I review recent progress on exclusive hadronic $B$ meson decays in the perturbative QCD approach, with focus on puzzles in the branching ratios and the CP asymmetries of the $B \rightarrow \pi K$ and $B \rightarrow \pi\pi$ modes, and polarization fractions in $B \rightarrow VV$ modes.

1. Introduction

$B$ factory experiments have accumulated a lot of data and have reported many interesting results [1]. Some observables, mixing-induced CP asymmetries for $b \rightarrow s$ penguin modes, branching ratios and direct CP asymmetries for $B \rightarrow \pi K$ and $\pi\pi$, and polarization fractions for penguin-dominated $B \rightarrow VV$ modes, have exhibited some deviations from naive expectations in the Standard Model.

It is necessary to go beyond naive estimations for understanding the observed deviations. The perturbative QCD (PQCD) approach [2, 3] is one of the theoretical attempts to include subdominant contributions, such as spectator and annihilation diagrams, and higher-order corrections. PQCD has applied to various two-body $B$ decays at leading order (LO) in $\alpha_s$ and has made reasonable predictions for various decay modes. Recently, the important next-to-leading-order (NLO) contributions were evaluated in the $B \rightarrow \pi K$, $\pi\pi$, and $\rho\rho$ decays to investigate the discrepancies between the LO PQCD predictions and the data [4, 5].

This talk is organized as follows: In Sec. 2, I briefly review the PQCD factorization formula. I discuss the branching ratios and the CP asymmetries of the $B \rightarrow \pi K$ and $\pi\pi$ decays with the NLO corrections in Sec. 3. The LO PQCD predictions of the polarization fractions are presented in Sec. 4. Section 5 is a summary.

2. PQCD Factorization Theorem

Most of the calculations of $B$ decay amplitudes rely on the factorization of decay amplitudes into a product of short-distance and long-distance physics. QCD-improved factorization (QCDF) [6] and soft-collinear effective theory (SCET) [7] are based on collinear factorization theorem, but PQCD is based on $k_T$ factorization theorem.

Employing collinear factorization theorem, some decay amplitudes involve a singularity arising from the end-point region of parton momentum fractions. An end-point singularity implies that a decay amplitude is dominated by soft dynamics and cannot be factorized. Such soft contributions are regarded as phenomenological parameters, which can be fitted from the experimental data.

In the PQCD approach with $k_T$ factorization theorem, the Sudakov factor ensures the absence of the end-point singularities [8]. All amplitudes can be factorizable into parton distribution amplitudes $\Phi$, the Sudakov factors $e^{-s}$, and a hard kernel $H$:

$$A(B \rightarrow M_2 M_3) = \Phi_{M_2} \otimes \Phi_{M_3} \otimes H \otimes e^{-S} \otimes \Phi_B 
(1)$$

where $\otimes$ stands for convolutions in both longitudinal and transverse momenta of partons [9]. The schematic picture of the factorization theorem is given in Fig. 1. The distribution amplitudes, which are universal in the processes under consideration, are determined from experiments, the light-cone QCD sum rules, lattice calculations, or other theoretical methods. The hard kernel is characterized by a hard scale $Q \sim \sqrt{\Lambda m_b}$, where $\Lambda$ is a hadronic scale and $m_b$ the $b$ quark mass [10–12], and can be evaluated as an expansion in powers of $\alpha_s(Q)$ and $\Lambda/Q$. The hard kernels of the spectator and annihilation contributions, as well as the emission contribution, are calculable and start from $O(\alpha_s)$. PQCD predicts a large direct CP asymmetry in $B^0 \rightarrow \pi^\pm K^\mp$ as a result of a large strong phase arising from annihilation penguin diagrams [2].

3. $B \rightarrow \pi K$ and $\pi\pi$ Puzzles

The current data of the direct CP asymmetries of $B \rightarrow \pi K$ and the branching ratios of $B \rightarrow \pi\pi$ [1],

$$A_{CP}(B^\pm \rightarrow \pi^0 K^\pm) = (4 \pm 4)\%$$

Figure 1: Factorization of hadronic two-body $B$ meson decays in the PQCD approach.
After the text, there is a table titled "Table II Direct CP asymmetries for the $B \to \pi K$ and $\pi \pi$ decays in percentage."

