_i/M_c)^2 + x^2}\Big)/(\beta\Lambda xz/M_c)$$

Therefore using Eq.(6-11) the phase space parameters will be defined by,

$$\zeta^{eff}_{1ps} = 6xy.f_{ab}.(1 - h_{abc}), \quad \zeta^{eff}_{2ps} = 6xy.f_{bc}.(1 - h_{bca}),$$

$$\zeta^{eff}_{3ps} = 6xy.f_{ab}.h_{xa}.h_{yc} \qquad (12)$$

Now, we can integrate over the two mass cuts (two boson systems), and obtain the hadronic decay rates as follows,

$$\Gamma'_{Q_1,...,Q_6} = \int_{min}^{m_{cut}}\int_{min'}^{m'_{cut}} \frac{d^2\Gamma_{Q_1,...,Q_6}}{d(q_{si}/M_c)d(q_{k\bar{j}/M_c})} dm_{cut}dm'_{cut},$$

$$= \Gamma_{0c}\int_{min}^{m_{cut}}\int_{min'}^{m'_{cut}} \frac{8q_{si}q_{k\bar{j}}}{\sqrt{\pi}M_c}\frac{\beta^2}{\Lambda}\frac{\sqrt{(2m_i/M_c)^2 + x^2}}{x^2}$$

$$\int_0^1 dy \int_0^1 dz \zeta^{eff}_{ps(q,z)} z e^{-\beta^2 z^2} dm_{cut}dm'_{cut}. \qquad (13)$$

4. Decay Rates of Processes C-F plus C-S (F+S) of Effective Hamiltonian

Now we want to calculate the decay rates of Effective Hamiltonian $(Q_1, ..., Q_6)$ for F+S at quark-level and spectator model. The Effective Hamiltonian for F+S, is given by

$$H^{A+B}_{eff} = H^{b\to ik\bar{j}}_{eff} + H^{b\to j\bar{j}k}_{eff}. \qquad (14)$$

where$H^{c\to ik\bar{j}}_{eff}$is defined by Equations mentioned during in the last page, so we can obtain $H^{c\to i\bar{j}k}_{eff}$.The decay rates of current-current plus penguin for F+S is given by,

$$d^2\Gamma^{F+S}_{EH}/dxdy = \Gamma_{0c}(I_{1ps} + I_{2ps} + I_{3ps}) \qquad (15)$$

$$I_{1ps} = 6xy.f_{ab}.[\alpha_1((3/2) - h_{abc}) + \alpha_2 - \alpha_3 h_{xa}h_{yb}],$$

$$I_{2ps} = -6xy.f_{ac}.[\alpha_1 h_{acb} + \alpha_3 h_{xa}h_{yc}],$$

$$I_{3ps} = 6xy.f_{bc}.[(\alpha_1/2)h_{bca} + \alpha_2(h_{xb}h_{yc} - h_{bca})]. \qquad (16)$$

5. Numerical Results

We use the standard Particle Data Group [5] parameterization of the CKM matrix. Following Ali and Greub [2] we treat internal quark masses in tree-level loops with the values (GeV)$m_b = 4.88, m_s = 0.2, m_d = 0.01, m_u = 0.005, m_c = 1.5, m_e = 0.0005, m_\mu = 0.1, m_\tau = 1.777$and $m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} = 0$.Following G.Buccella [6] we choose the effective Wilson coefficients $C_i^{eff}$for the various $c \to q$ transitions.We have used in Spectator Quark Model the value$\Lambda = 0.6 GeV$[7].For the maximum mass of the quark-antiquark systems (m_{cut})we take a value midway between the lowest mass1^-state and the next most massive meson. The decay rates of c quark for Effective Hamiltonian and Effective Hamiltonian of F+S shown in the Table.1. Also the decay rates of c quark for F+S is given by

$$(c \to du\overline{d}) \qquad D^+ \to (\pi^0, \eta, \rho^0, \omega), (\pi^+, \rho^+)$$

$$BR_{EH}^{F+S} = 2.1023 \times 10^{-2},$$

$$(c \to su\overline{d}) \qquad D^+ \to (\pi^+, \rho^+), (\overline{K^0}, \overline{K^{*+}})$$

$$BR_{EH}^{F+S} = 51.2871 \times 10^{-2},$$

$$(c \to su\overline{s}) \qquad D^+ \to (\eta', \phi), (K^+, K^{*+})$$

$$BR_{EH}^{F+S} = 3.8671 \times 10^{-2}.$$

6. Conclusion

We used Effective Hamiltonian theory and spectator quark model for c quark and calculated hadronic decays of D mesons. In this model we added decays of channel hadronic decays of D mesons. For colour favoured and suppressed we consider the channel $c \to du\overline{d}$ (*e.g.* $D^+ \to \pi^0\pi^+$)and achieved theoretical values very close to experimental ones. Finally it has been shown the case, in which the theoretical values are better than the amplitude of all the decay rates have been calculated. In table 1 (below) it must be noted that columns 2 and 4 have to be multiplied by $10^{-15}$and columns 3 and 5 should be multiplied by 10^{-3}.

Process	Γ_{EH}	BR_{EH}	Γ_{EH}^{F+S}	BR_{EH}^{F+S}
$c \to du\overline{d}$	31.689	32.12	35.611	31.262
$c \to du\overline{s}$	1.0785	1.093	1.4608	1.2824
$c \to su\overline{d}$	409.44	414.95	554.45	486.74
$c \to su\overline{s}$	23.836	24.157	26.927	23.638

Table 1: Decay rates(Γ)and Branching Ratio (BR) of Effective Hamiltonian (EH) and F+S of effective hamiltonian of c quark.

7. Acknowledgments

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