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>$B^{\pm} \to \pi^{\mp} K^\pm$</td>
<td>-2.4 ± 1.9</td>
<td>-1.9 ± 0.6</td>
<td>0 ± 2.1</td>
</tr>
<tr>
<td>$B^{\pm} \to \pi^{\mp} K^\mp$</td>
<td>4 ± 1.7</td>
<td>-1.9 ± 0.1</td>
<td>-1.9 ± 0.6</td>
</tr>
<tr>
<td>$B^{0} \to \pi^{\mp} K^\mp$</td>
<td>-1.0 ± 0.1</td>
<td>-2.1 ± 0.1</td>
<td>-10 ± 6</td>
</tr>
<tr>
<td>$B^{0} \to \pi^{0} K^0$</td>
<td>2 ± 1.7</td>
<td>1.0 ± 0.5</td>
<td>-7 ± 3</td>
</tr>
<tr>
<td>$B^{0} \to \pi^{0} K^0$</td>
<td>37 ± 10</td>
<td>16.0 ± 30.0</td>
<td>18 ± 12</td>
</tr>
<tr>
<td>$B^{\pm} \to \pi^{\mp} \pi^0$</td>
<td>1 ± 0.5</td>
<td>0.0 ± 0.0</td>
<td>0 ± 0</td>
</tr>
<tr>
<td>$B^{0} \to \pi^{0} \pi^0$</td>
<td>28 ± 40</td>
<td>20.0 ± 40.0</td>
<td>63 ± 34</td>
</tr>
</tbody>
</table>

The text then continues with a discussion of CP asymmetries and the comparison of predicted and observed branching ratios. It is noted that the inclusion of NLO contributions in the PQCD approach may lead to deviations from the observed values, and that the combination of the LO and NLO predictions is necessary to account for all the observed effects. The text concludes with a discussion of the implications of these results for understanding the underlying physics of $CP$ violation in $b$-quark decays.
C for $B \to \pi \pi$, as well as $C'$ for $B \to \pi K$, is enhanced by the vertex corrections, but it is insufficient to accommodate the $B^0 \to \pi^0 \pi^0$ branching ratio to the measured value [4]. NLO PQCD predicts $[C/T] \approx 0.2$ for $B \to \pi \pi$, though a much larger $[C/T] \approx 0.8$ is required to explain the observed $B^0 \to \pi^0 \pi^0$ branching ratio [25].

The same NLO PQCD formalism was applied to the $B \to \rho \rho$ decays [5], which are sensitive to the color-suppressed tree contribution. The predicted $B \to \rho \rho$ branching ratios are listed in Table III. The LO results differ from those in the previous LO analyses [31, 32] slightly due to the different choices of the hard scale and parameters. The NLO PQCD predictions for the $B^0 \to \rho^+ \rho^-$ and $B^\pm \to \rho^0 \rho^0$ branching ratios are consistent with the data. Because the decay amplitudes for $B \to \rho \rho$ are similar to those for $B \to \pi \pi$, the branching ratio of $B^0 \to \rho^0 \rho^0$ is expected to be larger than that of $B^0 \to \pi^0 \pi^0$ due to the meson decay constants $f_\rho > f_\pi$. In fact, the NLO predictions follow this expectation, and the central value of the predicted $B^0 \to \rho^0 \rho^0$ branching ratio has almost reached the experimental upper bound. The NLO PQCD analysis has thus confirmed that it is unlikely to accommodate both the $B^0 \to \pi^0 \pi^0$ and $B^0 \to \rho^0 \rho^0$ branching ratios to the data simultaneously. Hence, the $B \to \pi \pi$ puzzle is confirmed in the PQCD approach. All proposed resolutions to the $B \to \pi \pi$ puzzle should survive the constraints from the $B^0 \to \rho^0 \rho^0$ data.

I comment on $B \to \pi K$ and $\pi \pi$ results in other theoretical approaches. In QCDF, $C^{(i)}$ is enhanced by the NLO jet function obtained from SCET and the large $B^0 \to \pi^0 \pi^0$ branching ratio can be explained [34]. However, the inclusion of the NLO jet function over-shoots the $B^0 \to \rho^0 \rho^0$ branching ratio and deteriorates the predictions for the $B^0 \to \pi^0 K^\pm$ and $B^0 \to \pi^\pm K^\mp$ direct CP asymmetries [5]. In SCET, inculcable soft contributions are regarded as phenomenological parameters, which can be fitted from the experimental data. It was found that the charming penguin, which is one of the phenomenological parameters, is large as $[C^{(i)}/T^{(i)}] \approx 1$ [35–37]. Consequently, the large $B^0 \to \rho^0 \rho^0$ branching ratio can be realized. The $B^0 \to \rho^0 \rho^0$ branching ratio should be checked in the same formalism. The ratio $C^{(i)}/T^{(i)}$ is, however, real in the leading-power SCET formalism, and therefore the $B^\pm \to \pi^0 K^\mp$ and $B^0 \to \pi^\pm K^\mp$ direct CP asymmetries can not be explained at the same time [35–37].

4. Polarizations in $B \to V V$

In the naïve factorization approximation, the longitudinal and transverse polarization fractions, $R_L$ and $R_{\perp \perp}$, respectively, in $B \to V V$ modes obey the power-counting rules [38],

$$R_L \sim 1 - O(m^2_V/m^2_B), \quad R_{\perp \perp} \sim O(m^2_V/m^2_B),$$  \hspace{1cm} (9)

where $m_B$ is the mass of the $B$ meson and $m_V$ that of the emitted vector meson from the weak vertex. The data of the longitudinal polarization fractions are given by [1]

$$R_L(B^0 \to \rho^+ \rho^-) = 0.967^{+0.023}_{-0.028},$$

$$R_L(B^+ \to \rho^0 \rho^0) = 0.96 \pm 0.06,$$

$$R_L(B^+ \to K^{*+} \rho^0) = 0.91^{+0.23}_{-0.21},$$

$$R_L(B^+ \to K^{*0} \rho^+) = 0.48 \pm 0.08,$$

$$R_L(B^+ \to \phi K^+) = 0.50 \pm 0.07,$$

$$R_L(B^0 \to \phi K^{*0}) = 0.48 \pm 0.04. \hspace{1cm} (10)$$

The polarization fractions for tree-dominated modes, $B \to \rho \rho$, satisfy Eq. (9). However, those for penguin-dominated modes, $B \to \rho K^*$ and $K^{*0} \rho^+$, obviously conflict with the naïve expectations.

For the penguin-dominated modes, the polarization fractions could be modified by sub-leading contributions. The penguin annihilation contribution from the

Table III Branching ratios for the $B \to \rho \rho$ decays in units of $10^{-6}$ [5].

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>$B^0 \to \rho^+ \rho^-$</td>
<td>$30 \pm 4 \pm 5$</td>
<td>$22.8 \pm 3.8 \pm 2.3$</td>
<td>$27.8$</td>
<td>$25.3^{+2.5}_{-1.3}$</td>
</tr>
<tr>
<td>$B^\pm \to \rho^0 \rho^0$</td>
<td>$17.2 \pm 2.5 \pm 2.8$</td>
<td>$31.7 \pm 7.1^{+3.5}_{-6.7}$</td>
<td>$13.7$</td>
<td>$16.0^{+1.5}_{-0.8}$</td>
</tr>
<tr>
<td>$B^0 \to \rho^0 \rho^0$</td>
<td>$&lt; 1.1$</td>
<td>$&lt; 1.1$</td>
<td>$0.33$</td>
<td>$0.92^{+1.1}_{-0.56}$</td>
</tr>
</tbody>
</table>

Figure 2: $R, R_\perp$, and $R_\parallel$ as functions of $\phi_3$ from NLO PQCD with the bands representing the theoretical uncertainty [4]. The two dashed lines denote $1\sigma$ bounds from the data.
\( (S - P)(S + P) \) operators, which follows \( R_L \sim R_\| \sim R_{LL} \), could decrease the longitudinal fraction [38]. In LO PQCD, the spectator and penguin annihilation contributions help to reduce \( R_L \) as shown in Table IV, but it is not enough to explain the \( B \to \phi K^* \) and \( K^{*0} \rho^+ \) data [39-41]. \( R_L \) for \( B^+ \to K^{*+} \rho^0 \) remains as \( R_L \sim 0.85 \), which is consistent with the data, because this process involves additional tree amplitudes.

The tree-dominated modes, which are insensitive to the sub-leading corrections, follow the naïve counting rules in Eq. (9) [5, 31]. Therefore, only the penguin-dominated modes \( B \to \phi K^* \) and \( K^{*0} \rho^+ \) have exhibited anomalies in the measured polarization fractions.

There have been several mechanisms proposed to explain the observed \( B \to \phi K^* \) polarizations. In PQCD, it was proposed that the \( B \to K^* \) form factor \( A_0 \), associated with the longitudinal polarization, may be smaller than the central value of the LO PQCD prediction [42]. Postulating a smaller value, \( A_0 \approx 0.3 \), which does not contradict to any existing measurements, \( R_L \) for \( B \to \phi K^* \) decreases to \( 0.6 \). Another mechanism in PQCD was proposed in Ref. [32]. Adopting a modified definition of the hard scale, which is a source of theoretical uncertainty, \( R_L \) for \( B \to \phi K^* \) could approach to \( 0.6 \). However, \( R_L \) for \( B^+ \to K^{*0} \rho^+ \) becomes about \( 0.8 \), which is inconsistent with the data. This is because the sign of the real part of the annihilation amplitude \( B^+ \to K^{*0} \rho^+ \) is opposite to that for \( B \to \phi K^* \) [32]. Small \( R_L \) for the penguin-dominated modes might come from the complicated QCD dynamics, but it is important to explain both the \( B \to \phi K^* \) and \( K^{*0} \rho^+ \) data.

### Table IV Polarization fractions in \( B \to \phi K^* \), (I) without spectator and annihilation contributions, and (II) with spectator and annihilation contributions [39].

<table>
<thead>
<tr>
<th>Mode</th>
<th>( R_L )</th>
<th>( R_| )</th>
<th>( R_{LL} )</th>
<th>( \phi_L ) (rad)</th>
<th>( \phi_| ) (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B^0 \to \phi K^{*-} ) (I)</td>
<td>0.923</td>
<td>0.040</td>
<td>0.035</td>
<td>\pi</td>
<td>\pi</td>
</tr>
<tr>
<td>(II)</td>
<td>0.750</td>
<td>0.135</td>
<td>0.115</td>
<td>2.55</td>
<td>2.54</td>
</tr>
<tr>
<td>( B^+ \to \phi K^{*-} ) (I)</td>
<td>0.923</td>
<td>0.040</td>
<td>0.035</td>
<td>\pi</td>
<td>\pi</td>
</tr>
<tr>
<td>(II)</td>
<td>0.748</td>
<td>0.133</td>
<td>0.111</td>
<td>2.55</td>
<td>2.54</td>
</tr>
</tbody>
</table>

5. Summary

In this talk, I have summarized the recent works on exclusive hadronic \( B \) meson decays in the PQCD approach, concentrating on the observed deviations in the branching ratios and the CP asymmetries of the \( B \to \pi K, \pi \pi \) modes, and the polarization fractions of penguin-dominated \( B \to VV \) modes, which are sensitive to subdominant contributions.

Including the important NLO contributions, the color-suppressed tree amplitude is enhanced by the vertex corrections, and therefore the predicted direct CP asymmetries of the \( B \to \pi K \) modes become consistent with the experimental data. However, it is unlikely to accommodate both the \( B^0 \to \pi^0 \pi^0 \) and \( B^0 \to \rho^0 \rho^0 \) branching ratios to the measured ones simultaneously.

The polarization fractions of the penguin-dominated \( B \to VV \) modes deviate from the naïve power-counting rules, including the spectator and annihilation contributions. However, it is not enough to explain the observed data. A small longitudinal fraction for \( B \to \phi K^* \) might come from QCD uncertainty, but it is necessary to explain both the \( B \to \phi K^* \) and \( K^{*0} \rho^+ \) data. NLO corrections to the polarization fractions should be studied in future work.

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### References


